NATURAL CHAOTIC INFLATION

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We present a chaotic inflationary model, in which nonlinear interactions of dilaton and axion fields in the context of the superconformal theory can dynamically give rise to initial conditions for the inflation of the universe and a flat potential that can produce enough inflation. Our model is free from dangerous thermal effects and large energy density fluctuations.

The suggestion that the universe has undergone a phase of exponentially rapid expansion [1] created a lot of interest in the connection of cosmology to particle physics. This idea of an inflationary universe is extremely attractive, because it gave a solution to some of the long-standing problems of cosmology such as why the density of our universe is close to the critical value (the flatness problem) [‡1]. However, the specific models put forward so far, do always require a fine-tuning either in the potential $V(\phi)$ of the inflaton field ϕ, or in the initial conditions of the universe.

In this letter we construct a model for the inflationary universe, in which no specific fine-tuning is necessary. Our model is based on Linde's chaotic inflationary scenario [3]. In the original model of chaotic inflation, one must assume that at the Planck time $t = M_{\mathrm{p}}^{-1}$ the kinetic energy $E_{\mathrm{K}} = \frac{1}{2}\dot{\phi}^2$ of the inflaton ϕ is much smaller than the potential $V(\phi)$. However, since one needs a very flat potential such as $V = \frac{1}{2}m^2\phi^2$ with $m^2 \lesssim 10^{-10} M_{\mathrm{p}}^2$ to forbid large density fluctuations, this would imply that a fine-tuning $\dot{\phi}^2 \ll \phi^2$ is required at $t \sim M_{\mathrm{p}}^{-1}$ for the inflation to start. Clearly there is no reason why one should believe such a fine-tuning and hence inflation is ad hoc.

In our model, on the contrary, the desired situation $\dot{\phi} \ll V(\phi)$ emerges quite naturally on the onset of inflation ($t \sim 10 M_{\mathrm{p}}^{-1}$) as a consequence of gravitational dynamics. An important point in our model is the introduction of an extra scalar field ξ which has only derivative couplings to ϕ. The additional field plays a very important role for driving the situation $\dot{\phi}^2 \ll V(\phi)$ at a bit later time $t \sim 10 M_{\mathrm{p}}^{-1}$, even if $\dot{\phi}^2 \sim V(\phi)$ at the Planck time. As pointed out in ref. [4], because of the nonminimal couplings of ϕ and ξ fields, the solution of the equation of motion for the ξ field yields an effective potential term $V_{\mathrm{eff}}(\phi)$ at the classical level. In the presence of $V_{\mathrm{eff}}(\phi)$, one easily finds that $\dot{\phi}^2$ decreases with time more rapidly than $V(\phi)$. Although there are many possibilities of having such a ξ field (for example, it arises naturally in the no-scale models [5] as a part of an SU(1,1) symmetry, or simply it can be a Nambu–Goldstone boson associated with a global U(1) breaking), we consider here a broken superconformal theory as an illustration of our general idea. In this theory one will immediately understand not only why ξ has the specific form of coupling $g^{\mu\nu}\partial_\mu\xi\partial_\nu\xi f(\phi)$, but also the very slow time dependence of $V(\phi)$.

Superconformal invariance [6] is considered to play an important role in supersymmetric gauge theories and supergravity theories. For instance,

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the $N=4$ supersymmetric Yang–Mills theory has superconformal invariance and is known to be a finite theory [7]. In the no-scale supergravity models [5] there exists a certain spontaneously broken superconformal symmetry. Here, we shall consider a general class of spontaneously broken superconformal theories in which there appear the dilaton, the "axion" and the dilatino as the Nambu–Goldstone particles corresponding to the broken dilatation, chiral U(1) and conformal supersymmetry generators. The effective lagrangian which describes the interaction among these particles can be written, for example, in the framework of the nonlinear realization of superconformal symmetry [8]. When the system is coupled to Einstein (super-)gravity, the bosonic part of the lagrangian is given by

$$\mathscr{L}= -\tfrac{1}{2}\sqrt{-g}\,R + \sqrt{-g}\,g^{\mu\nu}\,\mathrm{e}^{-2\sigma}\left[\tfrac{1}{2}\partial_\mu\sigma\partial_\nu\sigma + \tfrac{1}{2}\partial_\mu\xi\partial_\nu\xi\right], \tag{1}$$

where σ and ξ denote the dilaton and "axion" fields, respectively. Note that the σ and ξ fields have noncanonical kinetic terms which are characteristic for the ξ-models on kählerian manifolds [4]. Of course, the gravity interaction breaks (super-)conformal symmetry explicity. However, we could have started from the system of conformal supergravity coupled to matter multiplets and let superconformal invariance spontaneously break down to super-Poincaré invariance. This would lead to an effective lagrangian of the same form.

We can regard the lagrangian (1) as coming from the spontaneously broken conformal ⊗ chiral U(1) invariance. However, the presence of chiral U(1) charge is a natural consequence of the supersymmetric extension of conformal invariance.

To the lagrangian (1) we now add a soft breaking term which is taken to be of the form of a mass term for the σ field

$$\mathscr{L}' = -\sqrt{-g}\,\epsilon\sigma^2, \tag{2}$$

where ϵ is a positive parameter with a mass dimension two. This term breaks conformal symmetry as well as supersymmetry explicitly but softly. The absence of a mass term for the ξ field is due to the chiral U(1) symmetry. If the U(1) current has the ABJ anomaly, the ξ acquires a mass through the instanton effects. However, for our discussion here these effects can be neglected.

We are now ready to analyze the lagrangian $\mathscr{L}_G=\mathscr{L}+\mathscr{L}'$ as given by (1) and (2). First we define the field ϕ by

$$\phi = \mathrm{e}^{-\sigma}. \tag{3}$$

Then, in the Friedman–Robertson–Walker background, the classical field equations for homogeneous (time dependent, spatially constant) $\phi(t)$ and $\xi(t)$ fields are

$$\ddot\phi + 3H\dot\phi - \phi\dot\xi^2 + 2\epsilon\ln(\phi)/\phi = 0, \tag{4a}$$

$$(\mathrm{d}/\mathrm{d}t)(\phi^2\dot\xi R^3) = 0. \tag{4b}$$

From the latter equation we have

$$\dot\xi = QR^{-3}/\phi^2, \tag{5}$$

where Q is an integration constant, R a cosmic scale factor and $H=\dot R/R$ the Hubble parameter. Substitution of (5) into (4a) gives the equation of motion for the properly normalized ϕ field:

$$\ddot\phi + 3H\dot\phi + \partial_\phi\left[\tfrac{1}{2}Q^2R^{-6}/\phi^2 + \epsilon(\ln\phi)^2\right] = 0. \tag{6}$$

Observe that the Q-dependent term behaves like an effective potential and therefore we can rewrite (6) as

$$\ddot\phi + 3H\dot\phi + \partial_\phi v_Q = 0, \tag{7}$$

where

$$v_Q = \tfrac{1}{2}Q^2R^{-6}/\phi^2 + \epsilon(\ln\phi)^2. \tag{8}$$

In the very early stages of the evolution of the universe, when R is very small, the Q-dependent term in (8) dominates over $\epsilon(\ln\phi)^2$ and the equation of motion for the ϕ field would be

$$\ddot\phi + 3H\dot\phi + \partial_\phi\left(\tfrac{1}{2}Q^2R^{-6}/\phi^2\right) = 0. \tag{9}$$

Supposing that R scales with time as $R=R_0t^n$, eq. (9) can be solved exactly. The first integration gives

$$(\mathrm{d}\phi/\mathrm{d}\tau)^2 + Q^2R_0^{-6}/\phi^2 = E^2, \tag{10}$$

where E is a positive constant, and

$$\tau = (1-3n)^{-1}t^{1-3n} \quad n\neq\tfrac{1}{3},$$
$$= \ln t \quad n=\tfrac{1}{3}. \tag{11}$$

Positivity of $\dot{\phi}^2$ requires $\phi^2 \geqslant Q^2 R_0^{-6}/E^2$. Eq. (10) can be integrated to give

$$\phi^2 = E^2\tau^2 + 2E\tau\left(\sqrt{\phi_0^2 - Q^2R_0^{-6}/E^2} - E\tau_0\right) + C, \tag{12}$$

where

$$C = E^2\tau_0^2 + \phi_0^2 - 2E\tau_0\sqrt{\phi_0^2 - Q^2R_0^{-6}/E^2}\,; \tag{13}$$

with R_0, τ_0, ϕ_0, we indicate the initial values of R, τ and ϕ, respectively.

The time evolution of $R(t)$ is controlled by the equation

$$\rho(t) = 3H^2 = \tfrac{1}{2}\dot{\phi}^2 + \tfrac{1}{2}Q^2R^{-6}/\phi^2, \tag{14}$$

where natural units have been used ($M_p/\sqrt{8\pi} = 1$). Then, one can easily verify that eq. (14) is also satisfied only if $n = 1/3$. Taking initially $\rho(t_p) \sim$ O(1) then, because of (12), the terms $\dot{\phi}^2$ and Q^2R^{-6}/ϕ^2 in (14) are initially of the same order of magnitude $\sim$ O(1). Note here, that, because of the relation $\phi^2 \geqslant Q^2R_0^{-6}/E^2$, any large initial value of the ϕ field is allowed. As the universe evolves with time, ϕ is moving under the influence of the force $F = Q^2R^{-6}/\phi^3$, and the scale factor R is increasing and therefore, there will be a time when the Q-dependent term becomes comparable with the potential term $\epsilon(\ln\phi)^2$. The ϕ field then will move under the force $F = Q^2R^{-6}/\phi^3 - 2\epsilon\ln(\phi)/\phi$ and there will be a time when F becomes zero. After that time, the Q-dependent term gradually decouples, the force becomes negative and, acting as a decelerating force, drives $\dot{\phi}$ to zero. Then inflation starts controlled by the potential $\epsilon(\ln\phi)^2$. Note that, because of the logarithmic dependence of this potential, there will be a large region where inflation takes place. Our mechanism of inflation, which we have just described, serves as a qualitative picture of the time evolution of $\dot{\phi}$, and a detailed numerical analysis of the equations of motion will be presented elsewhere

Our model is not based on supercooling [9]. If inflation starts at an early enough time, we do not have to worry about temperature effects [3]. The energy density fluctuations, however, impose a strong constraint on any model for inflation. In our model a simple calculation gives $\delta\rho/\rho \sim 2\sqrt{\epsilon}/\phi_{\rm inf}$, where $\phi_{\rm inf}$ is the value of ϕ at the onset of inflation (a more detailed study of the energy fluctuations is under progress). A large value of $\phi_{\rm inf}$ can easily give $\delta\rho/\rho \sim 10^{-4}$. Notice here, that, because of (12), $\phi \sim \ln t$ and therefore $\phi_{\rm inf}$ is not much different from the initial value of ϕ at the Planck scale. This indicates that, because of $\delta\rho/\rho \sim 2\sqrt{\epsilon}/\phi_{\rm inf}$, the energy density fluctuations depend not only on a parameter of the lagrangian but also on the initial value of ϕ.

In conclusion, we have presented a model for a chaotic inflationary scenario, in which superconformal invariance forced us to have a nonminimal coupling between the dilaton σ and the axion ξ field. This coupling results in a nontrivial Q-dependent effective potential. We find that, in the presence of such a Q-dependent term, a desirable initial condition for the inflation to start emerges naturally.

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