DEPARTURES FROM SCALING IN SU(2) LATTICE GAUGE THEORY

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High statistics Monte Carlo Data in SU(2) lattice gauge theory are presented. At $\beta=2.6$ and $\beta=2.7$ large deviations from scaling are observed for Creutz ratios, when 12^4 and 24^4 lattice data are compared. There is a trend towards a restauration of asymptotic scaling with increasing β, which vanishes if at the higher value of β larger loops are considered than at lower β. The static q$\bar{\text{q}}$-potential and an upper limit for the string tension are given.

How lattice gauge theories reach the continuum scaling behaviour in the limit $\beta\to\infty$ is an open problem. Both for SU(2) and SU(3) sizeable deviations from asymptotic scaling (AS) are by now well established [1-7] (we follow the convention to call AS the variation of physical dimensionful quantities proportional to the two-loop lattice scale parameter Λ_{latt}). Presently there are mainly Monte Carlo data [8,9] for the critical temperature in SU(3), which point towards a rapid restauration of asymptotic scaling at $\beta\geqslant 6.2$. Monte Carlo renormalization group studies (MCRG) suggest the same behaviour for $\beta>6.3$ in SU(3) [1,2] and for $\beta>2.5$ in SU(2) [6]. It is fair to say that the systematic errors in MCRG are not easy to pin down. Especially one has to rely on the assumption that there exists one universal β-function for objects of all sizes (in lattice units).

Doubts with respect to the latter point have been raised in ref. [10] for SU(2). There is evidence that short-distance quantities have a β-dependence closer to AS than large-distance quantities. It is worthwhile to extend the Monte Carlo measurements of refs. [10] to larger values of β to get a better understanding of the details of the approach to AS and its possible size dependence. Here I present new Monte Carlo data in SU(2), taken at $\beta_2=2.7$ on a lattice with size 24^4, together with extended results at $\beta_1=2.6$ on the same lattice. This paper will give only a summary of numerical results; for details ref. [10] has to be consulted. The analysis will show that there is indeed a tendency for a return to AS for increasing β, if generalized Creutz ratios [11] are taken as a probe. However:

- This approach to AS does not change the tendency observed at β_1, namely that ratios formed out of larger Wilson loops show a larger departure from AS than those from smaller ones. Deviations from AS at β_2 thus are as large as the deviations at β_1 of somewhat smaller loops.
- The pattern of scaling violations follows closely the fraction of the perturbative contributions in the Creutz ratios. If one considers the ratios as functions of this fraction, the deviations from AS at β_1 and at β_2 fall on a single curve within reasonable accuracy.

It is natural to suspect that at large β it is the smallness of the non-perturbative contributions to Creutz ratios which is responsible for the return to AS, and that there is no indication that the nonperturbative terms will obey AS themselves.

The analysis is based on the material listed in table 1. Throughout the standard single-plaquette action has been used, but both boundary conditions and the group varied somewhat. At low β occasionally the 120-dimensional icosahedral subgroup of SU(2) has been used. The data at β_2 have been taken on a VP200 vector computer. In order to obtain short update times, helical boundary conditions [12] with a shift of $s=2$ lattice units have been used at β_2. They are defined by the relation between the link variables $U_\mu(x, y, z, t)$, valid cyclically for all directions,

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Table 1
Survey of statistics collected on various lattices.

β	L	No. of sweeps	No. of sweeps discarded	Group	Boundary conditions	Maximal size of measured loops
2.4	12	27000	1000	icosahedral	periodic	6×6
2.45	12	34000	4000	icosahedral	helical, $s=1$	6×6
	12	41000	4000	full SU(2)	helical, $s=1$	6×6
2.5	12	32000	4000	icosahedral	periodic	6×6
	12	30000	2000	full SU(2)	helical, $s=1$	6×6
	24	3000	1000	full SU(2)	periodic	5×7
2.6	24	6000	1000	full SU(2)	periodic	8×8
	24	4000	1000	full SU(2)	periodic	8×8
2.7	24	50000	9000	full SU(2)	helical, $s=2$	8×10

$$U_\mu(x+L, y, z, t) = U_\mu(x, y+s, z, t) . \quad (1)$$

The CPU time needed to update one link could thus be lowered to below 3 μs, which corresponds to a sustained rate of 210 MFLOPS. On the 12^4 lattice a shift of $s=1$ has been employed. For the full group I applied a vectorized heat-bath method and the multihit method [13], measuring Wilson loops in one out of four planes after 30 sweeps. At $\beta=2.5$ no significant differences between lattices of size $L \geqslant 12$ and with different boundary conditions were found, and all available data were averaged in order to reduce the statistical errors.

First of all I will present the static q$\bar{\text{q}}$-potential, defined in the usual way. The extrapolation of Wilson loop expectation values $W(R, T)$ to $T \to \infty$ was performed by the $2N$-parameter fit

$$W(R, T) = \sum_{i=1}^{N} c_i(R) \exp[-\lambda_i(R) T] . \quad (2)$$

At β_2 I used $N=3$ for $R \leqslant 4$ and $N=4$ for $R \geqslant 5$. In the latter case I included the point $T=0$ with $W(R, 0)=1$. The stability of the fit was judged positively by dividing the data into 10 subsamples, and the statistical errors for $\lambda_1(R)$ and $\lambda_2(R)$ were estimated in the same way. The potential $\lambda_1(R)$ is still 1.5 s.d. below the logarithmic ratio

$$V_T(R) = -\ln[W(R, T)/W(R, T-1)] , \quad (3)$$

at $R=8$ for $T=10$.

In order to obtain a potential $V(R)$ as closely related to the continuum potential as possible, $\lambda_1(R)$ has to be corrected for finite lattice spacing. I repeat the method applied in ref. [10], where the difference between the continuum propagator $1/R$ and the infinite-size lattice propatator is considered. This difference times the bare α_S describes scaling violations in tree approximation. Since there is a strong renormalization of α_S by higher-order terms, I multiply the above difference by an effective coupling constant deduced from the small-R behaviour of the potential and add the result as a correction to the lattice potential. An error of 20% of this correction is added to the negligible statistical errors at small R. In table 2 I give the values of $\lambda_1(R)$, $V(R)$ and $\lambda_2(R)$ with errors. The values of $\lambda_2(R)$ at β_1 are consistent with those of ref. [14]. The potential $V(R)$ is also shown in fig. 1 together with the parameter-free two-loop continuum potential [15]. Constants have been added to make the potentials at fixed β to agree at $R=1$.

The difference $\Delta V_P(R)$ between perturbation theory and lattice potential[‡1] is already quite small at β_2 and $R=2$, i.e. 10% of the potential difference $V(2)-V(1)$. One cannot represent $\Delta V_P(R)$ by a term linear in R, as it still contains a contribution varying like $1/R$. If, however, the Λ-parameter is increased by 20% beyond the value derived in ref. [15], $\Delta V_P(R)$ is linear in R (within the systematic errors due to the finite "a"-corrections). This presence of a linear term also in the perturbative region is consistent with bag-type models [16–18]. But to identify the coefficient of the linear piece with the string tension K is just

[‡1] In the preprint version DESY 86-065 the evaluation of the perturbative potential contained a programming error, which produced a somewhat too steep potential. The error is present also in ref. [10], where, however, no significant changes in the value of the string tension, eq. (11), arise after correction of the error. Only the curve V_B in fig. 1 is affected.

Table 2
Results on the lattice potentials. The $\lambda_i(R)$ are defined in eq. (2).

β	R	$\lambda_1(R)$	$V(R)$	$\Delta V(R)$	$\lambda_2(R)$	$\Delta\lambda_2(R)$
2.6	1	0.3076	0.3236	0.0032	1.43	0.04
	2	0.4327	0.4404	0.0016	1.38	0.02
	3	0.4951	0.4976	0.0006	1.45	0.02
	4	0.5360	0.5368	0.0005	1.44	0.02
	5	0.5682	0.5686	0.0009	1.42	0.03
	6	0.5946	0.5948	0.0016	1.38	0.05
	7	0.6168	0.6169	0.0021	1.32	0.07
	8	0.6341	0.6342	0.0033	1.26	0.09
2.7	1	0.2857	0.3003	0.0029	1.24	0.09
	2	0.3956	0.4026	0.0014	1.24	0.05
	3	0.4464	0.4487	0.0007	1.20	0.04
	4	0.4764	0.4771	0.0006	1.07	0.04
	5	0.4981	0.4985	0.0007	1.05	0.04
	6	0.5168	0.5170	0.0009	1.08	0.04
	7	0.5320	0.5321	0.0011	1.09	0.03
	8	0.5458	0.5459	0.0017	1.03	0.04

one of many possibilities. E.g., at β_2 in the so-called "log–log" model of ref. [18] a string tension $K\approx 0.002/a^2$ is required to fit the data at small R, whereas from $\Delta V_P(R)$ one derives $K\approx 0.0045/a^2$. The significance of the above representation lies more in

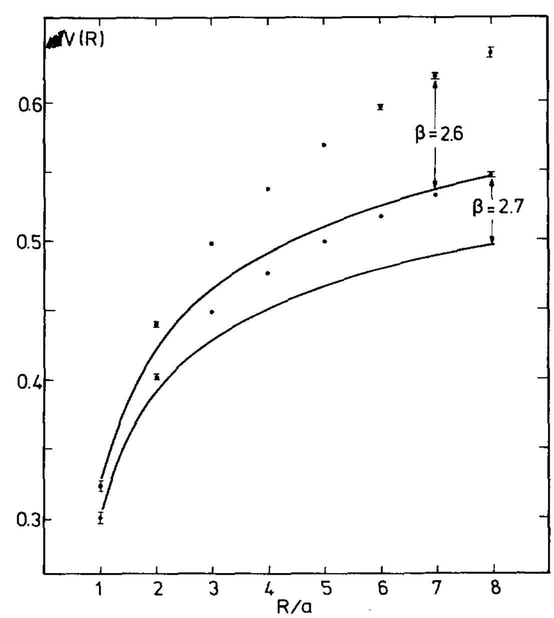

Fig. 1. The static q$\bar{\text{q}}$-potential at $\beta_1=2.6$ and $\beta_2=2.7$, corrected for finite lattice spacing. The curves are the two-loop perturbative potential [15].

what one can learn about the scaling behaviour of $V(R)$. Thus the "string tension" differs by a factor 2.3 ± 0.1 between β_1 and β_2, which is not in agreement with AS (which predicts 1.66 for this factor) and also not with scaling, since the perturbative piece obeys AS by construction.

At β_2 the potential $V(R)$ is inconsistent with a linear behaviour for $R\geqslant 5$, since such an assumption gives a $\chi^2/\text{d.f.}=4$. Various subtraction methods of nonleading terms for large R lead to a range of values for the string tension:

$$0.0055/a^2\leqslant K\leqslant 0.011/a^2\,. \quad (4)$$

The smallest value of K is obtained, if two-loop perturbation theory is subtracted, and the largest value follows from the standard linear + Coulomb fit. At β_2 a stringent upper limit for K is given by the potential difference:

$$K\leqslant (1/2a)\;[V(8)-V(6)]$$
$$=(0.0145\pm0.001)/a^2\,. \quad (5)$$

This upper limit is considerably higher than the quoted values for K, and this is true even for the lowest value for K at β_1 given in ref. [10]. The perturbative subtractions, which amount at least to $\Delta K=0.004/a^2$ at the presently obtainable range of R, will of course prevent any reliable determination of K for higher values of β unless the range of R can be extended considerably. The upper limit for K is 1 s.d. below the *value* quoted for K in refs. [19,20], where it was obtained from correlations of Polyakov loops on a 64×6^3 lattice. In view of the preceding remarks this may be a real discrepancy, and most likely such lattices are too small to measure K reliably.

I proceed to study the scaling behaviour of generalized Creutz ratios $\chi(l)$, where l stands for the 8 numbers describing the geometry of 4 rectangular Wilson loops. Ratios $\chi(l,\beta)$ on a 12^4 lattice with $\beta=\beta_i-\Delta\beta$ are compared with ratios $\chi(2l,\beta_i)$ on the 24^4 lattices. The former are multiplied by a correction factor $c_P(l,\beta)$ to suppress lattice artifacts. This factor is determined in such a way that perturbatively calculated ratios $\chi_P(l,\beta)$ will scale asymptotically:

$$c_P(l,\beta_A)\chi_P(l,\beta_A)=\chi_P(2l,\beta)\,, \qquad (6)$$

with $\beta_A\approx\beta-0.275$. The ratios $\chi_P(l,\beta)$ are taken from the $O(g^4)$ calculation [21], where periodic boundary conditions have been used. I do not expect that finite "*a*"-effects, which are predominantly a small-distance effect, depend crucially on boundary conditions. The perturbative ratios are actually used in a "Padéized" version [10], which includes higher-order terms in g^2. I now define $\Delta\beta$ by

$$c_P(l,\beta_A)\chi(l,\beta-\Delta\beta)=\chi(2l,\beta)\,, \qquad (7)$$

where for the l.h.s. quadratic interpolation between the data listed in table 1 is used.

In fig. 2 I show a selection of values $\Delta\beta$ for various sizes of ratios[‡2]. The errors are deduced from combining 3000 sweeps into 1 bin. Only ratios of quadratic or elongated form, which closely correspond to potential differences, are included. As illustration, the ratio denoted by $\frac{53}{43}\big|\frac{32}{42}$ is very close to the potential difference $V(3)-V(2)$ on the small lattice and to $2[V(6)-V(4)]$ on the large lattice. The standard Creutz ratios, where the largest loop is of quadratic form, differ more strongly from potential differences, since the perturbative background present for $R\approx T$ is not reduced as strongly as in the previous "oblique" ratio.

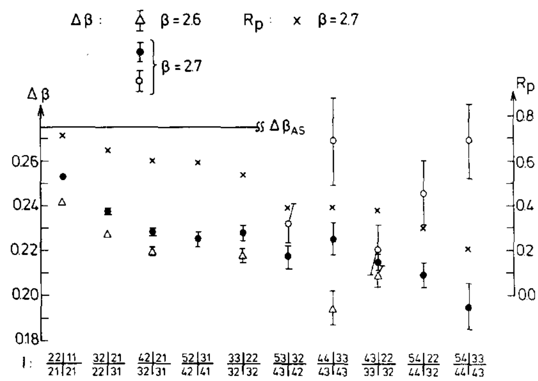

Fig. 2. Values of $\Delta\beta$ for square and for elongated Creutz ratios at $\beta_1=2.6$ and $\beta_2=2.7$. The symbol $\frac{32}{22}\big|\frac{11}{21}$ stands for the ratio $-\ln[W(3,2)W(1,1)/W(2,2)W(2,1)]$ etc. Open circles refer to $\Delta\beta$ derived from iteration 1500 to 6000 at β_2, and crosses denote the perturbative fraction in the measured ratios [see eq (8)]. The horizontal line gives the prediction of the two-loop β-function.

The ratios have been ordered according to the perturbative fraction

$$R_P(l)=\chi_P(l,\beta_A)/\chi(l,\beta_A)\,, \qquad (8)$$

which quantity is shown by crosses in fig. 2. The ordering with respect to $R_P(l)$ almost agrees with a naive geometrical ordering, where the ratio is interpreted as a potential difference, and for equal spatial distance the ordering is with respect to the timelike extension. Two observations are evident from fig. 2:

(1) For fixed l, $\Delta\beta$ is an increasing function of β, i.e., deviations from AS decrease with increasing β in the region under study.

(2) $\Delta\beta$ drops with increasing loop size, which means that there is only approximate scaling. Furthermore one sees that $\Delta\beta$ closely follows $R_P(l)$.

Property (1) does not necessarily imply that SU(2) lattice theory will return to AS in a way which is useful for physics. For this purpose objects of fixed physical size have to approach AS for increasing β, which means that we should look at the β-variation of $\Delta\beta$ for objects of increasing size in lattice units. Unfortunately there is still not enough information in the present data to test this behaviour in a straightforward way because of the many scales contained in a Creutz ratio. I therefore consider the fraction of the perturbative contribution as a variable playing the role of a size. In fig. 3 many more values

[‡2] Values of $\Delta\beta$ derived from the sweeps no. 1500 to 6000 at β_2 are also included. This material corresponds roughly to the amount of data collected in refs. [1,2]. Both the deviations from the results from the later iterations and the large errors for large ratios show that our statistics at β_2 is necessary to establish scaling violations.

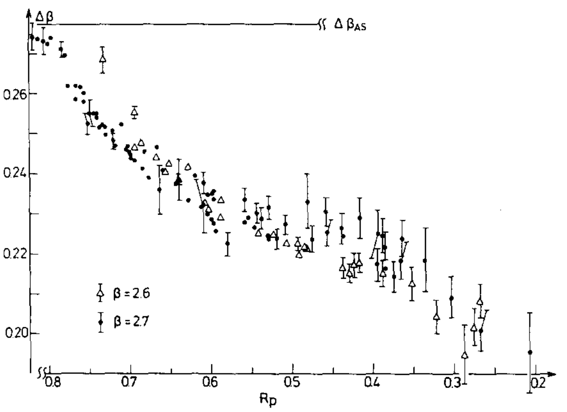

Fig. 3. Values of $\Delta\beta$ for all ratios allowed by the cuts given in the text, plotted as function of R_P [see eq. (8)]. Data are from β_1 and β_2. Many error bars have been omitted.

of $\Delta\beta$ than in fig. 2 are plotted, now as a function of $R_P(l)$. The following selection criteria were applied: The correction factor $c_P(l, \beta_A)$ should not deviate from 1 by more than 20%, and its value derived from the tree approximation should agree with the one-loop result within 5%. Only ratios with positive values and with an area difference between numerator and denominator of 1 and 2 were considered.

One notices that the ratios tend to fall on one curve with a spread of $\Delta\beta \approx \pm 0.06$. Property (2) is therefore likely to be a consequence of the fact that the $\chi_P(l, \beta)$ give a dominant contribution for ratios of small loops, but not for those of large loops. We have to conclude that there is evidence that nonperturbative quantities vanish faster than Λ_{latt} in the presently accessible region of β and that there is no evidence for a change in this trend.

Of course, it cannot be excluded that the multiplicative method of removing lattice artifacts is incorrect for the contributions of $O(g^6)$ and for the higher-order terms. We therefore should be especially cautious to infer scaling violations alone from the variation of $\Delta\beta$ of ratios including length 1. The difference between the ratios $\frac{54|33}{44|43}$ and $\frac{33|22}{32|32}$, however, amounts to more than 3 s.d. and should be considered as statistically significant. Even larger differences exist between ratios of essentially different shape. One argument in favour of the scaling violations being real is the fact that without the perturbative improvement of ratios the $\Delta\beta$'s would scatter around the band visible in fig. 3 with a spread larger by a factor of 10 or more than with improvement. The second argument is that the potential at short distances (after correction) agrees so well with two-loop perturbation theory. The quality of this agreement can certainly be checked for larger values of β in the near future without exorbitant computer resources. Finally, it has to be noticed that $\Delta\beta$ does not depend on how often the length 1 appears in the ratio. I.e., ratios like $\frac{22|22}{51|11}$, $\frac{31|22}{51|11}$, $\frac{32|11}{31|21}$ or $\frac{32|31}{51|21}$ show no deviation from the band in fig. 3, which contains many ratios with length 1 appearing only once. Nevertheless, for the ratios there does not seem to exist a rigorous method to get rid of finite "a"-effects except the expensive one to measure operators of larger extension on larger lattices. Although the claim for substantial scaling violations thus needs further corroboration, there is certainly no positive evidence for scaling from the present Monte Carlo data, which are much more precise than corresponding SU(3) data [1,2].

As to the apparent discrepancy between the scaling behaviour of the critical temperature on one side and of Creutz ratios on the other side, only a speculative remark can be made. From the potential it is clear that large violations of scaling occur only if we insist on all dimensions of ratios to become large (e.g., we begin to see large effects for loops of size 6×4 on the large lattice). For the critical temperature the situation is different. There large distances are enforced only in the timelike direction, whereas in the spacelike directions no scale is specified. It may be well that in spacelike directions short-distance contributions play a crucial role.

A qualitative statement on the magnitude of the departure from AS may be in order. We see that for the largest ratios a change in scale by a factor of 2 is accomplished by a change $\Delta\beta \approx 0.20$ both at β_1 and at β_2. Thus going from $\beta=2.4$ to $\beta=2.7$, the string tension will change a factor of 8, whereas AS predicts a change by a factor of 4.55. This is a substantial discrepancy. It has the consequence that the traditional way [22,23] to measure the static $q\bar{q}$-potential over a wide range of physical distances does not seem to be legitimate. If one keeps R small in lattice units but varies β over the indicated range and invokes AS, the asymptotic slope of $V(R)$ may be in error by about a factor 2. Thus the existence of a linear term in the potential at large β needs much better support than available up to now.

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