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Calculation of the $O(\alpha_s^2)$ Parity-Violating Structure Functions in $e^+e^- \to q\bar{q}g$

J.G. Körner, G. Schuler[1], G. Kramer[2], B. Lampe[2]

[1] Institut für Physik, Johannes Gutenberg-Universität, D-6500 Mainz, Federal Republic of Germany
[2] II. Institut für Theoretische Physik, Universität Hamburg, Luruper Chausse 149, D-2000 Hamburg 50, Federal Republic of Germany

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Abstract. We calculate the two nonvanishing $O(\alpha_s^2)$ parity-violating structure functions that contribute to $e^+e^- \xrightarrow{\gamma,Z} q\bar{q}g$. We discuss how these can be measured. We work with massless quarks and gluons and use dimensional regularization to regularize ultra-violet and infrared singularities. We carefully discuss how to deal with γ_5 in the dimensional regularization scheme when infrared singularities are present.

1. Introduction

Much experimental [1–6] and theoretical work [7–11] has been expended on the elucidation of the role that $O(\alpha_s^2)$ corrections play in the description of 3-jet events in e^+e^--interactions. These have been found to be nonnegligible and thus affect the α_s-determination from 3-jet event data.

The first $O(\alpha_s^2)$ calculations were done for the trace of the hadronic tensor, i.e. the $O(\alpha_s^2)$ corrections to space-direction averaged 3-jet events. With more data it will be desirable to check also on the $O(\alpha_s^2)$ corrections to oriented 3-jet events,* or, vice versa, it will be desirable to be able to generate $O(\alpha_s^2)$ oriented 3-jet events via Monte Carlo.

The general space-orientation of 3-jet events in e^+e^--annihilations is described by 5 parity-conserving (PC) and 4 parity-violating (PV) structure functions [14]. In massless QCD one of the PC and two of the PV structure functions can be shown to vanish identically in $O(\alpha_s^2)$ [15, 14]. A set of 4 linearly independent PC structure functions have recently been obtained by two of us [16]. In the present paper we complete our program of calculating oriented $O(\alpha_s^2)$ 3-jet events by presenting the results of a calculation of the two non-vanishing $O(\alpha_s^2)$ PV structure functions.

* For $O(\alpha_s)$ results see [12, 13]

The PV asymmetries that are induced by the PV hadronic structure are measurable for the present high energy machines and are sizeable in the energy range available to the next generation e^+e^--machines (TRISTAN, SLC, LEP). Off the Z these result from the usual electro-weak γ–Z interference effects, whereas a measurement on the Z requires longitudinally polarized Z's due to the smallness of the leptonic vector contribution proportional to $(1-4\sin^2\Theta_w)$. Longitudinally polarized Z's are planned to be available at the SLC using longitudinally polarized electrons and/or positrons for the annihilation process. The measurement of the PV asymmetries does, however, require quark flavour tagging, which will reduce the available data sample.

On the theoretical side the calculation of PV quantities involving parity-odd fermion traces requires a careful discussion of how to deal with γ_5 or the totally antisymmetric $\varepsilon_{\alpha\beta\gamma\delta}$ in n-dimensions. The n-dimensional parity-odd Dirac traces generate $O((n-4)^m$, $m>0)$ anomalous contributions, which in turn lead to *finite* anomalous terms when multiplied with ultraviolet (UV) or infrared (IR) divergent integrals.

Spurious UV anomalies can be and must be cancelled by taking the appropriate renormalization scheme [17–19]. In the IR case one encounters axial and charge conjugation anomalies. These cannot be removed by renormalization. We demonstrate in this particular application that the IR charge conjugation anomaly vanishes after IR integration and that the finite IR axial anomalies are spurious in the sense that they cancel among the real and virtual contributions to the PV structure functions as do the IR singular contributions.

We choose to work with massless quarks and gluons and use dimensional regularization to control UV and IR infinities.

We use the γ_5-scheme of Breitenlohner and Maison (BM) [20] which, to our knowledge, is the only n-dimensional γ_5-scheme free of internal inconsistencies

[17]. The BM scheme was originally developed for the regularization of UV singularities in the presence of γ_5. We found that the BM scheme is also well suited for the treatment of IR singularities in the presence of γ_5. For the purposes of the present calculation the BM γ_5-scheme can be implemented by observing the following two simple rules: i) Do not commute by γ_5 ii) The trace $-i/4\,\mathrm{Tr}\,\gamma_5\gamma_\alpha\gamma_\beta\gamma_\gamma\gamma_\delta$ equals the conventional antisymmetric ε-tensor $\varepsilon_{\alpha\beta\gamma\delta}$ in 4 dimensions ($\varepsilon_{0123}=1$) if the tensor indices $\alpha, \beta, \gamma, \delta$ are "4-dimensional" and equals zero otherwise. A more complete account of the BM γ_5-scheme is given in [21] and in Appendix A.

Concerning technical details we are very brief on those features which are similar to the corresponding PC case treated in detail in [8, 9]. However, the presence of γ_5 in the PV case brings in some novel features which require careful discussion. These are treated in more detail since we feel that the expertise gained from this first explicit calculation of a higher order QCD correction to a PV cross section will be quite valuable for the many higher order QCD calculations that have to be done for the interpretation of PV experiments at the high energy machines to be completed in the following years. Finally, our IR integrations are done up to $O(y^0)$, where y is the dimensionless invariant mass cut-off such that $(p_i+p_j)^2 \leq yq^2$.

Our presentation is organized in the following way. In Sect. 2 we write down the general structure of the PV hadron tensor and discuss how the PV structure functions are related to experimental observables. In Sect. 3 we treat the IR integrations of the $O(\alpha_s^2)$ 4-parton processes $e^+e^- \to q\bar{q}gg$ and $e^+e^- \to q\bar{q}q\bar{q}$. In Sect. 4 we deal with the $O(\alpha_s^2)$ virtual 1-loop contributions to the PV structure functions. In Sect. 5 we combine the results of Sects. 3 and 4 and show that the IR singular pieces and the finite anomalous pieces in the real and virtual contributions cancel. One remains with a finite, IR cut-off dependent contribution free of anomalies. Their contributions to some physical PV cross sections are evaluated numerically and are compared to the $O(\alpha_s)$ results.

In Appendix A we provide a brief introduction to the BM γ_5-scheme*. In Appendix B we calculate an n-dimensional integral of a 4-dimensional scalar that is needed in the main text. For the sake of completeness we present results on the $O(\alpha_s)$ corrections to the PC and PV cross sections in $e^+e^- \to q\bar{q}$ in Appendix C.

2. Angular Asymmetries

The differential cross section for $e^+e^- \to q\bar{q}g$ has the following form

$$d\sigma = \sigma_{\mu\mu}\, d\Omega^{(3)} L^{\mu\nu} H_{\mu\nu}. \tag{2.1}$$

Here $d\Omega^{(3)}$ is the invariant phase space for three massless particles in the final state, $L_{\mu\nu}$ is the lepton tensor and $H_{\mu\nu}$ the hadron tensor, respectively. $H_{\mu\nu}$ depends only on the final-state parton momenta and receives its contribution from the C-even, parity conserving current times current terms VV and AA, whereas the C-odd, parity-violating term originates from the $VA + AV$ interference. Therefore we can write

$$H_{\mu\nu} = H_{\mu\nu}^{\mathrm{PC}} + H_{\mu\nu}^{\mathrm{PV}} \tag{2.2}$$

where for massless QCD

$$H_{\mu\nu}^{\mathrm{PC}} = \tfrac{1}{2}(H_{\mu\nu}^{VV} + H_{\mu\nu}^{AA})$$
$$H_{\mu\nu}^{\mathrm{PV}} = \tfrac{1}{2}(H_{\mu\nu}^{VA} + H_{\mu\nu}^{AV}). \tag{2.3}$$

For unpolarized e^+e^- beams the parity-conserving lepton tensor is

$$L_{\mu\nu}^{\mathrm{PC}} = p_\mu^+ p_\nu^- + p_\mu^- p_\nu^+ - (p^+p^-)g_{\mu\nu} \tag{2.4}$$

and the parity-violating contribution is

$$L_{\mu\nu}^{\mathrm{PV}} = i\varepsilon_{\mu\nu\gamma\sigma} q^\rho p_-^\sigma \tag{2.5}$$

$p_+(p_-)$ is the positron (electron) momentum and $q=(p_+ + p_-)$. Then for unpolarized beams

$$L_{\mu\nu} = g_1(q^2) L_{\mu\nu}^{\mathrm{PC}} + g_5(q^2) L_{\mu\nu}^{\mathrm{PV}}. \tag{2.6}$$

All flavour dependence or all dependence on the particular neutral-current model used goes into the two functions $g_1(q^2)$ and $g_5(q^2)$. $\sigma_{\mu\mu}$ is the lowest order $e^+e^- \to \mu^+\mu^-$ cross section used for normalization as usual. The dependence on beam polarization is easily accounted for. The corresponding formulas for the general case of arbitrary transverse and longitudinal polarization of e^+ and e^- beams can be found in [13].

Before we go on let us recall some well known properties of (2.1), (2.2) and (2.3): (i) Since $H_{\mu\nu}^{\mathrm{PV}}$ arises from the C-odd VA interference it does not contribute to observables which do not distinguish quarks from antiquarks. (ii) The parity conserving hadron tensor $H_{\mu\nu}^{\mathrm{PC}}$ is the same as for pure one-photon exchange. This is because the quarks are assumed to be massless. So $v - a\gamma_5$ factors can be trivially moved through the traces, yielding only a common factor which is absorbed into $g_1(q^2)$. Therefore all angular distributions, thrust distributions etc. which do not distinguish quarks from antiquarks are not changed by the inclusion of the AA term. (iii) The C-odd asymmetries for unpolarized leptons are products of a pure QCD factor times the electroweak function $r_5(q^2) = g_5(q^2)/g_1(q^2)$. For the GWS-model $g_5(q^2)$ can be found in many papers [13, 22]. It is large over a wide range of energies and stays large above the Z energy.

The tensor structure of $H_{\mu\nu}$ is fixed by the requirement $q^\mu H_{\mu\nu} = q^\nu H_{\mu\nu} = 0$ and the fact that besides q it can depend only on two more momenta, for instance p_1 and p_2, where $p_1(p_2)$ is the quark (antiquark) momentum ($q = p_1 + p_2 + p_3, p_3 =$ gluon momentum). Then $H_{\mu\nu}^{\mathrm{PC}}$ depends in general on five structure function

* For more details see [20, 21]

$H_i (i=1, 2, 3, 4, 5)$ in the following form [14, 15]

$$H_{\mu\nu}^{PC} = H_1(g_{\mu\nu} - q_\mu q_\nu/q^2) + H_2 \hat{p}_{1\mu}\hat{p}_{1\nu}/q^2 + H_3 \hat{p}_{2\mu}\hat{p}_{2\nu}/q^2 + H_4(\hat{p}_{1\mu}\hat{p}_{2\nu} + \hat{p}_{2\mu}\hat{p}_{1\nu})/q^2 + H_5(\hat{p}_{1\mu}\hat{p}_{2\nu} - \hat{p}_{2\mu}\hat{p}_{1\nu})/q^2. \quad (2.7)$$

The parity-violating tensor $H_{\mu\nu}^{PV}$ has the following general structure [14]

$$H_{\mu\nu}^{PV} = H_6 q^{-2} i\varepsilon_{\mu\nu\alpha\beta} q^\alpha p_1^\beta + H_7 q^{-2} i\varepsilon_{\mu\nu\alpha\beta} q^\alpha p_2^\beta + H_8 q^{-4}(\hat{p}_{1\mu}F_\nu + \hat{p}_{1\nu}F_\mu) + H_9 q^{-4}(\hat{p}_{2\mu}F_\nu + \hat{p}_{2\nu}F_\mu) \quad (2.8)$$

where $\hat{p}_\mu = p_\mu - p\cdot q/q^2 q_\mu$ and $F_\mu = i\varepsilon_{\mu\nu\alpha\beta} p_1^\alpha p_2^\beta q^\nu$. From $H_{\mu\nu} = H_{\nu\mu}^*$ one concludes that H_1, H_2, H_3, H_4, H_6 and H_7 are real and H_5, H_8 and H_9 are imaginary. We note that H_1, H_2, H_3, H_4, H_8 and H_9 are associated with symmetric tensors and H_5, H_6 and H_7 with antisymmetric tensors, respectively. Therefore contributions to H_1, H_2, H_3, H_4, H_8 and H_9 can be detected only with the symmetric part of the lepton tensor and H_5, H_6 and H_7 only with the antisymmetric part of the lepton tensor present.

To fix our normalizations it is appropriate to present the formulas for $e^+e^- \to q\bar{q}g$ in lowest order α_s. For this purpose we write (2.1) in the form

$$d\sigma = d\sigma^{(e)} + d\sigma^{(o)} \quad (2.9)$$

with the C-even part of $e^+e^- \to \gamma, Z \to q\bar{q}g$ denoted by $d\sigma^{(e)}$ and the C-odd part as $d\sigma^{(o)}$. After specifying the usual coordinate system for the orientation of the $q\bar{q}g$ momenta with respect to the e^- beam direction [13] the angular distribution for $d\sigma^{(o)}$ has the following form

$$\frac{d\sigma^{(o)}}{dx_1 dx_2 d\cos\vartheta d\chi/2\pi} = \tfrac{3}{4}\cos\vartheta \frac{d\sigma^P}{dx_1 dx_2} - \frac{3}{\sqrt{2}}\sin\vartheta\cos\chi \frac{d\sigma^A}{dx_1 dx_2}. \quad (2.10)$$

$x_i = 2p_{io}/\sqrt{q^2}$ $(i=1,2,3)$ denote the scaled energies of the outgoing quark, antiquark and gluon. ϑ is the polar angle of the parton z axis with respect to the e^- beam direction. The parton z-axis may be either the direction of the quark, the antiquark or the gluon momentum. χ is the azimuthal angle of the $q\bar{q}g$ production plane with respect to the z-axis-e^--beam plane [13]*. χ is defined by the antiquark (quark) momentum if the z-axis is in the direction of the quark (antiquark); if the gluon defines the z direction χ is marked by the quark. The angles ϑ and χ vary between $0 \leq \vartheta \leq \pi$, $0 \leq \chi \leq 2\pi$.

Depending which of the quark, antiquark or gluon are chosen to define the z- and x-axis the partial cross sections $d\sigma^P (d\sigma^A)$ differ. In $O(\alpha_S)$ these partial cross sections have been calculated before [13]. They are

$$\frac{d\sigma^P}{dx_1 dx_2} = g_5 \sigma^{(1)} \frac{\alpha_s}{2\pi} C_F \frac{x_1^2 \hat{p}_{1z} - x_2^2 \hat{p}_{2z}}{(1-x_1)(1-x_2)}$$
$$\frac{d\sigma^A}{dx_1 dx_2} = g_5 \sigma^{(1)} \frac{\alpha_s}{2\pi} C_F \frac{x_1^2 \hat{p}_{1x} - x_2^2 \hat{p}_{2x}}{(1-x_1)(1-x_2)} \quad (2.11)$$

with

$$\hat{p}_{ix} = \sin\Theta_{ik}, \quad \hat{p}_{iz} = \cos\Theta_{ik}, \quad i=1,2$$
$$\cos\Theta_{ik} = \hat{p}_i \hat{p}_k = 1 + \frac{2}{x_i x_k}(1 - x_i - x_k) \quad (2.12)$$

$\hat{p}_k$ always refers to the parton which is used to define the z direction, i.e. $\hat{p}_k = (0,0,1)$. $\sigma^{(1)} = 3\sigma_{\mu\mu}$, $C_F = 4/3$ and α_s is the QCD coupling. g_5 contains all constants of the GWS model and the dependence on the longitudinal polarization of the electron ($\xi^{(-)}$) and the positron ($\xi^{(+)}$), respectively.

$$g_5 = 2\,\mathrm{Re}\,\beta(a_e(1+\xi^{(-)}\xi^{(+)}) - v_e(\xi^{(-)}+\xi^{(+)}))a_q Q_e Q_q + |\beta|^2[2v_e a_e(1+\xi^{(-)}\xi^{(+)}) - (v_e^2 + a_e^2)(\xi^{(-)}+\xi^{(+)})]2v_q a_q. \quad (2.13)$$

In (2.13)

$$\beta = \frac{g m_Z^2 q^2}{q^2 - m_Z^2 + i m_Z \Gamma_Z} \quad (2.14)$$

and the electromagnetic and weak coupling constant are specified as follows:

γ coupling: $eQ\gamma_\mu$ with $Q = -1$ for electron
$Q = 2/3, \ -1/3$ for quarks (2.15)

Z coupling: vector coupling $= e\sqrt{g} m_Z v\gamma_\mu$
axial-vector coupling $= e\sqrt{g} m_Z a\gamma_\mu\gamma_5$. (2.16)

m_Z and Γ_Z are the mass and the width of the Z boson and $g = G_F(8\sqrt{2}\pi\alpha)^{-1}, (\alpha = e^2/4\pi)$, G_F being the Fermi weak coupling constant. In the GWS model $m_Z\sqrt{g} = (2\sin^2\Theta_W)^{-1}, v = \mp 1 - 4Q\sin^2\Theta_w, a = \mp 1$, where the upper sign holds for electrons and quarks with charge $-1/3$ and the lower sign with charge 2/3. The contributions (2.11) to the cross section give (i) the forward-backward asymmetry of the Oz axis with respect to the e^- beam direction (the term $\sim \cos\vartheta$) and (ii) the azimuthal asymmetry of the event plane with respect to the scattering plane (the term $\sim \sin\vartheta\ \cos\chi$), both coupled with a charge asymmetry of quark versus antiquark distribution (sign change for $1 \leftrightarrow 2$). One should note from (2.13) that g_5 deviates substantially from zero at the Z resonance energy only if electron and/or positron are longitudinally polarized.

Since later on we shall present our higher order results for the structure functions H_6 and H_7 the relation of $d\sigma^P$ and $d\sigma^A$, as defined in (2.10), with H_6 and H_7 are of interest. For the case that the quark momentum is along the z-axis, and the antiquark

* In (2.10) we have omitted cross section contributions with $\sin^2\vartheta\ \sin 2\chi$ and $\sin 2\vartheta \sin\chi$ angular dependencies. In massless QCD one finds these to be zero at $O(\alpha_s^2)$ [15, 14]

momentum in the (pos. x; z)-half-plane, we have

$$\frac{d\sigma^P}{dx_1 dx_2} = \frac{g_5}{64\pi^2}\sigma_{\mu\mu} 2(H_{++} - H_{--}) = \frac{g_5}{64\pi^2}\sigma_{\mu\mu}(x_1 H_6 + x_2 \cos\Theta_{12} H_7) \tag{2.17}$$

$$\frac{d\sigma^A}{dx_1 dx_2} = \frac{g_5}{64\pi^2}\sigma_{\mu\mu}\,\mathrm{Re}(H_{+0} + H_{-0}) = \frac{g_5}{64\pi^2}\sigma_{\mu\mu}\frac{x_2}{2\sqrt{2}}\sin\Theta_{12} H_7. \tag{2.18}$$

Comparing (2.17) and (2.18) with (2.11) allows one to read off the lowest $O(\alpha_s)$ contribution to H_6 and H_7.

So far we have looked at the VA interference terms solely in connection with their appearance in e^+e^- annihilation. Up to now Z's (and similarly W's) have been produced only in $p\bar{p}$ collisions at the collider at CERN. Depending on the polarization of the produced Z's and W's the VA interference terms can also be seen in their angular decay distributions. The PV contributions to the decays $Z \to q\bar{q}g$ and $W \to q\bar{q}'g$ are described by the same 2 PV structure functions H_6 and H_7 calculated in this paper to $O(\alpha_s^2)$. Thus our results also apply to the decay of hadronically produced Z's and W's although we do not work out the details of the spin kinematics of these decay processes.

3. Integration of Four-Parton Cross Section

(i) General Remarks

Our aim is to integrate the 4-parton processes $e^+e^- \to q\bar{q}gg$ and $e^+e^- \to q\bar{q}q\bar{q}$ over the various 2-parton sub-phase-spaces up to an invariant mass cut-off $s_{ij} < yq^2$. Since we desire an accuracy of $O(y^0)$ we need to consider only those regions of phase space in which the 4-parton processes become IR singular.

The techniques of IR integration have been presented in detail in [8, 9] where the trace of the hadronic tensor was integrated. In our case we are dealing with a more complex case. Firstly we are integrating a tensor quantity $H_{\mu\nu}^{\mathrm{PV}}$ and secondly, we have the added complexity of odd-parity fermion loops. These added complexities bring in new technical features which have to be carefully discussed.

First note that the PV hadron tensor $H_{\mu\nu}^{\mathrm{PV}}$ defined in (2.3) is $(\mu \leftrightarrow \nu)$—antisymmetric for (real!) tree graph contributions, i.e.

$$H_{\mu\nu}^{\mathrm{PV}}(\text{tree}) = -H_{\nu\mu}^{\mathrm{PV}}(\text{tree}). \tag{3.1}$$

The antisymmetry (3.1) is true regardless of the space-time dimension n since it does not depend on the commutation property of γ_5.

The 4-parton tensor $H_{\mu\nu}^{(4)\mathrm{PV}}$ involves odd-parity fermion traces. These n-dimensional Dirac traces generate $O((n-4)^m; m>0)$ anomalous contributions.

Thus the above two processes develop chiral (or axial) anomalous terms for $n \neq 4$, i.e. $H_{\mu\nu}^{AV} \neq H_{\mu\nu}^{VA}$ and $q^\mu H_{\mu\nu}^{\mathrm{PV}} \neq 0$, $q^\nu H_{\mu\nu}^{\mathrm{PV}} \neq 0$. These anomalous terms have to be taken into account explicitly when the IR integrations are done.

Also due to the fact that $\{\gamma_\mu, \gamma_5\} \neq 0$ for $n \neq 4$ one finds $C(j_\mu^A)C^{-1} \neq j_\mu^A$. This charge conjugation anomaly, however, cancels after IR integration as shown after (3.5). In addition, the charge conjugation anomaly is of no physical relevance since the physical cross sections derive from the contraction of the hadron tensor and the lepton tensor $L_{\mu\nu}^{\mathrm{PV}} H^{\mu\nu\mathrm{PV}}$ as in (2.1). Therefore the (μ, ν)-indices are constrained to be 4-dimensional [see (A7)] and thus the proper charge conjugation properties of $H_{\mu\nu}^{(4)\mathrm{PV}}$ are restored in physical cross sections.

The sum over the various 2-parton IR integrations leads to the 3-parton hadron tensor $H_{\mu\nu}^{(3)\mathrm{PV}}(q_1, q_2, q_3, y)$ which is a function of the 3-parton momenta and the invariant mass cut y. The IR integrations are best performed in the CMS-system of the various two-body phase-space regions $\{ij\}$ integrated over [9]. At the $O(y^0)$ level this may be expressed by writing

$$\frac{q^2}{16\pi^2}\left(\frac{4\pi}{q^2}\right)^{\varepsilon}\frac{\Gamma(1-\varepsilon)}{\Gamma(2-2\varepsilon)}\sum_{\{ij\}}\int_0^y dy_{ij}\, y_{ij}^{-\varepsilon} \cdot \{\int d\tilde{\Omega}_{ij}^{n-1}\} H_{\mu\nu}^{(4)\mathrm{PV}}(p_1, p_2, p_3, p_4) = H_{\mu\nu}^{(3)}(q_1, q_2, q_3, y). \tag{3.2}$$

$d\tilde{\Omega}^{n-1}$ is the spherical element in $(n-1)$ dimensions normalized to unity, i.e. $\int d\tilde{\Omega}^{n-1} = 1$ and $\varepsilon = (4-n)/2$. The curly brackets in $\{d\tilde{\Omega}_{ij}^{n-1}\}$ mean that proper care has to be taken to avoid double counting for overlapping singularities. We shall deal with this problem in the manner of [16] which has been referred to as the direct dressing approach in [11] [see also (C2)]. $H_{\mu\nu}^{(4)\mathrm{PV}}(p_i)$ denotes the PV 4-parton hadron tensor. The $p_i (i = 1, \ldots, 4)$ are the 4-parton momenta. They are on-shell $p_i^2 = 0$. They have to be chosen as n-dimensional vectors when integrated over. The 3-parton momenta $q_l (l = 1, 2, 3)$ can be taken as on-mass-shell ($q_l^2 = 0$) in the $O(y^0)$ approximation. The coordinate system must be chosen such that the q_l have only 4-dimensional components [see e.g. (B1–B3)]. Finally we introduced $q^2 y_{ij} = s_{ij} = 2p_i p_j (q^2 z_{ij} = t_{ij} = 2q_i q_j)$. The densities $H_{\mu\nu}^{(4)\mathrm{PV}}(p_i)$ have been calculated in $n = 4$ dimensions in [22] for the various 4-jet cross sections. Of course we need them for $n \neq 4$. So they had to be recalculated.

There are two possible ways to proceed with the angular integration in (3.2). Using the representation (A5) for γ_5 one can calculate

$$H_{\mu\nu}^{(4)\mathrm{PV}}(p_i) = \varepsilon_{\alpha\beta\gamma\delta} T_{[\mu\nu]}^{(4)[\alpha\beta\gamma\delta]}(p_i). \tag{3.3}$$

The angular integration on the n-dimensional rank 6 antisymmetric tensor $T_{[\mu\nu]}^{(3)[\alpha\beta\gamma\delta]}(q_i)$ can then be done by the standard methods, viz.

$$\int d\tilde{\Omega}^{n-1} T_{[\mu\nu]}^{(4)[\alpha\beta\gamma\delta]}(p_i) = T_{[\mu\nu]}^{(3)[\alpha\beta\gamma\delta]}(q_i). \tag{3.4}$$

The γ_5-substitution in (3.3) necessitates the calculation of very long traces. Also the tensor integration in (3.4) is not simple.

Instead of doing the tensor integration (3.4) we use a short cut and directly scalarize the integrand in (3.2) by writing down the most general expansion for the angle integrated 4-parton tensor*

$$\int d\tilde{\Omega}^{n-1} H_{\mu\nu}^{(4)\mathrm{PV}}(p_i) = H_6^{\mathrm{PV}}\,\varepsilon(\mu\nu q q_1) + H_7^{\mathrm{PV}}\,\varepsilon(\mu\nu q q_2) + H'^{\mathrm{PV}}\varepsilon(\mu\nu q_1 q_2). \tag{3.5}$$

The invariants H_6^{PV} and H_7^{PV} obtain the conserved current contributions whereas H'^{PV} carries the anomalous contribution. Use has been made of the antisymmetry (3.1). Further covariants as e.g. $q_\mu \varepsilon(\nu q_1 q_2 q_3) - q_\nu \varepsilon(\mu q_1 q_2 q_3)$ are not independent and can be related to the above set by using the Schouten identity (A4). This is legitimate since we always choose our n-dimensional coordinate systems such that the outer momenta q_i have only 4-dimensional components [see e.g. (B1–B3)].

Since the RHS of (3.5) carries only 4-dimensional tensor indices μ and ν it is clear that the anomalous charge conjugate pieces in $H_{\mu\nu}^{(4)\mathrm{PV}}$ resulting from $C(\gamma_\mu\gamma_5)C^{-1} \neq (\gamma_\mu\gamma_5)^T$ vanish after the angular integration (3.5).

The integrand in (3.5) can be scalarized by using the three parity-odd projection tensors

$$\begin{aligned} &H_6^{\mathrm{PV}}: q_{2\nu}\varepsilon(\mu q_1 q_2 q_3)\\ &H_7^{\mathrm{PV}}: q_{1\nu}\varepsilon(\mu q_1 q_2 q_3)\\ &H'^{\mathrm{PV}}: q_\nu \varepsilon(\mu q_1 q_2 q_3)\end{aligned} \tag{3.6}$$

which at the same time project out the three invariants H_6^{PV}, H_7^{PV} and H'^{PV} as indicated in (3.6). Note that the contractions (3.6) can be exchanged with the angle integration in (3.5) since the q_l do not depend on the angular integration variables.

The action of the parity-odd projectors (3.6) on the 4-parton integrand in (3.5) bring into play 4-dimensional scalars via the products of ε-tensors [see (A6)]. These 4-dimensional scalars have to be treated separately from the n-dimensional scalars resulting from the trace manipulations. However, this does not pose a big problem, since there is only one relevant 4-dimensional scalar for every IR region.

Consider for example the (3–4) IR region in the (3–4) CMS system ($\mathbf{p}_3 + \mathbf{p}_4 = 0$). An explicit representation of the 4-parton momenta p_i in this system is given in (B1–B3). The relevant 3-parton momenta $q_1 = p_1$, $q_2 = p_2$ and $q_3 = p_3 + p_4$ are 4-dimensional. All 4-dimensional scalars can be expressed e.g. in terms of $\hat{p}_4^2$. Thus $\hat{p}_3\hat{p}_4 = p_4(p_3+p_4) - \hat{p}_4^2$, $\hat{p}_3^2 = \hat{p}_4^2$ and $\hat{p}_1\hat{p}_3 = p_1\hat{p}_3 = p_1 p_3$ etc. The n-dimensional angular integration of $\hat{p}_4^2$ is done in Appendix B. $\hat{p}_4^2$ is proportional to s_{34} as (B3) or (B5) show. This means that all $\hat{p}_4^2$ contributions can be dropped except for the true double pole contributions s_{34}^{-2} as long as one is calculating at $O(y^0)$.

* The 3-parton momenta q_l in (3.5) are in general off-shell

After the angular integration (3.5) one still has to do the invariant mass integration in (3.2). Apart from the y_{ij}-dependence in the scalar functions H_6^{PV}, H_7^{PV} and H'^{PV} there is also the y_{ij} dependence in the tensors themselves, since the q_i are in general off-mass-shell. However, this need not concern us at the $O(y^0)$ level as can be shown as follows. It is convenient to do the y_{ij}-integration in the e^+e^--CMS-system. Define the z-axis by the off-mass-shell parton $q_l = p_i + p_j$, and $q_l^2 = y_{ij}q^2$. Then one has $q_{l\mu} = (q_{l0}, 0, 0, \sqrt{q_{l0}^2 - s_{ij}}) = (q_{l0}, 0, 0, q_{l0}(1 - \frac{1}{2}s_{ij}/q_{l0}^2 + \cdots))$. Thus the s_{ij} dependence of the tensors in (3.4) can be neglected at the $O(y^0)$ level since no true double-pole s_{ij}^{-2} contributions remain after the angular integrations (3.5)*.

Finally, let us remark, that we did not find it more difficult to implement the PV calculation on the computer than the corresponding PC calculation, despite the fact that the BM γ_5-scheme looks formidable at first sight. First, note that in the parity-odd traces there is no need to use the "ugly" commutation rules (A3) since all trace manipulations can be performed without commuting by γ_5 because of the cyclic property of a trace. Second, we did not need to use the γ_5-substitution (A5) since the action of the projectors (3.7) reduced the 4-parton traces to traces with γ_5 and four γ's. Lastly, the appearance of 4-dimensional scalars brought in by the projection method causes only minimal additional problems as discussed above.

(ii) Integration of $e^+e^- \to q(p_1)\bar{q}(p_2)g(p_3)g(p_4)$

An inspection of the relevant Feynman diagrams (see e.g. [8, 9]) shows that the cross section becomes singular in the regions where s_{13}, s_{14}, s_{23}, s_{24} and s_{34} approach their mass-shell values. As discussed in detail in [8, 9] for the PC case there are symmetry relations obeyed by the 4-parton cross section under $1 \leftrightarrow 2$ and $3 \leftrightarrow 4$ exchange which allows one to consider only the (1–3) and (3–4) phase space regions in detail. Care has to be taken in that the PV cross section is effectively antisymmetric under $1 \leftrightarrow 2$ exchange** compared to the symmetry in the PC case due to the replacement of a vector current by an axial vector current.

Following the classification of [8, 9] we first consider the QED-QED contributions class A and class B and the QED-QCD interference contributions of class C. The latter will be denoted by C_I. Since there is no true double-pole singularity in these contributions, no care has to be taken in tagging 4-dimensional scalars as discussed after (3.6). One obtains

$$\begin{aligned} &H_{\mu\nu}^{(3)\mathrm{PV}}\,(\text{real}; A, B, C_I)\\ &\quad = g^4 N_C C_F (C_F A + N_C B) A_{\mu\nu}^{(3)\mathrm{PV}}(\text{Born})\end{aligned} \tag{3.7}$$

* It should be noted though that the IR singular pieces are picked up at the on-mass-shell limit $q_l^2 = 0$ regardless of the $O(y^0)$ approximation. This is of course quite necessary in order to cancel the IR singular pieces of the loop contributions which occur for on-mass-shell q_l's

** See discussion of the charge conjugation property of the 4-parton hadron tensor after (3.7) where

$$A = C\left(1+\frac{\pi^2}{3}\varepsilon^2\right)\frac{\Gamma(1-2\varepsilon)}{\Gamma^2(1-\varepsilon)}\int_0^y dy_{13}y_{13}^{-1-\varepsilon}$$
$$\cdot\left(2\int_0^1 dv - \int_{1-y/z_{12}}^1 dv\right)v^{-\varepsilon}(1-v)^{-\varepsilon}$$
$$\cdot\left[(1-v)(1-\varepsilon)+\frac{2v}{1-v}\right] \quad (3.8)$$

$$B = C\left(1+\frac{\pi^2}{3}\varepsilon^2\right)\frac{\Gamma(1-2\varepsilon)}{\Gamma^2(1-\varepsilon)}\left\{\int_0^y dy_{13}y_{13}^{-1-\varepsilon}\right.$$
$$\cdot\int_{1-y/z_{12}}^1 dv v^{-\varepsilon}(1-v)^{-\varepsilon}\frac{v}{1-v}$$
$$+\int_0^y dy_{34}y_{34}^{-1-\varepsilon}\left(\int_0^{1-y/z_{13}} dv + \int_0^{1-y/z_{23}} dv\right)v^{-\varepsilon}$$
$$\left.\cdot(1-v)^{-\varepsilon}\left(\frac{v}{1-v}+\frac{1}{2}\right)\right\} \quad (3.9)$$

and where

$$C = \frac{1}{8\pi^2}\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left(\frac{4\pi\mu^2}{q^2}\right)^\varepsilon\left(1-\frac{\pi^2}{3}\varepsilon^2\right). \quad (3.10)$$

v is related to the polar angle Θ of one of the partons in the 2-parton subsystem as described in Appendix B. The peculiar v-integration limits in (3.8)–(3.10) arise from the direct dressing approach of [16] and take care of the double counting problem. In the v-integration limits we have introduced the 3-parton variables z_{ij} via i) (1–3) system; $y_{23}=(1-v)z_{12}$ ii) (3–4) system; $y_{13}=(1-v)z_{13}$ and $y_{23}=(1-v)z_{23}$. This is legitimate to $O(y^0)$.

$A_{\mu\nu}^{(3)\mathrm{PV}}$(Born) refers to the space-time structure of the 3-parton Born term amplitude squared and is given by

$$A_{\mu\nu}^{(3)\mathrm{PV}}(\mathrm{Born}) = -\frac{8}{q^2}\left[\left(\frac{1-z_{23}}{z_{13}z_{23}}-\frac{\varepsilon}{z_{23}}\right)i\varepsilon(\mu\nu q q_1)\right.$$
$$\left.-(1\leftrightarrow 2)-\varepsilon\frac{1-z_{12}}{z_{13}z_{23}}i\varepsilon(\mu\nu q_1 q_2)\right]. \quad (3.11)$$

Equation (3.7) shows that the PV contributions of class A, B, C_1, when evaluated at the $O(y^0)$ level, exhibit the same factorization into Born term and Altarelli-Parisi type kernel as the PC contributions [8, 9, 16].

The contribution from the QCD-QCD graphs* in class C (denoted by C_{3g}) is more subtle due to the occurrence of the double pole singularity s_{34}^{-2}. In this case one obtains in addition also contributions from terms proportional to the 4-dimensional scalar $\hat{p}_4^2$ as explained after (3.6). One has

$$H_{\mu\nu}^{(3)\mathrm{PV}}(\mathrm{real}; C_{3g})$$
$$= g^4 N_C C_F^2 I_{3g} A_{\mu\nu}^{(3)\mathrm{PV}}(\mathrm{Born}) + g^4 N_C C_F^2 I_R$$
$$\cdot\left(\frac{1}{z_{13}q^2}\varepsilon(\mu\nu q q_1)-(1\leftrightarrow 2)+\frac{1-z_{12}}{z_{13}z_{23}q^2}\varepsilon(\mu\nu q_1 q_2)\right) \quad (3.12)$$

where

$$I_{3g} = C\left(1+\frac{\pi^2}{3}\varepsilon^2\right)\frac{\Gamma(1-2\varepsilon)}{\Gamma^2(1-\varepsilon)}\int_0^y dy_{34}y_{34}^{-1-\varepsilon}$$
$$\cdot\int_0^1 dv v^{-\varepsilon}(1-v)^{-\varepsilon}[v(1-v)-1] \quad (3.13)$$

and

$$I_R = C\left(1+\frac{\pi^2}{3}\varepsilon^2\right)\frac{\Gamma(1-2\varepsilon)}{2\Gamma^2(1-\varepsilon)}\int_0^y dy_{34}y_{34}^{-1-\varepsilon}$$
$$\cdot\int_0^1 dv v^{-\varepsilon}(1-v)^{-\varepsilon}\left[\varepsilon v(1-v)-(1-\varepsilon)\frac{1}{N_{\Theta_1}}\right.$$
$$\left.\cdot\int_0^\pi d\Theta_1 \sin^{-2\varepsilon}\Theta_1\frac{1}{N_{\Theta_2}}\int_0^\pi d\Theta_2\sin^{-1-2\varepsilon}\Theta_2\frac{\hat{p}_4^2}{s_{34}}\right]. \quad (3.14)$$

The normalization factors N_{Θ_1} and N_{Θ_2} are given by

$$N_{\Theta_1} = \sqrt{\pi}\,\Gamma(\tfrac{1}{2}(1-2\varepsilon))/\Gamma(1-\varepsilon) \quad (3.15)$$

and in (B6).

As is evident from (3.12) the PV triple-gluon contribution does not manifestly factorize into the Born term contribution and an universal Altarelli–Parisi type kernel as in the PC case [9, 16]. However, doing the $\hat{p}_4^2$ integration in (3.14), as described in Appendix B, the nonfactorizing terms in I_R(3.14) can be seen to cancel and we remain with a universally factorizing triple-gluon contribution as in the PC case.

(iii) Integration of $e^+e^-\to q(p_1)\bar{q}(p_2)q(p_3)\bar{q}(p_4)$

Following again the notation of [9] we found that the contributions of diagrams class F vanish identically because of their charge conjugation property. Also the contributions from class E vanish at the $O(y^0)$ level. The singular regions of the diagrams from class D can all be mapped into the (3–4) singular region. Since one has a double-pole singularity s_{34}^{-2} one obtains a nonvanishing $\hat{p}_4^2$-contribution as in the triple-gluon contribution to $e^+e^-\to q\bar{q}gg$. One obtains

$$H_{\mu\nu}^{(3)\mathrm{PV}}(\mathrm{real}; q\bar{q}q\bar{q})$$
$$= g^4 N_C C_F\frac{N_f}{2}I_D A_{\mu\nu}^{(3)\mathrm{PV}}(\mathrm{Born}) + g^4 N_C C_F\frac{N_f}{2}\frac{I_R}{1-\varepsilon}$$
$$\cdot\left(\frac{\varepsilon(\mu\nu q q_1)}{z_{13}q^2}-(1\leftrightarrow 2)+\frac{1-z_{12}}{z_{13}z_{23}q^2}\varepsilon(\mu\nu q_1 q_2)\right)$$
$$(3.16)$$

* In calculating the ghost contribution to the QCD-QCD graphs one cannot use the poor man's ghost prescription [23] since this prescription is no longer valid for $n\neq 4$

where

$$I_D = C\left(1 + \frac{\pi^2}{3}\varepsilon^2\right)\frac{\Gamma(1-2\varepsilon)}{\Gamma^2(1-\varepsilon)}\int_0^y dy_{34} y_{34}^{-1-\varepsilon} \cdot \int_0^1 dv\, v^{-\varepsilon}(1-v)^{-\varepsilon}\frac{v^2+(1-v)^2-\varepsilon}{1-\varepsilon} \quad (3.17)$$

and where the integral I_R containing the $\hat{p}_4^2$ contribution is identical to the corresponding contribution in the triple-gluon case (3.14). Since $I_R = 0$ we find again the factorization into the Born term contribution and an Altarelli-Parisi type kernel as in the PC case [9, 16].

(iv) Integrated 4-Parton Cross Sections

In this subsection we present the final results for the 3-parton hadron tensor after integration of the remaining variables y_{ij} and v in the 4-parton hadron tensor integrands (3.8, 3.9, 3.13, 3.17).

After a bit of regrouping the final result is

$$H_{\mu\nu}^{(3)\mathrm{PV}}(\text{real}) = g^4 N_C C_F\left(C_F H^C + \frac{N_C}{2}H^N + \left(\frac{N_f}{3} - \tfrac{11}{6}N_C\right)H^f\right)A_{\mu\nu}^{(3)\mathrm{PV}}(\text{Born}) \quad (3.18)$$

where

$$H^C = C\left(\frac{2}{\varepsilon^2} - \frac{1}{\varepsilon}(2\ln z_{12} - 3) + 7 - 2\ln^2 y + 4\ln y \ln z_{12} - \ln^2 z_{12} - 3\ln y + \frac{\pi^2}{3}\right)$$

$$H^N = C\left(\frac{2}{\varepsilon^2} - \frac{2}{\varepsilon}\ln\frac{z_{13}z_{23}}{z_{12}} + \frac{4}{3} - \ln^2 z_{13} - \ln^2 z_{23} + \ln^2 z_{12} - 2\ln^2 y + 4\ln y \ln\frac{z_{13}z_{23}}{z_{12}} + \frac{\pi^2}{3}\right)$$

$$H^f = C\left(-\frac{1}{\varepsilon} - \frac{5}{3} + \ln y\right). \quad (3.19)$$

One notes from (3.18) that the $O(y^0)$ IR result factorizes into a Born term contribution and an universal IR factor as in the PC case discussed in [9, 16]. The universal IR factor is the same for the PC and PV contributions.

Note that the hadron tensor (3.18) contains anomalous pieces of $O(\varepsilon^{-1})$ and $O(\varepsilon^0)$ due to the anomalous piece in the Born term structure $A_{\mu\nu}^{(3)\mathrm{PV}}(\text{Born})$ proportional to $\varepsilon(\mu\nu q_1 q_2)$ [see (3.11)].

4. One-Loop Contributions

In [14] two of us have calculated the $O(\alpha_s^{3/2})$ one-loop contributions to the vector current amplitude in $e^+e^- \to q\bar{q}g$. Writing*

$$J_\mu^V = \langle g(q_3)\bar{q}(q_2)q(q_1)|j_\mu^V(0)|0\rangle = \bar{u}(q_1)T_{\mu\beta}^V v(q_2)\varepsilon_\beta^*(q_3) \quad (4.1)$$

we expanded the vector current amplitude $T_{\mu\beta}^V$ along a complete set of seven covariants, c.f.

$$T_{\mu\beta}^V = N_i^V C_{\mu\beta}^{Vi} \quad (i = 1, \ldots, 7). \quad (4.2)$$

Our one-loop results were then in terms of the invariants N_i^V.

To begin with we concentrate on the IR singular pieces of the one-loop amplitude. These were shown to occur only in N_7^V, where $C_{\mu\beta}^{V7}$ has the covariance structure of the Born term amplitude

$$C_{\mu\beta}^{V7} = \gamma_\mu\frac{\not{q}_2 + \not{q}_3}{t_{23}}\gamma_\beta - \gamma_\beta\frac{\not{q}_1 + \not{q}_3}{t_{13}}\gamma_\mu \quad (4.3)$$

and where $t_{ij} = 2q_i q_j$.

For our later discussion we also need the corresponding axial vector current Born term amplitude

$$C_{\mu\beta}^{A7} = \gamma_\mu\gamma_5\frac{\not{q}_2 + \not{q}_3}{t_{23}}\gamma_\beta - \gamma_\beta\frac{\not{q}_1 + \not{q}_3}{t_{13}}\gamma_\mu\gamma_5. \quad (4.4)$$

It is clear that $C_{\mu\beta}^{A7}$ is not a conserved quantity, i.e. $q^\mu C_{\mu\beta}^{A7} \neq 0$, even if the outer momenta q_i and the outer index μ are 4-dimensional because of the n-dimensionality of the inner gluon index β.

The $O(\alpha_s^2)$ PV one-loop hadron tensor $H_{\mu\nu}^{(3)\mathrm{PV}}$ (1-loop) is given by

$$H_{\mu\nu}^{(3)\mathrm{PV}}(\text{1-loop}) = \tfrac{1}{2}[\langle J_\mu^V(\text{Born})J_\nu^{A*}(\text{1-loop})\rangle + \langle J_\mu^A(\text{Born})J_\nu^{V*}(\text{1-loop})\rangle + \langle J_\mu^A(\text{1-loop})J_\nu^{V*}(\text{Born})\rangle + \langle J_\mu^V(\text{1-loop})J_\mu^{A*}(\text{Born})\rangle] \quad (4.5)$$

where the symbol $\langle \ldots \rangle$ stands for the spin summation as in Sect. 2.

The contribution $\langle J_\mu^A(\text{Born})J_\nu^{V*}(\text{1-loop})\rangle$ can be calculated without much difficulty using the vector current 1-loop results of [14]. For the axial vector current 1-loop contribution to the hadron tensor (4.5) we postulate the ultraviolet chiral invariance relation

$$\langle J_\mu^V(\text{Born})J_\nu^{A*}(\text{1-loop})\rangle + \langle J_\mu^A(\text{1-loop})J_\nu^{V*}(\text{Born})\rangle = \langle J_\mu^A(\text{Born})J_\nu^{V*}(\text{1-loop})\rangle + \langle J_\mu^V(\text{1-loop})J_\mu^{A*}(\text{Born})\rangle. \quad (4.6)$$

The renormalization of the 1-loop axial vector current contribution must include appropriate counter terms to cancel spurious ultraviolet anomalies such that the chiral relation (4.6) holds [18, 19].

For the real part of the IR singular contributions to $H_{\mu\nu}^{(3)\mathrm{PV}}$(1-loop) we obtain

$$H_{\mu\nu}^{(3)\mathrm{PV}}(\text{sing.}) = \langle \mathrm{Re}\, N_7^V(\text{sing.})\rangle g^4\, \mathrm{Tr}\,(\not{p}_1 C_{\mu\beta}^{V7}\not{p}_2\bar{C}_{\nu\beta}^{A7} + (V \leftrightarrow A)) = 2g^4\langle \mathrm{Re}\, N_7^V(\text{sing.})\rangle A_{\mu\nu}^{(3)\mathrm{PV}}(\text{Born}) \quad (4.7)$$

where $A_{\mu\nu}^{(3)\mathrm{PV}}(\text{Born})$ is the Born term amplitude squared given in (3.11). The colour and flavour space

* We are suppressing colour indices for the present discussion

summations are denoted by the symbol $\langle \cdots \rangle$. Using the results of [14] one has*

$$2g^4 \langle \mathrm{Re}\, N_7^V(\mathrm{sing.}) \rangle = g^4 N_C C_F \left[C_F H_v^C(\mathrm{sing.}) + \frac{N_C}{2} H_v^N(\mathrm{sing.}) + \left(\frac{N_f}{3} - \frac{11}{6} N_C \right) H_v^f(\mathrm{sing.}) \right]. \tag{4.8}$$

where

$$H_v^C(\mathrm{sing.}) = C\left(\frac{2}{\varepsilon^2} + \frac{1}{\varepsilon}(2\ln z_{12} - 3) \right)$$
$$H_v^N(\mathrm{sing.}) = C\left(-\frac{2}{\varepsilon^2} + \frac{2}{\varepsilon}\ln\frac{z_{13}z_{23}}{z_{12}} \right) \tag{4.9}$$
$$H_v^f(\mathrm{sing.}) = C\frac{1}{\varepsilon}$$

and C defined in (3.10) ($z_{ij}q^2 = 2q_iq_j$).

Comparing the virtual one-loop contributions in (4.9) with the real tree graph contributions in (3.19) we see that the IR singularities as well as the finite anomalous pieces cancel.

What remains to be done is to list the nonsingular contributions of $H_{\mu\nu}^{(3)\mathrm{PV}}$(1-loop). The real parts of the one-loop amplitude contribute only to the invariants H_6^{PV} and H_7^{PV} in the expansion (2.8). Writing

$$H_i^{\mathrm{PV}}(\mathrm{nonsing.}) = g^4 N_C C_F \left[C_F H_{vi}^C(\mathrm{nonsing.}) + \frac{N_C}{2} H_{vi}^N(\mathrm{nonsing.}) \right] \tag{4.10}$$

one obtains [14]**

$$\begin{aligned} H_{v6}^C(\mathrm{nonsing.}) = \frac{C}{z_{13}z_{23}} \Big\{ &- 8(1-z_{23})(9 + \ln^2 z_{12}) \\ &+ 4\frac{z_{12}(1-z_{12})^2 + 2z_{23}[(1-z_{23})(1+z_{12}) - z_{12}(1-z_{12})]}{(1-z_{23})(1-z_{12})} \\ &+ 8\ln z_{12}\frac{z_{12}(1-z_{13}) + z_{23} - z_{12}}{(1-z_{12})^2} \\ &+ 4\ln z_{13}\frac{3z_{13}z_{23} + 2z_{12}(1-z_{13})}{1-z_{13}} \\ &+ 4\ln z_{23}\frac{z_{13}z_{23}(z_{23}-z_{12}) - 2z_{12}^2}{(1-z_{23})^2} \\ &+ 8r(z_{12}, z_{13})\frac{z_{12}(z_{12}-z_{23})}{z_{23}} \\ &- 8r(z_{12}, z_{23})\frac{2z_{13}(1-z_{23}) + z_{12}(1-z_{13})}{z_{13}} \Big\} \end{aligned} \tag{4.11}$$

$$H_{v7}^C(\mathrm{nonsing.}) = -H_{v6}^C(\mathrm{nonsing.}; 1 \leftrightarrow 2) \tag{4.12}$$

and

$$\begin{aligned} H_{v6}^N(\mathrm{nonsing.}) = \frac{C}{z_{13}z_{23}} \Big\{ &- 8\frac{z_{12}(1-z_{12}) + z_{23}(1+z_{12})}{1-z_{12}} \\ &+ 8(1-z_{23})(2 + \ln^2 z_{12} - \ln^2 z_{13} \\ &- \ln^2 z_{23} - 2r(z_{13}, z_{23})) \\ &+ 8z_{13}\ln z_{13} - 8z_{23}\ln z_{23}\frac{1-z_{13}}{1-z_{23}} \\ &- 8r(z_{12}, z_{13})\frac{z_{12}(z_{12}-z_{23})}{z_{23}} \\ &- 8\ln z_{12}\frac{z_{12}(1-z_{13}) + z_{23} - z_{12}}{(1-z_{12})^2} \\ &+ 8r(z_{12}, z_{23})\frac{2z_{13}(1-z_{23}) + z_{12}(1-z_{13})}{z_{13}} \Big\} \end{aligned} \tag{4.13}$$

$$H_{v7}^N(\mathrm{nonsing.}) = -H_{v6}^N(\mathrm{nonsing.}: 1 \leftrightarrow 2). \tag{4.14}$$

We used the abbreviation

$$r(x, y) = \ln x \ln y - \ln x \ln(1-x) - \ln y \ln(1-y) - L_2(x) - L_2(y) + \pi^2/6 \tag{4.15}$$

where

$$L_2(x) = -\int_0^x dz \frac{\ln(1-z)}{z}. \tag{4.16}$$

Up to now we have only discussed the real part of the 1-loop amplitude. This is in fact sufficient, since the imaginary part of the 1-loop amplitude does not contribute to the $O(\alpha_s^2)$ structure functions as can be shown as follows. The imaginary part of the 1-loop amplitude can be shown to be proportional to the Born term [14, 15]. Since the Born term squared contributes only to the $\mu \leftrightarrow \nu$ antisymmetric structure functions [see (3.11)], the invariants H_8^{PV} and H_9^{PV} which multiply ($\mu \leftrightarrow \nu$) symmetric covariants remain unpopulated [see (2.8)].

5. Results

Adding up the real tree graph contributions from Sect. 3 and the virtual one-loop contributions from Sect. 4 we obtain our final $O(\alpha_s^2)$ result. To this we add the $O(\alpha_s)$ contribution and obtain

$$H_6^{\mathrm{PV}}(q_1, q_2, q_3, y) = 64\pi^2 C_F N_C \frac{1-z_{23}}{z_{13}z_{23}} \frac{\alpha_s}{2\pi}$$

---

* g is the strong coupling constant in the $\overline{\mathrm{MS}}$ scheme

** There are several misprints in [14]. They are corrected by the following replacements
i) $\bar{H}_6^B \to 4\bar{H}_6^B$ in (13) ii) $H_{\mu\nu}^{\mathrm{PV}} \to 4H_{\mu\nu}^{\mathrm{PV}}$ in (16) iii) $\tilde{H}_6 \to -4\tilde{H}_6$ in (17) iv) $\hat{H}_6 \to 4\hat{H}_6$ in (18) with the uncorrected $\bar{H}_6^B$ in the latter two cases

$$\cdot\left\{1+\frac{\alpha_s}{2\pi}\left(C_F H_6^C+\frac{N_C}{2}H_6^N+\left(\frac{N_f}{3}-\frac{11}{6}N_C\right)H_6^f\right)\right\} \quad (5.1)$$

where

$$H_6^C = 7-2\ln^2 y+4\ln y\ln z_{12}-\ln^2 z_{12} -3\ln y+\frac{\pi^2}{3}+\frac{1}{8C}H_{v6}^C(\text{nonsing.})\frac{z_{13}z_{23}}{1-z_{23}}$$

$$H_6^N = \tfrac{4}{3}-\ln^2 z_{13}-\ln^2 z_{23}+\ln^2 z_{12}-2\ln^2 y +4\ln y\ln\frac{z_{13}z_{23}}{z_{12}}+\frac{\pi^2}{3} +\frac{1}{8C}H_{v6}^N(\text{nonsing.})\frac{z_{13}z_{23}}{1-z_{23}}$$

$$H_6^f = -\tfrac{5}{3}+\ln y \quad (5.2)$$

where the $H_{v6}^{C,N}$ (nonsing.) are given in (4.11) and (4.13).

H_7^{PV} can be obtained from (5.1) by the C-conjugation relation

$$H_7^{PV} = -H_6^{PV}(1\leftrightarrow 2). \quad (5.3)$$

From H_6^{PV} and H_7^{PV} we obtain the PV cross sections σ^P and σ^A as given by (2.17) and (2.18). This is in complete analogy to what has been done in connection with the $O(\alpha_s)$ $q\bar{q}g$ cross section. Several possibilities have been considered in [13]. Experimentally the difficulty is that in order to measure the PV contributions the quark (or antiquark) jet has to be detected. This can be done in various ways and seems to be easier for heavy, i.e. charm and bottom, quarks than for light quarks. For total energies around the Z mass neglect of the quark mass is certainly a good approximation for c quarks and presumably also for b quarks. Therefore our results should be applicable in this region.

In order to get an idea about the corrections originating from the higher order terms we have considered the special case that the quark jet is detected and that the quark momentum determines the thrust axis.

For this case the lowest-order thrust distributions for the $\cos\vartheta$-part in (2.16) which is obtained by integrating over the remaining variable x_2 ($x_1 \geqq 2/3$; $x_1 = T$) is the following expression

$$\frac{d\sigma^P}{dT} = g_5\sigma^{(1)}\frac{\alpha_s}{2\pi}C_F\frac{1}{1-T}\left\{(1+T^2)\ln\frac{2T-1}{1-T} -\tfrac{3}{2}T^2+4T-8+\frac{4}{T}\right\} \quad (5.4)$$

This distribution is plotted in Fig. 1 as the curve labelled $O(\alpha_s)$. The other two curves in Fig. 1 give $d\sigma^P/dT$ including the $O(\alpha_s^2)$ corrections for cut-values $y=0{\cdot}04$ and $y=0{\cdot}01$. We have taken $N_f=5$ and $\alpha_s=0.16$ which is a reasonable value obtained from analysis of experimental distribution of $\sigma_U+\sigma_L$ with $O(\alpha_s^2)$ corrections included [5]. We see that for $y=0.04$ the $O(\alpha_s^2)$ correction enlarge $d\sigma^P/dT$ by as much as 50%. This differs from what was found for $d\sigma_U/dT$ and $d\sigma_L/dT$ at $y=0.04$ where the corrections were fairly small, less than 20% [16]. For $y=0.01$ the $O(\alpha_s^2)$ corrections made $d\sigma^P/dT$ smaller as to be expected from the $\ln^2 y$ term in (5.2). Here the corrections are even larger. But $y=0.01$ lies outside the perturbative region as was already observed in connection with $d(\sigma_U+\sigma_L)/dT$. It is also generally agreed upon the cut-values as low as $y=0.01$ are in a region where

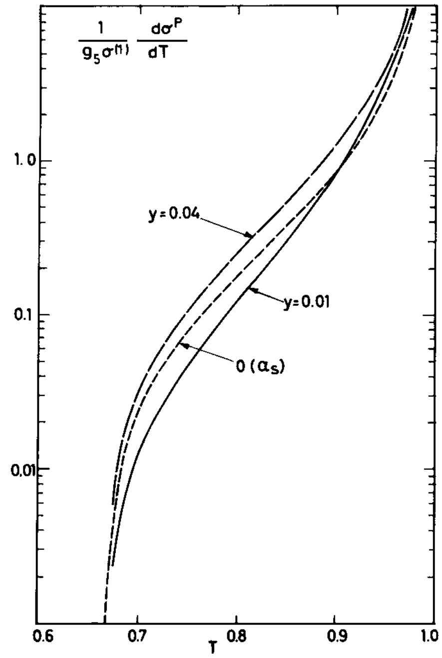

Fig. 1. $O(\alpha_s^2)$ three-jet cross section $1/g_5\sigma^{(1)}d\sigma^P/dT$ for $y=0.04$ and 0.01 together with the Born cross section ($O(\alpha_s)$) as a function of thrust T. The thrust axis is the quark momentum

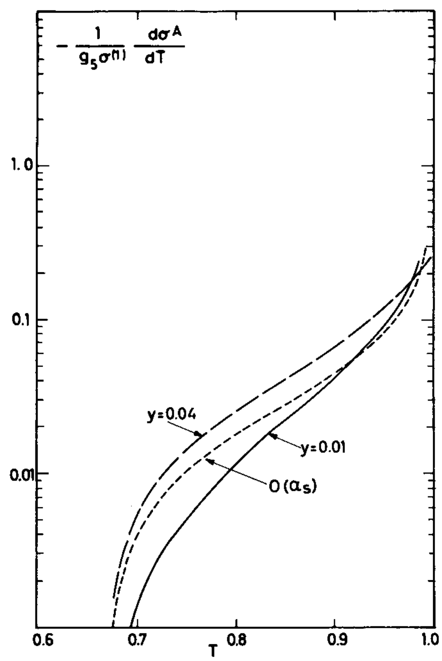

Fig. 2. $O(\alpha_s^2)$ thrust cross section $1/g_5\sigma^{(1)}d\sigma^A/dT$ for $y=0.04$ and 0.01 together with the Born cross section ($O(\alpha_s)$) as a function of thrust T. The thrust axis is the quark momentum. The χ axis is the in the direction of the antiquark

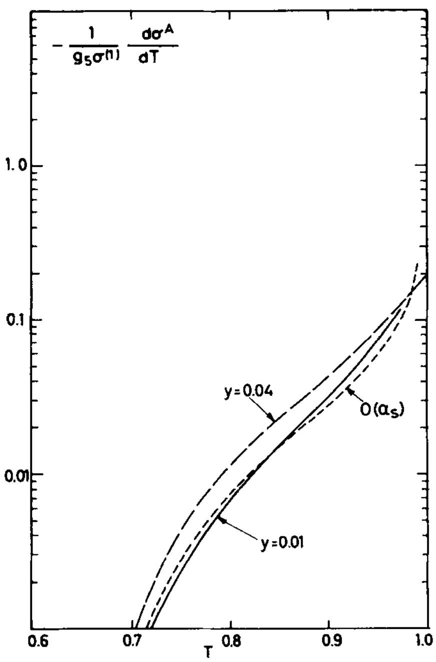

Fig. 3. $O(\alpha_s^2)$ thrust cross section $1/g_5\sigma^{(1)}d\sigma^A/dT$ for $y=0.04$ and 0.01 together with the Born cross section ($O(\alpha_s)$) as a function of thrust T. The thrust axis is the quark momentum. The χ axis is in the direction of the second most energetic jet

fragmentation effects make a separation of jets impossible and therefore should not be used [26].

The situation is similar for the cross section $d\sigma^A/dT$. Here the results are shown in Figs. 2 and 3. In Fig. 2 the coordinate system is chosen in such a way that the angle χ is defined by the antiquark momentum. In this case the $O(\alpha_s)$ distribution is

$$\frac{d\sigma^A}{dT} = -g_5\sigma^{(1)}\frac{\alpha_s}{2\pi}C_F\frac{1}{2\sqrt{2}\sqrt{1-T}\,T} \cdot\left\{(2-3T)\sqrt{1-T}\sqrt{2T-1}+\tfrac{1}{2}T(4-T)\cdot\arcsin\frac{3T-2}{T}\right\}. \quad (5.5)$$

We see that the $y=0.04$ curve is again appreciably bigger than the lowest order curve. In Fig. 3 we have considered the case that the angle χ is defined by the second most energetic jet which may be the antiquark or the gluon. Here it is not necessary to tag also the antiquark as is necessary for the asymmetry in Fig. 3. The lowest order distribution is

$$\frac{d\sigma^A}{dT} = g_5\sigma^{(1)}\frac{\alpha_s}{2\pi}C_F\frac{1}{\sqrt{2T}}\left(2\sqrt{2T-1}-\frac{T}{\sqrt{1-T}}\right). \quad (5.6)$$

This is smaller than the $O(\alpha_s)$ distribution in Fig. 3. Concerning the $O(\alpha_s^2)$ corrections the result is as follows. The $y=0.04$ distribution lies again appreciably higher than the $O(\alpha_s)$ distribution. The $y=0.01$ curve almost coincides with the $O(\alpha_s)$ result.

In total we observe that the $O(\alpha_s^2)$ corrections for $y=0.04$ for $d\sigma^P/dT$ and the two cases of $d\sigma^A/dT$ are somewhat larger than those obtained for $d\sigma_U/dT$ and $d\sigma_L/dT$. Since the $O(\alpha_s^2)$ terms involved in the IR cancellation procedure are rather universal, i.e. do not depend on the kind of cross section considered, we trace back this difference to a difference in the contribution of the virtual corrections to the $O(\alpha_s^2)$ terms.

Of course our results could have been presented as forward-backward asymmetries as was done earlier for the $O(\alpha_s)$ result [13]. We have not done so since we are interested to see the higher order corrections in the PV contributions without the influence of $O(\alpha_s^2)$ corrections of the PC contributions which are present in the denominator if asymmetries are formed. The order of magnitude of such asymmetries can be obtained from $O(\alpha_s)$ results in [13].

Instead of thrust distributions also other single-variable distributions could be considered as for example $x_\perp$ distributions with respect to the thrust axis or simply x_1 distributions.

The results presented in this paper are valid only as long as terms $O(y)$ and higher can be neglected. So to be sure that this is the case one must choose y small enough. Then our formulas should be valid. The results for a larger y value y_1 where our approximation might be questionable can be obtained by choosing first a small y_0, below $y=0.001$ say and by using our analytical formulae there. The contributions to the y-region between y_0 and y_1 can then be treated numerically by adding the appropriate 4-jet contribution in the desired y bin with two partons lying in this bin averaged over. From more general considerations it is clear that such corrections, say for $y=0.04$, are important only for thrust values above 0.8. Below these thrust values the $y=0.04$ distributions presented in Figs. 2–4 could be in error by several percent. Of course this can always be checked by calculating the correction terms numerically.

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Appendix A: The Breitenlohner–Maison (BM) γ_5-Scheme

It is well known that an anticommuting γ_5 is not compatible with dimensional continuation [17, 20, 24]. As an example consider the trace $\mathrm{Tr}(\gamma_5\gamma_\alpha\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5})$ with an anticommuting γ_5. Anticommute γ_α once around the trace to obtain

$$\varepsilon_{\mu_1\mu_2\mu_3\mu_4}g_{\mu_5\alpha}+\mathrm{cycl.}(\mu_1,\ldots,\mu_5)=0 \quad (A1)$$

where we introduce the totally antisymmetric tensor via $\mathrm{Tr}(\gamma_5\gamma_\alpha\gamma_\beta\gamma_\gamma\gamma_\delta)=4i\varepsilon_{\alpha\beta\gamma\delta}$. Contracting (A1) with $g^{\alpha\mu_5}$ gives

$$(n-4)\varepsilon_{\mu_1\mu_2\mu_3\mu_4}=0 \quad (A2)$$

which shows that $\varepsilon_{\alpha\beta\gamma\delta}$ or $\mathrm{Tr}(\gamma_5\gamma_\alpha\gamma_\beta\gamma_\gamma\gamma_\delta)$ cannot be analytically continued from $n=4$ to $n\neq 4$ with an anticommuting γ_5.

A related problem is that an anticommuting γ_5 leads to a nonunique result in the evaluation of traces of type $\mathrm{Tr}(\gamma_\alpha\gamma_5\gamma_\alpha\gamma_\beta\gamma_\delta\gamma_\gamma\gamma_\varepsilon)$ at the $O(n-4)$ level [24, 25].

A cure to the above problems has been suggested by 't Hooft and Veltman [24] and worked into a consistent scheme by Breitenlohner and Maison [20]. Split up a n-dimensional a_μ into its 4-dimensional component $\hat{\hat{a}}_\mu$ and the remaining component $\hat{a}_\mu$. Thus $\gamma_\mu = \hat{\hat{\gamma}}_\mu + \hat{\gamma}_\mu$ and a consistent γ_5-scheme is arrived at by postulating

$$\hat{\hat{\gamma}}_\mu\gamma_5 + \gamma_5\hat{\hat{\gamma}}_\mu = 0$$
$$\hat{\gamma}_\mu\gamma_5 - \gamma_5\hat{\gamma}_\mu = 0. \tag{A3}$$

The correct version of (A1) can now be obtained by considering the trace $\mathrm{Tr}(\gamma_5\hat{\hat{\gamma}}_\alpha\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5})$. One obtains

$$\varepsilon_{\mu_1\mu_2\mu_3\mu_4}\hat{\hat{g}}_{\mu_5\alpha} + \mathrm{cycl.}(\mu_1,\ldots,\mu_5) = 0 \tag{A4}$$

where we have used $\hat{\hat{\gamma}}_\mu\gamma_\nu + \gamma_\nu\hat{\hat{\gamma}}_\mu = 2\hat{\hat{g}}_{\mu\nu}$ and $\hat{\hat{g}}_{\mu\nu}$ is the 4-dimensional metric tensor. Equation (A4) will be referred to as the Schouten identity.

A suitable representation for γ_5 is [20]

$$\gamma_5 = \frac{i}{4!}\varepsilon_{\alpha\beta\gamma\delta}\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\delta. \tag{A5}$$

Using the identity

$$\varepsilon_{\mu_1\mu_2\mu_3\mu_4}\varepsilon_{\nu_1\nu_2\nu_3\nu_4} = -\det(\hat{\hat{g}}_{\alpha\beta}) \quad \begin{matrix}\alpha=\mu_1\cdots\mu_4\\ \beta=\nu_1\cdots\nu_4\end{matrix} \tag{A6}$$

one can prove that

$$\varepsilon_{\alpha\beta\gamma\delta}\hat{g}^{\delta\varepsilon} = 0 \tag{A7}$$

and

$$g_\alpha^\beta\hat{\hat{g}}_{\beta\gamma} = \hat{\hat{g}}_{\alpha\gamma}. \tag{A8}$$

Equation (A7) shows that the ε-tensor projects out the 4-dimensional components of any n-dimensional tensor it acts on. From (A7) it is clear that the Schouten identity (A4) must involve the 4-dimensional metric tensor $\hat{\hat{g}}_{\mu\nu}$ when comparing the different tensor components in (A4).

The appearance of the 4-dimensional tensor $\hat{\hat{g}}_{\mu\nu}$ on the RHS of (A6) can also be appreciated by considering the identity

$$12(\gamma_\mu\gamma_5+\gamma_5\gamma_\mu) = i\varepsilon_{\alpha\beta\gamma\delta}\gamma_\mu\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\delta + 4i\varepsilon_{\alpha\beta\gamma\mu}\gamma^\alpha\gamma^\beta\gamma^\gamma \tag{A9}$$

which can be derived using the representation (A5). Contracting (A8) with $\varepsilon^{\alpha'\beta'\gamma'\mu}$ and using (A6) with the *wrong* n-dimensional generalization $\hat{\hat{g}}_{\alpha\beta}\to g_{\alpha\beta}$ leads to a vanishing RHS of (A8) and thereby to an anticommuting γ_5 which was shown to be inconsistent in (A2).

Appendix B: n-Dimensional Integral of the Four Dimensional Scalar $\hat{\hat{p}}_4^2$

Consider the explicit representation of the n-dimensional momenta in the (3–4) CMS system. One has

$$p_1 = \frac{s_{134}-s_{34}}{2\sqrt{s_{34}}}(1,0,\ldots,\sin\beta,\cos\beta) \tag{B1}$$

$$p_2 = \frac{s_{234}-s_{34}}{2\sqrt{s_{34}}}(1,0,\ldots,0,1) \tag{B2}$$

$$\begin{aligned} p_{3,4} = \tfrac{1}{2}\sqrt{s_{34}}(1, &\pm\sin\Theta\sin\Theta_1\cdots\sin\Theta_{n-3},\ldots,\\ &\pm\sin\Theta\sin\Theta_1\cos\Theta_2,\\ &\sin\Theta\cos\Theta_1, \pm\cos\Theta)\end{aligned} \tag{B3}$$

where the dots in (B1) and (B2) denote $(n-4)$ zeros and in (B3) $(n-5)$ equal and opposite angular factors.

From (B3) one has

$$\begin{aligned}\hat{\hat{p}}_4^2 &= \frac{s_{34}}{4}(1-\sin^2\Theta\sin^2\Theta_1\cos^2\Theta_2\\ &\quad -\sin^2\Theta\cos^2\Theta_1-\cos^2\Theta)\\ &= \frac{s_{34}}{4}\sin^2\Theta\sin^2\Theta_1\sin^2\Theta_2\\ &= s_{34}v(1-v)\sin^2\Theta_1\sin^2\Theta_2\end{aligned} \tag{B4}$$

where we have set $v=\frac{1}{2}(1-\cos\Theta)$.

Compared to the case where n-dimensional scalars have to be integrated, $\hat{\hat{p}}_4^2$ introduces an additional Θ_2-dependence in the integrand. This can be taken into account by the replacement

$$1 \to \frac{1}{N_{\Theta_2}}\int_0^\pi d\Theta_2 \sin^{-1-2\varepsilon}\Theta_2 \tag{B5}$$

in the integrand where $N_{\Theta_2} = \sqrt{\pi}\,\Gamma(-\varepsilon)/\Gamma(\frac{1}{2}(1-2\varepsilon))$.

The Θ_2-integration on $\hat{\hat{p}}_4^2$ gives

$$\begin{aligned}&\frac{1}{N_{\Theta_2}}\int_0^\pi d\Theta_2\sin^{-1-2\varepsilon}\Theta_2\hat{\hat{p}}_4^2\\ &= s_{34}v(1-v)\sin^2\Theta_1\frac{-2\varepsilon}{1-2\varepsilon}\end{aligned} \tag{B6}$$

which can be used in (3.12) and (3.16) to show that the nonfactorizing IR parts in the integrated 4-parton cross sections do indeed vanish.

The complete angular integration of $\hat{\hat{p}}_4^2$ can of course be obtained more directly. Consider the integral

$$\begin{aligned}&\int d\tilde{\Omega}_{34}^{n-1}p_{4\alpha}p_{4\beta}\\ &= \frac{1}{4(1-n)}(s_{34}g_{\alpha\beta}-n(p_3+p_4)_\alpha(p_3+p_4)_\beta)\end{aligned} \tag{B7}$$

where the RHS is easily obtained remembering that $\int d\tilde{\Omega}^{n-1}=1$ (see Sect. 3). Then noting that $\hat{\hat{p}}_4^2 = \hat{\hat{g}}^{\alpha\beta}p_{4\alpha}p_{4\beta}$ one obtains

$$\int d\tilde{\Omega}_{34}^{n-1}\hat{\hat{p}}_4^2 = -\frac{\varepsilon}{2(3-2\varepsilon)}s_{34} \tag{B8}$$

Appendix C: The $O(\alpha_s)$ Parity-Violating Two-Parton Cross Section $e^+e^- \xrightarrow{\gamma,Z} q\bar{q}$

It is the purpose of this Appendix to give a brief presentation of the calculation of the $O(\alpha_s)$ corrections to the two-parton process $e^+e^- \to q\bar{q}$. As in the main text we employ an invariant mass cut $s_{ij} < yq^2$ and work to $O(y^0)$. We emphasize that the latter approximation is implicit in many steps of the following discussion.

(i) Real $O(\alpha_s)$ Contributions (Tree Graph Contributions)
The two contributing Feyman diagrams are shown in Fig. 4. As in the main text we denote the momenta occurring in the (3-parton)$\to$(2-parton) reduction by $(p_1, p_2, p_3) \to (q_1, q_2)$, where the p_i are n-dimensional and the q_i are 4-dimensional.

The 2-parton hadron tensor $H_{\mu\nu}^{(2)}$ (real) is obtained from the 3-parton hadron tensor $H_{\mu\nu}^{(3)}$ (p_1, p_2, p_3) by integration, cif.

$$H_{\mu\nu}^{(2)}(\text{real}) = \frac{q^2}{16\pi^2}\left(\frac{4\pi\mu^2}{q^2}\right)^{\varepsilon} \frac{1}{\Gamma(1-\varepsilon)} \int_0^y dy_{23}\, y_{23}^{-\varepsilon} \cdot \{\textstyle\int dv\} v^{-\varepsilon}(1-v)^{-\varepsilon} H_{\mu\nu}^{(3)}(p_1, p_2, p_3) \tag{C1}$$

where $y_{23} = 2p_2p_3/q^2$ and $v = y_{13}/(1-y_{23}) = \frac{1}{2}(1-\cos\Theta)$, where Θ is the polar angle between the gluon (p_3) and the quark (p_1). The integration symbol $\{\int dv\}$ stands for

$$\{\textstyle\int dv\} \equiv 2\int_0^1 dv - \int_0^{y/(1-y_{23})} dv. \tag{C2}$$

In (C2) one adds up the contributions from the two symmetric singular regions $s_{23} \to 0$ and $s_{13} \to 0$ with the appropriate proviso concerning double counting.

Inserting the appropriate PC and PV tree graph contributions in the integrand in (C1) one then obtains the desired singular $O(\alpha_s)$ PC and PV real contributions to the 2-parton cross sections after integration. Here we concentrate on the PV case, since the PC case is known from the work of [26].

For the PV tree graph contribution Fig. 4 one obtains

$$H_{\mu\nu}^{(3)\mathrm{PV}}(p_1, p_2, p_3) = g^2 C_F N_C (A_{\mu\nu}^{(3)\mathrm{PV}}(\text{Born}) + \varepsilon R_{\mu\nu}) \tag{C3}$$

where $A_{\mu\nu}^{(3)\mathrm{PV}}$(Born) is given in (3.11) (with $q_i \to p_i$) and

$$R_{\mu\nu} = \frac{8i}{s_{13}s_{23}}[2p_{3\mu}\varepsilon(\nu p_1 p_2 p_3) - 2p_{3\nu}\varepsilon(\mu p_1 p_2 p_3) + (s_{13}+s_{23})\varepsilon(\mu\nu p_1 p_2) + s_{23}\varepsilon(\mu\nu q p_1) - s_{13}\varepsilon(\mu\nu q p_2)]. \tag{C4}$$

Since $R_{\mu\nu} = 0$ in 4 dimensions by use of the Schouten identity (A4) one recovers (3.11) which was in fact derived for 4-dimensional outer momenta.

For the purpose of this calculation this assumption can no longer be maintained a priori, since the p_i are to

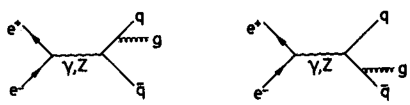

Fig. 4. $O(\alpha_s^{1/2})$ tree diagrams for $e^+e^- \to q\bar{q}g$

be taken as n-dimensional integration momenta. However, the error in using the Schouten identity for (C4) will be proportional to $\hat{p}_i^2$. From Appendix B it is clear that therefore the contribution of $R_{\mu\nu}$ can be at most $O(\varepsilon)$ after infrared integration. Thus $R_{\mu\nu}$ can be safely dropped.

In order to avoid having to do tensor integrations we again make an ansatz for the angular integral in (C1)

$$\{\textstyle\int dv\} v^{-\varepsilon}(1-v)^{-\varepsilon} H_{\mu\nu}^{(3)\mathrm{PV}} = \tilde{H}^{\mathrm{PV}} \varepsilon(\mu\nu q q_1). \tag{C5}$$

Note that on the RHS there appears only one covariant and thereby one invariant. The covariant is necessarily conserved. Thus one has

$$\{\textstyle\int dv\} v^{-\varepsilon}(1-v)^{-\varepsilon} q_\mu H_{\mu\nu}^{(3)\mathrm{PV}} = 0. \tag{C6}$$

Although the integrand in (C6) is nonzero since $H_{\mu\nu}^{(3)\mathrm{PV}}$ is nonconserved, the integral vanishes. This means that the anomalous pieces (or the gauge dependence) of $H_{\mu\nu}^{(3)\mathrm{PV}}$ vanish after integration.

Contracting both sides of (C5) with $\varepsilon(\mu\nu q q_1)$ we finally arrive at

$$\tilde{H}^{\mathrm{PV}} = 4g^2 C_F N_C \{\textstyle\int dv\} v^{-\varepsilon}(1-v)^{-\varepsilon} \frac{2}{q^2}\frac{1}{y_{23}} \cdot \left[(1-\varepsilon)v + 2\frac{1-v}{v}\right]. \tag{C7}$$

Repeating the same steps as in (C5)–(C7) for the PC case one arrives at the same integrand as in (C7). Finally, after doing the integrations (C1) we obtain

$$H_{\mu\nu}^{(2)\substack{\mathrm{PC}\\\mathrm{PV}}}(\text{real}) = A_{\mu\nu}^{(2)\substack{\mathrm{PC}\\\mathrm{PV}}}(\text{Born}) g^2 N_C C_F C \cdot \left(\frac{2}{\varepsilon^2} + \frac{3}{\varepsilon} + 7 - 2\ln^2 y - 3\ln y + \frac{\pi^2}{3}\right) \tag{C8}$$

where the 2-parton Born term contributions are given by

$$A_{\mu\nu\mathrm{Born}}^{(2)\mathrm{PC}} = 4\left(q_{1\mu}q_{2\nu} + q_{2\mu}q_{1\nu} - \frac{q^2}{2}g_{\mu\nu}\right) \tag{C9}$$

and

$$A_{\mu\nu}^{(2)\mathrm{PV}} = 4\varepsilon(\mu\nu q q_1) \tag{C10}$$

and C is given in (3.10).

Note that the IR factor in (C8) is the same as the QED-type IR factor in (3.19) for $z_{12} \to 1$. Vice versa the QED-type IR factor in (3.19) can be obtained from (C8) by the substitution $q^2 = s_{12}$.

(ii) Virtual $O(\alpha_s)$ Contributions (Loop Graph Contributions)
The three contributing loop diagrams are drawn in Fig. 5. The vector and axial vector current contri-

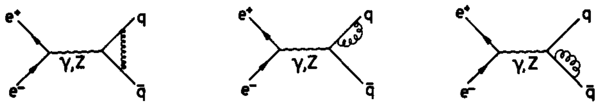

Fig. 5. $O(\alpha_s)$ one-loop diagrams for $e^+e^- \to q\bar{q}$

butions to the respective amplitudes are given by $C_V \bar{u}(q_1)\gamma_\mu v(q_2)$ and $C_A \bar{u}(q_1)\gamma_\mu\gamma_5 v(q_2)$. The vector current contribution C_V can be evaluated by using standard n-dimensional loop integrals as e.g. in [27]. To obtain the axial vector contribution C_A we impose the chiral invariance relation $C_V = C_A$.

One finally obtains

$$H_{\mu\nu}^{(2)\mathrm{PC}}_{\mathrm{PV}} = A_{\mu\nu}^{(2)\mathrm{PC}}_{\mathrm{PV}}(\mathrm{Born})\, C g^2 N_C C_F \left(-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} - 8 \right). \quad \text{(C11)}$$

Finally, adding up the two (real plus virtual) contributions (C8) and (C11), we obtain the Sterman-Weinberg type $O(y^0)$ correction to the 2-parton hadron tensor

$$H_{\mu\nu}^{(2)\mathrm{PC}}_{\mathrm{PV}} = A_{\mu\nu}^{(2)\mathrm{PC}}_{\mathrm{PV}}(\mathrm{Born}) \frac{\alpha_s}{2\pi} N_C C_F \cdot \left(-1 - 2\ln^2 y - 3\ln y + \frac{\pi^2}{3} \right) + O(y) \quad \text{(C12)}$$

The PC case agrees with the result given in [26].

If we integrate the PV $q\bar{q}g$ contribution over the 3-jet region ($y_{13}, y_{23} \geqq y$) we obtain

$$\sigma^P = \sigma_0^P \frac{\alpha_s}{2\pi} C_F \left(1 + 2\ln^2 y + 3\ln y - \frac{\pi^2}{3} \right) + O(y) \quad \text{(C13)}$$

Here σ^P denotes the cross section $d\sigma^P$ of (2.10) integrated over x_1 and x_2 up to the boundary y_{13}, $y_{23} \geqq y$. σ_0^P is the corresponding cross section for the $q\bar{q}$ final state. We see that the sum of 2- and 3-jet contribution to σ^P vanishes*. This agrees with the $m_q \to 0$ result in [28] where σ^P was calculated as a function of the quark mass.

* This result is of course independent of the $O(y^0)$ approximation

References

1. CELLO Collab. H.-J. Behrend et al.: Phys. Lett. **138B**, 311 (1984)
2. JADE Collab. W. Bartel et al.: Phys. Lett. **119B**, 239 (1982); Z. Phys. C—Particles and Fields **25**, 231 (1984)
3. MARK-J Collab. A. Adeva et al.: Phys. Rev. Lett. **50**, 2051 (1983); Phys. Rep. **109**, 131 (1984)
4. PLUTO Collab. Ch. Berger et al.: Z. Phys. C—Particles and Fields **28**, 365 (1985)
5. TASSO Collab. M. Althoff et al.: Z. Phys. C—Particles and Fields **26**, 157 (1984)
6. H. Aihara et al.: Z. Phys. C—Particles and Fields **28**, 31 (1985); E. Fernandez et al.: SLAC-PUB-3385 (1984)
7. A. Ali, B. Barreiro: Phys. Lett. **118B**, 155 (1982); Nucl. Phys. **B236**, 269 (1984)
8. K. Fabricius, I. Schmitt, G. Schierholz, G. Kramer: Phys. Lett. **97B**, 431 (1980); Z. Phys. C—Particles and Fields **11**, 315 (1982); H.N. Schneider, G. Kramer, G. Schierholz: Z. Phys. C—Particles and Fields **22**, 201 (1984)
9. R.K. Ellis, D.A. Ross, A.E. Terrano: Phys. Rev. Lett. **45**, 1226 (1980); Nucl. Phys. **B178**, 421 (1981)
10. J.A.M. Vermaseren, K.J.F. Gaemers, S.J. Oldham: Nucl. Phys. **B187**, 301 (1981)
11. T.D. Gottschalk: Phys. Lett. **109B**, 331 (1982); T.D. Gottschalk, M.P. Shatz: CALTEC preprint CALT-68-1172 (1984); F. Gutbrod, G. Kramer, G. Schierholz: Z. Phys. C—Particles and Fields **21**, 235 (1984)
12. G. Kramer, G. Schierholz, J. Willrodt: Phys. Lett. **78B**, 249 (1978); erratum ibid **80B**, 433 (1979)
13. G. Schierholz, D.H. Schiller: DESY 79-29 (1979); E. Laermann, K.H. Streng, P.M. Zerwas: Z. Phys. C—Particles and Fields **3**, 289 (1980); G. Schierholz, D.H. Schiller: DESY 80-88 (1980); J.G. Körner, D.H. Schiller: DESY 81-043 (1981)
14. J.G. Körner, G. Schuler: Z. Phys. C—Particles and Fields **26**, 559 (1985)
15. J.G. Körner, G. Kramer, G. Schierholz, K. Fabricius, I. Schmitt: Phys. Lett. **94B**, 207 (1980)
16. G. Kramer, B. Lampe: Commun. Math. Phys. **97**, 257 (1985)
17. G. Bonneau: Phys. Lett. **96B**, 147 (1980)
18. D. Maison: Phys. Lett. **150B**, 139 (1985)
19. T.L. Trueman: Phys. Lett. **88B**, 331 (1979)
20. P. Breitenlohner, D. Maison: Commun. Math. Phys. **52**, 11 (1977)
21. J.G. Körner, G. Schuler, G. Kramer, B. Lampe: Phys. Lett. **164B**, 136 (1985)
22. L. Clavelli, G.v. Gehlen: Phys. Rev. **D27**, 1495 (1983)
23. B.L. Combridge: Nucl. Phys. **B151**, 429 (1979)
24. G.'t Hooft, M. Veltman: Nucl. Phys. **B44**, 189 (1972)
25. M. Chanowitz, M. Furman, I. Hinchliffe: Phys. Lett. **78B**, 285 (1978); Nucl. Phys. **B159**, 225 (1979)
26. G. Kramer: Springer Tracts in Modern Physics, Vol. 102. Berlin, Heidelberg, New York: Springer 1984
27. G. Kramer, B. Lampe: Phys. Scr. **28**, 585 (1983)
28. J. Jeršak, E. Laermann, P.M. Zerwas: Phys. Rev. **D25**, 1218 (1982)