LINE OF SECOND-ORDER PHASE TRANSITIONS IN THE FOUR-DIMENSIONAL Z_2 GAUGE THEORY WITH MATTER FIELDS

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Strong evidence is presented that the phase transition between the free-charge and the screening region of the four-dimensional Z_2 lattice gauge theory with Z_2 matter fields is second order with mean field exponents. The quantity best suited for the analysis is an order parameter that tests the existence of charged states. Both its scaling and finite-size scaling properties are determined by performing a Monte Carlo simulation.

1. Introduction. The question of the existence of non-gaussian fixed points in four-dimensional quantum field theories has motivated various investigations of phase diagrams of lattice gauge theories [1].

The Z_2 theory with Z_2 Higgs fields is defined by the action [2]

$$S = -\beta_g \sum_p \delta\tau(p) - \beta_h \sum_\ell \delta\sigma(\ell)\tau(\ell), \tag{1.1}$$

where $\tau(\ell)$ is the Z_2 gauge field at the link ℓ, $\sigma(x)$ is the Z_2 Higgs field at the point x, $\delta\tau(p)$ is the product of the $\tau(\ell)$ around the plaquette p, $\delta\sigma(\ell)$ is the product of the two $\sigma(x)$ at the endpoints of ℓ, and β_g and β_h are couplings. The phase diagram in four dimensions (fig. 1) has been known from Monte Carlo [3] and mean field calculations [4,5]. The existence of charged states in the free-charge phase was proven in ref. [6].

The phase transition between the free-charge and the screening phase was predicted to be second order by mean field methods [5]. On the other hand, a field theoretical argument [7] predicts a first-order transition in the region close to the pure matter limit of gauge–matter theories with continuous groups.

In order to investigate this transition in the Z_2 model we used the order parameter introduced in refs. [6,8] which tests the existence of charged states. In the pictorial notation of refs. [8,9] we define $\rho_n(r)$ in terms of expectation values of gauge invariant functions (x and y are points in the time-zero hyperplane and the vertical lines point in euclidean time direction):

$$\rho_n(r) = \Big\langle \underset{x \qquad y}{\sqcap^{\,r}_{\,n}} \Big\rangle \Big/ \Big\langle \boxed{2n \; r} \Big\rangle^{1/2}, \quad r = |x - y| . \tag{1.2}$$

The order parameter is $\rho_\infty(\infty)$, with the limit taken such that n is proportional to r as $r \to \infty$. Different choices of the proportionality constant represent different energy regularizations. On a finite hypercubic lattice of size L with periodic boundary conditions the largest distance r is $L/2$ if $n = r/2$ and $L/4$ if $n = r$.

The pure-matter theory obtained for $\beta_g \to \infty$ is the Ising model. In this limit $\rho_n(r)$ becomes the spin–spin correlation function. In section 2 we give a theoretical argument which suggests that the numerics of the screening–free-charge transition for a fixed value of β_g

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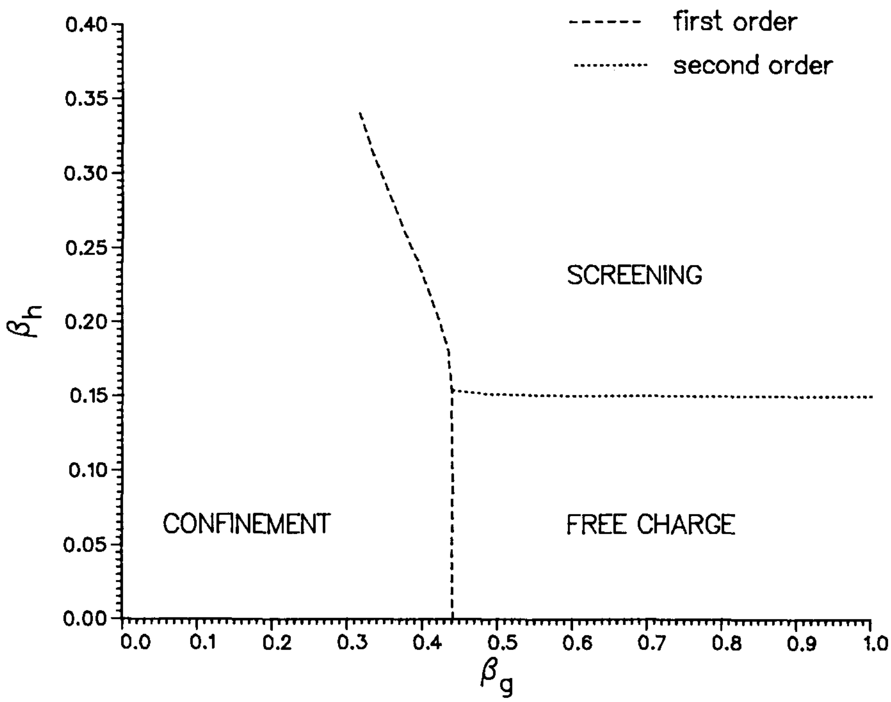

Fig. 1. The phase diagram of the four-dimensional Z_2 theory; the first-order line (with a second-order endpoint [4]) was computed from thermal sweeps on a lattice of size L = 10; the second-order line (which continues up to $\beta_g = \infty$) is eq. (2.3).

is very similar to that of the $\beta_g \to \infty$ case. This argument predicts the critical coupling $\beta_{h,c}$ as a function of β_g and of the critical coupling of the Ising model.

We checked the hypothesis that this transition is second order by performing a Monte Carlo simulation with fixed $\beta_g = 0.5$ and variable β_h. L was chosen between 4 and 16. The main result is that the order parameter $\rho_\infty(\infty)$ obeys the same scaling (section 3) and finite-size scaling (section 4) laws as the square of the magnetization in the Ising model. In section 5 we present the results for the second derivative of the free energy with respect to β_h, which show that this quantity behaves in the same way as the specific heat in the Ising case. In section 6 we summarize our results. In particular we argue that the triple point is the only point in the phase diagram with a possible nongaussian continuum limit.

2. Location of the screening–free-charge phase boundary. Let us consider the convergent expansion for $\rho_n(r)$ in the free-charge region [6,10]. We denote by M a path connecting the points x and y such that it contains no link more than once, and by m_q the coefficient in the exponent of the perimeter law for the expectation value of a Wilson loop,

$$\langle\Box\rangle = \exp(-m_q P) \tag{2.1}$$

(P is the perimeter of the rectangle). m_q is the infimum of the energy spectrum in the presence of an external source (see the discussion in ref. [6] (p. 105) and in ref. [9]).

A good approximation for $\rho_n(r)$ is the expansion in paths M ($|M|$ is the number of links in M):

$$\rho_n(r) = \sum_M [\tanh(\beta_h)\exp(-m_q)]^{|M|} \tag{2.2}$$

(see ref. [6] for details). Obviously in the Ising limit $m_q = 0$. Even though in general the value of m_q depends on both β_q and β_h, the dependence on the latter is weak in both the free-charge and the screening phase. Close to the phase-transition line, m_q can be accurately estimated using the Monte Carlo results for the Wilson loops.

For β_g fixed the expansion (2.2) breaks down at the phase-transition line. Let us denote by $\beta_{h,c}(\beta_g)$ and by $m_{q,c}(\beta_g)$ the values taken by β_h and m_q on this line, and by $\beta_{I,c}$ the critical coupling in the Ising limit. The sum in (2.2) becomes divergent for some critical value of the square bracket which does not depend on β_g. Therefore

$$\tan(\beta_{h,c}(\beta_g)) \exp[-m_{q,c}(\beta_g)] = \tanh(\beta_{I,c}). \qquad (2.3)$$

A careful consideration of the convergent expansion in the free-charge phase shows that eq. (2.3) holds even if we improve the approximation (2.2) by taking into account more terms occurring in the full expansion of $\rho_n(r)$ (see ref. [11] for a detailed discussion).

The value of $\beta_{I,c}$ obtained from high temperature expansions [12] is 0.14965 ± 0.00003. Our Monte Carlo result for $m_{q,c}(0.5)$ is 0.00753 ± 0.00003. Eq. (2.3) now leads to $\beta_{h,c}$ (0.5) = 0.15081 ± 0.00003. This is in perfect agreement with the results of section 3 and 4.

3. Scaling for the order parameter. In four dimensions the Ising model scaling laws have logarithmic corrections [12–16]. For a temperature T close to the critical point T the magnetization m obeys:

$$m^2 = Ct|\ln t|^{2/3},$$

$$t = 1 - T/T_c, \quad T < T_c, \quad C = \text{const.} \qquad (3.1)$$

This result was obtained using convergent-expansion

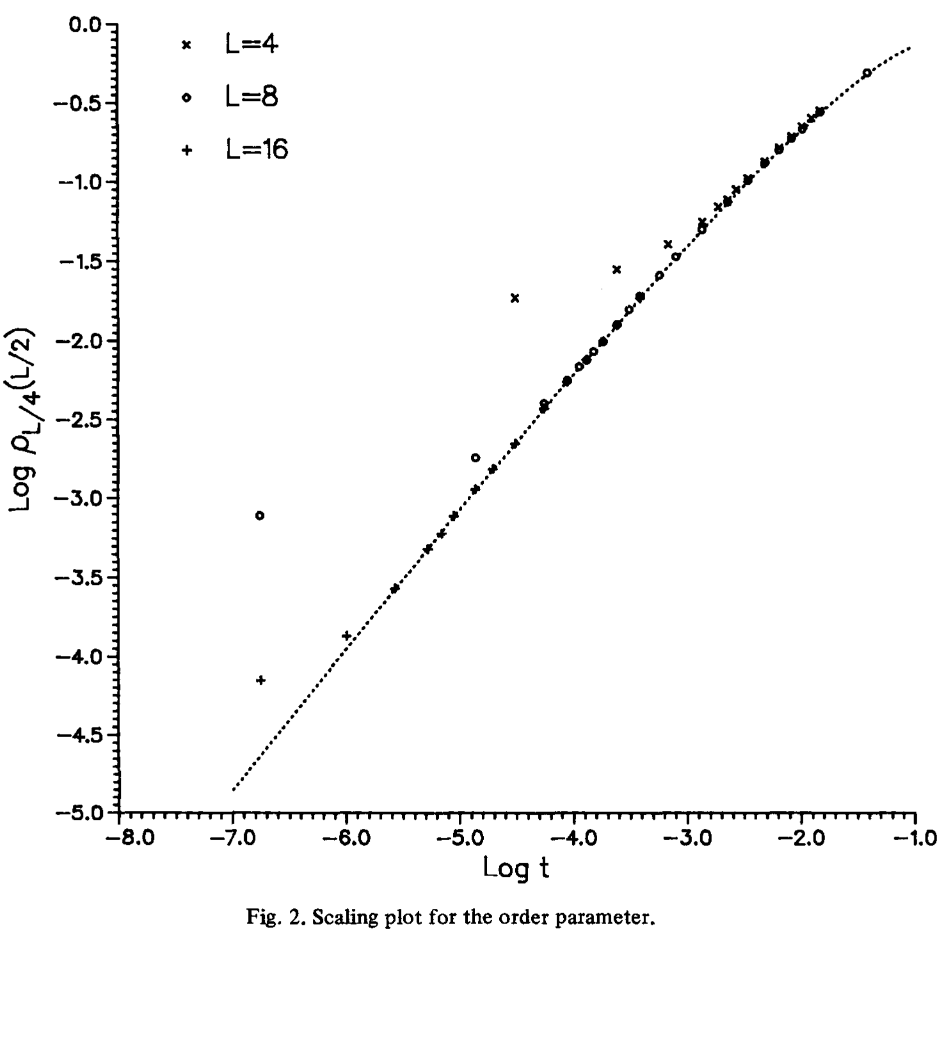

Fig. 2. Scaling plot for the order parameter.

[13,12], renormalization-group [14,15] and Monte Carlo [16] methods. The discussion in section 2 suggests that the order parameter $\rho_\infty(\infty)$ might obey a similar law for a fixed value of β_g. In our Monte Carlo simulation for $\beta_g = 0.5$ we computed $\rho_n(r)$ for two different regularizations [8,9] : $n = r/2$ and $n = r$. The results were independent of the choice of n (in general [9] one expects $\rho_n(r)$ at fixed r to increase with n until an asymptotic value is reached).

In fig. 2 we show a log–log plot of $\rho_n(L/2)$ against $t = 1 - \beta_{h,c}(0.5)/\beta_h$. The dotted curve is the best fit using the function in eq. (3.1) (for the fit we used only the datapoints we expect to be in the thermodynamic limit). Our estimate for the fit parameters is $\beta_{h,c}(0.5) = 0.15082 \pm 0.00003$ and $C = 2.35 \pm 0.01$ (in ref. [16] the Ising model estimate for C is 2.38 ± 0.05).

4. *Finite-size scaling*. The correct way of doing a finite-size scaling analysis for spin systems in four dimensions was recently discussed in ref. [17]. In the Ising model one expects the magnetization on a lattice of size L to obey

$$m_L^2 = [g(t)/L^2]\, f(tL^2 g(t)),$$

$$g(t) = |\ln t|^{1/3}, \quad t = 1 - T/T_c, \tag{4.1}$$

f is a universal scaling function.

Again, we expect the order parameter to behave in a similar way for a fixed value of β_g. For $\beta_g = 0.5$ we show in fig. 3 a plot of $\rho_n(L/2)L^2/g(t)$ against $tL^2g(t)$ ($t = 1 - \beta_{h,c}(0.5)/\beta_h$). The datapoints for $L = 8$ and $L = 16$ collapse on the same curve f. For $L = 4$ we observe small deviations from the universal curve. The

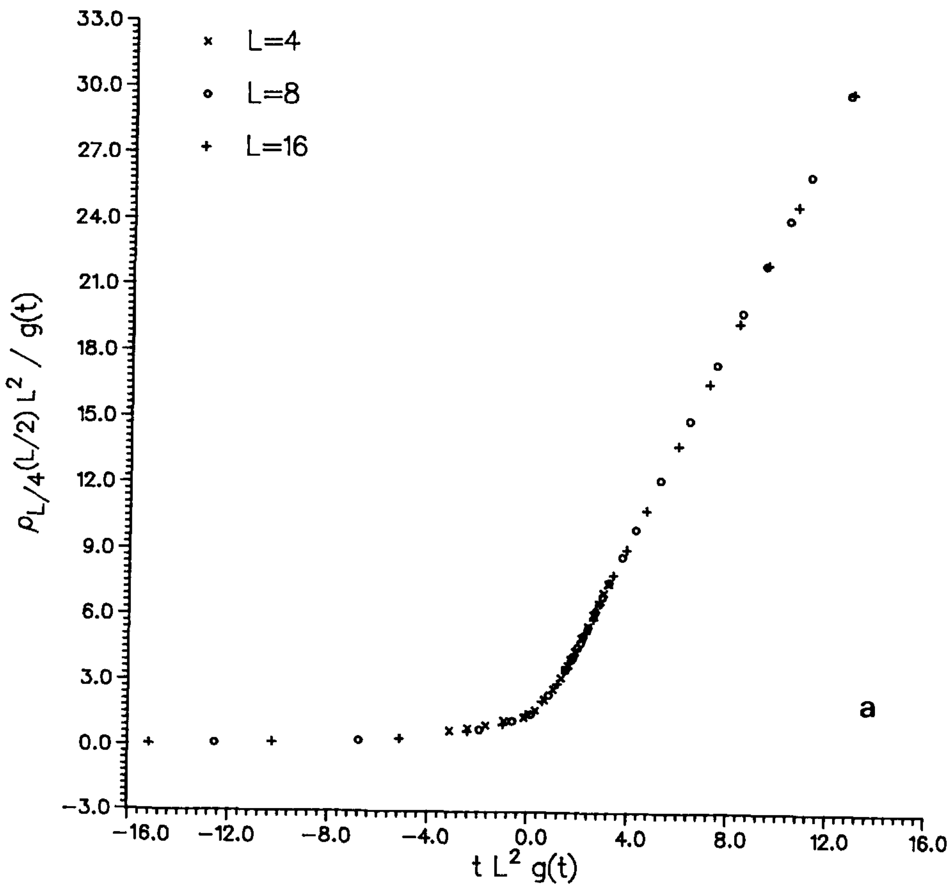

Fig. 3. Finite-size scaling plot for the order parameter; (a) is the entire range considered, (b) is a smaller region around the critical point.

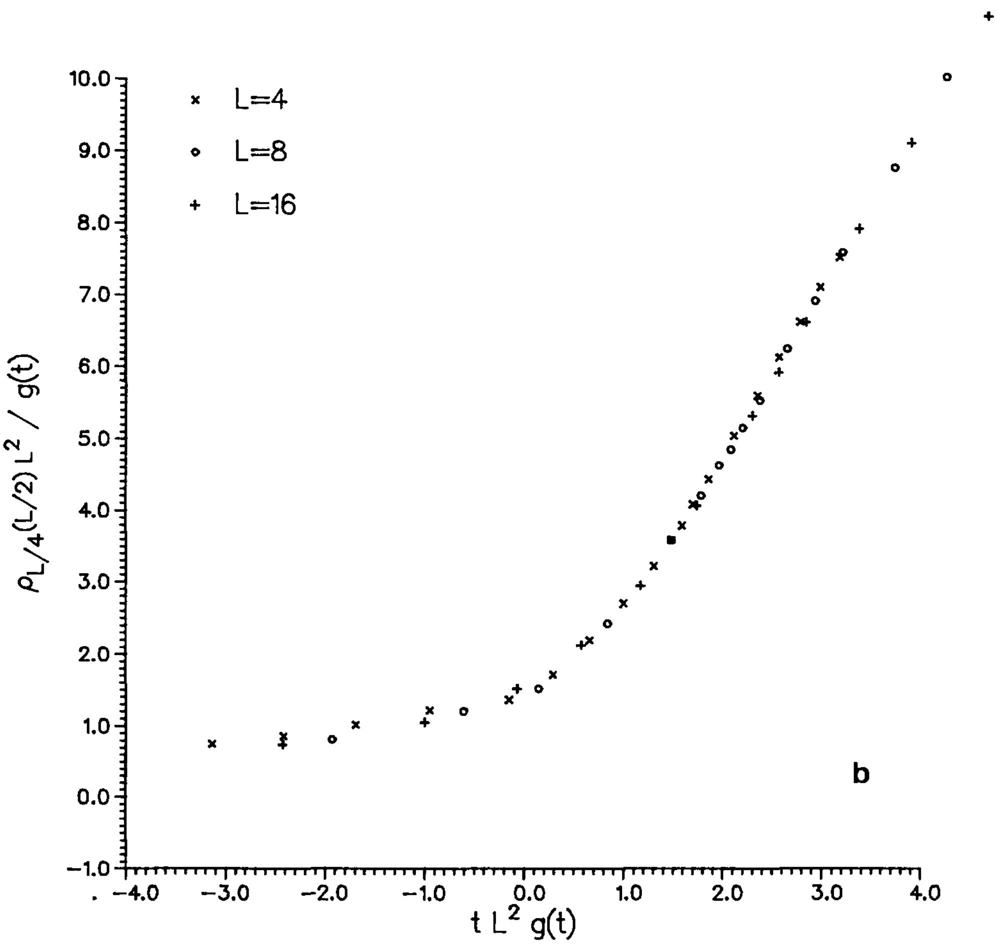

Fig. 3. (Continued).

best finite-size scaling plots were obtained for $\beta_{h,c}(0.5)$ chosen between 0.15081 and 0.15085.

5. Generalized susceptibilities. The critical behaviour of the generalized susceptibilities (second derivatives of the free energy) is in general a good indicator for the order of a phase transition. For $\beta = 0.5$ we measured

$$\chi_{hh} = \partial \ln Z/\partial\beta_h^2, \quad \chi_{gg} = \partial \ln Z/\partial\beta_g^2,$$

$$\chi_{gh} = \partial \ln Z/\partial\beta_h\partial\beta_g \tag{5.1}$$

(Z is the partition function) for lattice sizes $L = 4, 6, 8, 12$ and 16. Fig. 4 shows that the peaks in χ_{hh} become higher and sharper and approach $\beta_{h,c}(0.5)$ as L increases. The whole picture suggests an application of finite-size scaling theory. However, we were not able to find for χ_{hh} a relation similar to (4.1). This is not surprising since χ_{hh} becomes the specific heat in the Ising limit for which the finite-size scaling properties have not been worked out.

For the infinite lattice, the specific heat in the critical region of the Ising model has been calculated in ref. [18]:

$$c = A|\ln t|^{1/3}(1 - \tfrac{25}{81}|\ln|\ln t||/|\ln t| + B/|\ln t|),$$

$$t = |1 - T/T_c|. \tag{5.2}$$

In fig. 5 we plotted those data for χ_{hh} we expect to be in the thermodynamic limit (for $\beta_h < \beta_{h,c}(0.5)$ we did not have enough points in the thermodynamic limit).

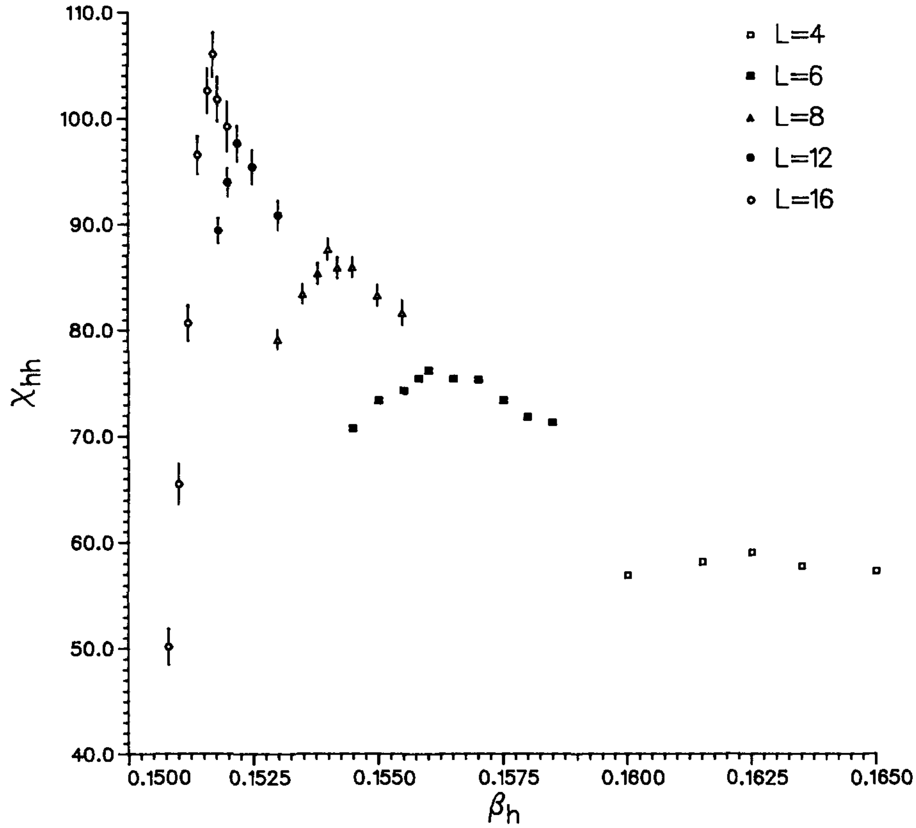

Fig. 4. The peaks in the generalized susceptibility χ_{hh} for finite L.

The dotted line is the function (5.2) (here $t = |t - \beta_{h,c}(0.5)/\beta_h|$), with the fit parameters estimated to be $\beta_{h,c}(0.5) = 0.1509 \pm 0.0002$, $A = 82 \pm 1.5$ and $B = -1 \pm 0.01$.

The behaviour of χ_{gh} and χ_{gg} will be discussed in ref. [11]. Here we only remark that they are much less sensitive to changes in β_h than χ_{hh}.

6. Conclusions and discussion. We showed to a high degree of accuracy that the screening–free-charge phase transition in the four-dimensional Z_2 lattice gauge theory with Z_2 matter fields is second order with mean field exponents. From the Monte Carlo simulation for $\beta_g = 0.5$ we determined independently 3 critical exponents: for the order parameter (in section 3), the mass gap (in section 4) and the specific heat (in section 5).

The order parameter $\rho_\infty(\infty)$ [6,8,9] turned out to be very useful numerically. In particular the determination of $\beta_{h,c}(0.5)$ using the order parameter is one order of magnitude more accurate than using χ_{hh}.

We gave a theoretical argument relating critical properties of the screening–free-charge phase transition to those of the Ising transition. We expect this argument to hold in any dimension $d > 2$.

Let us consider in the free-charge phase the "lines of constant physics" [19] defined as lines of constant ratio between the photon and the charged particle mass (the photon is massive because of the discrete gauge group). A rough estimate using the convergent expansion shows that these lines point in the direction of the triple point rather than of the Ising critical point. Therefore we expect block-spin transformations on the critical line to flow from the triple point to the Ising limit. In the region where we did our Monte Carlo simulation, the photon mass is much higher than twice

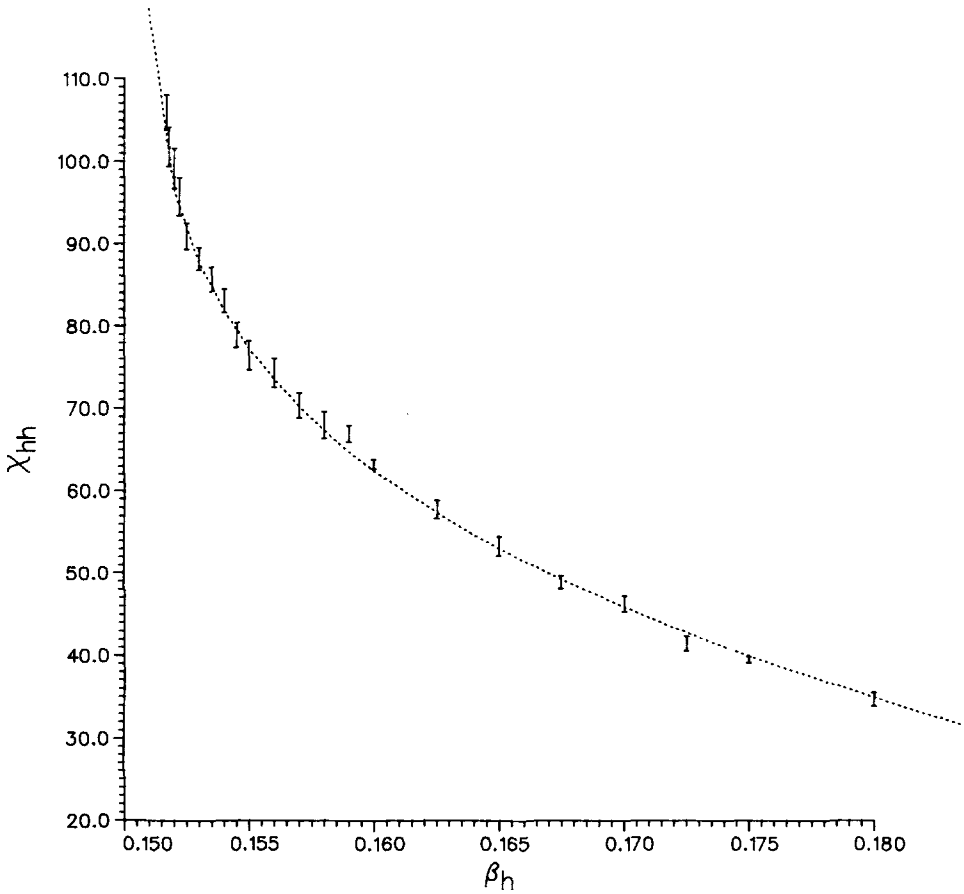

Fig. 5. Critical behaviour of the generalized susceptibility χ_{hh}.

the mass of the charged particle (in fact there most probably is no stable photon in this region). This means that the physics should be very similar to that of the Ising limit, which is in agreement with our numerical results. Thus the only possible continuum limit with different physics is at the triple point.

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