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Non-Standard Higgs Bosons in $SU(2)\times U(1)$ Radiative Corrections

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Abstract. The 1-loop renormalization of the $SU(2)\times U(1)$ electroweak gauge theory with two Higgs doublets is performed in the on-shell scheme with finite self energies and vertices. Assuming different vacuum expectation values for the scalar doublets, which yield enhanced Yukawa couplings to fermions, we calculate the effects of the additional Higgs bosons in the radiative corrections to the leptonic processes: μ-decay, $\nu_\mu e$-scattering, and $e^+e^-\to\mu^+\mu^-$, $\tau^+\tau^-$ with longitudinal polarization at PETRA and LEP/SLC energies. It is found that large effects occur in the M_W-M_Z mass relation, the determination of $\sin^2\theta_W$ from $\sigma(\nu_\mu e)/\sigma(\bar{\nu}_\mu e)$ and the e^+e^- forward-backward and polarization asymmetries, if either the charged Higgs or the additional neutral scalar/pseudoscalar are heavy. Enhancement effects and effects of light neutral bosons can better be observed in the $e^+e^-\to\tau^+\tau^-$ integrated cross section.

1. Introduction

The standard model of the electroweak interaction based on the gauge group $SU(2)\times U(1)$ describes successfully charged and neutral current reactions at low energies. It has achieved further strong support by the discovery of the predicted vector bosons in the correct mass range [1]. From the standard point of view the only missing object is the Higgs particle. In the standard model the Higgs appears as a fundamental field which describes neutral scalar particles without a substructure. The rôle of the standard Higgs is twofold: Through its non-vanishing vacuum expectation value $v\neq 0$ it is responsible for

- the masses of the weak gauge bosons, induced by the gauge-Higgs field couplings
- the masses of the charged fermions, induced by Yukawa couplings.

Thus the masses of vector bosons and fermions are set by the same scale $v\approx 250$ GeV. In order to obtain light fermions the Yukawa coupling constants g_f must be sufficiently small. Typically the couplings of the Higgs to fermions are suppressed by a factor m_f/M_W compared to the gauge coupling. As a consequence, Higgs effects in fermionic processes are very small unless heavy fermions like the top quark would be involved.

In $SU(2)\times U(1)$ the left-handed fermions are doublets and the right-handed singlets. Therefore Higgs doublets can couple to fermions and give them their masses. The minimal standard version with a single Higgs doublet predicts the ratio

$$\rho=\frac{M_W^2}{M_Z^2\cos^2\theta_W}$$

to be unity. But the converse is not true: $\rho=1$ remains valid for an arbitrary number of Higgs doublets automatically. Higher dimensional representations give in general $\rho\neq 1$ if no additional restrictions are imposed. Experimental data for ρ are close to 1, which favours the doublet character of the Higgs field (s).

Models with more than one doublet have attained interest e.g. in the context of CP symmetry breaking [2], the Peccei-Quinn solution of the strong CP problem [3], and supersymmetric extensions of the standard model [4], which need at least two scalar doublets.

The minimal extension of the standard model is a conventional $SU(2)\times U(1)$ gauge theory with two scalar complex doublets Φ_1, Φ_2 [5-9]. Three of their eight degrees of freedom form the longitudinal polarization states of the $W^\pm$ and Z and five remain as physical particles. These consist of two charged $\phi^\pm$ and three neutral states H_0, H_1, H_2 as mass eigenstates of the Higgs potential. One of the neutral scalars (e.g. H_0) behaves similarly to that of the standard model, whereas the additional ones may yield effects which are different from those of the conventional Higgs. From e^+e^- experiments at PETRA a lower limit for the charged Higgs mass can be deduced [10]

$$M_\phi \gtrsim 18\,\text{GeV}$$

and for the scalar/pseudoscalar pair H_1, H_2 the mass range can be excluded (95% c.l.) where one of them is below 0.2 GeV and the other one between 1 and 21 GeV [11].

In two-doublet models two vacuum expectation values

$$\langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ \frac{v_1}{\sqrt{2}} \end{pmatrix}, \quad \langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ \frac{v_2}{\sqrt{2}} \end{pmatrix}$$

are available to generate the vector boson and fermion masses. Their very different mass scales could be traced back to different Higgs vacuum expectation values $v_1 \gg v_2$ if only Φ_2 would have Yukawa couplings to fermions. The masses of W and Z are then essentially determined by v_1

$$M_W \approx \tfrac{1}{2} g_2 v_1, \quad M_Z \approx \tfrac{1}{2}\sqrt{g_1^2+g_2^2}\, v_1$$

(g_2 is the $SU(2)$, g_1 the $U(1)$ gauge coupling constant), whereas fermion masses arise as

$$m_f = g_f v_2.$$

An attractive phenomenological consequence in models with different vacuum expectation values is the enhancement of the Yukawa coupling constants by a factor v_1/v_2 compared to the minimal model, which can (partly) compensate the small m_f/M_W ratios.

In order to have flavor conservation in the neutral current sector the quark couplings have to be arranged in the way that Φ_1 couples to $I_3 = \frac{1}{2}$ and Φ_2 to $I_3 = -\frac{1}{2}$ quarks only. $v_2 > v_1$ enhances the u-like and $v_1 > v_2$ the d-like coupling constants. Existing constraints to v_1/v_2 are not yet very stringent:* In the first case [8]

$$\left(\frac{v_2}{v_1}\right)^2 < \frac{2M_\phi}{m_c} \simeq 109 \quad \text{for } M_\phi \simeq M_W$$

and for the second case [9]

$$\frac{v_1}{v_2} < \frac{4M_\phi}{m_b} \simeq 72 \quad \text{for } M_\phi \simeq M_W.$$

* More restrictive bounds from heavy quark systems have been obtained recently in [23]

If leptons couple to Φ_2 with $v_2 < v_1$ then a restriction resulting from the anomalous magnetic moment of the muon would be [6]

$$v_2/v_1 > 0.015^{**} \quad (\text{for } M_1 = 6\,\text{GeV}).$$

In this paper we extend the on-shell renormalization scheme of the standard model in [12] to the $SU(2)\times U(1)$ theory with two Higgs doublets and different vacuum expectation values $v_1 \gg v_2$. In particular we discuss the effects of the additional Higgs bosons in the 1-loop radiative corrections to the leptonic low energy processes

(i) μ decay

(ii) $\overset{(-)}{\nu}_\mu e$-scattering

and to the leptonic e^+e^- processes

(iii) $e^+e^- \to \mu^+\mu^-$, $\tau^+\tau^-$ for PETRA energies and on the Z^0.

A combined analysis of leptonic processes and direct M_W, M_Z mass measurements give the cleanest tests of electroweak theories, avoiding hadronic uncertainties as far as possible. The basic assumption that only Φ_2 with $v_2 < v_1$ couples to leptons yields enhanced couplings between charged leptons and the additional neutral scalar/pseudoscalar as well as between $l - \nu_l$ and $\phi^\pm$. The natural continuation of this picture to the quark sector would lead to $g_d > g_u$, but this is by no means necessary. As far as we restrict our discussion to leptons only we can renounce to assumptions about the hadronic sector. The obtained results therefore would give independent possibilities to explore the validity range of two-doublet models. In practice, the enhancement factor

$$\beta = v_1/v_2$$

will be considered as an additional parameter, which besides the Higgs masses enters the radiative corrections. According to the leptonic constraints from [6], which we will use as a guide, the Higgs couplings to e and μ still remain small, but they can get the normal gauge coupling strength for τ leptons. Technically this leads to scalar exchange contributions in the electromagnetic and neutral current vertex corrections, which are negligible in the standard model. Additional scalars with enhanced couplings could therefore be observed in terms of differences between μ and τ final states in e^+e^- annihilation.

The paper is organized as follows: Section 2 contains the basic Lagrangian and its renormalization, from which the Feynman rules and the counter

** For mass degenerate H_1 and H_2 this limit can be significantly lowered [6] terms are deduced. The unrenormalized 2- and 3-point functions of vector bosons ad fermions are presented in Sect. 3. In Sect. 4 we perform the renormalization, which yields the renormalized propagators and vertices, listed in Sect. 5. Section 6 contains the discussion of the leptonic processes specified above.

2. Lagrangian, Feynman Rules, and Counter Terms

We write for the two scalar doublets, splitting off the vacuum expectation values v_1 and v_2:

$$\Phi_1=\begin{pmatrix}\phi_1^+\\ \dfrac{v_1+\eta_1+i\chi_1}{\sqrt{2}}\end{pmatrix},\quad \Phi_2=\begin{pmatrix}\phi_2^+\\ \dfrac{v_2+\eta_2+i\chi_2}{\sqrt{2}}\end{pmatrix}. \tag{2.1}$$

The Higgs part of the Lagrangian is

$$\mathscr{L}_H=\mathscr{L}_{GH}+\mathscr{L}_{FH}-V(\Phi_1,\Phi_2). \tag{2.2}$$

It contains the Higgs-gauge field couplings:

$$\mathscr{L}_{GH}=|D_\mu\Phi_1|^2+|D_\mu\Phi_2|^2 \tag{2.3}$$

with

$$D_\mu=\partial_\mu-i\frac{g_2}{2}\boldsymbol{\sigma}\mathbf{W}_\mu+i\frac{g_1}{2}B_\mu, \tag{2.4}$$

the fermion-Higgs couplings, where we consider only the case of leptons, coupling to Φ_2:

$$\mathscr{L}_{FH}=-\sum_{f=e,\mu,\tau}g_f(\bar\psi_L^f\Phi_2\psi_R^f+\bar\psi_R^f\Phi_2^+\psi_L^f) \tag{2.5}$$

with

$$\psi_R=e_R,\mu_R,\tau_R$$

and

$$\psi_L=\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L,\ \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L,\ \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}_L, \tag{2.6}$$

and the Higgs potential with quadratic and quartic couplings. These can be chosen (assuming CP symmetry) that v_1 and v_2 are real [7]:

$$\begin{aligned}V=&-\mu_1^2\Phi_1^+\Phi_1-\mu_2^2\Phi_2^+\Phi_2+\lambda_1(\Phi_1^+\Phi_1)^2+\lambda_2(\Phi_2^+\Phi_2)^2\\&+\lambda_3(\Phi_1^+\Phi_1)(\Phi_2^+\Phi_2)+\lambda_4(\Phi_1^+\Phi_2)(\Phi_2^+\Phi_1)\\&+\frac{\lambda_5}{2}[(\Phi_1^+\Phi_2)^2+(\Phi_2^+\Phi_1)^2].\end{aligned}$$

The charged eigenstates following from V are

$$\phi^\pm=(v_2\phi_1^\pm-v_1\phi_2^\pm)/v \tag{2.7}$$

with

$$v=\sqrt{v_1^2+v_2^2},\quad \phi^-=(\phi^+)^+,$$

and the neutral mass eigenstates are

$$\begin{pmatrix}H_1\\ H_0\end{pmatrix}=\begin{pmatrix}\cos\zeta & \sin\zeta\\ -\sin\zeta & \cos\zeta\end{pmatrix}\begin{pmatrix}\eta_1\\ \eta_2\end{pmatrix} \tag{2.8}$$

$$H_2=(v_2\chi_1-v_1\chi_2)/v \tag{2.9}$$

ζ is a function of the parameters in V

$$\tan 2\zeta=\frac{v_1v_2(\lambda_3+\lambda_4+\lambda_5)}{\lambda_2v_2^2-\lambda_1v_1^2}. \tag{2.10}$$

The orthogonal combinations $\psi^\pm$, χ to (2.7, 9) form the unphysical components, which enter the gauge fixing and Faddeev-Popov Lagrangian. These are specified in a 't Hooft gauge in the same way as in [12] and we do not repeat them here.

The calculation of radiative corrections to fermionic processes where at least one fermion pair is light (e^+e^-, νe, ...) the Higgs exchange is suppressed already at the tree level (otherwise enhancement $>10^3$ has to be assumed). Hence also loop corrections to Higgs propagators can be neglected, which means that we do not need the details of the Higgs self couplings in $V(\Phi_1,\Phi_2)$. Only the masses M_ϕ (charged Higgs) and M_0, M_1, M_2 (neutral) and the couplings to fermions and gauge bosons enter the loop diagrams for W, Z propagators and vertices with internal Higgs lines.

The situation of enhanced Yukawa couplings $v_1/v_2=\beta\gg 1$ leads to

$$\begin{aligned}\phi^\pm&\simeq-\phi_2^\pm+\frac{v_2}{v_1}\phi_1^\pm\simeq-\phi_2^\pm,\\ H_2&\simeq-\chi_2+\frac{v_2}{v_1}\chi_1\simeq-\chi_2.\end{aligned} \tag{2.11}$$

The mixing angle ζ for the scalar fields in (2.8) makes in general the couplings of the neutral scalars different from the charged Higgs couplings. If the quartic couplings λ_i in V are all of the same order, $\tan\zeta$ is of the order v_1/v_2 for $v_1\gg v_2$ according to (2.10). In this situation we have equal enhancement for $\phi^\pm$, H_2 and H_1 and a minimum set of additional parameters beyond the standard model. For a first view on the effects caused by a second Higgs doublet we choose $\tan\zeta=v_1/v_2$ for concrete calculations in order to keep the number of further parameters as low as possible. In this case we get from (2.8):

$$H_0\simeq-\eta_1,\quad H_1\simeq\eta_2. \tag{2.12}$$

The masses of the weak bosons are essentially determined by v_1:

$$M_W=\tfrac{1}{2}g_2\sqrt{v_1^2+v_2^2},\quad M_Z=M_W\frac{\sqrt{g_1^2+g_2^2}}{g_2} \tag{2.13}$$

whereas $m_f = g_f v_2$ causes enhanced couplings for ϕ^+, H_1, H_2; H_2 has a pseudoscalar γ_5 coupling to the fermions.

With the definition of the weak mixing angle

$$\cos\theta_W = \frac{M_W}{M_Z} \tag{2.14}$$

the Feynman rules for the interaction between the Higgs and gauge bosons/leptons can be derived from (2.2). They are listed in Appendix D for the model with $v_1 \gg v_2$ as specified above.

The neutral scalar H_0 has the same couplings as the standard model Higgs. Also the behaviour of the unphysical Higgs and ghosts is that of the minimal version. Consequently, the only place where they become relevant in radiative corrections are the 2-point functions of the vector bosons.

Renormalization

The formal procedure of multiplicative renormalization is similar to that of [12]: each multiplet of fields achieves a renormalization constant Z_2 via

$$\begin{aligned} &W_\mu^a \to \sqrt{Z_2^W}\, W_\mu^a, && B_\mu \to \sqrt{Z_2^B}\, B_\mu \\ &\psi_L^f \to \sqrt{Z_L^f}\, \psi_L^f, && \psi_R^f \to \sqrt{Z_R^f}\, \psi_R^f \\ &\Phi_1 \to \sqrt{Z_{\Phi_1}}\, \Phi_1, && \Phi_2 \to \sqrt{Z_{\Phi_2}}\, \Phi_2. \end{aligned} \tag{2.15}$$

The coupling constants get renormalization constants Z_1:

$$\begin{aligned} &g_2 \to Z_1^W (Z_2^W)^{-3/2} g_2 \\ &g_1 \to Z_1^B (Z_2^B)^{-3/2} g_1 \\ &g_f \to Z_1^f (Z_{\Phi_2})^{-1/2} g_f. \end{aligned} \tag{2.16}$$

Here we drop further details of the Higgs renormalization concerned with $V(\Phi_1, \Phi_2)$ since we do not need loop corrections to Higgs propagators and vertices.

Expanding $Z_i = 1 + \delta Z_i$ yields the renormalized Lagrangian $\mathscr{L}$ which can now be re-written in terms of the physical fields $W^\pm$, Z, A, $\phi^\pm$, $H_{0,1,2}$ and the parameter set

$$\alpha;\ M_W, M_Z;\ M_\phi, M_0, M_1, M_2;\ \beta$$

($\alpha = 1/137.036$ is the usual fine structure constant) and the counter term Lagrangian $\delta\mathscr{L}$, which can also be expressed by the same fields and parameters. The counter terms which we need for our calculation are put together in Appendix C.

3. Unrenormalized Self Energies and Vertices

The masses of the additional scalar and pseudoscalar neutral Higgs $H_{1,2}$ are denoted by M_1 and M_2; M_ϕ denotes the mass of the charged Higgs particle ϕ^+.

In the following sections we list only those contributions to the 2- and 3-point functions (and consequently in the renormalization constants) that go beyond the standard model set. In exceptional cases where also the standard contributions are included this will be mentioned explicitly. All calculations are performed in the 't Hooft-Feynman gauge.

3.1. Vector Boson Self- and Mixing Energies

The 2-point functions for the vector fields can be decomposed into their transverse and longitudinal parts according to

$$\Delta_{\mu\nu}^\alpha(k) = \left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\right)\Delta_T^\alpha + \frac{k_\mu k_\nu}{k^2}\Delta_L^\alpha \tag{3.1}$$

where $\alpha = \gamma, Z, W, \gamma Z$.

For our purpose of calculating radiative corrections to fermionic processes where at least one fermion pair is light it is sufficient to deal with the transverse parts only. These define the self energies in the following way:

$$\begin{aligned} &\Delta_T^\alpha = \frac{i}{k^2 - M_\alpha^2} - \frac{i}{k^2 - M_\alpha^2}\Sigma^\alpha(k^2)\frac{1}{k^2 - M_\alpha^2}, \qquad \alpha = \gamma, W, Z \\ &\Delta_T^{\gamma Z} = -\frac{i}{k^2}\Sigma^{\gamma Z}(k^2)\frac{1}{k^2 - M_Z^2}. \end{aligned} \tag{3.2}$$

In particular we have as extra Higgs contributions:

Photon self energy (Fig. 1):

$$\begin{aligned} &\Sigma^\gamma(k^2) = \frac{\alpha}{12\pi}k^2\left(\Delta - \ln\frac{M_W^2}{\mu^2}\right) + \Sigma_{\text{fin}}^\gamma(k^2), \\ &\Sigma_{\text{fin}}^\gamma(k^2) = \frac{\alpha}{12\pi}\left[k^2 \ln\frac{M_W^2}{M_\phi^2} + \frac{2k^2}{3} \right. \\ &\qquad\left. + (k^2 - 4M_\phi^2)\, F(k^2, M_\phi, M_\phi)\right]. \end{aligned} \tag{3.3}$$

Photon-Z mixing energy (Fig. 2):

$$\begin{aligned} &\Sigma^{\gamma Z}(k^2) = \frac{\alpha}{4\pi}\frac{s_W^2 - c_W^2}{6 s_W c_W}k^2\left(\Delta - \ln\frac{M_W^2}{\mu^2}\right) + \Sigma_{\text{fin}}^{\gamma Z}(k^2), \\ &\Sigma_{\text{fin}}^{\gamma Z}(k^2) = \frac{\alpha}{4\pi}\frac{s_W^2 - c_W^2}{6 s_W c_W}\left[k^2 \ln\frac{M_W^2}{M_\phi^2} + \frac{2k^2}{3} \right. \\ &\qquad\left. + (k^2 - 4M_\phi^2)\, F(k^2, M_\phi, M_\phi)\right]. \end{aligned} \tag{3.4}$$

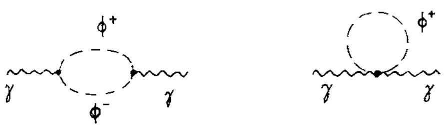


Fig. 1–9. Non-standard contributions to self energies and vertices

Fig. 1. Photon self energy

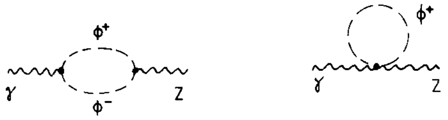


Fig. 2. Photon-Z mixing energy

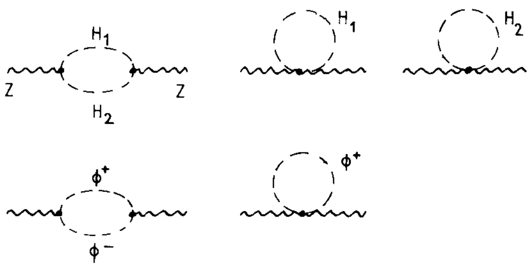


Fig. 3. Z boson self energy

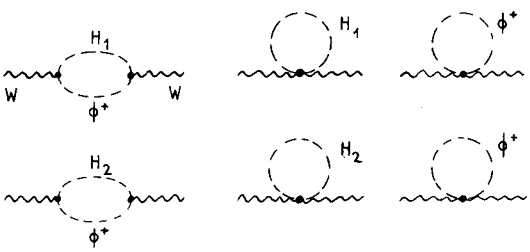


Fig. 4. W boson self energy

Z boson self energy (Fig. 3):

$$\Sigma^Z(k^2)=\frac{\alpha}{4\pi}\,\frac{1+(c_W^2-s_W^2)^2}{12c_W^2 s_W^2}\,k^2\left(\Delta-\ln\frac{M_W^2}{\mu^2}\right)+\Sigma^Z_{\text{fin}}(k^2),$$

$$\begin{aligned}\Sigma^Z_{\text{fin}}(k^2)=\frac{\alpha}{4\pi}\Bigg\{&\frac{1}{12c_W^2 s_W^2}\Bigg[k^2\ln\frac{M_W^2}{M_1M_2}+\frac{2k^2}{3}\\&+2(M_1^2-M_2^2)\ln\frac{M_2}{M_1}+\frac{(M_1^2-M_2^2)^2}{k^2}\\&\cdot F(k^2,M_1,M_2)+(k^2-2M_1^2-2M_2^2)\\&\cdot\left(1+\frac{M_1^2+M_2^2}{M_1^2-M_2^2}\ln\frac{M_2}{M_1}+F(k^2,M_1,M_2)\right)\Bigg]\\&+\frac{(c_W^2-s_W^2)^2}{12c_W^2 s_W^2}\Bigg[k^2\ln\frac{M_W^2}{M_\phi^2}+\frac{2k^2}{3}\\&+(k^2-4M_\phi^2)\,F(k^2,M_\phi,M_\phi)\Bigg]\Bigg\}.\end{aligned}\tag{3.5}$$

W-boson self energy (Fig. 4):

$$\Sigma^W(k^2)=\frac{\alpha}{4\pi}\,\frac{k^2}{6s_W^2}\left(\Delta-\ln\frac{M_W^2}{\mu^2}\right)+\Sigma^W_{\text{fin}}(k^2),$$

$$\begin{aligned}\Sigma^W_{\text{fin}}(k^2)=\frac{\alpha}{4\pi}\,\frac{1}{12s_W^2}\Bigg\{&k^2\left(\ln\frac{M_W^2}{M_\phi^2}+\ln\frac{M_W^2}{M_1M_2}-\frac{4}{3}\right)\\&+2(M_1^2-M_\phi^2)\ln\frac{M_\phi}{M_1}+2(M_2^2-M_\phi^2)\ln\frac{M_\phi}{M_2}\\&+\frac{(M_1^2-M_\phi^2)^2}{k^2}F(k^2,M_1,M_\phi)\\&+\frac{(M_2^2-M_\phi^2)^2}{k^2}F(k^2,M_2,M_\phi)\\&+(k^2-2M_1^2-2M_\phi^2)\\&\cdot\left[1+\frac{M_1^2+M_\phi^2}{M_1^2-M_\phi^2}\ln\frac{M_\phi}{M_1}+F(k^2,M_1,M_\phi)\right]\\&+(k^2-2M_2^2-2M_\phi^2)\\&\cdot\left[1+\frac{M_2^2+M_\phi^2}{M_2^2-M_\phi^2}\ln\frac{M_\phi}{M_2}+F(k^2,M_2,M_\phi)\right].\end{aligned}\tag{3.5}$$

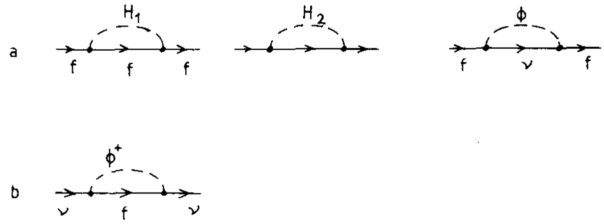


Fig. 5. a Charged lepton self energy, **b** neutrino self energy

In these formulae s_W and c_W are used as abbreviations for

$$s_W=\sin\theta_W,\qquad c_W=\cos\theta_W.\tag{3.7}$$

All other quantities in (3.3–6) are defined in the Appendix A.

3.2. Fermion Self Energy

The diagrams of Fig. 5 contain the Higgs scalars with enhanced couplings to the charged lepton. The self energy Σ^f of the fermion f, defined via

$$S_F(k)=\frac{i}{\not k-m_f}-\frac{i}{\not k-m_f}\,\Sigma^f(k)\,\frac{1}{\not k-m_f}\tag{3.8}$$

can be decomposed as

$$\Sigma^f(k)=\not k\,\Sigma^f_V(k^2)+\not k\,\gamma_5\,\Sigma^f_A(k^2)+m_f\,\Sigma^f_S(k^2).\tag{3.9}$$

The diagrams of Fig. 5 give for a charged lepton:

$$\begin{aligned}\Sigma^f_V&=-\frac{\alpha}{4\pi}\,G_f[B_1(k^2,m_f,M_1)+B_1(k^2,m_f,M_2)\\&\quad+B_1(k^2,0,M_\phi)]\\\Sigma^f_A&=-\frac{\alpha}{4\pi}\,G_f\,B_1(k^2,0,M_\phi)\\\Sigma^f_S&=\frac{\alpha}{4\pi}\,G_f[B_0(k^2,m_f,M_1)-B_0(k^2,m_f,M_2)]\end{aligned}\tag{3.10}$$

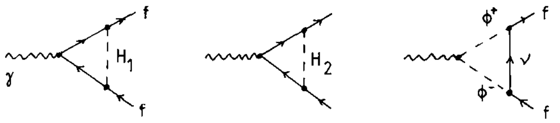


Fig. 6. Electromagnetic vertex of charged leptons

and for a neutrino:

$$\Sigma_V^{\nu_f} = -\Sigma_A^{\nu_f} = -\frac{\alpha}{4\pi} G_f B_1(k^2, m_f, M_\phi)$$
$$\Sigma_S^{\nu_f} = 0. \tag{3.10'}$$

In these equations we have used G_f as an "effective" coupling

$$G_f = \frac{1}{4 s_W^2}\left(\frac{\beta m_f}{M_W}\right)^2 \tag{3.11}$$

that contains the enhancement factor β.

The functions B_0 and B_1 are given in Appendix A.

3.3. Vertex Corrections

We calculate the contributions of the extra Higgs scalar with enhanced couplings to the leptonic electromagnetic and neutral current vertex. Thereby the fermion are taken on-shell; k^2 denotes the momentum transfer at the vertex.

Electromagnetic Vertex. The results from Fig. 6 can be summarized in the form*

$$\begin{aligned}\Gamma_\mu^{\gamma f f}(k^2) &= i e \gamma_\mu \\ &- i e \gamma_\mu \frac{\alpha}{4\pi} G_f\left[-\frac{1}{2}\left(\Delta - \ln\frac{M_1^2}{\mu^2} + \frac{1}{2}\right) + \bar{\Lambda}_1(k^2, M_1, m_f)\right] \\ &- i e \gamma_\mu \frac{\alpha}{4\pi} G_f\left[-\frac{1}{2}\left(\Delta - \ln\frac{M_2^2}{\mu^2} + \frac{1}{2}\right) + \bar{\Lambda}_1(k^2, M_2, m_f)\right] \\ &+ i e \gamma_\mu(1+\gamma_5)\frac{\alpha}{4\pi} G_f\left[\frac{1}{2}\left(\Delta - \ln\frac{M_\phi^2}{\mu^2} + \frac{1}{2}\right) + \bar{\Lambda}_2(k^2, M_\phi)\right].\end{aligned} \tag{3.12}$$

The UV-finite functions $\bar{\Lambda}_1, \bar{\Lambda}_2$ are given in Appendix B.

Weak Neutral Current Vertex. With the axial and vector coupling constants

$$a = -\frac{1}{4 s_W c_W}, \quad v = a(1 - 4 s_W^2) \tag{3.13}$$

we write for the sum of the diagrams in Fig. 7:

* Terms $\sim m_f^2$ are neglected in (3.12, 14, 17)

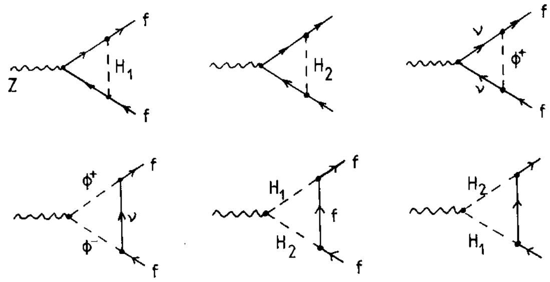


Fig. 7. Weak neutral current vertex for charged leptons

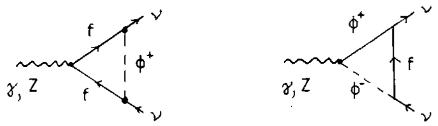


Fig. 8. Electromagnetic vertex of neutrinos and neutral current neutrino vertex

$$\begin{aligned}\Gamma_\mu^{Z f f}(k^2) &= i e \gamma_\mu (v - a\gamma_5) \\ &- i e \gamma_\mu (v + a\gamma_5)\frac{\alpha}{4\pi} G_f\left[-\frac{1}{2}\left(\Delta - \ln\frac{M_1^2}{\mu^2} + \frac{1}{2}\right) + \bar{\Lambda}_1(k^2, M_1, m_f)\right] \\ &- i e \gamma_\mu (v + a\gamma_5)\frac{\alpha}{4\pi} G_f\left[-\frac{1}{2}\left(\Delta - \ln\frac{M_2^2}{\mu^2} + \frac{1}{2}\right) + \bar{\Lambda}_1(k^2, M_2, m_f)\right] \\ &- i e \frac{c_W^2 - s_W^2}{2 c_W s_W}\gamma_\mu(1+\gamma_5)\frac{\alpha}{4\pi} G_f\left[\frac{1}{2}\left(\Delta - \ln\frac{M_\phi^2}{\mu^2} + \frac{1}{2}\right) + \bar{\Lambda}_2(k^2, M_\phi)\right] \\ &- i \frac{e}{2 c_W s_W}\gamma_\mu(1+\gamma_5)\frac{\alpha}{4\pi} G_f\left[-\frac{1}{2}\left(\Delta - \ln\frac{M_\phi^2}{\mu^2} + \frac{1}{2}\right) + \bar{\Lambda}_3(k^2, M_\phi)\right] \\ &+ i \frac{e}{2 c_W s_W}\gamma_\mu \gamma_5 \frac{\alpha}{4\pi} G_f\left[\Delta - \ln\frac{M_1 M_2}{\mu^2} + \frac{1}{2} + \bar{\Lambda}_4(k^2, M_1, M_2, m_f)\right].\end{aligned} \tag{3.14}$$

The neutral current neutrino vertex reads (Fig. 8)

$$\begin{aligned}\Gamma_\mu^{Z\nu\nu}(k^2) &= i \frac{e}{4 c_W s_W}\gamma_\mu(1-\gamma_5) \\ &+ i e \gamma_\mu (1-\gamma_5)\frac{\alpha}{4\pi} G_f\left\{\frac{1}{4 c_W s_W}\left(\Delta - \ln\frac{M_\phi^2}{\mu^2} + \frac{1}{2}\right) - \frac{s_W}{c_W}\bar{\Lambda}_5(k^2, M_\phi, m_f) + \frac{c_W^2 - s_W^2}{2 c_W s_W}\bar{\Lambda}_6(k^2, M_\phi, m_f)\right\}.\end{aligned} \tag{3.15}$$

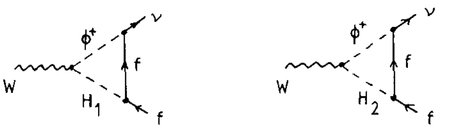


Fig. 9. Charged current leptonic vertex

Finally we give the electromagnetic vertex for the neutrino (Fig. 8):

$$\Gamma_\mu^{\gamma\nu\nu}(k^2)=i\,e\,\gamma_\mu(1-\gamma_5)\,F_\nu^\gamma(k^2), \tag{3.16}$$

$$F_\nu^\gamma(k^2)=-\frac{\alpha}{4\pi}\,G_f[\Lambda_5(k^2,M_\Phi,m_f)+\Lambda_6(k^2,M_\Phi,m_f)].$$

Weak Charged Current Vertex. The leptonic charged current vertex gets contributions from Fig. 9:

$$\Gamma_\mu^{W\nu f}(k^2)=i\frac{e}{2\sqrt{2}\,s_W}\gamma_\mu(1-\gamma_5)\Big\{1+\frac{\alpha}{4\pi}G_f\Big[\Delta-\ln\frac{M_\Phi^2}{\mu^2}+\frac{1}{2}-\frac{1}{2}\ln\frac{M_1M_2}{M_\Phi^2}+\tfrac{1}{2}\Lambda_7(k^2,M_1,M_\Phi)+\tfrac{1}{2}\Lambda_7(k^2,M_2,M_\Phi)\Big]\Big\}. \tag{3.17}$$

For the invariant functions $\Lambda_1,\ldots,\Lambda_7$ again see Appendix B. G_f is defined in (3.11).

4. Renormalization

We follow the on-shell renormalization scheme as worked out in detail in [12].

The procedure for obtaining the renormalized 2- and 3-point functions by adding counter terms is specified in Appendix C. We restrict ourselves to the renormalization in the physical sector (without longitudinal vector boson, ghost and Higgs-ghost self energy renormalization) that enters the radiative 1-loop corrections of the fermionic processes (i)-(iii), Sect. 1. The physical sector can be treated separately from the unphysical one by the method of [12]. A complete renormalization would need the whole information about the 2-doublet Higgs potential.

We denote the renormalized quantities by the same symbol as the unrenormalized ones in Sect. 3 but with an extra $\hat{}$.

The conditions which fix the renormalization constants in the counter terms of Appendix C are:

$$\mathrm{Re}\,\hat\Sigma^Z(M_Z^2)=\mathrm{Re}\,\hat\Sigma^W(M_W^2)=\mathrm{Re}\,\hat\Sigma^f(k\!\!\!/=m_f)=0 \tag{4.1a}$$

$$\hat\Sigma^{\gamma Z}(0)=0 \tag{4.1b}$$

$$\frac{\partial\hat\Sigma^\gamma}{\partial k^2}(0)=0 \tag{4.1c}$$

$$\frac{1}{k\!\!\!/-m_f}\hat\Sigma^f(k)\Big|_{k\!\!\!/=m_f}=0 \tag{4.1d}$$

$$\hat\Gamma_\mu^{\gamma ee}(0)=i\,e\,\gamma_\mu. \tag{4.1e}$$

The last condition involves the electrons on-shell. It is only a condition for the vector part; the vanishing axialvector in the Thomson limit is already a consequence.

From the set of (4.1), together with (C.2), (C.3) from Appendix C, the following expressions for the gauge field and gauge coupling renormalization constants are derived:

$$\left(\Delta_W:=\Delta-\ln\frac{M_W^2}{\mu^2}\right)$$

$$\delta Z_2^\gamma=\delta Z_1^\gamma=-\frac{\alpha}{12\pi}\left(\Delta_W+\ln\frac{M_W^2}{M_\Phi^2}\right), \tag{4.2}$$

$$\delta Z_2^Z=\delta Z_1^Z=-\frac{\alpha}{4\pi}\left[\frac{1+(c_W^2-s_W^2)^2}{12c_W^2s_W^2}\Delta_W+\tfrac{1}{3}\ln\frac{M_W^2}{M_\Phi^2}\right]+\frac{c_W^2-s_W^2}{s_W^2}\left(\frac{\delta M_Z^2}{M_Z^2}-\frac{\delta M_W^2}{M_W^2}\right)_{\mathrm{fin}},$$

$$\delta Z_2^W=-\frac{\alpha}{4\pi}\left(\frac{1}{6s_W^2}\Delta_W+\tfrac{1}{3}\ln\frac{M_W^2}{M_\Phi^2}\right)+\frac{c_W^2}{s_W^2}\left(\frac{\delta M_Z^2}{M_Z^2}-\frac{\delta M_W^2}{M_W^2}\right)_{\mathrm{fin}},$$

$$\delta Z_2^{\gamma Z}=\delta Z_1^{\gamma Z}=\frac{\alpha}{4\pi}\,\frac{s_W^2-c_W^2}{6s_Wc_W}\Delta_W+\frac{c_W}{s_W}\left(\frac{\delta M_Z^2}{M_Z^2}-\frac{\delta M_W^2}{M_W^2}\right)_{\mathrm{fin}}$$

with

$$\left(\frac{\delta M_Z^2}{M_Z^2}-\frac{\delta M_W^2}{M_W^2}\right)_{\mathrm{fin}}=\mathrm{Re}\left(\frac{\Sigma_{\mathrm{fin}}^Z(M_Z^2)}{M_Z^2}-\frac{\Sigma_{\mathrm{fin}}^W(M_W^2)}{M_W^2}\right) \tag{4.3}$$

and Σ_{fin}^Z, Σ_{fin}^W from (3.5) and (3.6).

For the charged leptons we give the expressions for the vector and axialvector renormalization constants which enter the counter terms for the vertices. They follow from (4.1d) and (C.6):

$$\delta Z_V=\frac{\alpha}{4\pi}G_f\left\{-\frac{1}{2}\left(\Delta-\ln\frac{M_1^2}{\mu^2}+\frac{1}{2}\right)-\frac{1}{2}\left(\Delta-\ln\frac{M_2^2}{\mu^2}+\frac{1}{2}\right)-\frac{1}{2}\left(\Delta-\ln\frac{M_\Phi^2}{\mu^2}+\frac{1}{2}\right)+\delta Z_V^{\mathrm{fin}}\right\}, \tag{4.4}$$

$$\delta Z_A=\frac{\alpha}{4\pi}G_f\cdot\frac{1}{2}\left(\Delta-\ln\frac{M_\Phi^2}{\mu^2}+\frac{1}{2}\right) \tag{4.5}$$

δZ_V^{fin} in (4.4) denotes the finite part

$$\delta Z_V^{\text{fin}}=\bar{B}_1(m_f^2, m_f, M_1)+\bar{B}_1(m_f^2, m_f, M_2) + 2m_f^2[B_1'(m_f^2, m_f, M_1)-B_0'(m_f^2, m_f, M_1) + B_1'(m_f^2, m_f, M_2)+B_0'(m_f^2, m_f, M_2)]. \quad (4.6)$$

For the functions $\bar{B}_1, B_1', B_0'$ see Appendix A.

Two limiting situations are of particular interest:

heavy Higgs ($M_{1,2}^2 \gg m_f^2$): $\delta Z_V^{\text{fin}} \simeq 0$

light Higgs ($M_{1,2}^2 \ll m_f^2$): $\delta Z_V^{\text{fin}} \simeq \frac{7}{2}+3\ln\frac{M_1}{m_f}-\ln\frac{M_2}{m_f}$.

The condition (4.1d) ensures that we do not need an external wave function renormalization in calculations of matrix elements with external charged leptons on their mass shell. External neutrino lines, however, get a wave function renormalization constant

$$1-\frac{1}{2}\cdot\frac{\alpha}{4\pi}G_f\left(\ln\frac{M_1 M_2}{M_\phi^2}+\delta Z_V^{\text{fin}}\right). \quad (4.7)$$

5. Renormalized Self Energies and Vertices

The formulae of Appendix C together with the explicit form of the renormalization constants from the previous section allow us to give the following list of the relevant boson self and mixing energies and vertex corrections.

Boson self energies:

$$\hat{\Sigma}^\gamma(k^2)=\Sigma_{\text{fin}}^\gamma(k^2)-\frac{\alpha}{12\pi}k^2\ln\frac{M_W^2}{M_\phi^2}, \quad (5.1)$$

$$\hat{\Sigma}^{\gamma Z}(k^2)=\hat{\Sigma}_{\text{fin}}^{\gamma Z}(k^2)-k^2\frac{c_W}{s_W}\left(\frac{\delta M_Z^2}{M_Z^2}-\frac{\delta M_W^2}{M_W^2}\right)_{\text{fin}} + \frac{\alpha}{4\pi}\cdot\frac{c_W^2-s_W^2}{6 s_W c_W}k^2\ln\frac{M_W^2}{M_\phi^2},$$

$$\hat{\Sigma}^Z(k^2)=\Sigma_{\text{fin}}^Z(k^2)-\text{Re}\,\Sigma_{\text{fin}}^Z(M_Z^2) + (k^2-M_Z^2)\left[\frac{c_W^2-s_W^2}{s_W^2}\left(\frac{\delta M_Z^2}{M_Z^2}-\frac{\delta M_W^2}{M_W^2}\right)_{\text{fin}} - \frac{\alpha}{12\pi}\ln\frac{M_W^2}{M_\phi^2}\right],$$

$$\hat{\Sigma}^W(k^2)=\Sigma_{\text{fin}}^W(k^2)-\text{Re}\,\Sigma_{\text{fin}}^W(M_W^2) + (k^2-M_W^2)\left[\frac{c_W^2}{s_W^2}\left(\frac{\delta M_Z^2}{M_Z^2}-\frac{\delta M_W^2}{M_W^2}\right)_{\text{fin}} - \frac{\alpha}{12\pi}\ln\frac{M_W^2}{M_\phi^2}\right].$$

Electromagnetic vertex:

$$\hat{\Gamma}_\mu^{\gamma ff}(k^2)=ie\gamma_\mu+ie\gamma_\mu(F_V^\gamma-F_A^\gamma\gamma_5) \quad (5.2)$$

with the formfactors

$$F_V^\gamma=-\frac{\alpha}{4\pi}G_f\left[\bar{\Lambda}_1(k^2, M_1, m_f)+\bar{\Lambda}_1(k^2, M_2, m_f) -\bar{\Lambda}_2(k^2, M_\phi)-\delta Z_V^{\text{fin}}\right], \quad (5.3)$$

$$F_A^\gamma=-\frac{\alpha}{4\pi}G_f\bar{\Lambda}_2(k^2, M_\phi). \quad (5.4)$$

δZ_V^{fin} is defined in (4.6), G_f in (3.11) and the $\bar{\Lambda}$-functions in Appendix B. F_A vanishes for $k^2=0$ so that real photons have no axial coupling.

Weak neutral current vertex:

$$\hat{\Gamma}_\mu^{Zff}=ie\gamma_\mu(v-a\gamma_5)+ie\gamma_\mu(F_V^Z-F_A^Z\gamma_5) \quad (5.5)$$

with the form factors

$$F_V^Z=-\frac{\alpha}{4\pi}G_f\left\{v[\bar{\Lambda}_1(k^2, M_1, m_f) +\bar{\Lambda}_1(k^2, M_2, m_f)-\delta Z_V^{\text{fin}}] +\frac{c_W^2-s_W^2}{2c_W s_W}\bar{\Lambda}_2(k^2, M_\phi) +\frac{1}{2c_W s_W}\bar{\Lambda}_3(k^2, M_\phi)\right\} \quad (5.6)$$

$$F_A^Z=\frac{\alpha}{4\pi}G_f\left\{a[\bar{\Lambda}_1(k^2, M_1, m_f)+\bar{\Lambda}_1(k^2, M_2, m_f) +2\bar{\Lambda}_4(k^2, M_1, M_2, m_f)+\delta Z_V^{\text{fin}}] +\frac{c_W^2-s_W^2}{2c_W s_W}\bar{\Lambda}_2(k^2, M_\phi)+\frac{1}{2c_W s_W}\bar{\Lambda}_3(k^2, M_\phi)\right\}. \quad (5.7)$$

For the neutrino vertex one finds:

$$\hat{\Gamma}_\mu^{Z\nu\nu}(k^2)=ie\gamma_\mu(1-\gamma_5)\left[\frac{1}{4c_W s_W}+F_\nu^Z(k^2)\right],$$

$$F_\nu^Z(k^2)=\frac{\alpha}{4\pi}G_f\left\{\frac{1}{4c_W s_W}\left(\ln\frac{M_1 M_2}{M_\phi^2}+\delta Z_V^{\text{fin}}\right) -\frac{s_W}{c_W}\bar{\Lambda}_5(k^2, M_\phi, m_f) +\frac{c_W^2-s_W^2}{2c_W s_W}\bar{\Lambda}_6(k^2, M_\phi, m_f)\right\}. \quad (5.8)$$

Since there is no counter term for the $\nu\nu\gamma$ vertex the form (3.16) is identical to the renormalized $\hat{\Gamma}_\mu^{\gamma\nu\nu}$.

Weak charged current vertex:

$$\hat{\Gamma}_\mu^{W\nu f}(k^2)=i\frac{e}{2\sqrt{2}s_W}\gamma_\mu(1-\gamma_5)[1+F^W(k^2)],$$

$$F^W(k^2)=\frac{\alpha}{4\pi}G_f\{\tfrac{1}{2}\bar{\Lambda}_7(k^2, M_1, M_\phi) +\tfrac{1}{2}\bar{\Lambda}_7(k^2, M_2, M_\phi)+\delta Z_V^{\text{fin}}\}. \quad (5.9)$$

For the functions $\bar{A}_1, \ldots$ we again refer to Appendix B.

6. Discussion of Leptonic Processes

The results of Sect. 5 enable us to discuss the effects of non-standard Higgs particles on those observables from which s_W^2 and the vector boson masses can be determined. We restrict the investigation to the following electroweak processes:

(i) μ decay: the lifetime τ_μ yields a relation between M_Z and M_W;

(ii) $\overset{(-)}{\nu}_\mu e$ scattering, which allows a determination of the weak mixing angle;

(iii) $e^+e^-\to\mu^+\mu^-$, $\tau^+\tau^-$ for PETRA and LEP energies.

6.1. Relation Between M_W and M_Z

For a given value of M_Z the mixing angle resp. $M_W = c_W M_Z$ is fixed in terms of the well known μ lifetime τ_μ and the theoretical expression

$$\frac{1}{\tau_\mu}=\frac{1}{\tau_\mu^0}\left[1+\frac{\alpha}{2\pi}\left(\tfrac{25}{4}-\pi^2\right)\right](1-\delta_{\text{weak}})^{-2} \tag{6.1}$$

with

$$\frac{1}{\tau_\mu^0}=\frac{\alpha^2}{384\pi}m_\mu\left(1-\frac{8m_e^2}{m_\mu^2}\right)\left(\frac{m_\mu}{M_W s_W}\right)^4.$$

δ_{weak} depends on M_Z and s_W^2; therefore (6.1), together with $M_W=M_Z c_W$, can be solved numerically yielding values s_W^2, M_W for a given M_Z.

δ_{weak} is the sum of the standard weak corrections δ_{weak}^S (including the standard single Higgs contribution) and a non-standard part $\delta_{\text{weak}}^{NS}$ due to the extra scalars. The standard part is specified in [13].

The non-standard contribution allows the following approximation: All the Higgs couplings to the fermions involved in μ decay contain at least a factor $(m_\mu/M_W)^2$ in the matrix elements, that suppresses single Higgs exchange and box diagrams with Higgs exchange so much that even $\beta\sim 10^3$ would not be sufficient a make their contribution physically significant. Also the insertion of vertex corrections and ν wave function renormalization do not give larger effects. Therefore the only relevant part in $\delta_{\text{weak}}^{NS}$ is the W self energy generated by the extra Higgses:

$$\delta_{\text{weak}}=\delta_{\text{weak}}^S+\frac{\hat{\Sigma}^W(0)}{M_W^2} \tag{6.2}$$

with $\hat{\Sigma}^W$ from (5.1).

The results for s_W^2 and M_W obtained numerically from (6.1) and (6.2) for a given M_Z are listed in Table 1 for some values of the extra Higgs masses. In this analysis also the standard correction δ_{weak}^S is incorporated with $M_{H_0}=100$ GeV. The results can therefore directly be confronted with experimental data.

Table 1. $\sin^2\theta_W$ and M_W for $M_Z=93.2$ GeV. (Pure numbers for masses in GeV)

M_1	M_2	M_ϕ	$\sin^2\theta_W$	M_W (GeV)
M_Z	M_Z	M_Z	0.2208	82.27
10	10	M_Z	0.2194	82.35
0.1	0.1	M_Z	0.2194	82.35
M_Z	M_Z	$5M_Z$	0.1995	83.39
10	10	$5M_Z$	0.1916	83.80
1	1	$5M_Z$	0.1915	83.80
0.1	0.1	$5M_Z$	0.1915	83.80
$5M_Z$	$5M_Z$	M_Z	0.2005	83.33
$5M_Z$	M_Z	M_Z	0.2212	82.25
$5M_Z$	M_Z	$5M_Z$	0.2207	82.28
Standard			0.2208	82.27

A significant deviation from the standard result is obtained if either ϕ^+ or H_1, H_2 are heavy. All other cases lead only to small modifications. These results are in agreement with those of a similar analysis by Bertolini [14] performed in Sirlin's renormalization scheme [15] without field renormalization.

If the neutral H_1, H_2 become light the values for s_W^2 and M_W tend to become insensitive to their actual masses depending only on M_ϕ (besides M_Z). Precision measurements of M_W and M_Z may decide about the existence of additional Higgs bosons with large mass splittings, since a variation of M_H in the standard model between 10 and 500 GeV gives only $\Delta s_W^2=0.0035$ resp. $\Delta M_W=0.19$ GeV. The value for M_W in case of $M_\phi\simeq 5M_Z$ is about the $1-\sigma$ limit of M_W [16].

6.2. Neutrino Electron Scattering

The determination of $\sin^2\theta_W$ with help of a purely leptonic process has the advantage that it is free of theoretical uncertainties. A sensitive measurement can be obtained in terms of the ratio of neutrino and antineutrino cross sections

$$R_\nu=\frac{\sigma(\nu_\mu e)}{\sigma(\bar{\nu}_\mu e)} \tag{6.3}$$

which reads in lowest order:

$$R_\nu^0=\frac{1+\xi+\xi^2}{1-\xi+\xi^2},\qquad \xi=1-4s_W^2. \tag{6.4}$$

The standard model corrections to R_ν have also been discussed in [13] and turned out to be very small around $s_W^2=0.22$. This is agreement with an independent analysis by Bardin and Dokuchaeva [18]. In particular the standard corrections are nearly insensitive to the mass of the standard Higgs such that R_ν can be considered as a function of s_W^2 only, also in higher order.

Now let us discuss the extra Higgs contributions. They consist of

a) the γZ mixing energy

b) the neutrino charge radius (from the electromagnetic neutrino vertex)

c) box diagrams with exchange of one or two Higgses.

The $\nu\nu Z$ vertex contributions together with the neutrino wave function renormalization vanishes for $k^2\to 0$. Moreover, by the same argument as in 6.1 one can neglect all diagrams where a Higgs couples to the electron (small m_e/M_W factors). Therefore the only relevant part consists of a) and b) which lead to

$$R_\nu=\frac{1+\delta^{\gamma Z}+\xi(1+2\delta^{\gamma Z})+\xi^2}{1-\delta^{\gamma Z}-\xi(1-2\delta^{\gamma Z})+\xi^2} \tag{6.5}$$

where

$$\delta^{\gamma Z}=4c_W s_W\left.\frac{\hat{\Sigma}^{\gamma Z}(k^2)}{k^2}\right|_{k^2=0} + \frac{\alpha}{3\pi}\left(\frac{\beta m_\mu}{M_\phi}\right)^2\left(\ln\frac{M_\phi^2}{m_\mu^2}-\frac{5}{6}\right) \tag{6.6}$$

with $\hat{\Sigma}^{\gamma Z}$ from (5.1). The second term is the ν charge radius $\lim_{k^2\to 0} F_\nu^\gamma/k^2$ where F_ν^γ is the electromagnetic ν formfactor in (3.16). Figure 10 shows the dependence of R_ν on the mixing angle s_W^2 for various mass values of the extra Higgs bosons. In contrast to the standard situation there is now also a significant dependence on the scalar masses, which means that the extraction of s_W^2 from a measured R_ν value will lead to different s_W^2 for different masses of the extra Higgses. Again we encounter the situation that remarkable deviations from the standard model occur only if either M_ϕ or (M_1, M_2) are large. R_ν becomes independent of M_1 and M_2 for light neutral scalar/pseudoscalars.

The neutrino charge radius in (6.6) plays only a subordinate rôle, in particular for heavy ϕ^+. E.g., $M_\phi\sim M_Z$ and $\beta=300$ would change R_ν by less than 0.01. This is also different from the minimal model, where relatively large contributions from $\hat{\Sigma}^{\gamma Z}$ and the ν charge radius cancel each other.

The experimental value for R_ν is [19]

$$R_\nu^{\exp}=1.26\,{}^{+0.41}_{-0.28}.$$

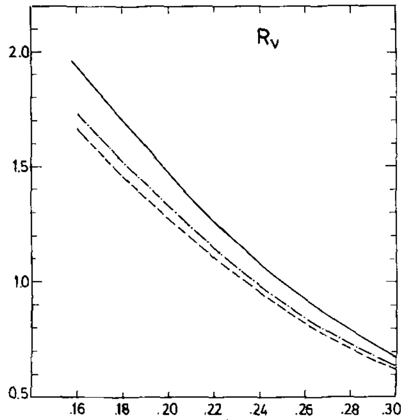


Fig. 10. R_ν, (6.3), in lowest order (———) and for different Higgs masses with radiative corrections due to additional Higgs bosons. $M_Z=93.2$ GeV. —·—·— $M_1=M_2=M_Z$, $M_\phi=5M_Z$, ------ $M_1=M_2=10$ GeV, $M_\phi=5M_Z$. $\beta=50$

This gives in the standard model:

$\sin^2\theta_W=0.221\pm0.031$.

The mean value of R_ν would give in the two doublet model:

M_1	M_2	M_ϕ	$\sin^2\theta_W$
10 GeV	10 GeV	M_Z	0.220
10 GeV	10 GeV	$5M_Z$	0.203
M_Z	M_Z	$5M_Z$	0.208
$5M_Z$	$5M_Z$	M_Z	0.208

The present accuracy does not allow to put tight restrictions on the possible mass range of extra scalars, but this will change with the expected improvement in the R_ν measurements aiming an accuracy of $\Delta s_W^2=0.005$.

There is also a second way to discuss the quantity R_ν:

For a fixed M_Z, $\sin^2\theta_W$ can be determined with help of τ_μ as done in 6.1. The theoretically predicted value for R_ν is then a function of the extra masses and can directly be compared with the experimental result. The theoretical R_ν values obtained in this way are listed in Table 2. Again the variation of R_ν is within the experimental uncertainty.

6.3. $e^+e^-\to l^+l^-$

The standard electroweak corrections in the on-shell scheme have already been presented in [13, 20] for the forward-backward asymmetry, and for the polarization asymmetry in [20, 21].

We want to discuss now the effect of the additional scalars in the 2-doublet extension of $SU(2)$

Table 2. R_ν for $M_Z=93.2$ GeV. (Pure numbers for masses in GeV)

M_1	M_2	M_ϕ	R_ν
M_Z	M_Z	M_Z	1.26
10	10	M_Z	1.27
0.1	0.1	M_Z	1.27
M_Z	M_Z	$5M_Z$	1.34
10	10	$5M_Z$	1.36
1	1	$5M_Z$	1.36
0.1	0.1	$5M_Z$	1.36
$5M_Z$	$5M_Z$	M_Z	1.32
$5M_Z$	M_Z	M_Z	1.26
$5M_Z$	M_Z	$5M_Z$	1.26
Standard			1.26

$\times U(1)$. Some simplifications can be made based on the small m_e/M_W ratio:

– vertex corrections with scalars in the e^+e^- vertices can be neglected because of the factor $(\beta m_e/M_W)^2$.

– Box diagrams with exchange of one and two scalar bosons can also be neglected since the Higgs has always to couple to the electron.

– The scalar-vector mixing propagators give also terms of order $(\beta m_e m_f/M_W^2)$ in the matrix element and can therefore also be neglected for β not essentially larger than 10^2.

Consequently we have to take into account

– the γ and Z self energies

– γZ mixing energy

– the final state vertex corrections.

In case of a $\mu^+\mu^-$ final state the latter one will also give a negligible contribution due to the factor $(m_\mu/M_W)^2$ in the vertex diagrams; for a $\tau^+\tau^-$ final state, however, m_τ/M_W can be (partly) compensated by the enhancement factor β. This different magnitude of the vertex corrections can give rise to an apparent violation of the $\mu-\tau$ universality in physical observables.

Since polarization experiments become feasible around the Z^0 we include the case where the electron is longitudinally polarized with polarization degree P_L. The following observables are of particular interest:

– Integrated cross section:

$$\sigma=\int d\Omega\,\frac{d\sigma}{d\Omega} \tag{6.7}$$

– Forward-backward asymmetry A_{FB}:

$$A_{FB}=\frac{1}{\sigma}\left(\int\limits_{\cos\theta>0} d\Omega\,\frac{d\sigma}{d\Omega}-\int\limits_{\cos\theta<0} d\Omega\,\frac{d\sigma}{d\Omega}\right) \tag{6.8}$$

– Longitudinal polarization asymmetry A_L:

$$P_L\cdot A_L=\frac{\sigma(P_L)-\sigma(-P_L)}{\sigma(P_L)+\sigma(-P_L)}. \tag{6.9}$$

The differential cross section has the form

$$\frac{d\sigma}{d\Omega}=\frac{\alpha^2}{4s}\left[\sigma_U(\theta)+P_L\,\sigma_L(\theta)\right] \tag{6.10}$$

where

$$s=(p_{e^-}+p_{e^+})^2,\qquad \theta=\sphericalangle(e^-,\mu^-).$$

With the propagator function

$$\chi(s)=\frac{s}{s-M_Z^2+\hat{\Sigma}^Z(s)} \tag{6.11}$$

σ_U and σ_L can be specified in the following way:

$$\begin{aligned}\sigma_j=&(A_1^j+A_2^j\,\mathrm{Re}\,\chi+A_3^j\,\mathrm{Im}\,\chi+A_4^j|\chi|^2)\cdot(1+\cos^2\theta)\\&+(B_1^j+B_2^j\,\mathrm{Re}\,\chi+B_3^j\,\mathrm{Im}\,\chi+B_4^j|\chi|^2)\cdot 2\cos\theta\end{aligned} \tag{6.12}$$

for $j=U$ and $j=L$.

The θ-independent coefficients A, B are put together in Table 3. The form factors F_V and F_A in the table are those defined in (5.3–7). For the numerical discussion we have used $\beta=50$. The quantities Π are the relative self energies

$$\Pi^\gamma=\frac{\hat{\Sigma}^\gamma(s)}{s},\qquad \Pi^{\gamma Z}=\frac{\hat{\Sigma}^{\gamma Z}(s)}{s}; \tag{6.13}$$

$\hat{\Sigma}^{\gamma,\gamma Z}$ are the renormalized functions of (5.1).

We divide the discussion into two parts:

a) PETRA Energies. At energies around 40 GeV the leptonic polarization asymmetry is small ($\sim 2\%$); higher order effects are $<1\%$. Therefore we concentrate our discussion on the unpolarized observables σ and A_{FB}.

Figure 11 shows how the relation between A_{FB} and s_W^2 (for fixed M_Z) is modified in case of a heavy ϕ^+. A heavy H_1, H_2 pair gives a similar effect. For light H_1, H_2 A_{FB} becomes independent of M_1, M_2. Values for s_W^2, if extracted from A_{FB}^{exp} for given M_Z, would be lower than in the standard model. This behaviour is just opposite to the tendency of the PETRA experiments [17]. On the other hand, the measured A_{FB} can be converted to restrict possible extra heavy Higgs states if s_W^2 is taken from $M_{W,Z}$ measurements as $s_W^2=1-M_W^2/M_Z^2$.

Differences between the μ and τ asymmetry are small in all cases ($\sim 0.1\%$). This is due to cancellations of the leading vertex corrections which are different for μ and τ.

The second point of view incorporates the results of 6.1 and relates σ and A_{FB} directly to M_1, M_2, M_ϕ

Table 3. Coefficients in the differential cross section for $e^+ e^- \to l^+ l^-$

	$j=U$	$j=L$
A_1^j	$1-2\,\mathrm{Re}\,\Pi^\gamma+2\,\mathrm{Re}\,F_V^\gamma$	0
A_2^j	$2v^2(1-\mathrm{Re}\,\Pi^\gamma)-4v\,\mathrm{Re}\,\Pi^{\gamma Z}$ $+2v^2\,\mathrm{Re}\,F_V^\gamma+2va\,\mathrm{Re}\,F_A^\gamma+2v\,\mathrm{Re}\,F_V^Z$	$2va(1-\mathrm{Re}\,\Pi^\gamma)-2a\,\mathrm{Re}\,\Pi^{\gamma Z}$ $+2a^2\,\mathrm{Re}\,F_A^\gamma+2va\,\mathrm{Re}\,F_V^\gamma+2a\,\mathrm{Re}\,F_V^Z$
A_3^j	$-2v^2\,\mathrm{Im}\,\Pi^\gamma+4v\,\mathrm{Im}\,\Pi^{\gamma Z}$ $+2v^2\,\mathrm{Im}\,F_V^\gamma+2va\,\mathrm{Im}\,F_A^\gamma-2v\,\mathrm{Im}\,F_V^Z$	$-2va\,\mathrm{Im}\,\Pi^\gamma+2a\,\mathrm{Im}\,\Pi^{\gamma Z}$ $+2a^2\,\mathrm{Im}\,F_A^\gamma+2va\,\mathrm{Im}\,F_V^\gamma-2a\,\mathrm{Im}\,F_V^Z$
A_4^j	$(v^2+a^2)^2-4v(v^2+a^2)\,\mathrm{Re}\,\Pi^{\gamma Z}$ $+2(v^2+a^2)(v\,Re\,F_V^Z+a\,\mathrm{Re}\,F_A^Z)$	$2va(v^2+a^2)-2a(3v^2+a^2)\,\mathrm{Re}\,\Pi^{\gamma Z}$ $+4va(v\,\mathrm{Re}\,F_V^Z+a\,\mathrm{Re}\,F_A^Z)$
B_1^j	0	$2\,\mathrm{Re}\,F_A^\gamma$
B_2^j	$2a^2(1-\mathrm{Re}\,\Pi^\gamma)$ $+2a^2\,\mathrm{Re}\,F_V^\gamma+2va\,\mathrm{Re}\,F_A^\gamma+2a\,\mathrm{Re}\,F_A^Z$	$2va(1-\mathrm{Re}\,\Pi^\gamma)-2a\,\mathrm{Re}\,\Pi^{\gamma Z}$ $+2va\,\mathrm{Re}\,F_V^\gamma+2v^2\,\mathrm{Re}\,F_A^\gamma+2v\,\mathrm{Re}\,F_A^Z$
B_3^j	$-2a^2\,\mathrm{Im}\,\Pi^\gamma$ $+2a^2\,\mathrm{Im}\,F_V^\gamma+2va\,\mathrm{Im}\,F_A^\gamma-2a\,\mathrm{Im}\,F_A^Z$	$-2va\,\mathrm{Im}\,\Pi^\gamma$ $+2va\,\mathrm{Im}\,F_V^\gamma+2v^2\,\mathrm{Im}\,F_A^\gamma-2v\,\mathrm{Im}\,F_A^Z$
B_4^j	$4v^2a^2-8va^2\,\mathrm{Re}\,\Pi^{\gamma Z}$ $+4va(v\,\mathrm{Re}\,F_A^Z+a\,\mathrm{Re}\,F_V^Z)$	$2va(v^2+a^2)-2a(3v^2+a^2)\,\mathrm{Re}\,\Pi^{\gamma Z}$ $+2(v^2+a^2)(v\,\mathrm{Re}\,F_V^Z+a\,\mathrm{Re}\,F_A^Z)$

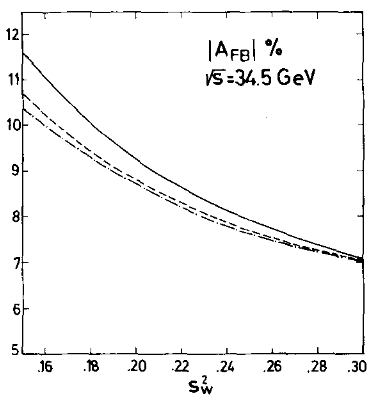


Fig. 11. Forward backward asymmetry as function of s_W^2 at $\sqrt{s} = 34.5\,\mathrm{GeV}$. $M_Z = 93.2\,\mathrm{GeV}$. ------ $M_1 = M_2 = M_Z$, $M_\phi = 5M_Z$, —·—·— $M_1 = M_2 = 10\,\mathrm{GeV}$, $M_\phi = 5M_Z$. $\beta = 50$

Table 4. $\sqrt{s} = 34.5\,\mathrm{GeV}$ $(\beta = 50)$

M_1	M_2	M_ϕ	$\sigma(\tau^+\tau^-)/\sigma_0$	$A_{FB}(\tau)$	$\sigma(\mu^+\mu^-)/\sigma_0$	$A_{FB}(\mu)$
M_Z	M_Z	M_Z	1.003	−8.52	1.002	−8.62
10	10	M_Z	0.999	−8.57	1.002	−8.63
0.1	0.1	M_Z	0.980	−8.57	1.002	−8.63
M_Z	M_Z	$5M_Z$	1.003	−8.69	1.002	−8.80
10	10	$5M_Z$	0.999	−8.79	1.002	−8.86
1	1	$5M_Z$	0.990	−8.79	1.002	−8.86
0.1	0.1	$5M_Z$	0.978	−8.79	1.002	−8.86
$5M_Z$	$5M_Z$	M_Z	1.002	−8.69	1.002	−8.80
$5M_Z$	M_Z	M_Z	1.003	−8.59	1.002	−8.62
$5M_Z$	M_Z	$5M_Z$	1.003	−8.59	1.002	−8.61
0.1	M_Z	M_Z	0.985	−8.53	1.002	−8.62
M_Z	0.1	M_Z	0.998	−8.54	1.002	−8.62
Standard			1.002	−8.53	1.002	−8.62

by means of (6.1-2). The results are listed in Table 4. Deviations from the standard model would be hard to detect experimentally ($\lesssim 0.3\,\%$). The reason is that the effect of the Z self energy in (6.12) and of the W self energy in (6.1) largely compensate each other.

In the cross section, however, there is a violation of the universality in the case of light neutral particles (1-3% effect). A light pseudoscalar gives a constant contribution for $M_2 \to 0$, whereas a light scalar yields a logarithmic increase for $M_1 \to 0$. Their contributions to σ are always negative. A 5% effect, which corresponds to the present experimental uncertainty for σ_τ/σ_0 [22] is obtained e.g. for $M_1 = M_2 = 10\,\mathrm{GeV}$ and $\beta = 200$ or $M_1 = M_2 = 5\,\mathrm{GeV}$ and $\beta = 140$. This is a tighter limit for β as from $g-2$ for muons [6] in the degenerate H_1, H_2 case.

b) On-resonance. We consider the experimentally most interesting case $\sqrt{s} = M_Z$ and include the longitudinal polarization asymmetry (6.9).

Figures 12 and 13 display the s_W^2-dependence of A_{FB} and A_L for the case of a heavy charged Higgs. The asymmetries for μ and τ are only slightly different due to the fact that the formfactors largely cancel in the asymmetries. Also a common limiting curve is reached for light neutral particles, which represents essentially the lower curve in the figures.

The case of a heavy H_1, H_2 pair and $M_\phi \sim M_Z$ practically coincides with the previous one (ϕ^+ heavy, $M_1 \sim M_2 \sim M_Z$) and is not displayed separately. Deviations from the standard model prediction in all other cases (no large mass splitting) are less sig-

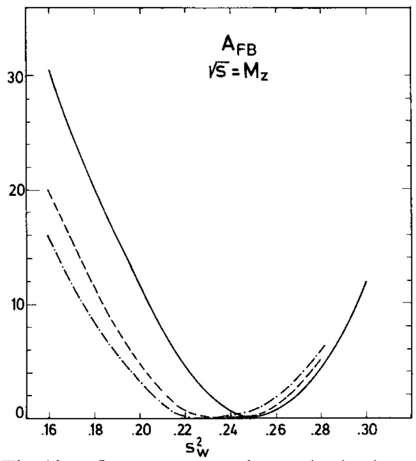


Fig. 12. On-resonance forward backward asymmetry, s_W^2-dependence. $M_Z = 93.2$ GeV. ------ $M_1 = M_2 = M_Z$, $M_\phi = 5 M_Z$, —·—·— $M_1 = M_2 = 10$ GeV, $M_\phi = 5 M_Z$. $\beta = 50$

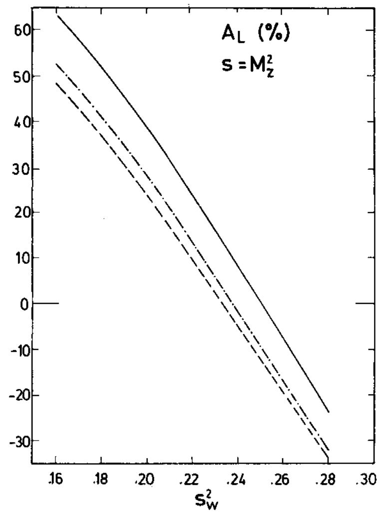


Fig. 13. On-resonance polarization asymmetry A_L. s_W^2-dependence. $M_Z = 93.2$ GeV. —·—·— $M_1 = M_2 = M_Z$, $M_\phi = 5 M_Z$, ------ $M_1 = M_2 = 10$ GeV, $M_\phi = 5 M_Z$. $\beta = 50$

nificant ($\lesssim 0.7\%$). Qualitatively, this behaviour is quite similar to that encountered in a).

Now we follow the lines of a) and incorporate the results of 6.1, which means that s_W^2 is no longer an independent quantity but already fixed if M_Z and the Higgs masses are specified.

The values of σ, A_{FB}, A_L obtained in this way are put together in Table 5 for various choices of the Higgs masses, both for μ and τ final states.

Let us first have a look at the asymmetries:

Table 5. $\sqrt{s} = M_Z$. ($\beta = 50$)

M_1	M_2	M_ϕ	$\tau^+\tau^-$			$\mu^+\mu^-$		
			σ/σ_0	A_{FB}	A_L	σ/σ_0	A_{FB}	A_L
M_Z	M_Z	M_Z	0.9989	3.95	22.90	0.9986	3.96	22.90
10	10	M_Z	0.9403	4.07	23.29	0.9398	4.09	22.90
1	1	M_Z	0.9287	4.07	23.28	0.9395	4.09	23.29
0.1	0.1	M_Z	0.9182	4.07	23.27	0.9395	4.09	23.29
M_Z	M_Z	$5M_Z$	0.9508	4.92	27.75	0.9506	4.92	27.75
10	10	$5M_Z$	0.8786	4.83	29.37	0.8783	4.89	29.36
1	1	$5M_Z$	0.8662	4.73	29.24	0.8745	4.88	29.38
0.1	0.1	$5M_Z$	0.8546	4.60	29.03	0.8744	4.88	29.37
$5M_Z$	$5M_Z$	M_Z	0.9526	4.86	27.45	0.9528	4.87	27.45
$5M_Z$	$5M_Z$	$5M_Z$	0.9982	3.89	22.70	0.9991	3.89	22.70
$5M_Z$	M_Z	$5M_Z$	0.9972	3.92	22.79	0.9981	3.92	22.97
M_Z	10	$5M_Z$	0.9424	4.92	28.45	0.9421	4.95	28.44
M_Z	1	$5M_Z$	0.9399	4.88	28.39	0.9420	4.93	28.41
M_Z	0.1	$5M_Z$	0.9395	4.88	28.38	0.9420	4.93	28.41
10	M_Z	$5M_Z$	0.9428	4.92	28.45	0.9421	4.95	28.44
1	M_Z	$5M_Z$	0.9371	4.86	28.35	0.9420	4.93	28.41
0.1	M_Z	$5M_Z$	0.9256	4.77	28.19	0.9420	4.93	28.41
$5M_Z$	$5M_Z$	M_Z	0.9562	4.86	27.45	0.9528	4.86	27.45
$5M_Z$	M_Z	M_Z	0.9982	3.89	22.70	0.9991	3.89	22.70
$5M_Z$	M_Z	$5M_Z$	0.9972	3.92	22.79	0.9981	3.92	22.79
M_Z	10	$5M_Z$	0.9424	4.92	28.45	0.9421	4.95	28.44
M_Z	1	$5M_Z$	0.9399	4.88	28.39	0.9420	4.93	28.41
M_Z	0.1	$5M_Z$	0.9395	4.88	28.38	0.9420	4.93	28.41
10	M_Z	$5M_Z$	0.9428	4.92	28.45	0.9421	4.95	28.44
1	M_Z	$5M_Z$	0.9371	4.86	28.35	0.9420	4.93	28.41
0.1	M_Z	$5M_Z$	0.9256	4.76	28.19	0.9420	4.93	28.41

Differences between A^{μ} and A^{τ} are not more than $\sim 0.4\%$; this is again a consequence of cancellation of the leading vertex corrections in A_{FB} as well as in A_L. Consequently, A_{FB} and A_L are not very sensitive to the enhancement factor.

Comparing the results with the standard model it becomes obvious that the on-resonance asymmetries are sensitive to the extra Higgs contributions, in particular when either ϕ^+ or H_1, H_2 are heavy. This is different from the off-resonance case. The reason for this is that W and Z self energies do not compensate each other for $s=M_Z^2$ (on-shell subtraction of Σ^Z). One can also learn that a light H_1, H_2 pair tends to a common limit in the asymmetries.

The integrated cross sections in Table 5 are given as ratios σ/σ_0, where σ_0 measures the lowest order standard cross section ($s_W^2=0.2208$). The sources for deviations from 1 are

– different coupling constants resulting from (6.1) and (6.2);
– contributions from $\hat{\Sigma}^{\gamma Z}$ and the formfactors; light neutral Higgs give 2-3% difference between μ and τ;
– different $\operatorname{Im}\hat{\Sigma}^Z(M_Z^2)$ in case of light neutral particles.

σ will therefore, in contrast to the asymmetries, show a dependence on light neutral particles and to enhancement effects. For a more realistic experimental discussion also the effect of light scalar bremsstrahlung has to be considered.

7. Conclusions

In the framework of a $SU(2)\times U(1)$ gauge theory with 2 Higgs doublets and enhanced Yukawa couplings we have calculated the 1-loop corrections to the leptonic processes μ decay, $\nu_\mu e$ scattering and $e^+e^-\to\mu^+\mu^-$, $\tau^+\tau^-$. The renormalization is performed in the on-shell scheme; field renormalization leads to finite self energies and vertex functions. Measurable effects on the $M_W - M_Z$ mass relation, $\sigma(\nu_\mu e)/\sigma(\bar{\nu}_\mu e)$ and A_{FB}, A_L in $e^+e^-\to l^+l^-$ appear if either the charged Higgs mass or the neutral Higgs masses are heavy. Effects of light scalars/pseudoscalars and the influence of the enhancement factor play a subordinate rôle in the asymmetries. They are better investigated in terms of cross sections. Present limits on $\sigma(e^+e^-\to\tau^+\tau^-)$ restrict the enhancement factor to ~ 140 for a neutral H_1, H_2 pair at 5 GeV and 200 for 10 GeV. The best place to look for heavy Higgs particles with large mass splittings will be the on-resonance polarization asymmetry in connection with precise vector boson mass measurements.

Acknowledgement. I want to thank M. Böhm and B. Naroska for helpful discussions.

Appendix A

Invariant Functions in 2-Point Integrals

With $\Delta=\frac{2}{\varepsilon}-\gamma+\ln 4\pi$, $\varepsilon=4-D$, and the mass scale μ introduced in dimensional regularization the function B_0 reads

$$B_0(k^2,m_1,m_2)=\Delta-\ln\frac{m_1 m_2}{\mu^2}+\bar{B}_0(k^2,m_1,m_2), \tag{A.1}$$

$$\bar{B}_0(k^2,m_1,m_2)=1-\frac{m_1^2+m_2^2}{m_1^2-m_2^2}\ln\frac{m_1}{m_2}+F(k^2,m_1,m_2). \tag{A.2}$$

An analytic expression for $F(k^2,m_1,m_2)$ is given in [12]. The integral representation for $\bar{B}_0$ is

$$\bar{B}_0(k^2,m_1,m_2) =\int_0^1 dx\,\ln\frac{x^2k^2-x(k^2+m_1^2-m_2^2)+m_1^2-i\varepsilon}{m_1\cdot m_2}. \tag{A.2'}$$

With help of

$$A(m)=-m^2\left(\Delta-\ln\frac{m^2}{\mu^2}+1\right)$$

one can write for the function B_1:

$$B_1(k^2,m_1,m_2)=\frac{m_2^2-m_1^2-k^2}{2k^2}B_0(k^2,m_1,m_2)+\frac{A(m_2)-A(m_1)}{2k^2}. \tag{A.3}$$

For the fermion renormalization constants we need the specific values

$$B_1(m_1^2,m_1,m_2) =-\frac{1}{2}\left(\Delta-\ln\frac{m_2^2}{\mu^2}+\frac{1}{2}\right)+\bar{B}_1(m_1^2,m_1,m_2) \tag{A.4}$$

$$\bar{B}_1(m_1^2,m_1,m_2) =-\frac{1}{4}+\frac{m_1^2}{m_2^2-m_1^2}\ln\frac{m_2}{m_1}+\frac{m_2^2-2m_1^2}{2m_1^2}F(m_1^2,m_1,m_2). \tag{A.5}$$

Furthermore we need the derivatives

$$B_0'(m_1^2,m_1,m_2)=\frac{\partial B_0}{\partial k^2}(k^2=m_1^2,m_1,m_2)$$

$$B_1'(m_1^2,m_1,m_2)=\frac{\partial B_1}{\partial k^2}(k^2=m_1^2,m_1,m_2).$$

They read:

$$B_1'(m_1^2, m_1, m_2)\cdot 2m_1^2$$
$$=\tfrac{1}{2}-\ln\frac{m_2}{m_1}-\bar{B}_0(m_1^2, m_1, m_2)$$
$$-2\bar{B}_1(m_1^2, m_1, m_2)+(m_2^2-2m_1^2)\,B_0'(m_1^2, m_1, m_2), \tag{A.6}$$

$$B_0'(m_1^2, m_1, m_2)$$
$$=-\frac{1}{m_1^2}+\frac{m_2^2-m_1^2}{m_1^4}\ln\frac{m_2}{m_1}-\frac{2m_2(m_2^2-3m_1^2)}{m_1^4\sqrt{|m_2^2-4m_1^2|}}$$
$$\cdot\begin{cases}\arctan\sqrt{\dfrac{2m_1-m_2}{2m_1+m_2}}, & (m_1-m_2)^2<m_1^2\\ \ln\dfrac{\sqrt{m_2+2m_1}+\sqrt{m_2^2-2m_1}}{2\sqrt{m_1}}, & (m_1-m_2)^2>m_1^2.\end{cases} \tag{A.7}$$

Appendix B

Invariant Functions in 3-Point Integrals

The finite parts of the γ_μ and $\gamma_\mu\gamma_5$ coefficients in Sect. 3 are:

$$\bar{A}_1(k^2, M, m)=-\tfrac{1}{4}+\tfrac{1}{2}\ln\frac{k^2}{M^2}$$
$$+\frac{M^2}{k^2}\left[1-\ln\frac{k^2}{M^2}+\frac{m^2}{M^2-m^2}\ln\frac{M^2}{m^2}+F(m^2, m, M)\right]$$
$$+\frac{M^4}{k^2}C_0(k^2, m, m, M; m)+i\pi\left(\frac{M^2}{k^2}-\frac{1}{2}\right); \qquad (k^2>0), \tag{B.1}$$

$$\bar{A}_2(k^2, M)=\frac{1}{4}+\frac{1}{2}\left(1-\frac{2M^2}{k^2}\right)F(k^2, M, M)+\frac{M^2}{k^2}$$
$$-4\left(\frac{M^2}{k^2}\right)^2\arctan^2\frac{1}{\sqrt{4M^2/k^2-1}}; \qquad (0<k^2<4M^2), \tag{B.2}$$

$$\bar{A}_3(k^2, M)=-\tfrac{1}{4}+\tfrac{1}{2}\ln\frac{k^2}{M^2}+\frac{M^2}{k^2}\left(1-\ln\frac{k^2}{M^2}\right)$$
$$+\left(\frac{M^2}{k^2}\right)^2\left[\ln\frac{k^2}{M^2}\ln\left(1+\frac{k^2}{M^2}\right)+Sp\left(-\frac{k^2}{M^2}\right)\right]$$
$$-i\pi\left[\frac{1}{2}-\frac{M^2}{k^2}+\frac{M^4}{k^4}\ln\left(1+\frac{k^2}{M^2}\right)\right]; \qquad (k^2>0), \tag{B.3}$$

$$\bar{A}_4(k^2, M_1, M_2, m)=\tfrac{1}{2}+\bar{B}_0(k^2, M_1, M_2)$$
$$+\frac{M_1^2}{k^2}\left[\ln\frac{M_2}{m}+\bar{B}_0(m^2, M_1, m)-\bar{B}_0(k^2, M_1, M_2)\right]$$
$$+\frac{M_2^2}{k^2}\left[\ln\frac{M_1}{m}+\bar{B}_0(m^2, M_2, m)-\bar{B}_0(k^2, M_1, M_2)\right]$$
$$+\frac{2M_1^2M_2^2}{k^2}C_0(k^2, M_1, M_2, m; m); \tag{B.4}$$

$$\bar{A}_5(k^2, M, m)=\tfrac{3}{4}+\ln\frac{m}{M}-\tfrac{1}{2}F(k^2, m, m)$$
$$+\frac{M^2-m^2}{k^2}[B_0(k^2, m, m)-B_0(0, m, M)]$$
$$+\left[m^2+\frac{(M^2-m^2)^2}{k^2}\right]C_0(k^2, m, m, M; 0);$$

$$\bar{A}_6(k^2, M, m)=\tfrac{1}{4}+\tfrac{1}{2}F(k^2, M, M)$$
$$-\frac{M^2-m^2}{k^2}[B_0(k^2, M, M)-B_0(0, m, M)]$$
$$+\left[m^2+\frac{(M^2-m^2)^2}{k^2}\right]C_0(k^2, M, M, m; 0);$$

$$\bar{A}_7(k^2, M_1, M_2)=\tfrac{1}{4}+\tfrac{1}{2}B_0(k^2, M_1, M_2)$$
$$+\frac{M_1^2}{2k^2}\left[\ln\frac{M_2}{M_1}-\bar{B}_0(k^2, M_1, M_2)\right]$$
$$+\frac{M_2^2}{2k^2}\left[\ln\frac{M_1}{M_2}-\bar{B}_0(k^2, M_1, M_2)\right]$$
$$+\frac{M_1^2M_2^2}{k^2}C_0(k^2, M_1, M_2, 0; 0).$$

C_0 denotes the scalar vertex integral with equal external masses $p_1^2=p_2^2=m^2$ and the momentum transfer $k^2=(p_1+p_2)^2$:

$$\frac{i}{16\pi^2}C_0(k^2, M_1, M_2, M_3; m)$$
$$=\int\frac{d^4q}{(2\pi)^4}\frac{1}{[(p_1+q)^2-M_1^2]\,[(p_2-q)^2-M_2^2]\,[q^2-M_3^2]}.$$

In our cases we do not need the full expression containing 12 Spence functions. Since we work in the approximation $m^2\ll k^2$ and since the C_0 functions in (B.1) and (B.4) appear with coefficients M_i^2/k^2 we need only their approximate form for $m^2\ll M_1^2, M_2^2$:

$$k^2\cdot C_0(k^2, m, m, M; m)$$
$$\simeq\left[\ln\left(\frac{k^2}{M^2}\right)-i\pi\right]\ln\left(1+\frac{k^2}{M^2}\right)+Sp\left(-\frac{k^2}{M^2}\right) \tag{B.5}$$

and

$$k^2\cdot C_0(k^2, M_1, M_2, m; m)$$
$$\simeq\frac{\pi^2}{6}-Sp\left(1-\frac{k^2}{M_2^2}\right)$$
$$+\sum_{j=1}^{2}\left\{Sp\left(\frac{M_2^2}{M_2^2-x_j}\right)-Sp\left(\frac{M_2^2-k^2}{M_2^2-x_j}\right)\right\} \tag{B.6}$$

with

$$x_{1,2}=\tfrac{1}{2}(M_2^2-M_1^2-k^2 \pm\sqrt{(M_2^2-M_1^2-k^2)^2-4M_2^2k^2+i\varepsilon}).$$

Sp means the Spence function or Dilogarithm

$$Sp(z)=-\int_0^1 dx\,\frac{\ln(1-xz)}{x}.$$

Appendix C

Counter Terms for Self Energies and Vertices

Here we collect the formulas for the renormalized 2- and 3-point functions which are composed by the unrenormalized quantities and their corresponding counter terms.

We expand the renormalization constants according to

$$Z_i=1+\delta Z_i.$$

It is convenient to introduce the following linear combinations of the $SU(2)$ and $U(1)$ field renormalization constants $\delta Z_2^{W,B}$ and the gauge coupling renormalization constants $\delta Z_1^{W,B}$

$$\begin{pmatrix}\delta Z_i^\gamma\\ \delta Z_i^Z\end{pmatrix}=\begin{pmatrix}s_W^2 & c_W^2\\ c_W^2 & s_W^2\end{pmatrix}\begin{pmatrix}\delta Z_i^W\\ \delta Z_i^B\end{pmatrix},\qquad i=1,2. \tag{C.1}$$

Denoting with Σ^γ, $\Sigma^{\gamma Z}$, Σ^Z, Σ^W the unrenormalized boson self energies, the corresponding renormalized ones are obtained via

$$\begin{aligned}\hat\Sigma^\gamma(k^2)&=\Sigma^\gamma(k^2)+\delta Z_2^\gamma k^2\\ \hat\Sigma^Z(k^2)&=\Sigma^Z(k^2)-\delta M_Z^2+\delta Z_2^Z(k^2-M_Z^2)\\ \hat\Sigma^W(k^2)&=\Sigma^W(k^2)-\delta M_W^2+\delta Z_2^W(k^2-M_W^2)\\ \hat\Sigma^{\gamma Z}(k^2)&=\Sigma^{\gamma Z}(k^2)-\delta Z_2^{\gamma Z}k^2+M_Z^2(\delta Z_1^{\gamma Z}-\delta Z_2^{\gamma Z}).\end{aligned} \tag{C.2}$$

In the last line the combinations

$$\delta Z_i^{\gamma Z}=\frac{c_W s_W}{c_W^2-s_W^2}(\delta Z_i^Z-\delta Z_i^\gamma),\qquad i=1,2 \tag{C.3}$$

have been introduced.

The mass counter terms $\delta M_{W,Z}^2$ (which get fixed by the on-shell conditions) fulfil the important relation

$$\frac{\delta M_Z^2}{M_Z^2}-\frac{\delta M_W^2}{M_W^2}=\frac{s_W}{c_W}(3\delta Z_2^{\gamma Z}-2\delta Z_1^{\gamma Z}). \tag{C.4}$$

This relation allows to express δZ_i^Z, δZ_i^W by means of the on-shell values of the unrenormalized vector boson self energies.

For fermion renormalization a field renormalization constant δZ_L is assigned to the left-hand lepton doublet and a δZ_R to the right-handed charged singlet. We make also use of the combinations

$$\delta Z_V=\frac{\delta Z_L+\delta Z_R}{2},\qquad \delta Z_A=\frac{\delta Z_L-\delta Z_R}{2}. \tag{C.5}$$

The renormalized fermion self energy can be written as

$$\begin{aligned}\hat\Sigma^f(k)=&\,k\!\!\!/(\Sigma_V^f(k^2)+\delta Z_V)+k\!\!\!/\gamma_5(\Sigma_A^f(k^2)-\delta Z_A)\\ &+m_f\left(\Sigma_S^f(k^2)-\delta Z_V-\frac{\delta m_f}{m_f}\right)\end{aligned} \tag{C.6}$$

with the unrenormalized $\Sigma_{V,A,S}^f$.

Finally we need the renormalized electromagnetic vertex of the leptons

$$\begin{aligned}\hat\Gamma_\mu^{\gamma ff}=&\,\Gamma_\mu^{\gamma ff}+ie\gamma_\mu(\delta Z_1^\gamma-\delta Z_2^\gamma+\delta Z_V-\delta Z_A\gamma_5)\\ &+ie\gamma_\mu(v-a\gamma_5)(\delta Z_1^{\gamma Z}-\delta Z_2^{\gamma Z})\end{aligned} \tag{C.7}$$

and the leptonic neutral current vertex:

$$\begin{aligned}\hat\Gamma_\mu^{Zff}=&\,\Gamma_\mu^{Zff}+ie\gamma_\mu(v-a\gamma_5)(\delta Z_1^Z-\delta Z_2^Z)\\ &-ie\gamma_\mu(\delta Z_1^{\gamma Z}-\delta Z_2^{\gamma Z})\\ &+ie\gamma_\mu(v\,\delta Z_V+a\,\delta Z_A)\\ &-ie\gamma_\mu\gamma_5(v\,\delta Z_A+a\,\delta Z_V).\end{aligned} \tag{C.8}$$

Γ_μ stands for the corresponding unrenormalized vertex.

The $\nu-Z$ vertex is given by

$$\hat\Gamma_\mu^{Z\nu\nu}=\Gamma_\mu^{Z\nu\nu}+i\frac{e}{4c_W s_W}\gamma_\mu(1-\gamma_5)(\delta Z_L+\delta Z_1^Z-\delta Z_2^Z) \tag{C.9}$$

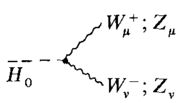
and the electromagnetic neutrino vertex:

$$\hat\Gamma_\mu^{\gamma\nu\nu}=\Gamma_\mu^{\gamma\nu\nu}-i\frac{e}{4c_W s_W}\gamma_\mu(1-\gamma_5)(\delta Z_1^{\gamma Z}-\delta Z_2^{\gamma Z}). \tag{C.10}$$

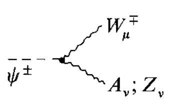
Appendix D

Feynman Rules for Gauge-Boson Higgs and Fermion-Higgs Interaction

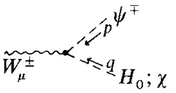
$\psi^\pm,\chi$ denote the unphysical Higgs states, $\phi^\pm$ and H_0, H_1, H_2 the charged and neutral physical states. Charges are always understood as incoming.



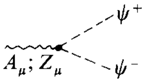
$$\left\{-i\frac{e}{s_W};\ -i\frac{e}{s_W c_W^2}\right\}M_W g_{\mu\nu}$$



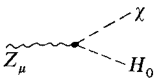
$$\left\{-ie;\ -ie\frac{s_W}{c_W}\right\}M_W g_{\mu\nu}$$



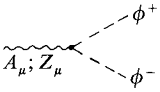
$$\left\{\pm i\frac{e}{2s_W};\ \frac{e}{2s_W}\right\}(p-q)_\mu$$



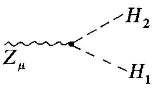
$$\left\{-ie;\ -ie\frac{s_W^2-c_W^2}{2c_W s_W}\right\}(p-q)_\mu$$



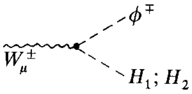
$$-\frac{e}{2c_W s_W}(p-q)_\mu$$



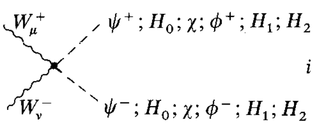
$$\left\{-ie;\ -ie\frac{s_W^2-c_W^2}{2c_W s_W}\right\}(p-q)_\mu$$



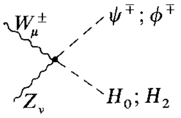
$$-\frac{e}{2c_W s_W}(p-q)_\mu$$



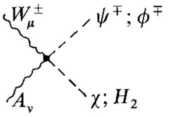
$$\left\{\pm i\frac{e}{2s_W};\ \frac{e}{2s_W}\right\}(p-q)_\mu$$



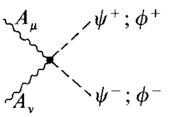
$$i\frac{e^2}{2s_W^2}g_{\mu\nu}$$



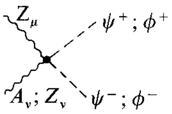
$$i\frac{e^2}{2c_W}g_{\mu\nu}$$



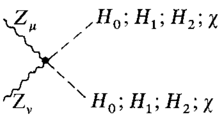
$$\pm\frac{e^2}{2s_W}g_{\mu\nu}$$



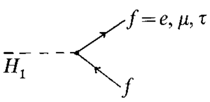
$$i2e^2 g_{\mu\nu}$$



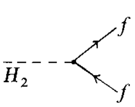
$$\left\{ie^2\frac{s_W^2-c_W^2}{s_W c_W};\ ie^2\frac{(s_W^2-c_W^2)^2}{2s_W^2 c_W^2}\right\}g_{\mu\nu}$$



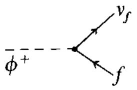
$$i\frac{e^2}{2s_W^2 c_W^2}g_{\mu\nu}$$



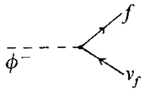
$$-i\frac{e}{2s_W}\cdot\frac{\beta m_f}{M_W}$$



$$-\frac{e}{2s_W}\cdot\frac{\beta m_f}{M_W}\gamma_5$$



$$i\frac{e}{s_W\sqrt{2}}\cdot\frac{\beta m_f}{M_W}\cdot\frac{1+\gamma_5}{2}$$



$$i\frac{e}{s_W\sqrt{2}}\cdot\frac{\beta m_f}{M_W}\cdot\frac{1-\gamma_5}{2}$$

Loop Integration: $\int\frac{d^D q}{(2\pi)^D}$

The matrix element $\mathcal{M}$ for $a+b\to 1+\ldots+N$ obtained by these rules is related to the differential cross section in the following way:

$$\sigma=\frac{(2\pi)^4\,\delta^4(p_1+\ldots+p_N-p_a-p_b)}{4\sqrt{(p_a\cdot p_b)^2-m_a^2 m_b^2}}|\mathcal{M}|^2 \cdot\prod_{i=1}^{N}\frac{d^3 p_i}{(2\pi)^3\,2p_i^0}.$$

References

1. G. Arnison et al.: Phys. Lett. **126B**, 398 (1983); **129B**, 273 (1983); P. Bagnaia et al.: Phys. Lett. **129B**, 130 (1983)
2. G.C. Branco, A.J. Buras, J.M. Gerard: MPI-PAE/PTh 4/85; A.J. Buras: Proceedings of EPS meeting, Bari 1985, and MPI-PAE/PTh 64/85
3. R.D. Peccei, H.R. Quinn: Phys. Rev. Lett. **38**, 1440 (1977); Phys. Rev. **D16**, 1791 (1977)
4. H.E. Haber, G.L. Kane: Phys. Rep. **117**, 75 (1985)
5. D. Toussaint: Phys. Rev. **D18**, 1626 (1978)
6. H.E. Haber, G.L. Kane, T. Sterling: Nucl. Phys. **B161**, 493 (1979)
7. R.A. Flores, M. Sher: Ann. Phys. (N.Y.) **148**, 95 (1983)
8. L.F. Abbott, P. Sikivie, M.B. Wise: Phys. Rev. **D21**, 1393 (1980)
9. M. Sher, D. Silverman: Phys. Rev. **D31**, 95 (1985)
10. S. Yamada: Proc. of Int. Symp. on Lepton and Photon Interactions at High Energies, Cornell University, Ithaca 595 (1983); S. Komamiya: Int. Symp. on Lepton and Photon Interactions, Kyoto 1985, Heidelberg preprint HD-PY 86/01
11. CELLO Coll. H.J. Behrend et al.: DESY 85-061; JADE Coll. W. Bartel et al.: Phys. Lett. **155B**, 288 (1985)
12. M. Böhm, W. Hollik, H. Spiesberger: DESY 84-027 (Progr. Phys., to be published)
13. M. Böhm, W. Hollik, H. Spiesberger: Z. Phys. C - Particles and Fields **27**, 523 (1985)
14. S. Bertolini: Preprint New York University, NYU/TR 10/85
15. A. Sirlin: Phys. Rev. **D22**, 971 (1980)
16. E. Radermacher: Progr. Part. Nucl. Phys. **14**, 231 (1985)
17. B. Naroska: DESY 85-090
18. D. Yu. Bardin, V.A. Dokuchaeva: Nucl. Phys. **B246**, 221 (1984)
19. J. Panman: First Aspen Winter Conference Series 1985, CERN-EP/85-35
20. M. Böhm, W. Hollik: Phys. Lett. **139B**, 213 (1984)
21. W. Hollik: Phys. Lett. **152B**, 121 (1985); B.W. Lynn, R. G. Stuart: Nucl. Phys. **B253**, 216 (1985)
22. E. Fernandez et al.: Phys. Rev. Lett. **54**, 1620 (1985); M.E. Levi et al.: Phys. Rev. Lett. **51**, 1941 (1983)
23. G.G. Athanasiu, P.J. Franzini, F.J. Gilman: SLAC-PUB 3648 (1985)