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Determination of the B Lifetime

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Abstract. We have determined the lifetime of B-hadrons produced in e^+e^- annihilation into multihadronic final states, by measuring the impact parameter of the track with respect to the centre of the interaction region. Two different analyses of the data were performed and gave compatible results. The lifetime is $\tau_B = 1.8^{+0.5}_{-0.4} \pm 0.4$ ps.

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1. Introduction

The B-lifetime, τ_B – the lifetime of hadrons containing a b-quark – depends on the weak couplings between the mass eigenstates of the b-quark and those of the c- and u-quarks. Thus a measurement of the B-lifetime constrains the values of the quark mixing angles in the Kobayashi-Maskawa scheme [1]. Three years ago the JADE collaboration [2] using data collected at the e^+e^- storage ring PETRA at DESY gave an upper limit $\tau_B < 1.4 \cdot 10^{-12}$ s. Thereafter the MAC [3], Mark II [4]. TASSO [5] and DELCO [6] groups obtained finite values of τ_B close to our upper limit. In the meantime our data sample has increased by a factor of seven. The measurement described here uses similar techniques to those of [2], and is based on 22,000 multihadronic events, corresponding to an integrated luminosity of 63 pb^{-1}, with an average centre of mass energy of about 35 GeV. As measured, this lifetime is an average over the lifetimes of the various types of B-flavoured hadrons produced and observed, and as such depends on the experimental conditions, and even on the cuts applied in the analysis, and may thus differ between experiments.

The JADE detector is described elsewhere [7]. Here we recall briefly the features of the apparatus relevant to this analysis. Charged tracks are measured in a cylindrical drift chamber (jet chamber). Up to 48 points are measured per track, with an accuracy of about 0.17 mm per point. Tracks can be extrapolated to the production vertex of the event with a resolution from these measurements of $\sigma_{xy} = 0.36$ mm in the plane transverse to the beams. This number has been determined by the analysis of μ pair events (for which the deviation from Coulomb scattering is negligible).

The centre of the interaction region (run vertex) is determined for the different running periods from Bhabha events, each determination using, on average, 150 to 200 Bhabhas, and is known with a typical statistical uncertainty of 0.04 mm. The half-width is approximately 0.4 mm horizontally and 0.02–0.04 mm vertically, including the time variation of the centre within the run period.

Electrons are identified by simultaneous measurement of (a) the momentum in the jet chamber (b) the electromagnetic shower energy deposited in a lead glass counter array and (c) the energy loss (dE/dx) in the jet chamber.

Muons are identified as penetrating charged particles in a system of hadron absorbers and drift chambers 6 absorption lengths thick, such that particles need a minimum momentum of 1.8 GeV/c to penetrate it.

The B-lifetime is determined by studying the displacement of tracks produced by the decay of the B-hadron from the production vertex of the event. The event configuration is considered in the x-y plane, perpendicular to the beam direction, as shown in Fig. 1. If the parent B-particle, which is assumed to be produced at the interaction point, travels a distance D in proper-time τ before decay, (thus $D = \beta\gamma c\tau$), then the projection of D in this plane is $D \sin\theta$, where θ is the polar angle of the B-particle. If the decay particle has an angle α with the trajectory of the B-particle in the x-y plane, then δ, the "distance of closest approach" to the vertex of the extrapolated track is

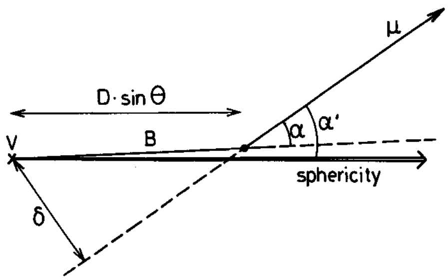


Fig. 1. Sketch of the method (projection onto the x-y plane perpendicular to the beam axis). The B-hadron is produced at the vertex and decays into a lepton. Symbols are defined in the text

$$\delta = D \sin\theta \sin\alpha .$$

The sign of δ is chosen such that a positive value corresponds to a positive value of the "decay length" D. In our analysis we use the sphericity axis to approximate the direction of flight of the parent B-particle, and the run vertex to estimate the interaction point of the event.

In what follows we present two different methods of analysis. Method 1 uses a system of cuts to isolate a sample of B-decay events of good purity, and obtains the δ-distributions for muons and electrons. The lifetime is determined from fits of the distributions using the maximum likelihood method. Method 2 uses a system of weights to enhance the effects of B-decay events statistically, and reconstructs the δ-distributions of muons and hadrons from B-decay on a statistical basis, obtaining the lifetime from the mean value of δ. These methods will be described in Sects. 2 and 3 respectively. The results are summarised in Sect. 4.

2. Method 1

In this method the aim is to select a sample of semileptonic B-decay events of high purity, with a well-measured lepton track.

The following sources of lepton candidates ("l") are considered in the analysis:

(i) $B \to$ "l",

(ii) $B \to C \to$ "l" (Indirect Charm production),

(iii) $C \to$ "l" (Direct Charm production),

(iv) Decays of long lived hadrons: $\pi^{\pm}$, $K^{\pm}$, K^0, Λ.

(v) Hadrons faking leptons originating from the production vertex of the event.

Muons and electrons were selected and analysed, as explained in Sects. 2.1 and 2.2. In Sect. 2.3 we describe the fitting of the distributions for δ and the selection of a sample of hadron tracks used to determine the resolution for δ and to confirm the absence of any systematic shift. Results are presented in Sect. 2.4.

2.1. Selection of Inclusive Muons from B-Decays

22,000 multihadronic events were selected using standard techniques [8] which remove τ lepton pair production and other backgrounds; of these, 1,600 satisfy the requirements of:

– At least 5 charged tracks extrapolate to within 10 mm of the origin in the x-y plane.

– $|\cos\theta_{sph}| < 0.85$, where θ_{sph} is the polar angle of the sphericity axis, as determined from charged and neutral particles.

– At least one μ-candidate track, using the standard selection procedure [9], which includes a requirement that the track momentum be greater than 1.8 GeV/c.

The following cuts were then imposed on the μ-candidate tracks in order to ensure that they extrapolate accurately to the vertex:

– At least 35 points are measured on the track, with an rms deviation from the fitted curve of $\sigma_{xy} < 0.22$ mm. This cut eliminates nearly all $K \to \mu$ decays inside the jet chamber.

– The polar angle of the track lies between 45° and 135°.

– The "distance of closest approach" δ to the production vertex of the event is less than 10 mm.

– The azimuthal angle between the track and the sphericity axis (α' in Fig. 1) is in the range $6° < \alpha' < 57°$. The purpose of the lower limit of this last cut is to reject tracks insensitive to the flight distance of the parent hadron. The upper limit is chosen to eliminate tracks which may be associated with the wrong jet, resulting in a negative flight distance.

293 μ-candidate tracks are left after these cuts.

Monte-Carlo calculations have been used to determine the different contributions to the categories (i)–(v). 38,000 multihadronic events were generated using the program [10] LUND 4.3, which includes the effects of initial state radiation, with the $B \to \mu$ branching ratio assumed to be 12%. The effect of these events in the detector was simulated by a Monte-Carlo procedure which includes the tracking of hadrons through the muon filter to determine the consequences of punch-through and decay in flight [11]. The generated events were then subjected to the same analysis and cuts applied to the real data.

Table 1. Contributions to the selected samples

Type	$b \to$ "l" (%)	$b \to c \to$ "l" (%)	$c \to$ "l" (%)	Decays (%)	Fake leptons (%)	Events MC	Events Data
μ events							
Before cuts	21	5	26	1	48	533	293
Cut 1	44	5	11	1	40	192	114
Cut 2	53	5	11	1	31	156	97
Cut 3	65	6	8	1	20	114	74
e events							
After cuts	80	8	10	0	< 3		34
hadron events							
After cuts	4	4	15	2	76		1,478

From this we obtain the fractional contributions of the 5 categories to the 293 μ-candidates selected. The results are shown in the first row of Table 1, and it can be seen that only 26% originate from B-decays.

Three cuts are then applied to enhance the B contribution without unnecessary loss of genuine $B \to \mu$ decays.

Cut 1. $p_T(\mu) > 0.9$ GeV/c, where $p_T(\mu)$ is the transverse momentum of the μ-candidate track with respect to the sphericity axis. This selects tracks with high transverse momenta, as are produced in $B \to \mu$ decays. Cut 1 also serves another purpose. If the angle ϕ between the direction of the non observed parent particle and the sphericity axis (see Fig. 1) is larger than the decay angle α, then the sign of δ is determined incorrectly if the parent particle decays towards the sphericity axis, washing out the shift in the δ-distribution towards positive values caused by the non-zero lifetime. For B-decays the effect is small, as in general α is large and the B particles are produced well aligned with the sphericity axis so ϕ is small (from the Monte-Carlo events we find $\phi < 100$ mrad.). But because of the cut on p_T, for the π and K decay background, on the other hand, the parent pions and kaons are not aligned with the sphericity axis, and as the decay angles are small, any shift in the δ-distribution from π and K decays is obscured and it is symmetric about zero.

Cut. 2. To reduce the contribution of events containing hard gluon radiation, the candidate μ track is required to be isolated from other high energy particles by the cut

$$\sum_i p^{\mu}_{L_i} < 2.0 \text{ GeV/c}$$

where $p^{\mu}_{L_i}$ is the longitudinal momentum of track i with respect to the μ direction; the sum runs over all charged and neutral particles (other than the muon) with $\cos\gamma_{i\mu} > 0.9$, where $\gamma_{i\mu}$ is the angle between the particle and the muon, and with $p^{\mu}_{T_i} < 0.32$ GeV/c, where $p^{\mu}_{T_i}$ is the transverse momentum of the particle with respect to the muon.

Cut 3. Events with a pronounced three jet structure were eliminated by restricting the width of the event in the event plane. Each event is assumed to contain two jets, and the principle axes of the sphericity tensor $(\hat{q}_1, \hat{q}_2, \hat{q}_3)$ are calculated for each jet separately. The projections of the particle momenta on the $\hat{q}_2$ axis (p_T^{in}) are used for the following cut:

$$0.20 < \left(\frac{\sum |p_T^{\text{in}}|}{\sum |p|}\right)_{\mu\text{-jet}} < 0.60$$

and

$$0.10 < \left(\frac{\sum |p_T^{\text{in}}|}{\sum |p|}\right)_{\text{opp.-jet}} < 0.45$$

where the sum runs over all particles of the μ-jet or the opposite jet. The μ-jet contains the $b \to \mu$ decay with an isolated muon of large transverse momentum, as already selected by cuts 1 and 2 whereas the opposite jet usually does not contain a leptonic decay. Therefore the average transverse momentum is slightly larger for the μ-jet than for the opposite jet, necessitating slightly different cuts for the two jets.

These 3 cuts reduce the data sample to 74 μ-tracks; the Monte-Carlo data sample is reduced from 533 to 114. The distributions of the cut variables for the experimental and generated data have been compared and found to be in good agreement, and the reduction factors achieved by the 3 cuts are the same within errors for the real and for the Monte-Carlo data. This is shown in Table 1, from which it can also be seen that (71 ± 4) % of the accepted μ-tracks originate from B-decays.

The δ-distribution for the remaining 74 tracks is shown in Fig. 2a. There is an excess of entries with positive values of δ, and it cannot be described by a Gaussian of mean zero (shown by the dashed line). The average is $\langle\delta\rangle = (0.282 \pm 0.078)$ mm. The sensitivity of $\langle\delta\rangle$ to the C- and B-lifetimes, as estimated by Monte-Carlo calculations, is $\langle\delta\rangle/\tau_C = 0.10$ mm/ps for C decays and $\langle\delta\rangle/\tau_B = 0.20$ mm/ps for B-decays. Combining these with the fractions shown in Table 1, one obtains $\tau_B = 1.92 \pm 0.57$ ps. This error incorporates the effect of the broadening of the δ distribution due to a non-zero lifetime.

It should be noted that there are no tracks in the range $-5\text{ mm} < \delta < -1.5$ mm, or $2.5\text{ mm} < \delta < 5$ mm. This supports the Monte Carlo calculation that the

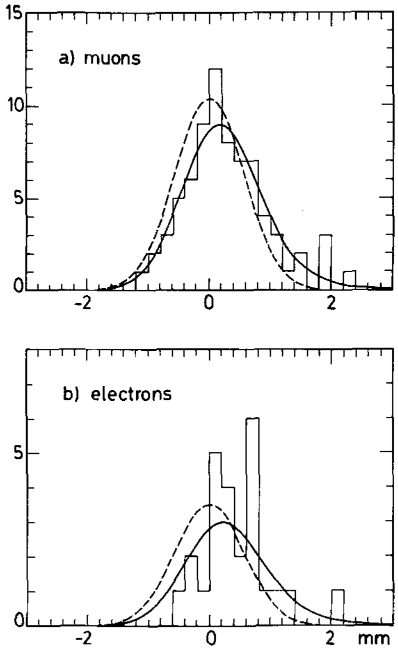


Fig. 2a, b. δ-distribution of **a** selected muons and **b** selected electrons. Dashed lines are Gaussian fits with zero mean. The full line uses a B lifetime of 1.8 ps

background from π and K decays in the range $-5\text{ mm} < \delta < 5$ mm is negligible (less than one event).

All events with $\delta > 1.0$ mm were visually scanned to ensure that they did not arise from some effect other than a decay, but no such effect has been found. Furthermore, the correlation plots of δ with the cut variables have been examined and they do not show any systematic trends.

2.2. *Selection of Inclusive Electrons from B Decays*

Energetic electrons are expected to provide a clean signal for B- and C-decays, since electron production from π and K decays is negligible, as is the contribution of Dalitz decays of the π^0, and hadrons faking electrons can be eliminated by appropriate cuts. Only electrons from photon conversions require a careful study.

The sample of events is subjected to the same B-enhancement cuts as applied to the muon sample. Electrons were identified by the following criteria:

– A momentum (p) above 1.5 GeV/c.

– The momentum compatible with the energy (E) deposited in the lead glass counter array. Explicitly we require:

$$E/p > 0.85.$$

– An energy loss in the jet chamber gas volume (dE/dx) greater than 8.6 keV/cm, to be consistent

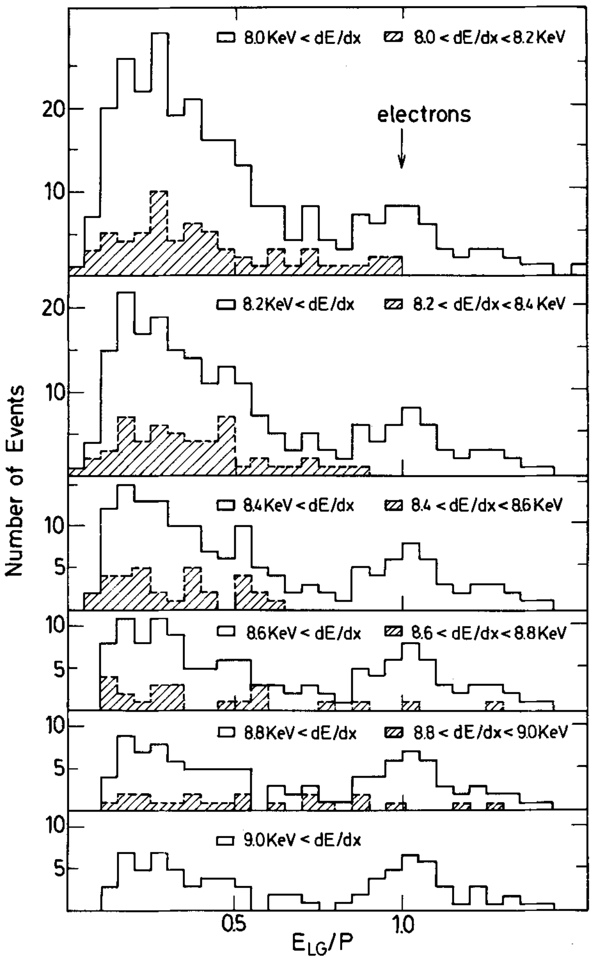


Fig. 3. E/p distribution of the electron candidates for different cuts of dE/dx. The shaded histograms show the entries that would be lost by the next higher dE/dx cut

with the value of (10.0 ± 0.8) keV/cm expected for an electron.

In addition the electron tracks are required to satisfy the same track quality cuts as those applied to the μ-candidates. The quality of the electron selection is shown in Fig. 3, where the E/p distributions are shown for various cuts of dE/dx; the peak at $E/p = 1.0$ is attributed to electrons. The signal becomes cleaner by increasing the dE/dx cut up to $dE/dx > 8.6$ keV/cm. At higher values the cut becomes unselective for electrons. This is demonstrated by the shaded histograms of Fig. 3, which indicate those entries which would be lost by the next higher cut.

After this selection there are 34 tracks left, which have been scanned visually and 9 candidates eliminated as they could be possibly attributed to photon conversions. Of the remaining sample of 25 events, the expected contamination by hadrons faking electrons is less than one event, as determined from interpolation of the E/p distributions near the cut. This background is assumed to consist of 1 track, originating from the event vertex; the remaining sample is assumed to come from $C \to e$, $B \to e$ and $B \to C \to e$ decays in the same proportions as determined by Monte-Carlo events for the muons (see Table 1). From this we estimate that 88% of the electrons can be attributed to B-decays.

The δ-distribution of the electron tracks is shown in Fig. 2b. Again there is an excess of tracks with positive values of δ, and it is not well fitted by a Gaussian of zero mean (dashed curve). The average value is $\langle \delta \rangle = (0.457 \pm 0.107)$ mm.

2.3. *Fits of the δ-Distributions*

The observed δ-distributions for leptons have been fitted using the maximum likelihood method with the B-lifetime τ_B as a free parameter. An average of 0.6 ps has been used for the C-lifetime.

The impact parameter distribution for use in the likelihood function is given by:

$$P(\delta; \tau_B) = f_B P_B(\delta; \tau_B) + f_{BC} P_{BC}(\delta; \tau_B, \tau_C) + f_C P_C(\delta; \tau_C) + f_l P_l(\delta; \tau_l) + f_0 P_0(\delta)$$

where the f's, the fractions of lepton candidates, and the P's, the probability densities, corresponding to the classes (i)–(v) are estimated by Monte-Carlo techniques.

For the process $B \to$ "l", for example, the impact parameter of a track, before smearing by the resolution is proportional to the lifetime with a proportionality factor $(\delta/\tau_B) = \beta\gamma c \sin\alpha \sin\theta$. The exponential impact parameter distribution folded with the Gaussian measurement error $P_0(\delta)$ is then averaged for the B-decays passing all cuts:

$$P_B(\delta; \tau_B) = \frac{1}{N_B} \sum_i \frac{1}{\sigma_\delta \sqrt{2\pi}} \cdot \int_0^\infty \exp\left(-x - \frac{(\delta - x\tau_B(\delta/\tau_B)_i)^2}{2\sigma_\delta^2}\right) dx.$$

The integral is calculated analytically for the different values of δ and τ_B, as well as the corresponding double integral for the $B \to C \to$ "l" cascade. $(\delta/\tau_B)_i$ is the impact parameter to lifetime ratio for event i.

The impact measurement error σ_δ includes the extrapolation error and the beam size averaged for the azimuthal angle. To determine its value experimentally, we studied inclusive hadronic tracks from the full hadronic event sample. These tracks originate predominantly from the production vertex of the event. In addition, the study also verified that

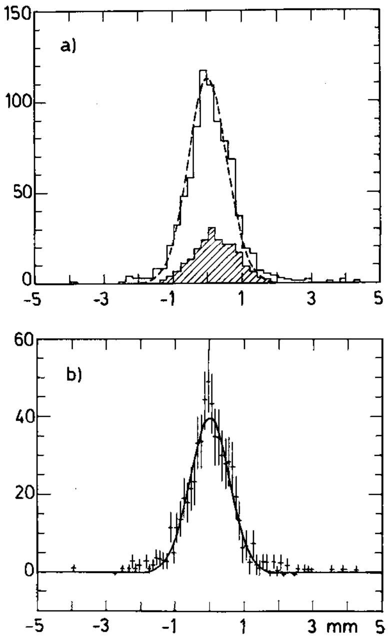


Fig. 4 a, b. δ-distribution of **a** all selected hadrons and **b** when estimated decay contributions have been subtracted. The subtracted contribution is also shown in **a** as a shaded histogram. The curves represent fits of a Gaussian with zero mean in the range $-1.5<\delta<1.5$ mm

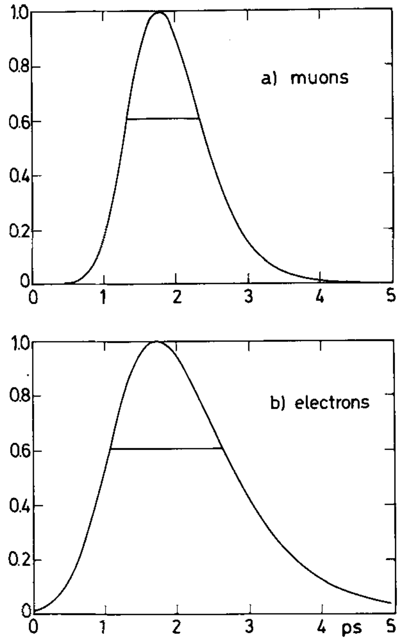


Fig. 5 a, b. Likelihood as a function of the *B*-lifetime for **a** muons and **b** electrons. The 1 standard-deviation range is indicated

there was no systematic shift of the average away from zero (apart from a small contribution from *B*- and *C*-decays which was corrected for by Monte-Carlo simulation).

Tracks were assumed to be hadrons if the corresponding lead glass energy did not exceed 0.5 GeV/c. The momentum of the hadron candidate tracks was required to exceed 2.0 GeV/c, in order to match the average momenta of the leptons. In addition, the same event criteria and track quality cuts were applied as were used for the μ-candidates. Cut 2 of the *B*-enrichment cuts was also applied, ensuring that there were no other close high-momentum tracks which might prevent an accurate track measurement. This leaves a sample of 1,478 hadron tracks.

The δ-distribution is shown in Fig. 4a. The average $\langle\delta\rangle=(0.042\pm0.021)$ mm is much smaller than that of the muon and electron samples. This excess can be accounted for from the composition of the hadron sample which has been investigated using Monte-Carlo events. As shown in Table 1, 8 % come from *B* decay and 15 % from *C*-decays. The expected contributions from *B*-decays, and the decays of long-lived hadrons (π, K, Λ) are shown as the shaded histogram in Fig. 4a. After subtraction one obtains a distribution (Fig. 4b) with mean $\langle\delta\rangle=(-0.006\pm0.024)$ mm. It is well fitted by a Gaussian of mean zero and standard deviation 0.57 mm; the χ^2 is 22.2 for 28 degrees of freedom.

This resolution of $\sigma_\delta=0.57$ mm, which is used in the distribution functions, can be compared to a value of 0.52 mm which has been calculated from the resolution for $\mu^+\mu^-$ pairs and the beam resolution, taking multiple scattering into account. The difference indicates an additional contribution to σ_δ which is not understood in detail. It is ascribed to the more complicated nature of multihadronic events where the tracks can easily be distorted by other close tracks. If σ_δ is allowed to vary in the fit, the resulting value is consistent with the one used.

2.4. Results of Method 1

The likelihood function

$$L=\prod_k P(\delta_k;\tau_B)$$

is calculated with τ_B as a free parameter. The product runs over the accepted tracks in the experimental data. The function is shown in Fig. 5 for the muon (Fig. 5a) and electron (Fig. 5b) samples. The one standard deviation range is indicated for each. From these we obtain:

$\tau_B=1.78^{+0.55}_{-0.45}$ ps from the muon sample, and

$\tau_B=1.73^{+0.90}_{-0.67}$ ps from the electron sample.

Table 2. Contributions to the systematic error

Uncertainty of δ distribution of $B \to$lepton decays	0.15 ps
Uncertainty of fraction of $B \to$lepton decays	0.30 ps
Uncertainty of average C lifetime: $\tau_C = 0.6 \pm 0.2$ ps	0.05 ps
Uncertainty of resolution: $\sigma_\delta = 0.570 \pm 0.075$ mm	0.15 ps
Uncertainty from π, K, and Λ decays	0.10 ps
Total systematic error	0.38 ps

Note that the purity is very different for the two samples, as shown in Table 1. This is why the lifetimes are similar though the values of $\langle \delta \rangle$ differ.

The combined value is

$$\tau_B = 1.76^{+0.45}_{-0.38} \text{ ps}.$$

The full line in the δ-distributions of muons and electrons (Figs. 2a and b) shows the probability distributions using a B-lifetime of 1.8 ps, normalised to the number of events. The agreement with the data is good: one obtains $\chi^2 = 2.4$ for 8 degrees of freedom for the μ data and $\chi^2 = 3.8$ for 3 degrees of freedom for the electron data.

The statistical error originating from the Monte-Carlo calculations is negligible. It is a factor of 3.6 smaller than the statistical error on τ_B from the data alone, and also smaller than the systematic uncertainties which essentially arise from the uncertainty in the δ distribution and in the fraction of $B \to$lepton decays. It should however be stressed that these uncertainties do not affect the significance of the measurement, but only the scale of the value of the B-lifetime. The error estimates are shown in Table 2, as well as the uncertainties due to the assumed average C-lifetime and due to the resolution σ_δ. The different systematic errors add in quadrature to a total of 0.38 ps.

The following checks have been performed to ascertain that the observed non-zero lifetime is a genuine effect. No significant difference was found between the lifetime estimates from subsamples such as positive, negative, horizontal and vertical tracks, or tracks on the left or right hand side of the sense wires in the jet chamber segments. The variation of the lifetime for different cuts in the impact parameter is consistent with statistics. Detailed studies show that errors in the determination of the beam position cannot lead to a bias in the lifetime estimation. Taking into account the shape of the beam spot in the $r-\phi$ plane affects our result only by ≈ 0.02 ps. Similarly Monte-Carlo investigations have shown that using the average extrapolation error instead of individual track errors has a negligible effect.

The analysis has been repeated using a later version of the LUND Monte-Carlo, version 5.2 [10]. The resulting lifetime is somewhat higher – $\tau_B = 2.0$ ps, which accords with the systematic error we quote on the B-fraction.

As an additional check, we have also determined with the same method the τ lepton lifetime from a sample of 1,865 clean $\tau^+\tau^-$ pair events. The same cuts as applied to the leptons of the enriched B-sample (apart from the B-enrichment cuts) reduced the sample to 284 tracks. The δ-distribution of these tracks is shifted away from zero towards positive values of δ. The average is (0.112 ± 0.035) mm. The τ-lifetime is determined from the maximum likelihood fit to be $\tau_\tau = (0.35 \pm 0.11)$ ps. This result is in good agreement with the world average value [12] of: $\tau_\tau = (0.290 \pm 0.017)$ ps.

3. Method 2. Weighted Distributions

An alternative way to isolate the b-quark signal is to use a weighting method [13]. Instead of using cuts to select a purified sample, the whole sample is used, and the effects of the signal are enhanced when histogramming quantities of interest (such as the impact parameter) by using a weight function which favours signal events. Then, as the effect of the increase is known from Monte-Carlo studies, the pure signal histogram can be reconstructed as the appropriate linear combination of the weighted and unweighted histograms. Comparing this analysis with the method of cuts used above has the advantage that the systematic errors come largely from different sources in the two cases.

B events are characterised by (1) a fat event shape, as measured by the aplanarity A, and (2) a muon with high transverse momentum p_T. (This may be produced indirectly, via $B \to C$ decay, but it still serves to indicate the presence of a B event.) The properties of b-quark events in the sample are thus enhanced by preferentially weighting events with high A and/or p_T. As is shown in [13], the best weight function to use for this purpose is the probability $P(p_T, A)$ that an event of given p_T and A belongs to the *signal* – i.e. $b\bar{b}$ events – rather than the *background* – i.e. $u\bar{u}$, $d\bar{d}$, $s\bar{s}$ and $c\bar{c}$ events. This function is determined from Monte-Carlo studies of the forms of the distributions in p_T and A for the two classes of events. In the next section we describe how the probability function was obtained. It is also possible to use the hadrons from the B-decay to determine the lifetime, and this is described in Sect. 3.2. Results are given in Sect. 3.3.

3.1. The Weighting Method and Muon Tracks

To determine the form of $P(p_T, A)$ a sample of 35,000 LUND 5.2 [10] Monte Carlo events was

Table 3. Quark flavours

	up	down	strange	charm	bottom
Unweighted	23.5 ± 1.4	5.4 ± 0.7	5.6 ± 0.7	46.9 ± 1.9	18.6 ± 1.2
Weighted	18.3 ± 1.4	3.8 ± 0.6	4.0 ± 0.7	38.6 ± 2.1	35.4 ± 2.7
Signal	-4.1 ± 5.2	-3.2 ± 2.2	-2.9 ± 2.6	2.8 ± 7.7	107.4 ± 10.1
Background	29.4 ± 4.8	7.3 ± 2.3	7.4 ± 2.4	56.4 ± 6.7	-0.5 ± 3.0

This shows the flavour content as percentage fractions for the various distributions. The errors come from the Poisson statistics on the total numbers. The unweighted total ratio is not 4:1:1:4:1 because these events are already selected as containing a muon

used. These include 2nd order QCD with Λ set to 0.5 GeV, and electromagnetic radiative corrections. They were tracked and analysed as described previously (Sect. 2.1). 2,351 penetrating tracks were found, of which 1,570 survived a cut of $|\cos \theta_{\rm sph}| < 0.7$, which was also applied to the data.

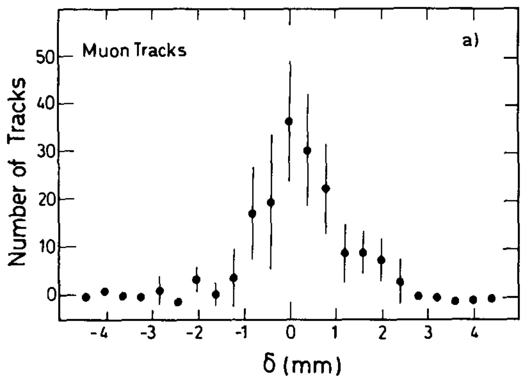
We parametrise the dependence of P on A and p_T by a polynomial of the form:

$$P(p_T, A) = \sum_{i=0}^{n} \sum_{j=0}^{n} a_{ij} A^i p_T^j.$$

A satisfactory fit was obtained for $n=3$; an increase to $n=4$ did not yield significant improvement. Using the same weight function for all bins of the δ distribution, ignoring the variation of the signal fraction with δ, makes the weight function only slightly less than optimal.

To assure the applicability of the weighting method, it was checked that for Monte-Carlo events the correlation between δ and the indicator variables p_T, A is, as one would expect, negligible, both for signal and background. Applying the method to the flavour of the original $q\bar{q}$ pairs in the Monte-Carlo data, the reconstructed signal and background distributions are consistent with the corresponding pure subsamples. These are shown in Table 3.

The polynomial functions were now used to weight the data. The "distance of closest approach" δ was found for the muon tracks, as described in the previous method. Tracks with less than 30 hits, or with $|\delta| > 2.5$ mm were not used, and each value was weighted (in addition to the flavour-enhancing weight) by the inverse square of the combined error from track measurement, Coulomb scattering, and the beam spot size. The signal and background distributions obtained from the appropriate linear combinations of the weighted and unweighted histograms are shown in Figs. 6a, b. The signal histogram shown in Fig. 6a is noticably asymmetric, and has a mean value of (0.242 ± 0.148) mm. In addition to the statistical error there are systematic errors on this value due to the weight function and the cuts of 20%, as determined by varying the Monte-Carlo.



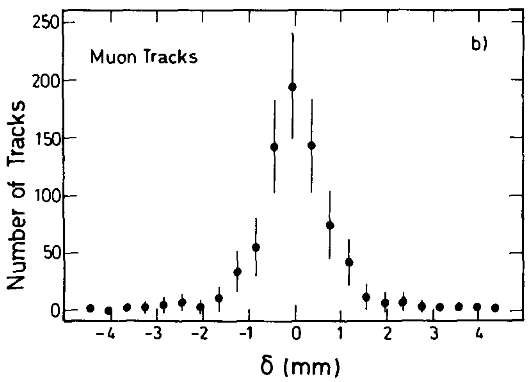


Fig. 6a, b. δ-distributions from muon tracks. **a** shows the "signal" distribution, extracted by the weights of method 2, and **b** is the "background" distribution

The background (non-b) sample is symmetric and has a mean consistent with and close to zero, being (0.047 ± 0.084) mm. This shows that the shift in δ is associated with b-quark events.

3.2. *Hadron Tracks from B-Decays*

If, as is generally accepted, b quarks have a hard fragmentation [14], the B-hadron containing the b-quark takes up a large fraction of the jet energy, and there is little left over for the hadronisation cascade: at the primary particle level a typical event contains two B-hadrons and only two or three others. Using the function of Peterson [15], with ε for b quark fragmentation set to 0.02, the LUND model predicts

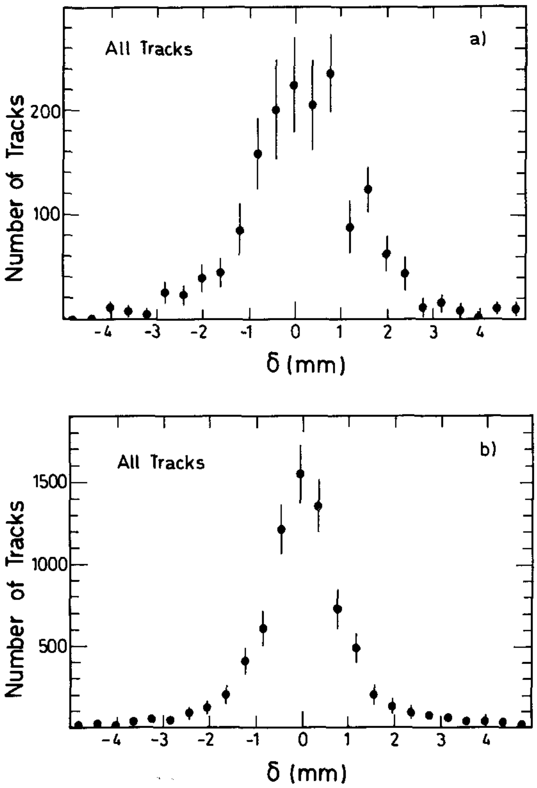


Fig. 7 a, b. As Fig. 6, but using all tracks instead of only the muons

that in $b\bar{b}$ events, 71 % of all charged tracks come from B-hadrons. Changing ε within broad limits alters this figure only slightly, by ± 5 %. This suggests that the analysis can be repeated using all the charged tracks (i.e. hadrons and leptons) in the signal sample to find the mean δ (though the muon is still used to indicate the b quark).

Results are shown in Figs. 7a, b. These use 9,233 charged tracks from the selected events: again, tracks were weighted by their measurement errors, and those with less than 30 hits were not used. A clear asymmetry is visible in the signal, while the background curve is symmetric. The average δ for the signal is (0.195 ± 0.062) mm, and for the background (0.074 ± 0.039) mm. When corrected for the B-fraction, this becomes (0.275 ± 0.087) mm.

A further correction is due to interactions in the beam pipe. The probability of a hadronic track interacting is calculated as 5 %, producing on average 1.7 charged tracks, so only 92 % of the observed tracks come from the production vertex. The size of the signal is thus (0.299 ± 0.095) mm.

3.3. Results of Method 2

The proper lifetime of a decay process is related to δ as described in Sect. 2. The conversion factor $\langle \delta \rangle / \tau_B$ was accordingly evaluated using Monte-Carlo events, giving 0.14 mm/ps for the muons (with a momentum cut at 1.8 GeV/c) and 0.15 mm/ps using all tracks, with an error of 15 %. These differ from the conversion factors used by Method 1 because of the different cuts imposed.

Part of the average δ is due to the finite charm lifetime. Based on the Monte-Carlo estimate, we expect 46 % of the final state charged particles from b quarks to come via a cascade decay through the c quark. They thus produce an extra effective lifetime equal to that of the charm quark (as the angle between the b and c quark directions in the lab is very small), which we take as 0.6 ps. This is aggravated by the fact that the c quark, though of lower energy than the parent b quark, has a slightly higher velocity, the mean gamma factors being 2.6 and 2.2 respectively. The effect of the charm lifetime is thus 0.3 ps, and this is accordingly subtracted.

We obtain the value of τ_B directly from the average δ and the conversion factor, rather than by fitting the form of the distributions of Figs. 6a and 7a, as to perform such a fit, assumptions about the shape of the distribution are necessary. Thus although the statistical accuracy is improved by such a fit, sources of systematic error become more important. This procedure also has the advantage of complementing the analysis of Method 1.

The results from muons and from all tracks are compatible, the latter having a superior statistical accuracy; it is:

$$\tau_B = (1.7 \pm 0.6 \pm 0.4)\ \text{ps}.$$

The systematic error of 25 % arises from the uncertainties in the weight function (20 %) and the conversion factor (15 %).

The B-hadrons in the Monte-Carlo program were assumed to decay with a lifetime of 1 ps, and the value found by the analysis was (0.7 ± 0.4) ps, thus confirming its validity.

4. Conclusions

The results of Method 1 and Method 2 agree with and support each other. As Method 1 gives a more accurate determination, that is the number we adopt:

$$\tau_B = (1.8^{+0.5}_{-0.4} \pm 0.4)\ \text{ps}.$$

Our previous result [2], expressed as a measurement of τ_B, agrees with the current value within 2 standard deviations, even disregarding the systematic errors.

This result agrees with other measurements by PEP and PETRA experiments [3–6].

We expect to perform a more accurate determination in the future, with the addition of a precision vertex chamber to our apparatus.

Gaillard and Maiani have expressed the B-lifetime in terms of the K-M matrix elements [16]:

$$\tau_B = \frac{\tau_\mu (m_\mu/m_b)^5}{2.75\,|U_{bc}|^2 + 7.7\,|U_{bu}|^2}$$

where τ_μ is the μ lifetime and m_μ and m_b are the masses of the μ and the b-quark. Studies of the lepton spectrum from B-decays [17] show that

$$|U_{bu}| < 0.14\,|U_{bc}|$$

so the $|U_{bu}|$ term can be neglected. Assuming $m_b = 5$ GeV one obtains:

$$|U_{bc}| = 0.043.$$

The error on this is dominated by a $\approx 10\%$ uncertainty on m_b, therefore it is not quoted.

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