ON THE SENSITIVITY OF THE F_2 PHOTON STRUCTURE FUNCTION TO THE QCD SCALE PARAMETER Λ

J.H. FIELD

Deutsches Elektronen-Synchrotron DESY, D-2000 Hamburg 52, Fed. Rep. Germany

F. KAPUSTA and L. POGGIOLI

Laboratoire de Physique Nucléaire et des Hautes Energies, Université de Paris VI et VII, F-75230 Paris Cedex 05, France

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Different final state configurations contributing to F_2^{HAD} and $F_2^{\text{POINT-LIKE}}$ are discussed in a QCD improved parton model. The leading log part of the perturbatively calculable $F_2^{\text{POINT-LIKE}}$ is found to be independent of Λ at all orders in α_s. At experimentally relevant Q^2 values non-leading log terms are important, QCD corrections to the naive QPM prediction are small and the sensitivity of F_2^{γ} to Λ is weak.

The photon structure function as measured in the process $e^+e^- \to e^+e^-$ hadrons [1–3] has been the focus of considerable attention, both theoretical and experimental, during the last decade. The presence of an "anomalous" component [4] due to the point-like coupling of quarks to the target photon has been confirmed by the first [5] and many subsequent [6] experimental measurements of F_2.

Much interest was generated by the first QCD calculation of the F_2, F_L structure functions by Witten [7]. Using the OPE (operator product expansion) technique the leading logarithmic (LL) part of the anomalous structure function, summed to all orders in α_s was predicted absolutely in the Bjorken limit $Q^2 \to \infty, x = \text{const}$. Witten's result (valid for massless quarks) was confirmed using different calculational techniques by several authors [8–10] and generalised to sub-leading order (where terms $\approx \ln\ln Q^2$ in the Bjorken limit are accounted for) by Bardeen and Buras [11] and Duke and Owens [12].

The aim of this note is a critical discussion of the applicability of these QCD calculations to experimental measurements in the Q^2 range 5–100 $(\mathrm{GeV}/c)^2$, in particular the controversial question of the sensitivity of F_2 to Λ, the QCD scale parameter [13–15]. Our conclusions will be discussed in relation to other recent theoretical work at the end.

For clarity we consider first the light quark contribution in the naive parton model for F_2, and how this is expected, from general physical arguments, to be modified by QCD corrections. In the parton model the F_2 structure function is given by integrating over the p_T of an outgoing quark with respect to the target photon direction (fig. 1a) or, equivalently over the four-momentum transfer $-p^2$ of the virtual quark line. As has been pointed out by Peterson, Walsh and Zerwas [16] there is a direct relation between the p_T of the quark and the "anomalous" and "hadronic" parts of the photon structure function. For small values of p_T the $q\bar{q}$ state coupling to the real photon has a long lifetime $\simeq k/(p_T^2 + m_q^2)$ where k is the target photon energy and m_q the constituent quark mass. The coupling constant for gluon exchange between q and $\bar{q}$ is large $(\simeq \alpha_s(p_T^2 + m_q^2)$ favouring multiple gluon exchanges that give bound state effects (production of virtual ρ, ω, φ) which cannot be described perturbatively in QCD. For large values of p_T the $q\bar{q}$ state has a short lifetime giving a point-like $q\bar{q}\gamma$ coupling, gluon exchange effects are weaker $(\simeq \alpha_s(p_T^2), p_T^2 \gg \Lambda^2)$ and may be described perturbatively as a $q\bar{q}\gamma$ vertex correction. The hadronic (F_2^{HAD}) and point-like (F_2^{PL}) contributions to the F_2 structure function then come from distinct regions of the p_T integration:

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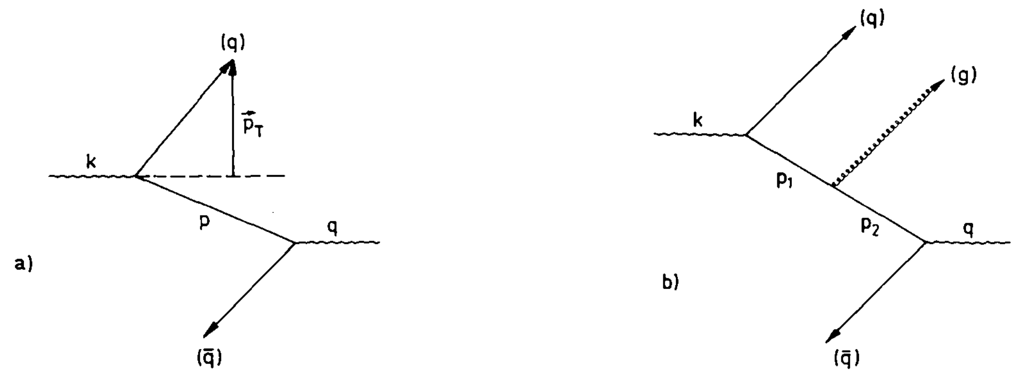

Fig. 1. Feynman diagrams for (a) parton model contribution (b) leading $O(\alpha_s)$ correction, to the photon structure function.

$$F_2 = \int_0^{(p_T^0)^2} \frac{dF_2^{HAD}}{dp_T^2} dp_T^2 + \int_{(p_T^0)^2}^{W^2/4} \frac{dF_2^{PL}}{dp_T^2} dp_T^2 , \tag{1}$$

where p_T^0 defines a boundary (which must be determined phenomenologically from experimental data) between the hadronic (non-perturbative) and point-like (perturbative) regions. Data on inclusively produced hadrons [17] or jets [18] indicate that $p_T^0 \simeq 1.5$ GeV for $\gamma\gamma$ masses W of $\simeq 6$ GeV in rough agreement with the naive expectation $p_T^0 \simeq m_\rho$. With a Λ value of 100–200 MeV/c this implies that $\alpha_s((p_T^0)^2)$ is sufficiently small that QCD corrections to F_2^{PL} can be estimated perturbatively. For $p_T > p_T^0$ scaling behaviour $\sim p_T^{-4}$ is observed [17,18]. In general p_T^0 is a function of W^2, the target photon (mass)^2($-P^2$) and the probe photon mass squared ($-Q^2$):

$$p_T^0 = p_T^0(W^2, P^2, Q^2) ,$$

p_T^0 is expected to be an increasing function of W^2 (due to phase-space effects [19]), a decreasing function of P^2 (due to suppression of F_2^{HAD} by the ρ form factor $\sim 1(1+P^2/m_\rho^2)^2$ and a decreasing function of Q^2 (due to the suppression of F_2^{HAD} at any finite x by logarithmic QCD scale breaking corrections).

We now discuss successively the first-order QCD correction to F_2^{PL} and the all-orders LL Bjorken limit QCD predictions to both F_2^{HAD} and F_2^{PL}. A suitable starting point for a discussion of the first-order correction is the Altarelli–Parisi equation (APE) [10,20] for the quark density in a photon $q(x, Q^2)$ where

$$F_2^{PL}(x, Q^2) = \sum e_q^2 x [q(x, Q^2) + \bar{q}(x, Q^2)] , \tag{2}$$

x is the Bjorken variable $Q^2/2(k \cdot q)$ (see fig. 1 for four-vector notation) and e_q is the quark charge in units of the electron charge e. The asymptotic LL, APE to first order in α_s is

$$Q^2\, dq/dQ^2 = (3\alpha e_q^2/2\pi)[x^2 + (1-x)^2] + \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} q(y, Q^2) P_{qq}(x/y) . \tag{3}$$

As we are interested in non-asymptotic Q^2 values (3) is re-written in a fully differential form with exact kinematics. This implies the replacements

$$dQ^2/Q^2 \to d\ln p_T^2 = dt_2/t_2 , \quad \alpha_s(Q^2) \to \alpha_s(t_{12}) , \quad q(y, Q^2) \to (3\alpha e_q^2/2\pi)[y^2 + (1-y)^2] \int_{t_1^{MIN}}^{t_2} dt_1/t_1 .$$

where (see fig. 1b)

$$t_i = m_q^2 - p_i^2 \ (i = 1,2)\,, \quad y = p_1^0/k\,, \quad Q^2 = -q^2\,, \quad t_{12} = t_1 \ \text{if}\ t_1 > t_2\,, \quad t_{12} = t_2 \ \text{if}\ t_2 > t_1\,.$$

The scale of α_s is given by the most virtual quark. Strictly the argument of α_s should be $-p_i^2$ [21]. We have verified by explicit computation that our results are unchanged if t_i is used instead. The fully differential form of (3), with exact kinematics, is

$$dq = dq^{(0)} + dq^{(1)}\,, \quad dq^{(0)} = (3\alpha/2\pi)e_q^2[x^2 + (1-x)^2]\,dt_2/t_2\,, \tag{4a,b}$$

$$dq^{(1)} = (\alpha/\pi^2)e_q^2[(x/z)^2 + (1-x/z)^2]\,[(1+z^2)/(1-z)]_+(dz/z)(dt_1/t_1)\alpha_s(t_{12})\,dt_2/t_2\,, \tag{4c}$$

where $z = x/y$. (The $[\ ]_+$ operator is defined in ref. [20].

It is instructive to consider the $dt_1\,dt_2$ integral in the first-order correction term $q^{(1)}$. As is well known [22] th LL correction comes only from the ordered $(t_2 > t_1)$ part of the domain of integration. The lower limit on t_1, t_0 is kinematically fixed by p_T^0:

$$t_0 = t_0(W^2, Q^2, P^2, y, m_q^2, (P_T^0)^2)\,.$$

For example if $k^2(1-y)^2 \gg P^2, m_q^2, (p_T^0)^2$:

$$t_0 = yP^2 + [m_q^2 + (p_T^0)^2]/(1-y)\,.$$

The LL term is then contained in the integral

$$I_t = \int_{t_0}^{t_2^{\mathrm{MAX}}} \frac{dt_2}{t_2}\alpha_s(t_2)\int_{t_0}^{t_2}\frac{dt_1}{t_1} = \frac{12\pi}{25}\left[\ln\left(\frac{t_2^{\mathrm{MAX}}}{t_0}\right) - \ln\left(\frac{t_0}{\Lambda^2}\right)\ln\left(\frac{\ln(t_2^{\mathrm{MAX}}/\Lambda^2)}{\ln(t_0/\Lambda^2)}\right)\right]\,, \tag{5}$$

where the four-flavour one-loop formula is used for α_s:

$$\alpha_s(t) = \tfrac{12}{25}\pi/\ln(t/\Lambda^2)\,. \tag{6}$$

It can be seen that the LL term in (5) (the first term in square brackets) *is independent of* Λ. As will be shown below this feature is retained in the all-orders QCD calculation. In the Bjorken limit this term gives a $\ln Q^2$ factor in $q(y, Q^2)$. However for experimentally relevant values of Q^2 the contribution of the "non-leading" term in (5) is large. For $t_2^{\mathrm{MAX}} = 20\ (\mathrm{GeV}/c)^2$, $t_0 = 1\ (\mathrm{GeV}/c)^2$; $\Lambda = 100$ MeV/c, the second term in the square brackets of (5) is 77% of the LL term, leading to strong cancellation. This is to be expected since $I_t \to 0$ as $\Lambda \to 0$, and actually $\Lambda^2 \ll t_0, t_2^{\mathrm{MAX}}$. For non-asymptotic Q^2 values the LL term strongly overestimates the correction.

A prediction for F_2^{PL}, including the first-order QCD correction due to the real gluon radiation graph in fig. 1a, is given by integrating dq over the full phase space of z, t_1, t_2. The integration was performed numerically using the recursive gaussian algorithm RGAUSS. The result, with $\Lambda = 100$ MeV/c is compared to the PLUTO experimental measurement of F_2 with $\langle Q^2\rangle = 5.3\ (\mathrm{GeV}/c)^2$ in fig. 2. Based on the experimental data reported in refs. [17–19] p_T^0 was taken, for the light quarks u, d, s as 1.5 GeV/c for $W > 6$ GeV and $\propto W$ for $W < 6$ GeV. For the charmed quark p_T^0 was set to zero. This first very rough estimate of p_T^0 can be refined when more precise experimental data is available. The numerically important non-leading terms in $F_2^{(0)}$ are included:

$$F_2^{(0)} = (3\alpha/\pi)e_q^4\{x[x^2 + (1-x)^2]\ln(t_2^{\mathrm{MAX}}/t_0) + 8x^2(1-x) - x\}\,, \tag{7}$$

where $t_2^{\mathrm{MAX}}/t_0 = W^2/[m_q^2 + (p_T^0)^2]$.

The contributions of u, d, s, c quarks with constituent masses 300, 300, 500, 1700 MeV/c^2 are summed. The prediction is almost independent of the light quark mass since the lower limits of the t integrals are determined by p_T^0. The usual phenomenological approximation (with a coherent sum over $u\bar{u}$, $d\bar{d}$, $s\bar{s}$ quark pairs) is taken for F_2^{HAD} [16]

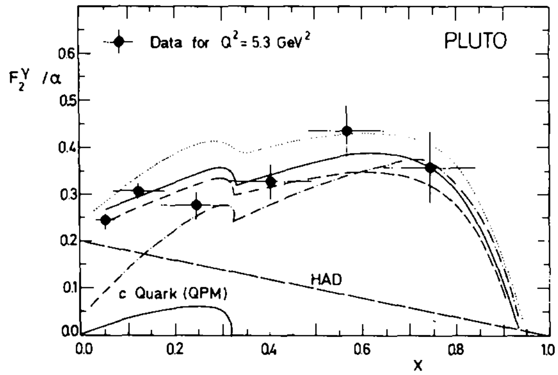

Fig. 2. PLUTO measurement of F_2 ($\langle Q^2\rangle = 5.3$ (GeV/c)^2) compared to $F_2^{\mathrm{HAD}} + F_2^{(0)} + F_2^{(1)}$ (solid line), $F_2^{\mathrm{HAD}} + F_2^{(0)}$ (dashed line), $F_2^{\mathrm{HAD}} + F_2^{(0)}$ with $p_{\mathrm{T}}^0 = 0$, $m_{\mathrm{u,d}} = 300$ MeV/c^2, $m_{\mathrm{s}} = 500$ MeV/c^2 (dotted line) and $F_2^{(0)}$ with $p_{\mathrm{T}}^0 = 0$, $m_{\mathrm{u,d}} =$ 300 MeV/c^2, $m_{\mathrm{s}} = 500$ MeV/c^2 (dot-dashed line).

$$F_2^{\mathrm{HAD}} = 0.2\,\alpha(1-x)\,. \tag{8}$$

Also shown in fig. 2a (dashed curve) is $F_2^{\mathrm{HAD}} + F_2^{(0)}$ (no QCD correction). This gives only a marginally worse fit than $F_2^{\mathrm{HAD}} + F_2^{(0)} + F_2^{(1)}$. The sensitivity of F_2 to Λ is very weak. varying Λ in the interval 50–200 MeV/c, makes only a tiny change to F_2. More quantitatively, at $Q^2 = 45$ GeV/c^2, where $F_2^{(1)}$ is proportionally much larger than for $Q^2 = 5.3$ GeV/c^2 a $(^{+100}_{-50})$% change in Λ at $\Lambda = 100$ MeV/c gives only a ±5% change in F_2. In contrast a 10% change in the value of p_{T}^0 at the same Q^2 value changes F_2 by 4%. Since p_{T}^0 is only crudely determined from experimental data, any possible sensitivity of F_2 to Λ is completely obscured by the "theoretical uncertainty" in p_{T}^0.

In fig. 2 the PLUTO data is also compared with (dotted curve) $F_2^{\mathrm{HAD}} + F_2^{(0)}$ where p_{T}^0 is set to zero for all quark flavours (i.e. for the light quarks, the lower limits of the t integrals are fixed by the constituent quark masses), and also with (dot-dashed curve) $F_2^{(0)}$ (again with p_{T}^0 zero). The dotted curve lies well above the data for small x values. This is because the hadronic contribution is "double counted" when p_{T}^0 is set to zero in $F_2^{(0)}$. $F_2^{(0)}$ alone however lies well below the data for $x < 0.2$, indicating that an additional hadronic contribution is required at small x values. As first observed in ref. [5] at large x values $F_2^{(0)}$ is completely dominant.

To discuss the all-orders LL QCD results we follow the derivation of Llewellyn-Smith [8] in summing ladder graphs in an axial gauge. Considering first only the non-singlet contribution then graphs such as fig. 1b but with N gluons emitted from the virtual quark line (fig. 3) must be summed. From (1) the hadronic part of F_2 comes from $p_{\mathrm{T}} < p_{\mathrm{T}}^0$, the point-like part from $p_{\mathrm{T}} > p_{\mathrm{T}}^0$. So summing ladder graphs, or equivalently, iterating the APE:

$$q_N^{\mathrm{HAD}}(Q^2,x) = \int_{y_2}^{1} q^{\mathrm{HAD}}(t_0,y_1)\frac{\mathrm{d}y_1}{y_1}C_N\,,\quad q_N^{\mathrm{PL}}(Q^2,x) = \frac{3e_{\mathrm{q}}^2\alpha}{2\pi}\int_{y_2}^{1}[y_1^2+(1-y_1)^2]\frac{\mathrm{d}y_1}{y_1}\int_{t_0}^{t_1^{\mathrm{MAX}}}\frac{\mathrm{d}t_1}{t_1}C_N\,, \tag{9a,b}$$

where

$$C_N = P_{\mathrm{qq}}(y_2/y_1)\frac{b}{\ln(t_2/\Lambda^2)}\left(\prod_{i=2}^{N}\int_{y_{i+1}}^{1}\frac{\mathrm{d}y_i}{y_i}\int_{t_i^{\mathrm{MIN}}}^{t_i^{\mathrm{MAX}}}\frac{\mathrm{d}t_i}{t_i}P_{\mathrm{qq}}(y_{i+1}/y_i)\frac{b}{\ln(t_{i+1}/\Lambda^2)}\right)\int_{t_{N+1}^{\mathrm{MIN}}}^{t_{N+1}^{\mathrm{MAX}}}\frac{\mathrm{d}t_{N+1}}{t_{N+1}}\,,\qquad b = 6/25\,.$$

The suffix N indicates the term where N gluons are radiated. To extract the LL term in the Bjorken limit $Q^2 \to \infty$ it is sufficient to evaluate the convolution integrals in the limited phase-space region [22]

$$t_i^{\mathrm{MIN}} = t_0\,,\quad t_i < t_{i+1}\,,\quad t_{N+1}^{\mathrm{MAX}} = Q^2\,,\quad y_{i+1} < y_i\,,\quad y_{N+1} = x\,.$$

Performing the t integrals and taking moments to decouple the y convolution gives the LL contributions

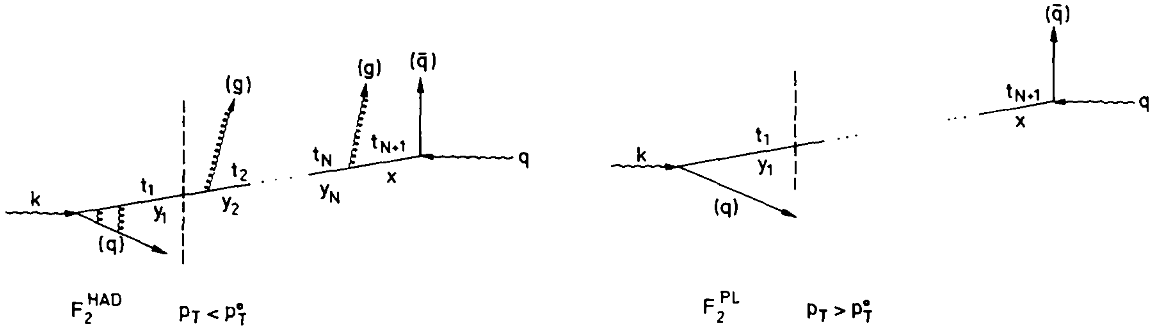

Fig. 3. Non-singlet ladder graphs contributing to the LL terms in F_2^{HAD} and F_2^{PL}.

$$q_N^{\mathrm{HAD}}(Q^2,n) = q^{\mathrm{HAD}}(t_0,n)\{d_{\mathrm{NS}}^n \ln[\ln(Q^2/\Lambda^2)/\ln(t_0/\Lambda^2)]\}^N/N!\ , \quad q_N^{\mathrm{PL}}(Q^2,n) = a(n)\ln(Q^2/t_0)(d_{\mathrm{NS}}^n)^N\ , \tag{10a,b}$$

here

$$a(x) = (3\alpha e_{\mathrm{q}}^2/2\pi)[x^2 + (1-x)^2]\ , \quad f(n) = \int_0^1 f(x)x^{n-1}\,\mathrm{d}x\ ,$$

and d_{NS}^n is the non-singlet anomalous dimension [20]:

$$d_{\mathrm{NS}}^n = bP_{\mathrm{qq}}(n)\ .$$

Summing (10a), (10b) to infinity gives for the all-orders LL non-singlet quark densities

$$q^{\mathrm{HAD}}(Q^2,n) = q^{\mathrm{HAD}}(t_0,n)[\ln(Q^2/\Lambda^2)/\ln(t_0/\Lambda^2)]^{d_{\mathrm{NS}}^n}\ , \quad q^{\mathrm{PL}}(Q^2,n) = a(n)\ln(Q^2/t_0)/(1-d_{\mathrm{NS}}^n)\ . \tag{11a,b}$$

The logarithmic term in (11b) is the same as in (5) $\ln(Q^2/t_0) = \ln(t_{N+1}^{\mathrm{MAX}}/t_0)$ and is independent of Λ.

The generalisation of (11) to include the singlet and "next to leading order" terms is straightforward. The full result for the moments of the quark densities is

$$q(Q^2,n) = \sum_i \{q^{\mathrm{HAD}}(t_0,n)r^{-d_i^n} + [a_i^n/(1-d_i^n)]\,[\ln(Q^2/\Lambda^2)](1-r^{1-d_i^n}) - (b_i^n/d_i^n)(1-r^{-d_i^n})\} + C^n\ , \tag{12}$$

where

$$r = \ln(t_0/\Lambda^2)/\ln(Q^2/\Lambda^2) = \alpha_s(Q^2)/\alpha_s(t_0)\ ,$$

and the sum i is over the components +, –, NS of the anomalous dimension matrix. The coefficients a_i^n, b_i^n were first derived using the OPE in refs. [7,11,12]. Eq. (12) agrees with the formula of Uematsu and Walsh [23] when $t_0 = P^2$, except that a hadronic term (which is important at finite Q^2 values) is included in (12). As the point-like contributions (the second and third terms in the curly brackets of (12)) are completely defined once t_0 is phenomenologically determined, a definite prediction for this contribution is possible. Eq. (12) has no singularities associated with the poles at $1 - d_-^n = 0$, $d_-^n = 0$ in the "asymptotic" LL and next-to-leading terms [24]. An ad hoc "regularisation" [14] of the associated divergences is therefore unnecessary. The importance of using the full solution (12) for finite Q^2 values has been stressed by Glück and Reya [13]. Knowledge of t_0 enables however an absolute prediction to be made for the point-like contribution to F_2. In ref. [13] only the evolution of F_2 from a scale Q_0^2 (where F_2 is assumed to be known) to Q^2, is predicted. We confirm, however, the lack of sensitivity of

F_2 to Λ pointed out in ref. [13].

It has become common practice to take the Bjorken limit $Q^2 \to \infty$ in eq. (12) and to compare the "leading" term in this limit

$$[a_i^n/(1-d_i^n)] \ln(Q^2/\Lambda^2) ,$$

with finite Q^2 data in an attempt to determine Λ^2. Such comparisons are meaningless because:

(i) At experimentally interesting Q^2 values the non-leading terms are always important.

(ii) Even if data with astronomically high Q^2 values were available Λ could still not be determined since $\ln \Lambda^2$ is non-leading, and by definition negligible as compared to $\ln Q^2$ in the Bjorken limit.

In this context it is interesting to note that in the limit $\Lambda^2 \to 0$ the second term in (12) reduces to

$$a_i^n \ln(Q^2/t_0) ,$$

i.e. the Born term, as expected from consistency of the perturbation expansion. If only the Bjorken-limit term is retained, the corresponding $\Lambda^2 \to 0$ limit diverges. In summary there is no sense in comparing asymptotic LL (or LL + subleading) terms with measurements of F_2 at finite Q^2.

In fact the predictions in refs. [7–12] are exact *asymptotic* predictions in the limit where all mass scales are (logarithmically) negligible as compared to Q^2. As already pointed out by Hill and Ross [25] this will not occur for any existing (or forseeable) experimental values of Q^2.

It may also be remarked that our result is consistent with the findings of ref. [25] that the QCD prediction is well defined at finite Q^2 for heavy quarks where $t_0 = m_q^2/(1-y) \gg \Lambda^2$.

In summary we have made a study of the F_2 photon structure function within the parton model and QCD, treating separately the final state configurations contributing to the hadronic and point-like parts. The LL part of the point-like contribution is independent of Λ. The non-leading terms are large compared to the LL term for Q^2 values of experimental interest and are only weakly sensitive to Λ. The sensitivity of F_2 to Λ is also obscured by uncertainties related to the unavoidable phenomenological cut-off p_T^0.

As the first-order QCD correction to the parton model prediction seems to be small, progress can perhaps be made by exact Feynman diagram calculations order-by-order in α_s as is typically done for 1γ e^+e^- annihilation into quarks and gluons. By restricting both the theoretical calculation and the experimental data to regions of phase space where perturbation theory is expected to be valid, meaningful tests of QCD should still be possible.

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