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The hadronic and point-like contributions to the F_2 photon structure function in perturbative QCD

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Abstract. A detailed discussion is given of the hadronic and point-like contributions to the F_2 photon structure function (F_2^γ) in both the naive parton model and QCD. The non-singlet part of the leading order solution, first found by Witten, is re-derived using the QCD improved parton model, enabling the hadronic and point-like terms to be clearly identified and correlated with observed jet structure in the final state. When important non leading-log terms are included, the sensitivity of the solution to Λ is found to be weak for all Q^2 values, and the all orders solution to be well approximated by the $O(\alpha_s^2)$ solution. The approximations made in deriving the leading order solution are critically examined, and an approach enabling more quantitative tests of QCD to be made from measurements of the point-like (perturbative) component of F_2^γ is suggested.

The existence of hadronic [1–3] and 'point-like' [4] parts in the F_2 photon structure was theoretically predicted many years ago. Both contributions have been confirmed by recent experimental measurements [5–9].

In a recent letter [10] we discussed the distinction between F_2^{HAD} and F_2^{PL} (HAD = hadronic, PL = point-like) in the parton model and QCD noting, following Peterson, Walsh and Zerwas [11], that the jet structure in the final state gives an experimental signature for F_2^{HAD} and F_2^{PL}. Introducing a cutoff on the jet p_T relative to the $\gamma\gamma^*$ axis to separate the hadronic (non-perturbative) and point-like (perturbative) contributions, the calculable value of F_2^{PL} was found to be very insensitive to the QCD scale parameter Λ. This is in contradiction to several claims in the published literature [12–15] but in agreement with the conclusions of Glück and Reya [16]. That QCD corrections to the naive parton model predictions of F_2^{PL} for a real photon target are expected to be small at experimentally accessible Q^2 values was first pointed out by Hill and Ross [17] and by Chase [18]. As the crucial point in this continuing controversy [10, 15, 19] seems to be different definitions of the terms 'hadronic' and 'point-like' in relation to the photon structure function, this paper examines the question in greater detail.

Further points discussed are the sensitivity of the complete leading order solution for F_2^{PL} (to be defined below) to Λ, and the effect of truncating the perturbation series in this solution at $O(\alpha_s)$ or $O(\alpha_s^2)$. Finally the approximations necessary to derive the solution are critically examined and proposals made for obtaining more quantitative comparisons of theory and experiment for F_2^γ.

Defining quark densities $q(Q^2, x)$ via the relation:

$$F_2^\gamma = \sum_q e_q^2 x [q(Q^2, x) + \bar{q}(Q^2, x)] \tag{1}$$

the complete leading order solution for the moments of q, defined by

$$q(Q^2, n) = \int_0^1 q(Q^2, x)\, x^{n-1}\, dx$$

is:

$$q(Q^2, n) = \sum_i \left\{ A_i(\mu^2, n)\, r^{-d_i^n} - \frac{a_i(n)}{1-d_i^n} \left[\ln\left(\frac{Q^2}{\Lambda^2}\right)\right]\left[1 - r^{1-d_i^n}\right]\right\} \tag{2}$$

Here $i = +, -$, NS labels the components of the one-loop anomalous dimension matrix, and d_i^n, given for example in [20], are proportional to the corresponding anomalous dimensions. Also:

$$r=\ln(\mu^2/\Lambda^2)/\ln(Q^2/\Lambda^2)=\alpha_s(Q^2)/\alpha_s(\mu^2)$$

where the one-loop formula for α_s is used and μ^2 is an arbitrary renormalisation scale. $a_i(n)$ gives the moment of the leading-log Born term contribution [10].

This formula has been derived using several different calculational techniques: The Operator Product Expansion (OPE) and Renormalisation Group Equation (RGE) [21], summing ladder graphs in an axial gauge [22, 23] or by seeking a solution of suitably modified Altarelli-Parisi [24] Equations [25]. Because μ^2 is arbitrary only the leading-log (LL) term:

$$q^{\mathrm{LL}}(Q^2, n)=\frac{a_i(n)}{1-d_i^n}\ln\left(\frac{Q^2}{\Lambda^2}\right) \tag{3}$$

is predicted by the theory. All other terms (which depend on μ^2) are usually called 'uncalculable'. In fact (2) may be rewritten as:

$$q(Q^2, n)=\sum_i\left\{\tilde{A}_i(\mu^2, n)\, r^{-d_i^n}+\frac{a_i(n)}{1-d_i^n}\ln\left(\frac{Q^2}{\Lambda^2}\right)\right\} \tag{4}$$

where:

$$\tilde{A}_i(\mu^2, n)=A_i(\mu^2, n)+\frac{a_i(n)}{1-d_i^n}\ln(\mu^2/\Lambda^2). \tag{5}$$

Since the first terms in the curly bracket in (2), (4) have the same Q^2 dependence as the proton structure function both terms are frequently referred to in the literature as 'uncalculable hadronic terms' [12, 14, 15]. For example in [12, 14, 15] the $\tilde{A}_i(\mu^2, n)$ are parametrised by poles with adjustable residues λ chosen to cancel singularities that occur in (3) for certain values of d_i^n [26, 27]. These poles do not occur in the complete solution (2) and become a problem only if (2) is written in the form (4) and one attempts to neglect, say at very high Q^2, the $\tilde{A}(\mu^2, n)$ term.

Below we re-derive (2) using the QCD improved parton model, where the Altarelli-Parisi Equations are iterated to arbitrary order in α_s. In this case we find that for a suitable choice of the scale μ^2 (determined experimentally by the jet structure of the final state) the identification:

$$A_i(\mu^2, n)=q^{\mathrm{HAD}}(\mu^2, n)$$

is possible, where q^{HAD} corresponds to the well-known phenomenologically determined [11] and experimentally measured [9] hadronic photon structure function:

$$F_2^{\mathrm{HAD}}=\alpha 0.2(1-x).$$

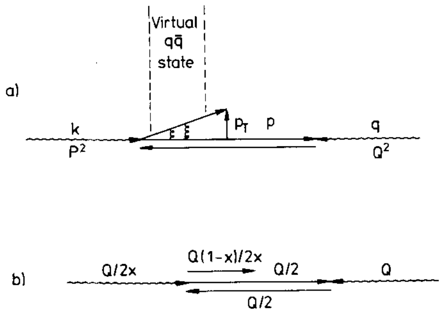

Fig. 1. a Definition of kinematic variables in the parton model description of the photon structure function. $k(q)$ are the 4-vectors of the target (probe) photons. **b** Momenta in the Breit frame of the probe photon, for massless partons and $p_T=0$

The second term in (5) then gives a calculable (since μ^2 is now known) non-leading log (NLL) but purely *point-like* contribution. There is then no freedom for an arbitrary parametrisation of the supposedly unknown 'hadronic' term $\tilde{A}_i(\mu^2, n)$ mainly with a view to removing the troublesome singularities which occur if the LL term (3) is considered in isolation [12, 14, 15]. The new element in the argument is the experimental knowledge of the phenomenological scale μ^2 which is specified by the boundary between the hadronic and point-like regions as seen [28, 29, 9] in the final state jet structure. We note in passing here that 'sub-leading' corrections to (2) contributing additional constant and lnln Q^2 terms have been calculated [30, 31]. These terms, which originate in two-loop corrections to α_s and the anomalous dimensions, should not be confused with the numerically very important NLL terms which are part of the leading order solution (2). The sub-leading terms are inessential to the general arguments presented below, and will not be considered further here. Before proceeding to the derivation of (2) we briefly review the physics of hadronic photon structure [32] to clarify the physical distinction between the hadronic and point-like contributions to F_2^γ.

It is convenient to work in the Breit frame of the probe photon with 4-vector momentum q (Fig. 1a). In this frame the momentum of the probe photon is $\sqrt{-q^2}=Q$, and its energy is zero. The 3-momentum vectors for the target and probe photons and the struck and spectator quarks are shown in Fig. 1b for a real photon target, massless quarks and $p_T=0$.

In this frame the characteristic struck-quark probe photon interaction time τ_i is $\simeq 1/Q$. Because of vacuum polarisation effects the target photon γ_t^* of $(\mathrm{mass})^2=-P^2$ evolves into virtual states:

$$\gamma_t^*\to q\bar{q},\ q\bar{q}g,\ \bar{q}qq\bar{q},\ \ldots$$

The lifetime τ^* of such states can be estimated by the Uncertainty Principle [32]:

$$\tau^* \simeq \frac{1}{\Delta E} = \frac{1}{E^* - E_{\gamma_t^*}} \simeq \frac{2k}{M^2+P^2} = \frac{Q}{x(M^2+P^2)} \quad (6)$$

where it is assumed that $M^2+P^2 \ll k^2$. E^* and M are the energy and mass of the virtual state. The parton model description of the structure function, with factorisation between the quark distribution function $q(Q^2, x)$ and the probe-photon quark hard scattering process $\gamma_p^* q \to q$, requires the condition:

$$\tau_i \ll \tau^*$$

or:

$$\frac{Q^2}{x(M^2+P^2)} \gg 1 \quad (7)$$

If the virtual state is $q\bar{q}$ then:

$$M^2 \simeq 4(p_T^2 + m_q^2) = 4m_T^2$$

where $p_T(m_T)$ are the transverse momentum (mass) of the quarks relative to the target photon direction and m_q their constituent mass.

If the $q\bar{q}$ are in a bound state:

$$M = M_v, \qquad v = \rho, \omega, \varphi, \rho', \ldots$$

(6) leads to the well-known Vector Meson Dominance (VMD) form factor. As the p_T of the spectator quark at the target photon vertex increases the struck quark becomes highly virtual with $(\text{mass})^2 = -p^2$. Gluon exchange between q and $\bar{q}$ is unfavoured by the smallness of the strong coupling constant ($\alpha_s(p^2) \ll 1$) and τ^* becomes small. In accordance with asymptotic freedom ideas, a point-like $\gamma_t^* q\bar{q}$ coupling then emerges. Similarly a point-like coupling becomes dominant when P^2 is large. Considering now the virtual $q\bar{q}$ state, for small values of p_T multiple gluon exchange (bound state) effects are dominant and the p_T distribution of the spectator quark is exponential, characteristic of a non-perturbative interaction. For larger values of p_T the $q\bar{q}$ lie well outside the wave-functions of light hadron systems and a scaling p_T distribution $\simeq p_T^{-4}$ corresponding to a point-like $\gamma_t^* q\bar{q}$ coupling is expected. Such a transition between the perturbative and non-perturbative regions is in fact observed in the p_T distribution of both final state hadrons [28] and jets [9, 29].

The hadronic structure of the photon may, alternatively, be probed in real-γ hadron collisions [32] or in almost-real virtual $\gamma\gamma$ collisions. In the first case the photon structure is probed by a hadronic collision in which the virtual vector meson itself, rather than its quark substructure, interacts. In almost real $\gamma\gamma$ collisions a similar interaction occurs with symmetry between 'probe' and 'target' photons. The total $\gamma\gamma$ cross section for $W > 2$ GeV/c^2 is well described by virtual vector meson-vector meson scattering [33].

In the above discussion only the lowest order (parton model) contribution to F_2^γ was considered. In perturbative QCD the parton model hard scattering process $\gamma_p^* q \to q$ is replaced by higher order QCD processes [20]:

$$\gamma_p^* q \to qg, qgg, qq\bar{q}, \ldots.$$

The distinction between the hadronic and point-like regions of phase space, signed by the p_T distribution at the target vertex, is however unchanged, enabling the separation of pointlike and hadronic terms according to the jet configuration to be maintained in the presence of QCD corrections [10].

The non-singlet (NS) hadronic and point-like quark densities for light (u, d, s) flavours are then given by the iterated Altarelli-Parisi equations [10]:

$$q_N^{\mathrm{HAD}}(Q^2, x) = \int_{y_2}^{1} q^{\mathrm{HAD}}(t_0, y_1) \frac{dy_1}{y_1} C_N \quad (8\text{a})$$

$$q_N^{\mathrm{PL}}(Q^2, x) = \frac{3\alpha e_q^2}{2\pi} \int_{y_2}^{1} [y_1^2 + (1-y_1)^2] \frac{dy_1}{y_1} \int_{t_0}^{t_1^{\mathrm{MAX}}} \frac{dt_1}{t_1} C_N \quad (8\text{b})$$

where:

$$C_N = P_{qq}\left(\frac{y_2}{y_1}\right) \frac{b}{\ln\left(\frac{t_2}{\Lambda^2}\right)} \left\{ \prod_{i=2}^{N} \int_{y_{i+1}}^{1} \frac{dy_i}{y_i} \int_{t_i^{\mathrm{MIN}}}^{t_i^{\mathrm{MAX}}} \frac{dt_i}{t_i} \cdot P_{qq}\left(\frac{y_{i+1}}{y_i}\right) \frac{b}{\ln\left(\frac{t_{i+1}}{\Lambda^2}\right)} \right\} \int_{t_{N+1}^{\mathrm{MIN}}}^{t_{N+1}^{\mathrm{MAX}}} \frac{dt_{N+1}}{t_{N+1}}.$$

Here $t_i = m_q^2 - p_i^2$ where p_i is the 4-vector momentum of the ith virtual quark, y_i is the energy splitting fraction $y_i = p_i^0/k$, $b = 6/25$ (corresponding to 4 quark flavours) and $P_{qq}(z)$ is the well-known [24] Altarelli-Parisi splitting function. For simplicity the NS label is dropped in (8).

The factorisation of the QCD correction is manifested in (8) by the convolution integral C_N which occurs in both hadronic and point-like parts and corresponds to the radiation of N real gluons. For $N=1$ the curly bracket in the expression for C_N is replaced by unity. The scale parameter t_0 is $-$ (4-momentum transfer)^2 of the virtual quark at the target photon vertex at the boundary between the 'hadronic' and 'point-like' regions of phase space. If $k^2(1-y_1)^2 \gg P^2$, m_q^2, $(p_T^0)^2$ then:

$$t_0 = y_1 P^2 + \frac{m_q^2 + (p_T^0)^2}{1-y_1}. \quad (9)$$

For light quarks t_0 is determined experimentally as the jet $p_T(p_T^0)$ at which point-like (p_T^{-4}) behaviour becomes evident. For highly virtual target photons $t_0 = y_1 P^2$ [34], while for heavy quarks $t_0 = m_q^2/(1-y_1)$ [17].

Since the hadronic structure function is non-perturbative $q^{\mathrm{HAD}}(t_0, y_1)$ in (8a) must be determined either phenomenologically [11] or by direct measurement [9]. $q^{\mathrm{HAD}}(t_0, y_1)$ implicity contains an integral over the p_T of the spectator quark from $0 < p_T < (1-y_1)\,t_0$. The corresponding integral over the range $t_0 < t_1 < t_1^{\mathrm{MAX}}$ is given explicitly for the point-like part in (8b). For large values of P^2, q^{HAD} is suppressed by a VMD form factor:

$$q^{\mathrm{HAD}}(t_0, P^2, x) = \frac{m_\rho^4}{(m_\rho^2 + P^2)^2}\, q^{\mathrm{HAD}}(t_0, 0, x). \tag{10}$$

To derive the 'leading order' solution from (8) several additional approximations must be made:

(*i*) The convolution integrals are restricted to the 'ordered' region of phase space [35]:

$$t_i^{\mathrm{MIN}} = t_0, \quad t_i < t_{i+1}, \quad t_{N+1}^{\mathrm{MAX}} = Q^2, \quad y_{N+1} = x$$

(*ii*) t_0 is assumed to be constant (independent of t_i, y_i)

These approximations result in the dropping of some 'non-leading' terms in the convolution integral.

With (*i*)–(*ii*) the y convolution in (8) can be decoupled by taking moments giving:

$$q_N^{\mathrm{HAD}}(Q^2, n) = q^{\mathrm{HAD}}(t_0, n)\, I_N^{\mathrm{HAD}} \tag{11a}$$

$$q_N^{\mathrm{PL}}(Q^2, n) = a(n)\, I_N^{\mathrm{PL}} \tag{11b}$$

where:

$$I_N^{\mathrm{HAD}} = \int_{t_0}^{Q^2} \frac{dt_{N+1}}{t_{N+1}} \frac{\beta}{\ln\left(\frac{t_{N+1}}{\Lambda^2}\right)} \int_{t_0}^{t_{N+1}} \frac{dt_N}{t_N} \frac{\beta}{\ln\left(\frac{t_N}{\Lambda^2}\right)} \cdots \int_{t_0}^{t_3} \frac{dt_2}{t_2} \frac{\beta}{\ln\left(\frac{t_2}{\Lambda^2}\right)}$$

$$I_N^{\mathrm{PL}} = \int_{t_0}^{Q^2} \frac{dt_{N+1}}{t_{N+1}} \frac{\beta}{\ln\left(\frac{t_{N+1}}{\Lambda^2}\right)} \int_{t_0}^{t_{N+1}} \frac{dt_N}{t_N} \frac{\beta}{\ln\left(\frac{t_N}{\Lambda^2}\right)} \cdots \int_{t_0}^{t_2} \frac{dt_1}{t_1}$$

$$a(x) = \frac{3\alpha e_q^2}{2\pi}\,[x^2 + (1-x)^2] \quad \text{and} \quad \beta = d_{\mathrm{NS}}^n = \frac{6}{25} P_{qq}(n).$$

A recurrence relation exists relating I_N^{HAD}, I_N^{PL}:

$$I_N^{\mathrm{PL}} = \beta I_{N-1}^{\mathrm{PL}} - \ln\left[\left(\frac{t_0}{\Lambda^2}\right)\right] I_N^{\mathrm{HAD}}. \tag{12}$$

Multiple use of (12) gives:

$$I_N^{\mathrm{PL}} = \beta^N I_0^{\mathrm{PL}} - \left[\ln\left(\frac{t_0}{\Lambda^2}\right)\right] \sum_{M=1}^{N} I_M^{\mathrm{HAD}}. \tag{13}$$

The integrals I_N^{HAD}, I_0^{PL} are:

$$I_N^{\mathrm{HAD}} = \frac{\{\beta \ln[\ln(Q^2/\Lambda^2)/\ln(t_0/\Lambda^2)]\}^N}{N!}$$

$$I_0^{\mathrm{PL}} = \ln\left(\frac{Q^2}{t_0}\right).$$

So (11) can be expressed as:

$$q_N^{\mathrm{HAD}}(Q^2, n) = q^{\mathrm{HAD}}(t_0, n)\, \frac{[-\beta \ln R]^N}{N!} \tag{14a}$$

$$q_N^{\mathrm{PL}}(Q^2, n) = a(n) \left\{ \beta^N \ln\left(\frac{Q^2}{t_0}\right) - \left[\ln\left(\frac{t_0}{\Lambda^2}\right)\right] \beta^N \sum_{M=1}^{N} \frac{(-\ln R)^M}{M!} \right\} \tag{14b}$$

where:

$$R = \ln(t_0/\Lambda^2)/\ln(Q^2/\Lambda^2) = \alpha_s(Q^2)/\alpha_s(t_0)$$

Summing up all orders in α_s gives:

$$\begin{aligned} q^{\mathrm{HAD}}(Q^2, n) &= \sum_{N=0}^{\infty} q_N^{\mathrm{HAD}}(Q^2, n) = q^{\mathrm{HAD}}(t_0, n) \exp(-\beta \ln R) \\ &= q^{\mathrm{HAD}}(t_0, n) R^{-d_{\mathrm{NS}}^n} \end{aligned} \tag{15a}$$

and

$$\begin{aligned} q^{\mathrm{PL}}(Q^2, n) &= a(n) \left\{ \ln\left(\frac{Q^2}{t_0}\right) \sum_{N=0}^{\infty} \beta^N - \left[\ln\left(\frac{t_0}{\Lambda^2}\right)\right] \sum_{N=1}^{\infty} \beta^N \sum_{M=1}^{N} \frac{(-\ln R)^M}{M!} \right\} \\ &= a(n) \left\{ \frac{\ln\left(\frac{Q^2}{t_0}\right)}{1-\beta} - \left[\ln\left(\frac{t_0}{\Lambda^2}\right)\right] \frac{(R^{-\beta} - 1)}{1-\beta} \right\} \\ &= \frac{a(n)}{1 - d_{\mathrm{NS}}^n} \left[\ln\left(\frac{Q^2}{\Lambda^2}\right)\right] \left[1 - R^{1 - d_{\mathrm{NS}}^n}\right]. \end{aligned} \tag{15b}$$

It can be seen that q^{HAD} has the logarithmic Q^2 evolution typical of a hadron (e.g. the proton) structure function. The origin of the well-known $\ln(Q^2/\Lambda^2)$ term in q^{PL} is evident in (15b). At each order in α_s the LL term is $\simeq \ln(Q^2/t_0)$ (independent of Λ) [10]. On performing the double sum to infinity in the first line of (15b) however the sum over the NLL terms generates a constant $\ln(t_0/\Lambda^2)/(1-\beta)$, which in combination with the LL term $\ln(Q^2/t_0)/(1-\beta)$ gives the $\ln(Q^2/\Lambda^2)/(1-\beta)$ dependence first found by Witten [21]. If the perturbation series is truncated at any finite power of α_s however, the leading term in the Bjorken limit is $\simeq \ln(Q^2/t_0)$, independent of Λ.

Comparing (2) and (15) it can be seen that $r=R$ when $\mu^2=t_0$. This QCD improved parton model derivation then shows that μ^2 should indeed be associated with the boundary of the perturbative and non-perturbative regions. If μ^2 is chosen $<t_0$ in (2) then the first term will underestimate the true hadronic contribution, and the second term will incorrectly represent as point-like a part of the hadronic contribution. On the other hand if $\mu^2>t_0$ a part of the point-like distribution will be incorrectly described by the first term in (2) as hadronic. The point-like term with lower cut off $\mu^2>t_0$ gives however a valid prediction, providing that a corresponding cut (in jet p_T) is made in the experimental distribution.

Simply from the condition that (2), (15) are derived perturbatively, the scales μ^2, t_0 cannot be too small. The condition $\alpha_s<1$ implies that actually $\Lambda^2 \ll \mu^2$ which is sufficient to establish that the NLL terms $\simeq r^{1-d_i^n}$ are never negligible as compared to the LL term $a_i(n)\ln(Q^2/\Lambda^2)/(1-d_i^n)$ for any experimentally interesting value of Q^2. In fact the experimental measurements of jet structure in the final state [9, 28, 29] indicate that $t_0 \simeq 1\,(\mathrm{GeV/c})^2$ so the condition $\Lambda^2 \ll t_0$ for the validity of perturbation theory is well satisfied for values of Λ in the range 50–200 MeV/c.

The importance of the NLL terms is shown in Figs. 2a, b, c where $q^{\mathrm{PL,NS}}(Q^2, n)/a(n)$ is plotted as a function of Q^2 in the range $10<Q^2<10^4\,(\mathrm{GeV/c})^2$ for $n=2$, 5, 10 and for different Λ values $\Lambda=0$, 50, 200 MeV/c, assuming that $t_0=1\,(\mathrm{GeV/c})^2$. $\Lambda=0$ corresponds to the Born term in the LL approximation:

$$a(n)\ln\left(\frac{Q^2}{t_0}\right)$$

which is the correct $\Lambda \to 0$ limit of (15b). If instead (4) is used with an arbitrarily parameterised 'hadronic' part $\tilde{A}(\mu^2, n)$ as in [12, 14, 15] the Born term is logarithmically divergent. The LL solution (3), with $i=\mathrm{NS}$, is also shown for $\Lambda=50$, 200 MeV/c (dashed lines) in Fig. 2. Even for Q^2 as large as $10^4 (\mathrm{GeV/c})^2$ the NLL terms are important. It can also be seen,

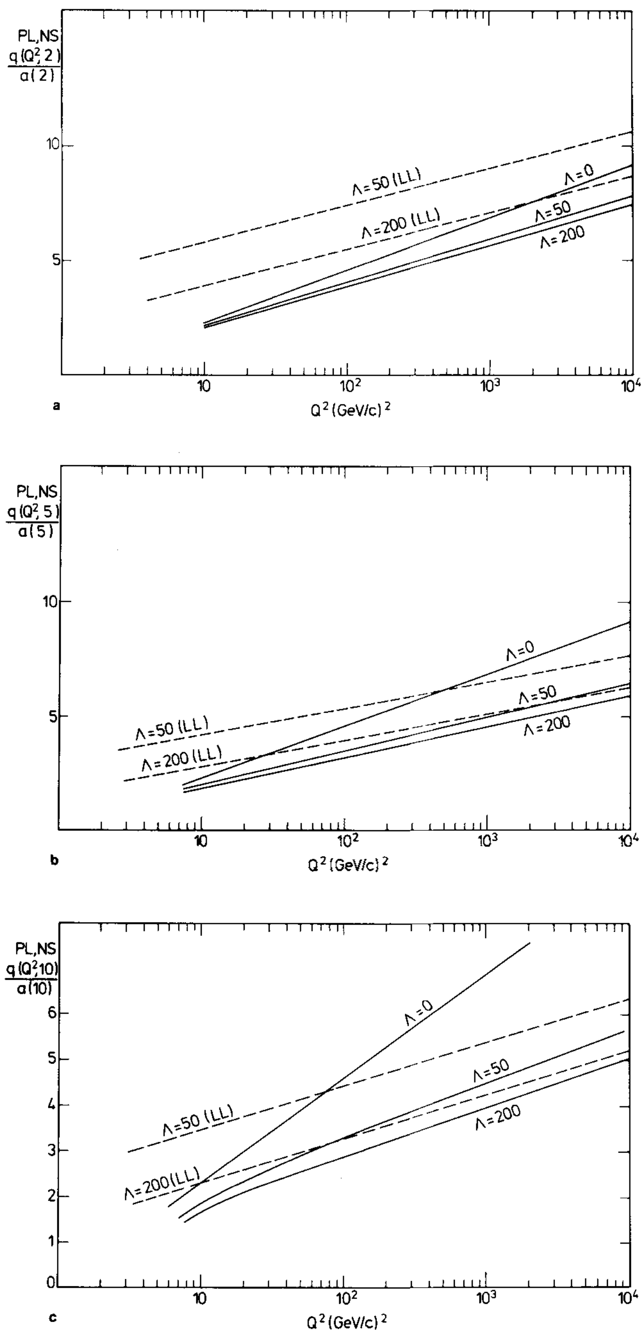

Fig. 2a–c. The leading order solution for the non-singlet moments of F_2^γ (solid lines) compared with the leading-log approximation (dashed lines) for various Λ values. $t_0=1\,(\mathrm{GeV/c})^2$ is assumed. **a** $n=2$, **b** $n=5$, **c** $n=10$

as pointed out previously [10, 16–18], that the sensitivity to Λ is very weak. In fact the sensitivity of the leading order solution (2), (15) to Λ varies very little with Q^2. For example $q^{\mathrm{PL,NS}}(Q^2, 5)/a(5)$ changes by 9% if Λ is increased from 50 to 200 MeV/c when $Q^2=10\,(\mathrm{GeV/c})^2$. The corresponding change when

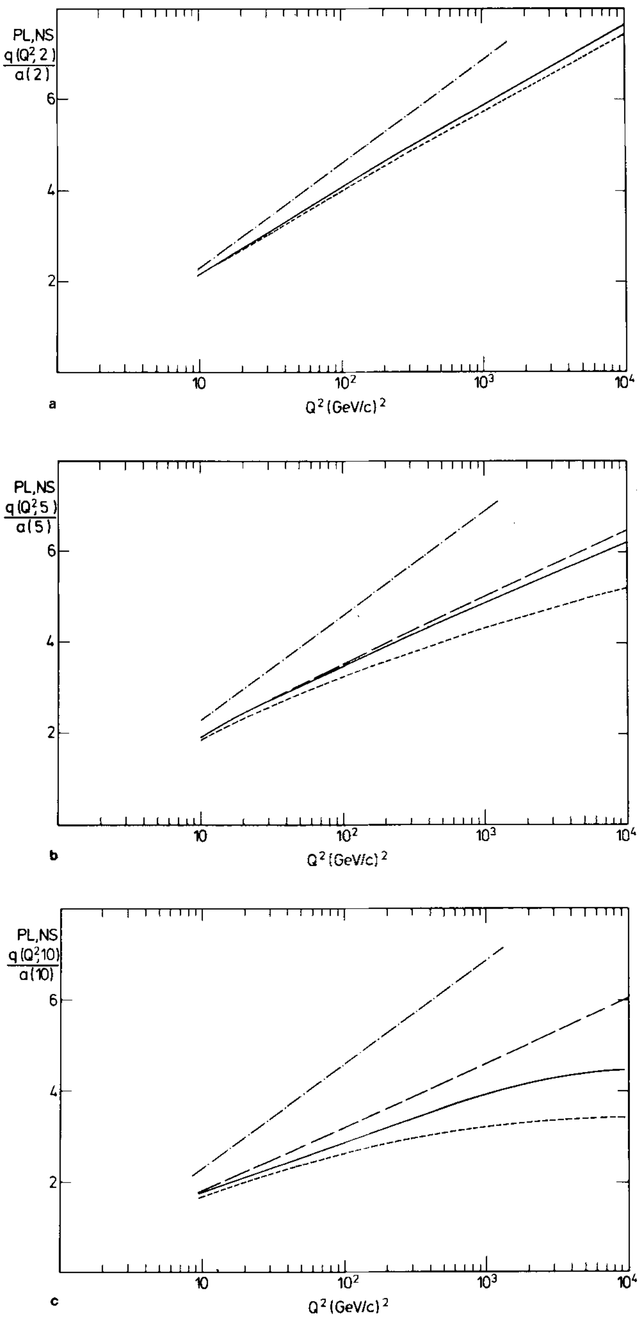

Fig. 3a–c. The leading order solution for the non-singlet moments of F_2^γ, summed to all orders in α_s (solid line) compared with solutions truncated at $O(\alpha_s)$ (fine dashed line) and $O(\alpha_s^2)$ (broad dashed line). $\Lambda = 100$ MeV/c, $t_0 = 1$ (GeV/c)^2. The dot-dashed line is the Born term ($\Lambda = 0$). **a** $n=2$, **b** $n=5$, **c** $n=10$

$Q^2 = 10^{12}$ (GeV/c)^2 is even less, 6.6%. So, although the LL term becomes a better approximation to the leading order solution at such astronomically high Q^2 values, the sensitivity to Λ actually becomes worse.

In Fig. 3a–c the all orders QCD prediction (15b) is compared with respectively the Born term, $O(\alpha_s)$, $O(\alpha_s^2)$ solutions given by summing up to $N=0$, 1, 2 in the first line of (15b). $q^{\mathrm{PL,NS}}(Q^2, n)/a(n)$ is plotted as a function of Q^2 for $n=2$, 5, 10 with $\Lambda = 100$ MeV/c and $t_0 = 1$ (GeV/c)^2. In Fig. 3a ($n=2$) only the $O(\alpha_s)$ curve is shown, as the $O(\alpha_s^2)$ curve is essentially identical. Figure 3 shows that the $O(\alpha_s)$ or $O(\alpha_s^2)$ solutions give a good approximation to the all orders result for $Q^2 \lesssim 100$ (GeV/c)^2.

Although only the non-singlet contribution has been discussed above, a similar separation of the hadronic and pointlike parts of the singlet moments can also be made. For the singlet case the convolution integral C_N in (8) is replaced by $C_{N,M}$ where N is the number of real gluons and M the number of real q, $\bar{q}$ (M is even) produced in the QCD evolution. Because of the complexity of the convolution integrals in the singlet case the solution is found more conveniently by OPE and RGE methods, leading to the $i=+, -$ terms in (2), which actually have a similar Q^2 dependence to the $i=\mathrm{NS}$ term. Clearly the identification $\mu^2 = t_0$ on the basis of the final state configuration holds also for the singlet contribution, as the hadronic/point-like separation depends essentially on the p_T of the spectator quark at target photon vertex (Fig. 1). This will not be affected by the existence (or not) of virtual gluons in the subsequent perturbative QCD evolution for the singlet (non-singlet) terms.

A critical discussion is now given of the approximations made in the derivation of the leading order solution (2). This solution is compared with an 'exact' one where, at each order in α_s, a gauge invariant set of Feynman diagrams is summed, exact kinematics is used and α_s is allowed to run with a scale defined by the four-momentum squared of the most off-shell parton at the relevant $q\bar{q}g$ or ggg vertex. This definition of the scale of α_s gives the correct correspondence between the arbitrary scale μ^2 of the OPE, RGE derivation of (2) and the cut-off t_0 of the QCD improved parton model formula (15). It also corresponds to the scale conventionally chosen in the perturbative calculation of the QCD beta function that gives the one or two loop formulae for the running coupling constant in QCD [20]. In fact from Lorentz invariance, a vertex function or coupling constant can depend only on this variable, up to an arbitrary multiplicative constant.

The comparison can of course only be made for the point-like contribution, and for configurations where perturbation theory should be valid. The necessary Feynman diagram calculation has so far not been done for $O(\alpha_s)$ and higher order terms in F_2^γ, so a precise comparison can only be done at $O(\alpha_s^0)$ i.e. for the Born term.

The leading order Born term is given by taking the $\Lambda = 0$ limit of (2). The corresponding asymptotic

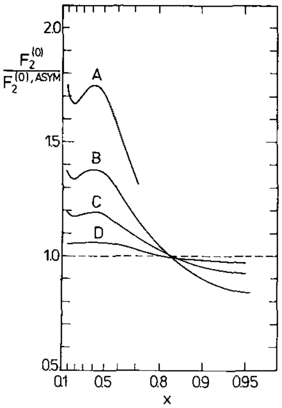

Fig. 4. The ratio of the parton model prediction (including non logarithmic terms) for F_2^γ, $F_2^{(0)}$ to the asymptotic leading-log parton model prediction $F_2^{(0),\mathrm{ASYM}}$. $t_0 = 1\ (\mathrm{GeV/c})^2$ is assumed. Curves A, B, C, D correspond to $Q^2 = 10, 100, 10^4, 10^{12}\ (\mathrm{GeV/c})^2$

F_2 structure function is:

$$F_2^{(0),\mathrm{ASYM}}(Q^2, x, t_0) = \frac{3\alpha}{\pi} \sum_q e_q^4 x [x^2 + (1-x)^2] \ln(Q^2/t_0) \tag{16}$$

where:

$t_0 = \mu^2$.

This may be compared with the result of a Feynman diagram calculation [36] where non-leading terms are also included. Neglecting terms of order m_q^2/Q^2 and m_q^2/t_0 the point-like structure function corresponding to the integration interval:

$t_0 < t < t_{\mathrm{MAX}}$

is:

$$F_2^{(0)}(Q^2, x, t_0) = \frac{3\alpha}{\pi} \sum_q e_q^4 x \left\{ [x^2 + (1-x)^2] \ln\left(\frac{t_{\mathrm{MAX}}}{t_0}\right) + [6x(1-x) - 1]\left[1 - \frac{t_0}{t_{\mathrm{MAX}}}\right]\right\} \tag{17}$$

Figure 4 shows the ratio $F_2^{(0)}/F_2^{(0),\mathrm{ASYM}}$ as a function of x taking a fixed $t_0 = 1\ (\mathrm{GeV/c})^2$ and $t_{\mathrm{MAX}} = Q^2/x$. Curves are shown for $Q^2 = 10, 100, 10^4, 10^{12}\ (\mathrm{GeV/c})^2$. Equation (17) gives F_2 to good accuracy for light quarks, except near the threshold region $x \simeq 1$. It can be seen that $F_2^{(0),\mathrm{ASYM}}$ is a bad approximation to $F_2^{(0)}$ for experimentally relevant Q^2 values $\lesssim 100\ (\mathrm{GeV/c})^2$ and $x \lesssim 0.7$. This is largely due to an important contribution of the non-logarithmic terms in (17).

If the LL term (3), plus a phenomenological hadronic term [11] is compared with experimental data in an attempt to extract a value for Λ the important non-logarithmic corrections to the Born term mentioned above and shown in Fig. 4 are not taken into account. It is easy to see why a value of 'Λ' of $\simeq 200$ MeV/c is found in such comparisons. As shown in [10] when the NLL terms and exact kinematics are taken into account the QCD correction for the PLUTO F_2 measurement [5] with $\langle Q^2 \rangle = 5.3\ (\mathrm{GeV/c})^2$ is small, the data being well described by $F_2^{(0)}$ added to a phenomenological hadronic contribution:

$F_2^{\mathrm{HAD}} = 0.2\alpha(1-x)$.

Suppose now that the same data are compared to the x-space version of (3):

$$F_2^{\mathrm{LL}}(Q^2, x) = \frac{3\alpha}{\pi} \sum_q e_q^4 f^{\mathrm{LL}}(x) \ln\left(\frac{Q^2}{\Lambda^2}\right). \tag{19}$$

As shown, for example, in [21]:

$f^{\mathrm{LL}}(0.5) = f^{\mathrm{BORN}}(0.5) = 0.25$

and since Λ is determined essentially by the normalisation of the theoretical curve (19) to the data, the expected value of 'Λ' for the PLUTO measurement is given by solving:

$F_2^{(0)}(x = 0.5) = F_2^{\mathrm{LL}}(x = 0.5)$

for $Q^2 = 5.3\ (\mathrm{GeV/c})^2$ using $F_2^{(0)}$ as given in (7) of [10] it is found that:

'Λ' $= 210$ MeV/c

which is certainly 'consistent with other experimental determinations of Λ'. This is however clearly a completely bogus determination of Λ and the 'good agreement' is purely fortuitous since both the numerically important NLL terms in (2) and the non-logarithmic contributions to the Born term, shown in (17), are neglected. The latter give a correction factor $\simeq 2$ for $Q^2 = 5\ (\mathrm{GeV/c})^2$.

Non-leading logarithmic and constant terms which give the very large corrections to the Born term shown in Fig. 4 may also give important corrections at $O(\alpha_s)$, $A(\alpha_s^2)$, ... Only exact Feynman diagram calculations as routinely done [37, 38] to estimate QCD corrections in the 1γ annihilation process $e^+e^- \to$ hadrons can estimate the importance of these terms. Since, as shown in Fig. 3, the $O(\alpha_s)$, $O(\alpha_s^2)$ corrections apparently saturate the leading order solution for $Q^2 < 100\ (\mathrm{GeV/c})^2$, the number of relevant diagrams to be evaluated may not be too large. Six diagrams must be summed for the $O(\alpha_s)$ correction. This calculation has in fact been done for the case of real $\gamma\gamma$ collisions [39].

Another reason why finite order QCD calculations may be the only ones that can be meaningfully compared with data at non-asymptotic energy scales, is that only in such cases can one have confidence in the applicability of perturbation theory. The sum to infinity which gives (15b) from (14b) involves, for the large N terms, many real, soft gluons. The spectra of such gluons is cut off by confinement effects (the colour singlet nature of the produced hadrons) at energy scales of $\simeq 1$–2 GeV. The hadronisation mechanism which cuts off the soft gluons can only at present be described by phenomenological methods. Similarly resumming soft gluons as in the calculation of the Sudakov form factor of a quark may be a perfectly well defined procedure mathematically, but in fact have little physical relevance in the presence of strong non-perturbative confinement effects [20]. Because of this the argument that only the lowest order ($O(\alpha_s^N)$ $N=1$, 2) terms are needed because they essentially saturate the solution (15b) is not on very firm ground. For the reasons just mentioned the higher order contributions to the 'all orders' solution which are calculated perturbatively, ignoring confinement effects, are probably unreliable, in consequence so is the conclusion that their contribution is small compared to the $O(\alpha_s)$, $O(\alpha_s^2)$ terms. As has always anyway been done in QED, the best approach may still be to calculate to a given order and compare with experiment, to see if higher orders are 'needed' or not.

The approximations which are made in going from an exact $O(\alpha_s^N)$ Feynman diagram calculation to a formula such as (14b) which gives the corresponding $O(\alpha_s^N)$ 'leading order' nonsinglet contribution are the following:

a) Purely kinematical approximations such as (i)–(ii) above, used to derive (15) starting from (8)

b) Non-leading terms in the Altarelli-Parisi splitting functions [20] are dropped

c) 'Sub-leading terms' due to two, three, ... loop corrections in both α_s and the Altarelli-Parisi functions are dropped

d) Certain terms in the full gauge invariant cross section are dropped when factorisation is assumed in the derivation [20, 24] of the Altarelli-Parisi Equations. These terms are unimportant only if p_T^2, $P^2 \ll Q^2/x$ (see (7)). If this condition is not satisfied $\tau_i \simeq \tau^*$ and the parton model description, which is the basis of the Altarelli-Parisi Equations, is no longer valid. This breakdown of the parton model description is bound to happen for some regions of phase space for F_2^{PL} since p_T can extend up to the kinematic limit $W/2$ in this case, violating the inequality (7).

Finally we should like to remark that the OPE, RGE approach may not be the best theoretical tool for studying the photon structure function in QCD. The rather abstract terminology employed makes it difficult to compare the results, on the one hand with phenomenological and experimental knowledge of non-perturbative contributions, and on the other with exact gauge invariant Feynman diagram calculations of the point-like part. The QCD improved parton model picture makes these comparisons much more straightforward.

An example of the lack of clarity of the OPE, RGE approach is the interpretion of the $\tilde{A}_i(\mu^2)$ term in (4) as a 'hadronic' or 'long distance' term [12, 14, 15] because it contains an arbitrary scale μ^2 and shows the same Q^2 evolution as a hadronic structure function. In fact, as shown above $\tilde{A}_i(\mu^2)$ contains a large purely point-like part which gives important contributions at all experimentally interesting Q^2 values. This part cannot be parameterised in a quite arbitrary fashion as suggested in [12, 14, 15].

The most serious disadvantage of the OPE, RGE approach is that many 'non-leading' terms are dropped when asymptotic solutions are derived (see for example Fig. 4). No estimate is available of the importance of these terms so comparisons of asymptotic predictions with non-asymptotic data are unable to quantitatively test the theory.

In conclusion it may be remarked that the OPE was originally proposed to extract asymptotic predictions when the underlying field theory is not precisely known ('Non-Lagrangian Models of Current Algebra' [40]). QCD has a Lagrangian and Feynman rules. Its predictive power in the perturbative domain is therefore only limited, in principle, by computational power. As Λ is found experimentally to be rather small very large scales are not necessary for perturbation theory to be valid ('precocious freedom'). Asymptotic or leading log solutions may in any case however have little relevance to *quantitative* experimental tests of QCD.

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Note added in proof

The part of F_2^γ calculated in this paper $\simeq \Sigma e_q^4$ is more correctly called the 'valence' [31] rather than the 'non-singlet' contribution. The latter is $\simeq \Sigma e_q^4 - f\langle e^2\rangle^2$ where f is the number of quark flavours.