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CONSTRAINTS ON VARIANT AXION MODELS

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After describing the general structure of variant axion models, we examine the theoretical predictions for, and the experimental bounds on, weak decay processes and nuclear de-excitations involving variant axions. Although no individual reaction alone can be used to rule out the existence of variant axions, we find that the recent bound on the decay, $\pi^+ \to a e^+ \nu_e$, in combination with a bound for a $\Delta T = 0$ transition in ^{10}B effectively exclude these excitations.

1. Introduction

The existence of a narrow positron line in the produced positron spectrum in heavy ion collisions at GSI [1], which appears to be correlated with an equally narrow electron line [2], has renewed interest in axions. In principle, if there existed an axion of mass near 1.7 MeV produced nearly at rest in the heavy ion collision, its dominant decay, $a \to e^+e^-$, would then provide both a positron peak and a correlated e^+e^- signal. There are a number of difficulties with this scenario. First of all, as has been pointed out by many authors [3], it is difficult to conceive of a dynamics which will produce axions nearly at rest and in sufficient quantity to fit the GSI observations. Secondly, very recent observations [4] appear to indicate two, not one, correlated e^+e^- signals whose origin, obviously, is difficult to reconcile with a single axion. Thirdly, the excitation observed at GSI cannot be a standard axion [5], since a standard axion with a mass as heavy as 1.7 MeV would have very enhanced couplings to either charm or bottom quarks and so would be in conflict with the existing bounds on $\psi \to \gamma a$ or $\Upsilon \to \gamma a$ [6].

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Although it is unclear whether one can invent axion models which can overcome the first two difficulties, it is possible to construct variant axion models in which axions with mass near 2 MeV can exist, without being in contradiction with the quarkonia bounds. These models are of a type first suggested by Bardeen and Tye [7] and recently rediscovered, motivated by the GSI phenomena, by Krauss and Wilczek [8] and by Peccei, Wu, and Yanagida [9]. In these variant axion models, the axion decays very rapidly into e^+e^- pairs (for the simplest model discussed in [8] and [9], $\tau(a \to e^+e^-) \approx 6 \times 10^{-13}$ sec). As a result most previous bounds on axions are irrelevant for variant axions [10]. This is true for most beam dump experiments done in the past, as well as for the ^{12}C de-excitation experiment of Calaprice et al. [11], which are sensitive only to relatively long lived axions. It is clearly important, therefore, to ascertain if these excitations really could have escaped detection up to now, irrespective of whether variant axions have anything to do with the effects seen at GSI.

The purpose of this paper is to examine the question of the viability of variant axion models in detail. After briefly discussing the structure of variant axion models in sect. 2, we examine in sect. 3 what bounds exist on variant axions from weak decay processes, notably $\pi^+ \to ae^+\nu_e$ and $K^+ \to a\pi^+$. To calculate these processes, we make use of an effective lagrangian technique in sect. 4 which incorporates correctly all the (approximate) symmetries present at the quark level. This section, which is the core of our paper, also serves to clarify certain important issues relating to the structure of axion interactions. In sect. 5 we discuss the expectation of variant axion models for nuclear de-excitation processes. There we comment particularly on the implications of a recent experiment involving ^{14}N [12] and on the reanalysis performed by the Princeton group [13] of the old nuclear internal pair correlation experiments of Warburton et al. [14]. Our conclusions, which cast serious doubt on the existence of variant axions, are given in sect. 6.

2. Variant axion models

In the $SU(2) \otimes U(1)$ electroweak theory, the Yukawa interactions between fermions and doublet Higgs fields are invariant under an additional global U(1) symmetry, provided one has at least two doublet Higgs fields [15]. Such a symmetry, when imposed also on the purely Higgs sector, allows one to solve the strong *CP* puzzle, since one can show that the effective *CP* violating parameter

$$\bar{\theta} = \theta + \arg\det M \tag{2.1}$$

vanishes [15]. If one has a theory where this additional PQ symmetry exists, then the breakdown of $SU(2) \otimes U(1)$ caused by the nonvanishing expectation values of the doublet Higgs fields also causes the extra global $U_{PQ}(1)$ to break down. The associated Goldstone boson is the axion. However, because the $U_{PQ}(1)$ symmetry is anomalous in the presence of the strong interaction, this excitation acquires a small mass.

The standard axion model [5] has precisely two Higgs doublets: Φ_1 and Φ_2. The model is constructed so that, automatically, there are no Higgs induced flavor changing neutral currents (FCNC). This requires that Φ_1 couple only to the right-handed charge $-\frac{2}{3}$ quark fields and Φ_2 couple only to the right-handed charge $-\frac{1}{3}$ quark fields. If i, j are family indices and we let Q_{Li} stand for the left-handed quark doublets, then the standard axion couplings are given by

$$\mathscr{L}^{\text{Yukawa}}(\text{std. axion}) = \Gamma^{\text{u}}{}_{ij}\bar{Q}_{\text{L}i}\Phi_1 u_{\text{R}j} + \Gamma^{\text{d}}{}_{ij}\bar{Q}_{\text{L}i}\Phi_2 d_{\text{R}j} + \text{h.c.} \tag{2.2}$$

Diagonalization of the quark mass matrices will automatically diagonalize the Higgs couplings. However, the above structure also implies that quarks of the same charge are treated in an identical fashion. Furthermore, all that distinguished charge $\frac{2}{3}$ from charge $-\frac{1}{3}$ couplings, apart from quark mass factors, is the ratio of the doublet vacuum expectation values:

$$x = \langle\Phi_2\rangle / \langle\Phi_1\rangle . \tag{2.3}$$

To get an axion mass as large as 2 MeV, it is necessary that x (or x^{-1}) be large [7]. This necessarily implies, therefore, that one has enhanced couplings to all the charge $-\frac{2}{3}$ quarks (or all the charge $-\frac{1}{3}$ quarks) and one runs into trouble with the quarkonia bounds [6].

Variant axion models [8, 9], to avoid the quarkonia bounds, must de-enhance the coupling of axions to both c and b quarks. Thus these models will not automatically prevent the appearance of Higgs induced FCNC. Retaining only two Higgs doublets [9], it is not possible to avoid altogether these interactions but one can minimize the effects by restricting them to the charm sector. If one is willing to complicate the Higgs sector sufficiently and impose certain discrete symmetries, one can construct models [8] where no FCNC occur at all. At any rate, the important property of variant axion models is that the axion has couplings to quarks which, besides the usual mass factor, can *differ* for quarks of the same charge. For example, one can enhance the coupling of axions to the u quark and de-enhance the coupling of axions to all other quarks. Indeed, this is precisely the situation for the simplest variant axion model considered both in [8] and [9].

For simplicity, here we shall consider variant axion models with only two Higgs fields, Φ_1 and Φ_2. Furthermore, to avoid the FCNC problems in the charge $-\frac{1}{3}$ sector, we shall couple all charge $-\frac{1}{3}$ right-handed fields to Φ_2 [9]. Then the various different axion models are characterized by the number N of charge $-\frac{2}{3}$ right-handed fields which are coupled to Φ_1 and they depend further on whether u_{R} couples or not to Φ_1. Thus the variant axion couplings are given by

$$\mathscr{L}^{\text{Yukawa}}(\text{var. axion}) = \Gamma^{\text{u}}{}_{ij}\bar{Q}_{\text{L}i}\Phi_j u_{\text{R}j} + \Gamma^{\text{d}}{}_{ij}\bar{Q}_{\text{L}i}\Phi_2 d_{\text{R}j} + \text{h.c.} \tag{2.4}$$

TABLE 1
Variant axion model assignments for three families

(1) Φ_j:	$\Phi_1, \tilde{\Phi}_2, \tilde{\Phi}_2$
(2) Φ_j:	$\Phi_1, \tilde{\Phi}_2, \Phi_1$
(3) Φ_j:	$\tilde{\Phi}_2, \tilde{\Phi}_2, \Phi_1$

and the different models are distinguished by what Higgs field Φ_j couples to $u_{\mathrm{R}j}$. To avoid the quarkonia problem however, one must always take the Higgs field coupled to c_{R} as $\tilde{\Phi}_2 = i\tau_2\Phi_2^*$. For three families of fermions, there are three possible variant axion models with the assignments detailed in table 1 for Φ_j.

It is convenient to isolate the axion field in Φ_1 and Φ_2 as an overall phase field, dropping the other quantum excitations. In the zero charge sector, the axion is orthogonal to the excitation that eventually gets eaten by the Z^0 and it is easy to see [7] that one should write

$$\Phi_1 = \sqrt{\tfrac{1}{2}}\, f_1 e^{iax/f}\begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$\Phi_2 = \sqrt{\tfrac{1}{2}}\, f_2 e^{ia/xf}\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \tag{2.5}$$

where $x = f_2/f_1$ is the ratio of the Higgs vacuum expectation values and f is the scale of the breakdown of the weak interaction symmetries,

$$f = \sqrt{f_1^2 + f_2^2} = \left(\sqrt{2}\, G_{\mathrm{F}}\right)^{-1/2} \approx 250 \text{ GeV}. \tag{2.6}$$

Under a PQ symmetry transformation the axion field should just translate,

$$a \to a + \xi f. \tag{2.7}$$

A particularly convenient definition of the PQ symmetry in the quark sector is one where $Q_{\mathrm{L}i}$ is left invariant and the right-handed quarks fields transform so as to insure that (2.4) is left invariant. Hence under a PQ transformation,

$$d_{\mathrm{R}j} \to e^{-i\xi/x} d_{\mathrm{R}j},$$

$$u_{\mathrm{R}j} \to e^{-i\xi z_j} u_{\mathrm{R}j}, \tag{2.8}$$

where $z_j = x$ if the corresponding Higgs field in (2.4) is $\Phi_j = \Phi_1$, but $z_j = -1/x$ if the corresponding Higgs field is $\Phi_j = \tilde{\Phi}_2$. The PQ symmetry current for the variant axion models is therefore

$$J_\mu^{\mathrm{PQ}} = f\partial_\mu a + (1/x)\Sigma_1^{N_f}\bar{d}_{\mathrm{R}i}\gamma_\mu d_{\mathrm{R}i} + x\Sigma_1^{N}\bar{u}_{\mathrm{R}i}\gamma_\mu u_{\mathrm{R}i}$$
$$+(-1/x)\Sigma_{N+1}^{N_f}\bar{u}_{\mathrm{R}i}\gamma_\mu u_{\mathrm{R}i}, \tag{2.9}$$

where N_f is the number of families and N is the number of charge $-\frac{2}{3}$ quarks coupled to Φ_1. Clearly this current has a color anomaly [16] which is only proportional to N

$$\partial^\mu J_\mu^{\mathrm{PQ}} = \tfrac{1}{2}N(x+1/x)(\alpha_s/4\pi)F^a_{\mu\nu}\tilde{F}^{a\mu\nu}. \tag{2.10}$$

Obviously, besides N, it is important for the physics of the model in the light quark sector to know whether the u quark couples proportional to x or $1/x$ in eq. (2.9).

Besides J_μ^{PQ}, it proves useful to define another current, $\tilde{J}_\mu$, which is anomaly free and contains the axion. In principle, an infinity of such currents exist. However, the only interesting such currents, as Bardeen and Tye have emphasized, are ones which have a soft divergence. That is, a divergence which vanishes in the limit as the light quark masses vanish. For our purposes it will suffice to consider here the case in which only the u and the d quarks are considered as light. Then it is easy to see that [7]

$$\tilde{J}_\mu = J_\mu^{\mathrm{PQ}} - \tfrac{1}{2}N(x+1/x)\left\{(m_{\mathrm{d}}/(m_{\mathrm{u}}+m_{\mathrm{d}}))\bar{u}\gamma_\mu\gamma_5 u + (m_{\mathrm{u}}/(m_{\mathrm{u}}+m_{\mathrm{d}}))\bar{d}\gamma_\mu\gamma_5 d\right\} \tag{2.11}$$

has precisely the desired property. Its divergence

$$\partial^\mu\tilde{J}_\mu = -N(x+1/x)(m_{\mathrm{u}}m_{\mathrm{d}}/(m_{\mathrm{u}}+m_{\mathrm{d}}))\left\{\bar{u}\left[i\gamma_5 \mathrm{e}^{iaz\gamma_5/f}\right]u + \bar{d}\left[i\gamma_5\mathrm{e}^{ia\gamma_5/xf}\right]d\right\} \tag{2.12}$$

vanishes as either m_{u} or m_{d} goes to zero. Here we have neglected the effects of axion mixing angles related to possible flavor changing neutral currents as they are constrained to be small from the analysis of charm decays [9].

For what follows, it is convenient to explicitly indicate the axial-vector current content of $\tilde{J}_\mu$. Let us define, as usual, the isoscalar and isovector axial currents as

$$A_{s\mu} = \tfrac{1}{2}\left[\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d\right],$$
$$A_{3\mu} = \tfrac{1}{2}\left[\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d\right]. \tag{2.13}$$

Then the axial piece of the current $\tilde{J}_\mu$ can be written as

$$\tilde{J}_\mu = f\partial_\mu a + \lambda_s A_{s\mu} + \lambda_3 A_{3\mu} + \text{heavy quark pieces}. \tag{2.14}$$

The constants, λ_s and λ_3, are model dependent and read

$$\lambda_s = \tfrac{1}{2}\{(z + 1/x) - N(x + 1/x)\},$$

$$\lambda_3 = \tfrac{1}{2}\{(z - 1/x) - N(x + 1/x)[(m_d - m_u)/(m_d + m_u)]\}. \tag{2.15}$$

Here, $z = x$, if the u_R quark in eq. (2.4) couples to Φ_1 or $z = -1/x$, if the u_R quark in eq. (2.4) couples to $\tilde{\Phi}_2$. Given that x will turn out to be large, for the axion to have a mass near 2 MeV, and that the light quark masses give [17]

$$[(m_d - m_u)/(m_d + m_u)] \approx 0.26, \tag{2.16}$$

it is possible to have models in which λ_s vanishes or λ_3 vanishes, but both can not vanish simultaneously. This remark will have important phenomenological consequences.

3. Experimental bounds on variant axions from weak decays

The decay $K^+ \to a\pi^+$ provides a strong constraint on the standard axion model. For $m_a < 2m_e$, the axion can only decay into two photons and its lifetime is very long, of order $(100\ \text{keV}/m_a)^5$ sec for the standard axion [7]. In these circumstances, the axion just gives, experimentally, a missing energy signal (a = nothing). The most stringent bound for these axions was obtained by KEK [18] with

$$B(K^+ \to \pi^+ + \text{nothing}) < 2.7 \times 10^{-8}. \tag{3.1}$$

Although, as we shall see, it is difficult to reliably compute the nonleptonic process $K^+ \to a\pi^+$, for the case of the standard axion one has a penguin contribution, which gives a relatively safe estimate [19]

$$B^{\text{penguin}}(K^+ \to a\pi^+) \approx 10^{-6} \times x^2. \tag{3.2}$$

Hence x must be small to survive (3.1). However, an $x \approx 10^{-1}$ would then lead one into contradiction with the $\Upsilon \to a\gamma$ bound [6]. So a combination of the K decay bound and the Υ decay bound rules out the standard axion.

For variant axions, since $m_a > 2m_e$, the main decay channel for the axion is now into e^+e^-. Furthermore, the lifetime of the axion is now very short, and it is no longer true that experimentally the axions give a missing energy signal. If $\tau_a \approx 6 \times 10^{-13}$ sec, as is the case for the models of refs. [8] and [9], then the decay distance for the KEK experiment would be around 2 cm. The produced e^+e^- pairs would have been vetoed in the setup of ref. [18], so that the bound in (3.1) is irrelevant for the variant axions. To the best of our knowledge, there is no relevant bound on the process $K^+ \to a\pi^+$ with $a \to e^+e^-$, as yet! A Berkeley experiment of a decade ago

[20], which measured the process $K^+ \to \pi^+ e^+ e^-$ and found

$$B(K^+ \to \pi^+ e^+ e^-) = (2.7 \pm 0.5) \times 10^{-7} \tag{3.3}$$

could in principle provide some information, if reanalyzed. However, the published data has a cut for $m_{e^+e^-} > 140$ MeV. Similarly, the experiment of Yamazaki et al. [21] at KEK, which obtained a bound

$$B(K^+ \to \pi^+ \text{anything}) < 2 \times 10^{-6} \tag{3.4}$$

also had a cut on the recoil mass of $m_{\text{anything}} > 5$ MeV. However, we understand [22] that experiments at BNL and KEK should in the near future be able to establish limits for the process $K^+ \to a\pi^+$ with $a \to e^+e^-$ at the branching ratio level of 10^{-6}–10^{-7}. Our theoretical estimates, to be discussed in the next section, should be compared, therefore, with branching ratios of this order of magnitude.

Suzuki [23] has pointed out another weak decay process which is of importance for variant axions: $\pi^+ \to ae^+\nu_e$ followed by $a \to e^+e^-$. A rather stringent bound for the process $\pi^+ \to e^+e^-e^+\nu_e$

$$B(\pi \to e^+e^-e^+\nu_e) < 5 \times 10^{-9} \tag{3.5}$$

was established a decade ago at Dubna [24]. From a reanalysis of this experiment, it would be possible to infer a bound on the $\pi^+ \to ae^+\nu_e$ decay, but this bound would be strongly dependent on the axion lifetime. Fortunately, very recently, in an elegant experiment at SIN, the process $\pi^+ \to e^+e^-e^+\nu_e$ has actually been seen [25]. The observed branching ratio

$$B(\pi \to e^+e^-e^+\nu_e) = (3.4 \pm 0.5) \times 10^{-9} \tag{3.6}$$

is slightly below the bound of eq. (3.5) and is in agreement with standard expectations. Furthermore, an analysis of the e^+e^- invariant mass distribution can be performed to set a bound on the $\pi^+ \to ae^+\nu_e$ decay mode. This analysis has now been completed, giving a branching ratio bound of the order of [26]

$$B(\pi^+ \to ae^+\nu_e) < (1-2) \times 10^{-10}, \tag{3.7}$$

provided the axion lifetime is sufficiently short ($\tau_a < 10^{-11}$ sec). As we shall see in the next section, this bound is very restrictive for variant axion models.

4. Theoretical considerations on weak decays involving axions

Branching ratios for the processes $K^+ \to a\pi^+$ and $\pi^+ \to ae^+\nu_e$ can be estimated rather simply by using the fact that the axion, at some level, "mixes" slightly with the π^0 [27]. Let us consider, for instance, the decay $\pi^+ \to ae^+\nu_e$, which is somewhat simpler to calculate because it is a semileptonic process. To compute the decay rate for this process, one needs to know the matrix element of the charged current, $J_{-\mu}$, between an axion and a π^+ state. If we denote the mixing angle between the π^0 and the axion as $\xi_{\pi a}$ and proceed naively, then we expect the relations

$$\langle a|J_{-\mu}|\pi^+\rangle \approx \xi_{\pi a}\langle \pi^0|J_{-\mu}|\pi^+\rangle = \sqrt{2}\,\xi_{\pi a}(P_a + P_\pi)_\mu . \tag{4.1}$$

The second line follows, since only the f_+ form factor is non-vanishing for the pion matrix element. A simple calculation then gives a formula for the rate

$$\Gamma(\pi^+ \to ae^+\nu_e) = (384\pi^3)^{-1}(G_F)^2(m_\pi)^5(\xi_{\pi a})^2 . \tag{4.2}$$

The mixing angle $\xi_{\pi a}$ can be taken as the fraction of the isovector axial current $A_{3\mu}$ present in $\tilde{J}_\mu$, modified by the ratio of the pion-to-axion decay constants, f_π/f.

$$\xi_{\pi a} = \lambda_3(f_\pi/f) . \tag{4.3}$$

Using eq. (4.3) in eq. (4.2), one obtains a sizable branching ratio for the process $\pi^+ \to ae^+\nu_e$ in variant axion models. For instance, in the simplest variant model considered in refs. [8] and [9], one has $N = 1$ and $z = x \approx 70$, so that $\lambda_3 \approx 26$. This value implies a branching ratio

$$B_{\text{simplest}}(\pi^+ \to ae^+\nu_e) \approx 2 \times 10^{-6} , \tag{4.4}$$

which is four orders of magnitude above the SIN bound (3.7)!! Clearly, if the above estimate of the $J_{-\mu}$ matrix element (eq. (4.1)) and of the $\pi^0 - a$ mixing (eq. (4.3)) are correct, then the only tenable variant axion models are ones where the isovector mixing parameter λ_3 is suppressed by about two orders of magnitude below that found for the simplest case. Although such models exist, they require one to have $N = 4$ and so one needs more families than we presently know with PQ couplings, or they require some other mechanism for producing a large color anomaly in the Higgs sector.

Because the bound obtained from the process, $\pi^+ \to ae^+\nu_e$, is so strong, it is imperative to make sure that the above estimate for the branching ratio is not in error. We will see that, in fact, the result obtained above is correct*. However, it is important to analyze this process (and also the process, $K^+ \to a\pi^+$) with some care, since there are a number of questions which tend to cast some doubt on the simple minded treatment used to obtain the above bound. Two such questions immediately come to mind:

(i) If it were not for the axial anomaly, the axion would be a massless Goldstone boson. Therefore, the axion should decouple at zero momentum. So, why is the

* After completion of this work we received a paper by Krauss and Wise [39] where the result (4.4) is also obtained.

matrix element in eq. (4.1) not simply proportional to the axion momentum, p_a, only?

(ii) The current, $\tilde{J}_\mu$, which contains the physical axion and is anomaly free, has a divergence which is purely isoscalar (c.f. eq. (3.12)). How is it possible that there should be any communication between the low-energy coupling of the physical axion and the physical π^0? Doesn't the mixing angle, $\xi_{\pi a}$, actually vanish?

To answer these questions, one can systematically study the low-energy theorems associated with the current algebra of the axion, or alternatively one can give a general solution to the current algebra by constructing an effective lagrangian involving pions and axions (or π's, K's, and axions) which reflects all the symmetries present at the quark level. This effective lagrangian can be used to compute all the decay amplitudes. Such an approach was used by Bardeen and Tye [7] and by Kandaswamy, Salomonson, and Schechter [28] to compute standard axion properties and, more recently, by Georgi, Kaplan, and Randall to compute some properties of invisible axion models [29]. We will find it also to be very useful to examine the variant axion models.

We want to construct an effective lagrangian for pions and axions, including the effects of the weak interactions, which reproduces the low-energy dynamics of the standard model augmented by a PQ symmetry [15]. Before constructing the effective lagrangian, it is important to understand the full global symmetry structure at the quark level. The variant axion models discussed in sect. 2 are described by the following fermion-axion lagrangian,

$$\begin{aligned}\mathscr{L} = {} & \bar{Q}_L\{i\gamma D\}Q_L + \bar{L}_L\{i\gamma D\}L_L + \bar{u}_R\{i\gamma D\}u_R + \bar{d}_R\{i\gamma D\}d_R + \bar{e}_R\{i\gamma D\}e_R \\ & - \bar{u}_L\{M_u e^{iza/f}\}u_R + \text{h.c.} - \bar{d}_L\{M_d e^{ia/xf}\}d_R + \text{h.c.} \\ & - \bar{L}_L\{M_e e^{iza/f}\}e_R + \text{h.c.} + \tfrac{1}{2}(\partial_\mu a)^2, \end{aligned} \tag{4.5}$$

where $\{D_\mu\}$ are the covariant derivatives for the $SU(3)_c \otimes SU(2) \otimes U(1)$ gauge interactions of the standard model and M_u, M_d, and M_e are fermion mass matrices with the family indices being suppressed. The axion couplings are model dependent with the elements of the matrices, z, being x or $-1/x$ for up quarks and $1/x$ or $-x$ for leptons. The axion couplings are clearly directly related to the structure of the right-handed fermions.

The physical structure of this lagrangian may be examined if we first diagonalize the mass matrices for the fermions. The mass matrices for the up and down quarks can be put in the form,

$$\begin{aligned} M_u &= B_u m_u C_u^+ , \\ M_d &= B_d m_d C_d^+ , \end{aligned} \tag{4.6}$$

where m_u and m_d are the diagonal quark mass matrices and B_k and C_k are the necessary rotations in flavor space. We diagonalize these matrices by making the following transformations on the quark fields,

$$u_R \to C_u u_R, \qquad u_L \to B_u u_L,$$
$$d_R \to C_d d_R, \qquad d_L \to B_d d_L. \tag{4.7}$$

In the standard axion model, all reference to the mixing matrices disappears from the Yukawa terms and the only physical dependence on these matrices is in the combination $B_u^+ B_d$ which is just the KM matrix for the $W^\pm$ interactions. In variant axion models, more of the mixing angles become physical as the Yukawa interactions are not independent of C_u. After diagonalization, we have

$$\mathscr{L}^{\text{Yukawa}} = -\bar{u}_L\{m_u C_u^+ e^{iza/f} C_u\} u_R + \text{h.c.} - \bar{d}_L\{m_d e^{ia/xf}\} d_R + \text{h.c.}$$
$$-\bar{L}_L\{m_e e^{iza/f}\} e_R + \text{h.c.}, \tag{4.8}$$

where we have presumed the lepton mass matrix to be diagonal. The physical mixing angles in C_u are responsible for FCNC interactions and are strongly constrained by the charm decays [9]. We will ignore their effects in our subsequent discussion.

The relevant symmetries can be seen by examining the lagrangian in (4.5). As in eq. (2.9), the PQ current may be written in terms of the right-handed fermion currents

$$J_\mu^{PQ} = f\partial_\mu a + \bar{u}_R\{z\gamma_\mu\} u_R + \bar{d}_R\{x^{-1}\gamma_\mu\} d_R + \bar{e}_R\{z\gamma_\mu\} e_R. \tag{4.9}$$

The PQ symmetry [15] is spontaneously broken at the same time as the SU(2) ⊗ U(1) weak symmetry breaking by the non-zero vacuum expectation value of the Higgs fields, $\langle \Phi_1 \rangle$ and $\langle \Phi_2 \rangle$. The concomitant Goldstone boson is massless in the absence of the non-perturbative QCD interactions. The axion lagrangian (4.5) has additional right-handed symmetries if some of the quarks may be considered as massless. Taking the first family of quarks to be massless, we have the freedom to rotate independently the corresponding u_R and the d_R fields

$$u_R \to e^{i\alpha} u_R,$$
$$d_R \to e^{i\beta} d_R, \tag{4.10}$$

and the currents which correspond to these symmetries may be written as isoscalar and isovector currents as in (2.3)

$$A_{s\mu} = \bar{u}_R \gamma_\mu u_R + \bar{d}_R \gamma_\mu d_R,$$
$$A_{3\mu} = \bar{u}_R \gamma_\mu u_R - \bar{d}_R \gamma_\mu d_R. \tag{4.11}$$

The effects of including the strong interactions are twofold. First, non-trivial condensates of the light quarks form in the physical vacuum of QCD,

$$\langle \bar{u}u \rangle \approx \langle \bar{d}d \rangle \neq 0 . \tag{4.12}$$

These condensates induce a spontaneous breaking of the global symmetries (4.10), producing (apparently) two additional Goldstone bosons in the neutral charge sector, the π^0 and an isosinglet excitation we shall call φ^0. Second, when the full strong interactions are included, the isoscalar current is no longer a good symmetry current. Indeed, this current has an Adler-Bell-Jackiw anomaly [16] associated with the color gauge fields,

$$\partial^\mu A_{s\mu} = (\alpha_s/4\pi) F^a{}_{\mu\nu} \tilde{F}^{a\mu\nu} . \tag{4.13}$$

This anomaly combined with the QCD vacuum structure implies there is no symmetry reason for the φ^0 meson to remain massless. In fact the situation is more complicated due to the presence of the PQ symmetry. The global PQ symmetry is also broken by the same anomaly as that given in (2.10). However, the strong anomaly cannot break independently both symmetries, and a linear combination of the PQ current and the isoscalar quark current remains conserved. It is easy to see that the current

$$J^*{}_\mu = J_\mu^{\mathrm{PQ}} - \tfrac{1}{2} N (x + 1/x) A_{s\mu} \tag{4.14}$$

does not have a strong anomaly and is conserved along with the isovector current in the limit that the light quarks remain massless. Hence, we expect to have only two true Goldstone bosons in the symmetry limit, the physical π^0 and the axion. The presence of mass terms for the light quarks breaks both of these remaining symmetries. However, this symmetry breaking is much weaker than the breaking caused by the strong anomaly, $(m_{\eta'} \gg m_{\pi^0})$. Hence, the mixing between the pion and the axion can be determined by studying the chiral limit, m_u, $m_d \to 0$, as emphasized in [7]. The interplay between these three symmetries will be evident in our formulation of the effective lagrangian to be discussed below.

The structure of the axion couplings can be explicitly exhibited at the quark level by making a local, right-handed gauge transformation to remove the axion field from the Yukawa interactions. This transformation is accomplished by rotating the quark and lepton fields,'

$$\begin{aligned} u_R &\to e^{-iza/f} u_R , \\ d_R &\to e^{-ia/xf} d_R , \\ e_R &\to e^{-iza/f} e_R . \end{aligned} \tag{4.15}$$

In making this transformation we must be careful to account for the anomaly structure of the fermions. The naive transformation removes the axion field from Yukawa interactions and generates derivative interaction from the kinetic terms for the fermions. The anomalies produce additional, non-derivative interactions which can be computed from the known anomaly structure of the fermion loops [16]. We obtain the following lagrangian equivalent to (4.5) ignoring the right-handed mixing angles associated with FCNC,

$$\begin{aligned}\mathscr{L}= & \bar{Q}_{\mathrm{L}}\{i\gamma D\}Q_{\mathrm{L}}+\bar{L}_{\mathrm{L}}\{i\gamma D\}L_{\mathrm{L}}+\bar{u}_{\mathrm{R}}\{i\gamma D\}u_{\mathrm{R}}+\bar{d}_{\mathrm{R}}\{i\gamma D\}d_{\mathrm{R}}+\bar{e}_{\mathrm{R}}\{i\gamma D\}e_{\mathrm{R}}\\ & -\bar{u}_{\mathrm{L}}\{m_{\mathrm{u}}\}u_{\mathrm{R}}+\mathrm{h.c.}-\bar{d}_{\mathrm{L}}\{m_{\mathrm{d}}\}d_{\mathrm{R}}+\mathrm{h.c.}-e_{\mathrm{L}}\{m_{\mathrm{e}}\}e_{\mathrm{R}}+\mathrm{h.c.}\\ & +f^{-1}\bar{u}_{\mathrm{R}}\{z\gamma^{\mu}\}u_{\mathrm{R}}\,\partial_{\mu}a+(xf)^{-1}\bar{d}_{\mathrm{R}}\{\gamma^{\mu}\}d_{\mathrm{R}}\,\partial_{\mu}a\\ & +f^{-1}\bar{e}_{\mathrm{R}}\{z\gamma^{\mu}\}e_{\mathrm{R}}\,\partial_{\mu}a+\tfrac{1}{2}(\partial_{\mu}a)^{2}\\ & +a\{\mathrm{tr}_{\mathrm{u}}[z/f]+\mathrm{tr}_{\mathrm{d}}[1/xf]\}(\alpha_{\mathrm{s}}/8\pi)\{F^{a}{}_{\mu\nu}\tilde{F}^{a\mu\nu}\}\\ & +a\{\tfrac{4}{3}\mathrm{tr}_{\mathrm{u}}[z/f]+\tfrac{1}{3}\mathrm{tr}_{\mathrm{d}}[1/xf]+\mathrm{tr}_{\mathrm{e}}[z/f]\}(16\pi^{2})^{-1}\{B_{\mu\nu}\tilde{B}^{\mu\nu}\},\end{aligned}\tag{4.16}$$

where $F^{a}{}_{\mu\nu}$ is the color gluon field strength and $B_{\mu\nu}$ is the field strength of the U(1) weak gauge field coupled to the right-handed fermions,

$$B_{\mu\nu}=e\left(F_{\gamma}{}^{\mu\nu}-(g'/g)F_{Z}{}^{\mu\nu}\right).\tag{4.17}$$

For the standard axion, it is this last term which is used to compute the decay of the axion to two photons. From the form of the lagrangian in (4.16), we see that the axion has only derivative coupling to hadrons except for the anomaly coupling to gluons which will obviously generate only flavor singlet interactions. We will see that it is, in fact, the derivative interactions which are responsible for the mixing with the pion and give the strong constraints from pion and kaon decay. We also remark that the lagrangian in (4.16) can be used to demonstrate the decoupling of the heavy quarks as their derivative interactions with the axion can only generate a small renormalization of the kinetic energy of axion, or terms which are highly suppressed by powers of the heavy quark mass. The real effect of the heavy quarks on the low-energy theory only comes through the contribution of the anomaly.

We now return to the formulation of our effective lagrangian. This lagrangian represents the full interactions of the axions and mesons as expanded to lowest order in the meson momenta or masses. The effective lagrangian contains three separate pieces. There is a chiral lagrangian term describing the U(2) ⊗ U(2) invariant strong interactions of the π and φ^0 fields plus a kinetic term for the axion,

$$\mathscr{L}_{\text{chiral}} = \tfrac{1}{4}(f_\pi)^2 \text{tr}\{\partial_\mu U^\dagger \partial^\mu U\} + \tfrac{1}{2}\partial_\mu a\, \partial^\mu a\,, \tag{4.18}$$

where the chiral field U is given by

$$U = \exp\{i(\boldsymbol{\tau}\cdot\boldsymbol{\pi} + \varphi^0)/f_\pi\}\,. \tag{4.19}$$

Clearly eq. (4.18) is invariant under global U(2) ⊗ U(2) transformations,

$$U \to g_{\text{L}} U g_{\text{R}}^\dagger \tag{4.20}$$

and under a global translation of the axion field,

$$a \to a + \xi f\,. \tag{4.21}$$

The electroweak interactions can be introduced into eq. (4.18) by replacing the derivatives by the appropriate covariant derivatives. According to our discussion at the quark level, even after this is done, the theory should still be invariant under three chiral U(1) symmetries. It is easy to check that the substitution

$$\partial_\mu U \to D_\mu U = \partial_\mu U + i\tfrac{1}{2}g\boldsymbol{\tau}\cdot \boldsymbol{W}_\mu U + i\tfrac{1}{6}g' Y_\mu U + ig' Y_\mu U\begin{bmatrix} -\frac{2}{3} & 0 \\ 0 & \frac{1}{3}\end{bmatrix}, \tag{4.22}$$

which introduces the electroweak interactions for the U field, still preserves the $(\text{U}(1))^3$ symmetry in the effective lagrangian,

$$\mathscr{L}_{\text{chiral}+\text{Wl}} = \tfrac{1}{4}(f_\pi)^2 \text{tr}\{(D_\mu U)^\dagger (D^\mu U)\} + \tfrac{1}{2}\,\partial_\mu a\,\partial^\mu a\,. \tag{4.23}$$

That is, eq. (4.23) is still invariant under the restricted set of transformations (4.20) where

$$g_{\text{R}} = \begin{bmatrix} e^{i\alpha} & 0 \\ 0 & e^{i\beta}\end{bmatrix}, \qquad g_{\text{L}} = 1\,, \tag{4.24}$$

and is obviously also invariant under (4.21).

In addition to (4.23), the effective lagrangian must contain terms which incorporate the effects of the chiral anomalies and terms which reflect the Yukawa interactions at the quark level. Let us look at this last term first. For the models under consideration, the interaction of eq. (2.4) for the light quark sector reads effectively,

$$\mathscr{L}_{\text{mass}} = -m_{\text{u}}(\bar{u}_{\text{L}} e^{iaz/f} u_{\text{R}}) + \text{h.c.} - m_{\text{d}}(\bar{d}_{\text{L}} e^{ia/xf} d_{\text{R}}) + \text{h.c.}\,, \tag{4.25}$$

where $z = x$ or $-1/x$ depending on the particular model considered. This interaction no longer preserves the two U(1) transformations of eq. (4.10), but is does preserve the PQ symmetry, provided that u_R and d_R respond appropriately (c.f. eq. (2.8)). Thus, we may include the effects of the Yukawa interactions in the effective lagrangian by adding a term which explicitly breaks the symmetry in an analogous way to (4.25). Since the U matrices are the unique, non-derivative fields which have the same chiral transformation properties as the quark mass operators, the Yukawa interactions are represented by

$$\mathscr{L}_{\text{mass breaking}} = \tfrac{1}{2}v\,\mathrm{tr}\{UAM + M^{\dagger}A^{\dagger}U^{\dagger}\}, \tag{4.26}$$

where

$$M = \begin{bmatrix} m_u & 0 \\ 0 & m_d \end{bmatrix}, \tag{4.27}$$

$$A = \begin{bmatrix} e^{-iza/f} & 0 \\ 0 & e^{-ia/xf} \end{bmatrix}. \tag{4.28}$$

The parameter v is related to the scale of the spontaneous chiral symmetry breaking. Clearly (4.26) is invariant under the PQ symmetry transformation,

$$a \to a + \xi f, \tag{4.29}$$

$$U \to U \cdot \begin{bmatrix} e^{iz\xi} & 0 \\ 0 & e^{i\xi/x} \end{bmatrix}. \tag{4.30}$$

This interaction, however, is not invariant under the transformations (4.24). Thus, two combinations of the Goldstone fields, a, φ^0, and π^0 will acquire masses from this term in the effective lagrangian. A linear combination of the neutral meson fields remains massless and is the Goldstone excitation associated with the naive PQ transformation.

The final piece to be added to the effective lagrangian is a term which incorporates the anomaly structure of the quark theory. For the heavy flavors, we have exhibited, in eq. (4.16), the axion anomalies which are induced by the quark theory. There are anomalies associated with both the weak and the strong gauge fields. For the processes we wish to consider, only the strong anomalies contribute and we will ignore the weak anomaly contributions. Using eq. (4.16), we obtain from the heavy quarks,

$$\begin{aligned} \mathscr{L}_{\text{anomaly}}(a) &= a\{\mathrm{tr}_u[z/f] + \mathrm{tr}_d[1/xf]\}_H(\alpha_s/8\pi)\{F^a{}_{\mu\nu}\tilde{F}^{a\mu\nu}\} \\ &= af^{-1}\{N_H(x + 1/x)\}(\alpha_s/8\pi)\{F^a{}_{\mu\nu}\tilde{F}^{a\mu\nu}\}, \end{aligned} \tag{4.31}$$

where N_H is the number of heavy quark families with PQ couplings, $z = x$. The effective lagrangian must also reflect the strong anomalies of the meson fields as indicated by the anomalous divergence of the isosinglet current in eq. (4.13). This anomaly may be determined in a manner similar to (4.31) with the result

$$\mathscr{L}_{\text{anomaly}}(\varphi^0) = \varphi^0(2/f_\pi)^{-1}(\alpha_s/8\pi)\{F^a{}_{\mu\nu}\tilde{F}^{a\mu\nu}\}. \tag{4.32}$$

The strong gauge fields may be integrated out with the effect that the strong anomaly contribution is effectively a mass term for the meson fields which multiply $F \cdot \tilde{F}$. For the case at hand, this procedure gives the anomaly term in the effective lagrangian

$$\mathscr{L}_{\text{anomaly}} = -\tfrac{1}{2}m_0^2\left[\varphi^0 + \tfrac{1}{2}(f_\pi/f)\{N_H(x+1/x)\}a\right]^2. \tag{4.33}$$

Since the mass parameter, m_0, must be large to produce the physical meson mass spectrum, the combination of fields appearing in (4.33) effectively decouples from the low-energy dynamics. The orthogonal combination of φ^0 and a

$$\tilde{a} = \left(fa - \tfrac{1}{2}f_\pi\{N_H(x+1/x)\}\varphi^0\right)/\tilde{f} \tag{4.34}$$

does not feel the effect of the strong anomaly. In the absence of the Yukawa interactions, but including the full weak interactions and the strong anomalies, both $\tilde{a}$ and π^0 – in the neutral sector – would be massless.

The physical meson states and the mixing parameters may be determined from the meson mass matrix which can be obtained by expanding the Yukawa interactions (4.26) to second order in the meson fields and adding the contribution of the strong anomalies (4.33). In the charged pion sector, we find

$$\mathscr{L}_{\text{mass}}(\text{charged}) = -(f_\pi)^{-2}(m_u + m_d)v\pi^+\pi^-, \tag{4.35}$$

which identifies the parameter, v, as

$$v = (f_\pi m_\pi)^2/(m_u + m_d). \tag{4.36}$$

The mass terms in the neutral sector read,

$$\begin{aligned}\mathscr{L}_{\text{mass}}(\text{neutral}) = &-\tfrac{1}{2}\left((f_\pi m_\pi)^2/(m_u+m_d)\right)\\ &\times\left\{m_u\left[\pi^0/f_\pi + \varphi^0/f_\pi - za/f\right]^2 + m_d\left[-\pi^0/f_\pi + \varphi^0/f_\pi - a/xf\right]^2\right\}\\ &-\tfrac{1}{2}m_0^2\left[\varphi^0 + \tfrac{1}{2}(f_\pi/f)\{N_H(x+1/x)\}a\right]^2\\ = &-\tfrac{1}{2}m_\pi^2(m_u/(m_u+m_d))\left[\pi^0+\varphi^0 - a(zf_\pi/f)\right]^2\\ &-\tfrac{1}{2}m_\pi^2(m_d/(m_u+m_d))\left[-\pi^0+\varphi^0 - a(f_\pi/xf)\right]^2\\ &-\tfrac{1}{2}m_0^2\left[\varphi^0 + \tfrac{1}{2}(f_\pi/f)\{N_H(x+1/x)\}a\right]^2.\end{aligned} \tag{4.37}$$

Since $m_0 \gg m_\pi$, the mass matrix can be easily diagonalized to give the axion mass,

$$m_a^2 = m_\pi^2 (f_\pi/f)^2 N^2 (x + 1/x)^2 \left(m_u m_d / (m_u + m_d)^2 \right) \tag{4.38}$$

and the axion mixing parameters,

$$\begin{aligned} \xi_{\pi a} &= \lambda_3 (f_\pi/f) \left[1 + m_a^2/m_\pi^2 \right], \\ \xi_{a\varphi} &= \lambda_s (f_\pi/f), \end{aligned} \tag{4.39}$$

where N is the total number of PQ families ($N = (N_H + 1)$ if $z = x$ and $N = N_H$ if $z = (-1/x)$) and λ_3 and λ_s are as given in eq. (2.15),

$$\begin{aligned} \lambda_3 &= \tfrac{1}{2} \{ (z - 1/x) - N(x + 1/x) \left[(m_d - m_u)/(m_d + m_u) \right] \}, \\ \lambda_s &= -\tfrac{1}{2} N_H (x + 1/x). \end{aligned} \tag{4.40}$$

These are essentially the results for the mass and mixing parameter as given by Bardeen and Tye [7]. The πa mixing parameter of eq. (4.39) is precisely that of eq. (4.3) apart from a tiny correction of order (m_a^2/m_π^2).

The principal strong and weak interactions of mesons are described by the interactions contained in the chiral lagrangian of eq. (4.23). The couplings involve only the π^0 and φ^0 fields and have no explicit dependence on the axion field. Therefore, the axion couplings are generated by the mixing with the meson fields as determined by the mixing parameters of (4.39) and the relations

$$\begin{aligned} \pi^0 &\approx \pi^0_{\text{phys}} + \xi_{a\pi} a_{\text{phys}}, \\ \varphi^0 &\approx \varphi^0_{\text{phys}} + \xi_{a\varphi} a_{\text{phys}}, \\ a &\approx a_{\text{phys}} - \xi_{a\pi} \pi^0_{\text{phys}} - \xi_{a\varphi} \varphi^0_{\text{phys}}. \end{aligned} \tag{4.41}$$

There will be corrections to the results obtained by this procedure of order (m_π^2/m_K^2). If we use the obvious generalization of this procedure to include the strange quark as one of the light quarks, then the predictions should then be good to order $(m_\pi^2/m_{\eta'}^2)$. The mixing with the π^0 is described with sufficient accuracy for our purposes by the calculation given above.

The calculation of the process $\pi^+ \to ae^+\nu_e$ is now straightforward using the effective lagrangian given in eq. (4.23) with definitions in (4.22). As we have discussed above, there is no direct coupling of the axion to the $W^\pm$ and the weak decay proceeds through the mixing with the π^0, as there is also no coupling for the

φ^0 in this amplitude. The mixing gives the following amplitude

$$A(\pi^+ \to \mathrm{ae}^+\nu_\mathrm{e}) = \xi_{\mathrm{a}\pi} A(\pi^+ \to \pi^0\mathrm{e}^+\nu_\mathrm{e})(p_{\pi^0} = p_\mathrm{a})$$

$$= \xi_{\mathrm{a}\pi} G_\mathrm{F}\left[(p_\pi + p_\mathrm{a})^\mu \bar{U}(p_\nu)\gamma_\mu(1-\gamma_5)V(p_\mathrm{e})\right]. \qquad (4.42)$$

Here, as usual G_F is the Fermi constant, $G_\mathrm{F} = g^2/8M_\mathrm{W}^2 = 1/\sqrt{2}f^2$. This amplitude gives the rate quoted in eq. (4.2).

We can now make some comments on aspects of the effective lagrangian solution to the current algebra. We first consider the role of the $\tilde{J}_\mu$ current for studying the properties of axions. It was constructed from the PQ current by using a particular combination of light quark currents which cancels the strong anomaly. It is dominated by the axion pole and its conservation implies a massless axion. The structure of the currents can be seen explicitly using the effective lagrangian to express them in terms of the meson currents. The PQ current becomes

$$J_\mu^{\mathrm{PQ}} = f\partial_\mu a + \tfrac{1}{2}\left[(z + 1/x)f_\pi\, \partial_\mu\varphi^0 + (z - 1/x)f_\pi\, \partial_\mu\pi^0\right]. \qquad (4.43)$$

This is just a transcription of eq. (2.9) in which the roles of the isoscalar and isovector currents are given by

$$A_{s\mu} = f_\pi\, \partial_\mu\varphi^0,$$

$$A_{3\mu} = f_\pi\, \partial_\mu\pi^0. \qquad (4.44)$$

Using these identifications the anomaly free, soft current $\tilde{J}_\mu$ of eq. (2.4) is simply

$$\tilde{J}_\mu = f\partial_\mu a + \lambda_s f_\pi\, \partial_\mu\varphi^0 + \lambda_3 f_\pi\, \partial_\mu\pi^0. \qquad (4.45)$$

Of course, any linear combination of the currents, $A_{3\mu}$ and $\tilde{J}_\mu$, is anomaly free, and both currents are conserved in the chiral limit $m_\mathrm{u}, m_\mathrm{d} \to 0$. However, when the chiral symmetry breaking from the Yukawa interaction is included, it is clear that $\tilde{J}_\mu$ is the axion current for two closely related reasons:

(i) The divergence of $\tilde{J}_\mu$ is soft, i.e. it vanishes in any of the symmetry limits for the axion, m_u or $m_\mathrm{d} \to 0$.

(ii) Expanding the $\tilde{J}_\mu$ current in terms of the physical fields, we see that it has essentially only an axion contribution.

$$\tilde{J}_\mu = f\partial_\mu a_{\mathrm{phys}} - (m_\mathrm{a}/m_\pi)^2 \lambda_3 f_\pi\, \partial_\mu\pi^0_{\mathrm{phys}}. \qquad (4.46)$$

The pion component is suppressed by the small axion mass.

This discussion hopefully clarifies an essential point raised at the beginning of this section. It is indeed true that the current, $\tilde{J}_\mu$, has an isoscalar divergence and that this current is dominated by the axion pole. This, however, does not mean that there is no πa mixing as described by the mixing parameter, $\xi_{a\pi}$. This mixing occurs for the chiral invariant interactions, while the properties of the divergence of the current relate to the interactions which involve symmetry breaking. The weak processes we are considering are all related to the chiral invariant couplings of the π^0 and the axion.

In view of the above discussion, there remains a small problem of principle to clarify connected with the first query raised in the beginning of the section: why is the pion decay amplitude (4.42) proportional to $(p_\pi + p_a)_\mu$ and not only to the axion momentum p_a as one might expect from the low-energy theorem associated with the almost Goldstone nature of the axion? We note that in the chiral limit, both the π^0 and the axion should be exact Goldstone bosons as the explicit weak interactions should not break the chiral symmetry. Hence, it should be sufficient to study these interactions at purely the pionic level.

To understand this point it is necessary to write out a bit more of the structure of the weak vertex for the pions as we have kept only the leading terms needed for our calculation. From the lagrangian given in (4.23), the full pion-W boson interactions are given by

$$\mathscr{L}_{\pi W} = \tfrac{1}{2} g f_\pi W^{-\mu} e^{i(\pi^0/f_\pi)} \left\{ \partial_\mu \pi^+ - i\pi^+ \partial_\mu \left(\pi^0/f_\pi\right)\right\}. \tag{4.47}$$

The non-derivative interaction term involving the π^0 appears as a phase which reflects the chiral structure of the left-handed current. This phase can be removed by a point transformation of the π^+ field

$$\pi^+ \to \pi^+ e^{-i(\pi^0/f_\pi)}. \tag{4.48}$$

With this transformation, the lagrangian really involves only the derivatives of the π^0 field

$$\mathscr{L}_{\pi W} = \tfrac{1}{2} W^{-\mu}\left[f_\pi \partial_\mu \pi^+ - 2i\pi^+ \partial_\mu \pi^0 \right]. \tag{4.49}$$

A similar transformation at the quark level in (4.16) was used to make all interactions of the axion into derivative coupling except for the anomaly terms which contribute effectively only to the meson mass terms. Of course, the transformation (4.48) also affects the purely strong interaction terms in the effective lagrangian (4.23), giving an additional term

$$\Delta\mathscr{L} = i\,\partial^\mu\left(\pi^0/f_\pi\right)\left(\pi^- \partial_\mu \pi^+ - \pi^+ \partial_\mu \pi^-\right). \tag{4.50}$$

One can check, explicitly, that (4.50) and (4.49) give the same physical amplitudes for W interactions as (4.47), as they should since the S-matrix elements are unaffected by piont transformations at tree level. Although the transformed lagrangian involves only $\partial_\mu \pi^0$, and therefore through mixing only $\partial_\mu a_{\rm phys}$, the presence of the trilinear coupling (4.50) gives an extra contribution to the amplitude for $\pi^+ \to ae^+\nu_e$, involving an intermediate pion pole. The charged pion propagator is also proportional to $p_a p_\pi$ and this cancels out the p_a factor in the numerator, yielding a result consistent with the previous calculation.

Let us turn now to the process $K^+ \to a\pi^+$. This reaction is considerably more difficult to estimate than the decay $\pi^+ \to ae^+\nu_e$, since the processes it is naturally related to, $K^+ \to \pi^+\pi^0$ and $K^+ \to \pi^+\eta_{\rm virtual}$, are both non-leptonic decays. A number of approaches exist already in the literature to compute this rate for standard axion [30]. Here we shall try to estimate the rate by using a chiral lagrangian for the weak decay involving the meson nonet, supplemented with appropriate mixings of the axion with the π^0, η, and η'. Although there is considerable uncertainty in our estimate, it is important to get at least an order of magnitude idea of the expected branching ratio as this process provides *complementary* information to the decay $\pi^+ \to ae^+\nu_e$. This latter process, as we have seen, measures essentially the isovector mixing of the axion, λ_3. This mixing, because of the SIN experiment [26], must be much below what is expected in the simplest axion models [8, 9] requiring a delicate cancellation to take place in eq. (2.15). However, if λ_3 nearly vanishes, then it is *not* possible to also get the mixing of the axion with the η or φ (essentially the λ_s coupling) to also be small. This means that the process $K^+ \to a\pi^+$, proceeding through the η, φ-axion mixing, could provide an additional independent constraint on variant axion models and indeed serve to rule out these models.

To proceed with our model calculation, we need the mixing of the axion with the pion and the eta and the singlet isoscalar $\varphi' \approx \eta'$,

$$\xi_{\pi a} = (f_\pi/f)\lambda_3\,,$$
$$\xi_{\eta a} = (f_\pi/f)\lambda_8\,,$$
$$\xi_{\varphi a} = (f_\pi/f)\lambda_0\,. \tag{4.51}$$

To compute the mixing angles, we can proceed in two alternative ways. Either we construct a U(3) ⊗ U(3) chiral lagrangian and proceed as before to compute the mixing by diagonalizing the relevent 3×3 mass matrix analogous to (4.37), an approach taken in the last reference of [30]. Or more simply, we can extract λ_3, λ_8, and λ_0 by considering the current $\tilde{J}_\mu$, appropriate to three light flavors, analogous to (2.11). This latter route is much more efficient, since the generalization of eq. (2.11) is immediate. One defines the current

$$\tilde{J}_\mu = J_\mu^{\rm PQ} - \tfrac{1}{2}N(x+1/x)(m_u m_s + m_d m_s + m_u m_d)^{-1}$$
$$\times \{ m_d m_s \bar{u}\gamma_\mu\gamma_5 u + m_u m_s \bar{d}\gamma_\mu\gamma_5 d + m_u m_d \bar{s}\gamma_\mu\gamma_5 s \}\,. \tag{4.52}$$

In terms of the light quark currents,

$$A_{3\mu} = \tfrac{1}{2}\left(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d\right),$$

$$A_{8\mu} = \tfrac{1}{2}\sqrt{\tfrac{1}{3}}\left(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s\right),$$

$$A_{0\mu} = \sqrt{\tfrac{1}{6}}\left(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s\right), \tag{4.53}$$

one may rewrite $\tilde{J}_\mu$ as

$$\tilde{J}_\mu = f\partial_\mu a + \lambda_3 A_{3\mu} + \lambda_8 A_{8\mu} + \lambda_0 A_{0\mu}. \tag{4.54}$$

The parameters, $\{\lambda\}$, are easily identified. Since $m_s \gg m_u, m_d$, one can read off the approximate values from the structure of the current

$$\lambda_3 = \tfrac{1}{2}\left[(z - 1/x) - N(x + 1/x)(m_d - m_u)/(m_d + m_u)\right],$$

$$\lambda_8 = \left[(z - 1/x) - N(x + 1/x)\right],$$

$$\lambda_0 = \sqrt{\tfrac{1}{6}}\left[z + 2/x - N(x + 1/x)\right]. \tag{4.55}$$

The SIN bound (3.7) puts a very strong constraint on λ_3. Since $(m_d - m_u)/(m_d + m_u) \approx 0.26$, it is clear that the only model consistent with the SIN data would be one with $N = 4$ and with $z = x$. The combination Nx, furthermore, is determined from the axion mass (c.f. eq. (4.38)),

$$m_a \approx 25Nx \text{ keV}. \tag{4.56}$$

Using the value inferred from the GSI experiment [1,2] would give an axion mass, $m_a \approx 1.7$ MeV, which implies for $N = 4$ that $x \approx 17$. This determines $\lambda_3 \approx -0.34$ which is marginally consistent with the SIN bound [26]. However, we are then able to predict the value of λ_8:

$$\lambda_8 \approx \sqrt{\tfrac{1}{2}}\lambda_0 \approx \sqrt{\tfrac{1}{3}}\lambda_s \approx -\left(\tfrac{1}{2}\sqrt{3}\right)x \approx -15, \tag{4.57}$$

which will give phenomenological troubles for variant axions.

With the mixing angles determined by (4.51) and (4.55), we must now find the appropriate effective lagrangian to describe the $\Delta S = 1$, nonleptonic K-meson decays. The construction of this effective lagrangian is complicated by the large enhancement of the $\Delta I = \frac{1}{2}$ component of the interactions. It is expected that this enhancement will also enhance the processes involving axions. The fundamental interaction is the current-current interaction generated by W-boson exchange.

However the strong interactions renormalize this interaction and cause a mixing of the operators participating in the interaction. When the short-distance QCD corrections are included, there is the expected enhancement effect which increases the $\Delta I=\frac{1}{2}$ components and decreases the $\Delta I=\frac{3}{2}$ components, but only by a factor of 2–3 generated by the usual mixing [31]. This effective lagrangian has two pieces, corresponding to operators that transform under $SU(3)\otimes SU(3)_{V-A}$ as an $\underline{8}$ and $\underline{27}$:

$$\mathscr{L}_{\text{eff}}(\Delta S=1)=g_8\mathscr{L}_8+g_{27}\mathscr{L}_{27}\ . \tag{4.58}$$

Here the operators in $\mathscr{L}_8$ and $\mathscr{L}_{27}$ can be represented in terms of currents, J_μ, which in turn are described in terms of the chiral 3×3 matrices, $U=\exp\{i\boldsymbol{\lambda}\cdot\boldsymbol{\pi}/f_\pi\}$. One has the current

$$J_\mu=i(f_\pi)^2\left[U\partial_\mu U^\dagger\right] \tag{4.59}$$

and the octet operator, $\mathscr{L}_8$, is given by

$$\mathscr{L}_8=\left(J_\mu^\dagger\cdot J^\mu\right)_{\text{ds}}=(f_\pi)^4\left[\partial_\mu U\cdot\partial^\mu U^\dagger\right]_{\text{ds}}. \tag{4.60}$$

This enhancement does not explain the large factors observed in the K-decays. However, additional operator mixing can occur through the exchange of gluons through the mechanism known as penguins [32]. The penguin interactions generate new operators of a different structure than the usual left-handed current-current operators. It is likely that the penguin contributions will explain much of the $\Delta I=\frac{1}{2}$ enhancement [32, 33]. For our analysis, it is sufficient to observe that both the enhanced current-current interactions and the penguins have exactly the same chiral structure and are both represented by effective lagrangian given by (4.60). In fact, this effective lagrangian is the unique operator giving the correct chiral structure for the $\Delta I=\frac{1}{2}$ amplitude, if we compute the amplitudes to lowest order in the meson momenta. Hence, we can compute the axion amplitudes in terms of the enhanced $\Delta I=\frac{1}{2}$ amplitudes directly from the structure of the operator given in (4.60). For the two-body decays, we can expand (4.60) as

$$\mathscr{L}_8=\tfrac{1}{2}i(f_\pi)^2\left\{\partial_\mu\boldsymbol{\pi}^2\partial^\mu\boldsymbol{\pi}-\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi}^2\right\}. \tag{4.61}$$

The necessary matrix elements elements are given by

$$\langle\pi^+\pi^-|\mathscr{L}_8|K^0\rangle=C\left\{2\sqrt{2}\,(P_K)^2-2\sqrt{2}\,(P_\pi)^2\right\}, \tag{4.62}$$

$$\langle\pi^+\pi^0|\mathscr{L}_8|K^+\rangle=C\left\{-2(P_{\pi^+})^2+2(P_{\pi^0})^2\right\}, \tag{4.63}$$

$$\langle\pi^+\eta|\mathscr{L}_8|K^+\rangle=C\left\{4\sqrt{\tfrac{1}{3}}\,(P_K)^2-6\sqrt{\tfrac{1}{3}}\,(P_\eta)^2+2\sqrt{\tfrac{1}{3}}\,(P_\pi)^2\right\}, \tag{4.64}$$

$$\langle\pi^+\varphi|\mathscr{L}_8|K^+\rangle=C\left\{4\sqrt{\tfrac{2}{3}}\,(P_K)^2-4\sqrt{\tfrac{2}{3}}\,(P_\pi)^2\right\}. \tag{4.65}$$

We may combine these amplitudes with our knowledge of the axion mixing angles to relate the axion amplitude to the $\Delta I = \frac{1}{2}$ K-decay amplitude. Using this relation and that $(P_a)^2 \approx 0$, we find

$$\langle \pi^+ a | H_{wk} | K^+ \rangle \approx \langle \pi^+ \pi^- | H_{wk} | K^0 \rangle \left(m_K^2 - m_\pi^2 \right)^{-1} \times \left\{ -\sqrt{\tfrac{1}{2}}\, \xi_{a\pi^0} m_\pi^2 + \sqrt{\tfrac{1}{6}}\, \xi_{a\eta} \left(2m_K^2 + m_\pi^2 \right) + 2\sqrt{\tfrac{1}{3}}\, \xi_{a\varphi'} \left(m_K^2 - m_\pi^2 \right) \right\}. \tag{4.66}$$

Neglecting terms of order (m_π^2/m_K^2), this result simplifies to

$$\langle \pi^+ a | H_{wk} | K^+ \rangle \approx \langle \pi^+ \pi^- | H_{wk} | K^0 \rangle \left\{ 2\sqrt{\tfrac{1}{6}}\, \xi_{a\eta} + 2\sqrt{\tfrac{1}{3}}\, \xi_{a\varphi'} \right\} \approx \langle \pi^+ \pi^- | H_{wk} | K^0 \rangle \sqrt{2} \left\{ \xi_{a\varphi} \right\}, \tag{4.67}$$

where $\xi_{a\varphi}$ is just the mixing with the two-flavor isoscalar previously considered. Hence, a bound on this amplitude directly complements the bound on the isovector mixing. From (4.51) and (4.55), we have

$$\langle \pi^+ a | H_{wk} | K^+ \rangle \approx \langle \pi^+ \pi^- | H_{wk} | K^0 \rangle \left\{ \left(\sqrt{\tfrac{2}{3}}\, \lambda_8 + 2\sqrt{\tfrac{1}{3}}\, \lambda_0 \right\} (f_\pi/f) \approx \langle \pi^+ \pi^- | H_{wk} | K^0 \rangle \left\{ \sqrt{2}\, \lambda_s \right\} (f_\pi/f). \tag{4.68}$$

Therefore we compute the result for the branching ratio for the axion amplitude

$$B(K^+ \to a\pi^+) = (P_a/P_\pi) \left\{ \Gamma(K^0 \to \pi^+\pi^-)/\Gamma(K^+ \to \text{all}) \right\} \left\{ \sqrt{2}\, \lambda_s \right\}^2 (f_\pi/f)^2 = 2.9 \times 10^{-5} \{ \lambda_s \}^2. \tag{4.69}$$

Using the mixing parameters given in (4.57), one sees that the branching ratio is very large. Our calculation is based on the chiral structure of the $\Delta I = \frac{1}{2}$ amplitudes and should be a good estimate for the expected rate. Even the prediction based only on the short-distance enhancement of the current-current amplitudes, which is weaker by two orders of magnitude, would give a strong bound on this amplitude.

Although there is as yet no real experimental bound on the process $K^+ \to a\pi^+$, $a \to e^+e^-$, it is clear that the situation for variant axions is extremely precarious. As we will see in the next section, nuclear de-excitation experiments give similar discouraging results for the existence of the variant axion.

5. Variant axions and nuclear de-excitations

Axions can cause the decay of an excited nuclear state, N*, to its ground state, N. A general discussion of the formalism for calculating the ratio of the rates of axion and photon de-excitation of a nuclear level is contained in the paper of Donnelly et al. [34]. Basically, because the axion is a 0^- excitation, it acts as a "magnetic" photon. Thus the axion rate, Γ_a, can be computed in an analogous way to the photon rate Γ_γ, by using standard multipole techniques [35]. Many of the details of the precise nuclear wave functions disappear when one considers the ratio Γ_a/Γ_γ. Furthermore, since the transition energies to be considered are much smaller than the typical nuclear Fermi momentum ($k_F \approx 250$ MeV), one may evaluate the multipole operators in the long-wavelength limit. In this case the ratio, Γ_a/Γ_γ, depends essentially only on some static quantities describing the coupling of axions and photons to nucleons.

We reproduce below, for the case of M1 transitions, the relevant formulas for Γ_a/Γ_γ obtained by Donnelly et al. [34]. One finds for isovector M1 transitions

$$\Gamma_a/\Gamma_\gamma = \tfrac{1}{2}(\tilde{\alpha}/\alpha)(k_a/k)^3\left[\rho^{(1)}/(\mu^{(1)} - \eta^{(1)})\right]^2, \qquad \Delta T = 1, \tag{5.1}$$

while for isoscalar M1 transitions one has

$$\Gamma_a/\Gamma_\gamma = \tfrac{1}{2}(\tilde{\alpha}/\alpha)(k_a/k)^3\left[\rho^{(0)}/(\mu^{(0)} - \eta^{(0)})\right]^2, \qquad \Delta T = 0. \tag{5.2}$$

Here k_a and k are the momentum of the axion and the photon in the transition and $\tilde{\alpha}$ is the relevant scaled effective coupling squared of axions to nucleons

$$\tilde{\alpha} = g_{\pi NN}^2 (f_\pi/f)^2/4\pi, \tag{5.3}$$

where $g_{\pi NN}$ is the pion nucleon coupling constant. Numerically, one has

$$\tilde{\alpha}/\alpha \approx 2.33 \times 10^{-4}. \tag{5.4}$$

The parameters $\mu^{(T)}$, $\eta^{(T)}$, and $\rho^{(T)}$ ($T = 0, 1$) are related to the coupling of photons and axions to nucleons. Since one is dealing with a magnetic photon transition, $\mu^{(T)}$ is related to the magnetic moment, while $\eta^{(T)}$ is related to the ratio of the convection current contribution to that of the magnetization current contribution [34, 35]. Specifically, one has

$$\mu^{(0)} = \mu_p + \mu_n \approx 0.88,$$

$$\mu^{(1)} = \mu_p + \mu_n \approx 4.70,$$

$$\eta^{(0)} = \tfrac{1}{2}, \tag{5.5}$$

while $\eta^{(1)}$ depends on the specific nuclear transition considered. However, typically $\eta^{(1)} \ll \mu^{(1)}$ and we shall neglect it in what follows. If one writes an effective axion-nucleon lagrangian as

$$\mathscr{L}_{\mathrm{aNN}} = \tfrac{1}{2} i \bar{N} \gamma_5 \left[g^{(0)} + g^{(1)} \tau_3 \right] N a \,, \tag{5.6}$$

then the parameters $\rho^{(T)}$ are given by the equation [34]

$$g^{(T)} = (f_\pi / f) \rho^{(T)} g_{\pi \mathrm{NN}} \,. \tag{5.7}$$

To compute nuclear de-excitations of variant axions, we need, therefore, to ascertain what the $\rho^{(T)}$ parameters are. We shall see that $\rho^{(0)}$ and $\rho^{(1)}$ are simply related to the mixing parameters $\lambda_s \approx \sqrt{3}\lambda_8$ and λ_3 of the preceding section, see eqs. (4.55), (4.57).

To compute the effective lagrangian (5.6), we remark that, neglecting terms of $O(m_a^2/m_\pi^2)$, the current $\tilde{J}$ contains only physical axion poles (recall the result (4.46)). Therefore the matrix element of $\tilde{J}_\mu$ between nucleon states will allow us to compute directly the coupling constants $g^{(0)}$ and $g^{(1)}$, since the pseudoscalar form factors will be dominated by just the axion pole. Let us write in all generality

$$\begin{aligned} \langle \mathrm{N} | \tilde{J}_\mu | \mathrm{N} \rangle = \bar{U}(p') \Big\{ & \left[i \gamma_\mu \gamma_5 \tilde{G}_{\mathrm{A}}^{(0)}(t) + i (p'-p)_\mu \gamma_5 G_{\mathrm{P}}^{(0)}(t) \right] \cdot \tfrac{1}{2} \\ & + \left[i \gamma_\mu \gamma_5 \tilde{G}_{\mathrm{A}}^{(1)}(t) + i (p'-p)_\mu \gamma_5 G_{\mathrm{P}}^{(1)}(t) \right] \left(\tfrac{1}{2} \tau_3 \right) \Big\} U(p) \,. \end{aligned} \tag{5.8}$$

The pseudoscalar form factors $G_{\mathrm{P}}^{(T)}$ are dominated by the axion pole and measure the coupling $g^{(T)}$

$$G_{\mathrm{P}}^{(T)}(t) = g^{(T)} f / \left(t + m_a^2 \right) . \tag{5.9}$$

The pseudovector form factors, given the form of $\tilde{J}_\mu$ in eq. (2.14), are nothing but the usual nucleon pseudovector form factors multiplied by the mixing parameters, λ_s and λ_3. That is

$$\begin{aligned} \tilde{G}_{\mathrm{A}}^{(0)}(t) &= \lambda_s G_{\mathrm{A}}^{(0)}(t) \,, \\ \tilde{G}_{\mathrm{A}}^{(1)}(t) &= \lambda_3 G_{\mathrm{A}}^{(1)}(t) \,. \end{aligned} \tag{5.10}$$

Using the fact that the divergence of the $\tilde{J}_\mu$ current is dominated by the axion pole one obtains, in the usual Goldberger-Trieman way [36], a relation for the couplings, $g^{(T)}$ in terms of $G_{\mathrm{A}}^{(T)}(0)$,

$$\begin{aligned} g^{(0)} &= 2\lambda_s G_{\mathrm{A}}^{(0)}(0) M/f \,, \\ g^{(1)} &= 2\lambda_3 G_{\mathrm{A}}^{(1)}(0) M/f \,. \end{aligned} \tag{5.11}$$

Using the Goldberger-Trieman relation [36]:

$$G_A^{(1)}(0)\,M = f_\pi g_{\pi NN}\,, \tag{5.12}$$

and eq. (5.7), we identify

$$\rho^{(0)} = 2\lambda_s\left[G_A^{(0)}(0)/G_A^{(1)}(0)\right],$$
$$\rho^{(1)} = 2\lambda_3\,. \tag{5.13}$$

There is no direct measurement for $G_A(0)$ experimentally. We shall therefore use a quark model estimate [36] for the ratio $G_A^{(0)}(0)/G_A^{(1)}(0)$

$$G_A^{(0)}(0)/G_A^{(1)}(0) \approx \tfrac{3}{5}\,. \tag{5.14}$$

Since, for the models of interest for variant axion, $\lambda_s \approx \sqrt{3}\,\lambda_8$, we obtain finally the result

$$\rho^{(0)} = \left(\tfrac{6}{5}\sqrt{3}\right)\lambda_8\,,$$
$$\rho^{(1)} = 2\lambda_3\,. \tag{5.15}$$

With these parameters fixed and eqs. (5.1) and (5.2), we are now ready to confront experiment.

As mentioned in the introduction, there are two recent nuclear de-excitation studies which have bearing on variant axions. Savage et al. [12] studied, in a very pretty experiment, the decay of the 9.17 MeV, 2^+, $T=1$ state of ^{14}N to the 1^+, $T=0$ ground state. Calaprice et al. [13] reanalyzed the pair correlation experiments of Warburton et al. [14], focussing in particular on the isoscalar, M1 transition from the 3.58 MeV, 2^+, $T=0$ state of ^{10}B to the 0.72 MeV, 1^+, $T=0$ state. In both cases, the presence of variant axions would give an additional source of prompt e^+e^- pairs, besides those expected from normal internal conversion. Furthermore, the angular distribution of the e^+e^- pairs for variant axions is significantly different from that of internal pair conversion, so that one can distinguish between the two sources of pairs even if the rates are comparable in magnitude.

Using eqs. (5.1) and (5.2), one predicts for variant axion models (assuming $m_a \approx 1.7$ MeV) the following rates:

(i) $9.17 \to 0$, $\Delta T=1$, ^{14}N transition: $\quad \Gamma_a/\Gamma_\gamma \approx 2\times 10^{-5}(\lambda_3)^2$, (5.16)

(ii) $3.58 \to 0.72$, $\Delta T=0$, ^{10}B transition: $\quad \Gamma_a/\Gamma_\gamma \approx 1.8\times 10^{-3}(\lambda_8)^2$. (5.17)

The main difference in these rates, apart from the λ factors, comes from the large isovector magnetic moment in the $\Delta T = 1$ transition. If the axion lifetime is less than 10^{-11} sec, then Savage et al. [12] gives a 90% confidence limit bound on the rate,

$$^{14}\mathrm{N}: \qquad (\Gamma_a/\Gamma_\gamma)_{\mathrm{exp}} < 4 \times 10^{-4}, \tag{5.18}$$

which implies

$$\lambda_3 < 4.5. \tag{5.19}$$

This bound on λ_3 is enough to rule out the simplest variant axion model of refs. [8] and [9], which had predicted $\lambda_3 \approx 26$. However, eq. (5.19) gives roughly an order of magnitude weaker bound on the isovector parameter, λ_3, than that obtained by the SIN experiment [26].

The reanalysis of the Warburton et al. experiment [14], as done by Calaprice and collaborators [13], gives a branching ratio limit for axion lifetimes shorter than 10^{-11} sec, at the 1σ level,

$$^{10}\mathrm{B}: \qquad (\Gamma_a/\Gamma_\gamma)_{\mathrm{exp}} < 0.75 \times 10^{-4}, \tag{5.20}$$

which implies that

$$|\lambda_8| < 0.18. \tag{5.21}$$

This value of λ_8 is about two orders of magnitude below what would be predicted by the model where λ_3 was tuned to be small enough to escape the isovector bound of the SIN experiment. That is, for the case of $N = 4$ and $x \approx 17$ where recall that we found the value for $\lambda_8 \approx -15$(c.f. eq. (4.57)). Thus the combination of both of these bounds excludes the existence of variant axions. Of course, the result (5.20) was obtained by reanalyzing an old experiment and one should be a bit cautious. However, if λ_8 were of the order of magnitude expected in the surviving $N = 4$ model, one would have expected a rate of $\Gamma_a/\Gamma_\gamma \approx 0.3$ which would have totally swamped the predicted internal pair rate $\Gamma_\pi/\Gamma_\gamma \approx 1.5 \times 10^{-4}$ [38]. So although the bound in eq. (5.21) may be too strong, a value of 15 should definitely be excluded*.

6. Conclusions

The narrow e^+e^- signal observed at GSI motivated the construction of variant axion models. Because, in these models, the axion decays very rapidly to e^+e^- pairs, many of the previous bounds on axions are rendered irrelevant. Furthermore, by assigning the same PQ charge for c and b quarks, one can suppress both the $\psi \to \gamma a$ and $\Upsilon \to \gamma a$ decays. Hence these models appear, at first sight, to provide a viable and interesting way to solve the strong *CP* puzzle.

In variant axion models the isoscalar and isovector properties of the axions, characterized by the mixing parameters λ_s and λ_3, respectively, are not universal

* After this paper was written, a report appeared of an experiment [40] looking for axions in ^{10}B in the decay of the 3.58 MeV state to the ground state. This direct experiment sets a bound $\lambda_8 \lesssim 1.75$.

but depend on the individual model considered. It is, in fact, possible to have models where either λ_s or λ_3 vanishes, so that no individual further experiment can be used *alone* to rule out the existence of variant axions. However, as one can see from eq. (4.38), (4.55), and (4.57), there is a model independent prediction for the difference between λ_s and λ_3. For large x, one has

$$(\lambda_s - \lambda_3)^2 \approx N^2(x+1/x)^2(m_u/(m_u+m_d))^2$$

$$\approx (f/f_\pi)^2(m_a/m_\pi)^2(m_u/m_d) \approx (25)^2, \tag{6.1}$$

where the numerical result applies for the case in which $m_a \approx 1.7$ MeV. Because of the relation (6.1) variant axion models are excluded by combining the recent results of π decay and nuclear de-excitation which require individually that λ_3 *and* λ_s be less than about 0.25. Note that since the constraint (6.1) is applicable for any variant axion model, the precise mass value inferred from GSI is not a particularly important factor in ruling out models with $m_a > 2m_e$. We wish to remark that the phenomenology of completely general axion models is sensitive to only three potentially independent parameters, the axion coupling to the up quark (z/f), the coupling to the down quarks $(1/xf)$, and the coupling to the color gauge fields $(r = N(x+1/x)/f)$ through the strong anomaly which determines the axion mass. Even models with no direct axion coupling to the quarks $(z/f = 1/xf = 0)$ are strongly constrained because of the mixing induced by the strong anomaly of the quarks.

The above considerations suggest that there is no window for axions to exist, whether of standard or variant type, if the breakdown of the PQ symmetry is intimately connected with that of the $SU(2) \otimes U(1)$ scale, i.e. $f \approx 250$ GeV. Thus, if the solution to the strong *CP* puzzle is to be found by using an additional chiral symmetry, this symmetry most likely must be broken at a large scale, and the axion is of the invisible type.

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