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Experimental Studies on Multijet Production in e^+e^- Annihilation at PETRA Energies

JADE Collaboration

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Abstract. Hadronic jet production by e^+e^--annihilation has been studied in the energy range of 14.0–46.7 GeV. The data have been analysed in terms of a cluster algorithm and other topological quantities. The results are compared with 2nd order QCD calculations which incorporate models for the fragmentation of quarks and gluons into hadrons. At the higher energies we observe more spherical and 4-jet like events than predicted by these calculations. We cannot achieve a simultaneous description of the observed 3- and 4-jet production by adjusting the strong coupling constant α_s or the fragmentation parameters of the 2nd order QCD models. The observed excess of spherical events can partially be explained by the production of multi-parton events expected from higher order QCD contributions. Consequences of the presented results for the value

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of α_s, determined in previous analyses, are discussed. QCD parton shower models, including soft gluon interference, are able to describe the observed number of spherical events.

In 1982 experimental evidence for the production of 4-jet like events in e^+e^- annihilation was published by the JADE collaboration [1]. This evidence was obtained at center of mass energies of 33 GeV comparing the data with perturbative QCD calculations including effects up to the order α_s^2. The analysis was carried out using topological event distributions especially sensitive to spherical, 4-jet like events, like acoplanarity and tripodity. The data showed a production rate of 4-jet events which was about 50% higher than the theoretical prediction. Later, these observations were qualitatively confirmed by other experiments [2], but no specific study of the production of multijet events in e^+e^--annihilation has been published.

Experimental results on the production rates of multijet events provide interesting tests of higher order QCD calculations, since QCD starts to reveal its full gauge structure only in second order. For example, such calculations provide a definite prediction of the ratio of the 4-jet to the 3-jet production rates, which shall be tested in this analysis.

In the first part of this paper we will present experimental jet multiplicities as defined by a cluster algorithm, and will compare the results to the predictions of QCD calculations which incorporate the fragmentation of quarks and gluons into hadrons. In the second part, studies of the acoplanarity distributions will be presented. These latter distributions will provide an alternative method of investigating higher order QCD effects in the data.

This analysis includes the data taken with the JADE detector until the end of 1985, covering center of mass energies of 14 GeV, 22 GeV, between 32.0 and 36.7 GeV and between 40.0 and 46.7 GeV. A description of the JADE detector, the trigger conditions and the selection of hadronic events is given in [3]. Both charged and neutral particles with momenta exceeding 100 MeV/c and 150 MeV/c, respectively, are used in the analysis. In order to further reject events with hard initial state photon bremsstrahlung or significant particle losses around the beampipe, we require the momentum sum of all particles of an event, $|\sum \mathbf{p}_i| \cdot c$, to be less than 30% of the center of mass energy, and the angle of the event thrust axis with the beam direction to satisfy the relation $|\cos\Theta_{\text{thrust}}| < 0.8$. Table 1 shows the integrated luminosities as well as the number of events which entered this analysis.

Table 1. Luminosities and the number of hadronic events

$\langle E_{\text{cm}}\rangle$	$\int L\,dt$	Number of events
14 GeV	1.57 pb^{-1}	2090
22 GeV	2.69 pb^{-1}	1666
34 GeV	71.3 pb^{-1}	13617
44 GeV	43.1 pb^{-1}	6636

We compare the data to the predictions of Monte Carlo models which are based on QCD calculations describing the production of quarks and gluons, and on different fragmentation schemes parametrizing the transition of quarks and gluons into final state hadrons. We mainly use the Lund string fragmentation model version 5.2 [4] together with the 2nd order perturbative QCD calculations of Gutbrod, Kramer and Schierholz (GKS) [5], which we will refer to as "$O(\alpha_s^2)$ model". Within the GKS calculation, the production of 2-, 3- and 4-parton final states is determined by the strong coupling constant α_s and a lower cutoff $y_{\min}$ for the squared invariant mass M_{ij}^2 of any pair of partons i and j of an event,

$$\frac{M_{ij}^2}{E_{\text{cm}}^2} > y_{\min}. \tag{1}$$

This cutoff is used in order to avoid divergences in the calculations, which appear in the cases of collinear and low energy gluon radiation[*]. In the GKS calculations, certain 2nd order correction terms proportional to $y_{\min}$ have been neglected. Recent studies by Gottschalk and Shatz [12] show that these terms give corrections to the 3-jet cross section of the order of 10%, even in the case of small $y_{\min}$ values. Recently, the complete second order calculations of the 3- and 4-jet matrix elements of Gottschalk and Shatz have become available [13] and have been incorporated into the Lund fragmentation model. The consequences of this QCD model will be discussed later in this analysis. Note that all available

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[*] The following parameters are used for the model calculations: The QCD scale parameter $\Lambda_{\overline{\text{MS}}} = 500$ MeV and $y_{\min} = 0.015$, as determined from an analysis of energy correlations [6]. For the ratio of vector mesons to the sum of vector plus pseudovector mesons $r = 0.5$ and for the production probability of secondary quarks u:d:s = 1:1:0.3 is taken, which is consistent with measurements of K^* and ρ^0 production [7–9]. The fragmentation functions for c- and b-quarks are parametrized by the function of Peterson et al. [10] using $\varepsilon_c = 0.050$ and $\varepsilon_b = 0.018$ [11]. The gaussian p_t-distribution with a width of $\sigma_q \cdot \sqrt{2} = 375$ MeV and the Lund fragmentation function for u-, d-, and s-quarks with $a = 1.0$ and $b = 0.6$ were used in order to describe the measured multiplicity and momentum distributions of the charged particles

QCD calculations in 2nd order perturbation theory have been carried out for massless partons only.

The second model to which we compare our data is the QCD shower model of Webber and Marchesini [14], based on the leading log approximation to QCD including soft gluon interference, which we will refer to as "LLA model"**. QCD shower models provide an alternative way of describing partonic final states, taking into account the leading logarithmic terms of all orders in perturbation theory. Such models permit the description of multiple gluon radiation in terms of a cascade-like process, but are found to underestimate the experimental rate of 3-jet like events.

All generated Monte Carlo events include the effects of photon initial state radiation. They are tracked through the JADE detector by a computer simulation program and undergo the same selection procedure as is applied to the data.

Cluster-Multiplicities

Hadronic events with multi-jet structures are visible at PETRA energies above 30 GeV. As an example, in Fig. 1 we show the energy flow of a hadronic event containing four separated jets of particles, measured with the JADE detector at the highest PETRA energy of 46.7 GeV.

By interpreting such events in terms of their underlying parton structure, it is possible to test the predictions of perturbative QCD. However, the fragmentation of partons into visible particles, which cannot be calculated theoretically, obscures the original parton structure of the events. Algorithms are therefore required to define and to reconstruct jets in the experimental data.

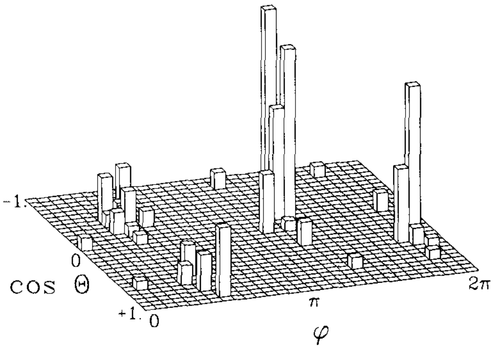

Fig. 1. Energy flow in the $\varphi-\cos\Theta$-plane of an event measured with the JADE detector at 46.7 GeV c.m. energy. φ is the angle in the plane perpendicular to the beamline and Θ is the angle with respect to the beamline. The highest bin corresponds to an energy of 6 GeV

There are many different types of such algorithms. After detailed studies in the model calculations we use the following cluster algorithm in order to achieve the closest resemblance between cluster- and parton-multiplicities:

For all pairs of particles k and l of an event, the scaled invariant mass squared $y_{kl}=M_{kl}^2/E_{\mathrm{vis}}^2$ is calculated, where E_{vis} is the total visible energy of an event*. The two particles with the smallest value of y_{kl} are replaced by a pseudoparticle or "cluster" of four-momentum (p_k+p_l). This procedure is repeated until all y_{kl} exceed a certain threshold value y_{cut}, and the resulting number of clusters is called the cluster- or the jet-multiplicity of the event. Calculating the invariant pair-masses M_{kl} we use the expression

$$M_{kl}^2=2\cdot E_k\cdot E_l\cdot(1-\cos\Theta_{kl}). \tag{2}$$

This choice of M_{kl} provides the closest agreement between cluster- and parton-multiplicities at comparable values of y_{cut} (the experimental cutoff in the cluster algorithm) and $y_{\min}$ (the QCD cutoff parameter for the massless partons in the $O(\alpha_s^2)$ model).

In Table 2 the resulting rates of n-cluster events (n = 2, 3, 4) are given for data and model calculations at $E_{\mathrm{cm}}=34$ GeV with $y_{\mathrm{cut}}=0.040$, which corresponds to a minimum invariant pairmass of 6.8 GeV/c^2 and is a reasonable choice for the definition of jets. At this value of y_{cut}, the rates of 5-cluster events are less than 0.1% and are not given separately, but are added to the number of 4-cluster events. The $O(\alpha_s^2)$ model, with the α_s value optimized to the data in an analysis of the energy-energy correlations [6], produces too many 3-cluster events and too few 4-cluster events; also the 2-cluster rate is too low. In particular, the ratio of 4- to 3-cluster events does not

Table 2. n-cluster event rates obtained with the described cluster-algorithm at $y_{\mathrm{cut}}=0.040$ (numbers in %) and the ratios of 4- to 3-cluster event rates ($E_{\mathrm{cm}}=34$ GeV)

	Data	$O(\alpha_s^2)$ model	LLA model
2-cluster	56.1 ±0.4	53.2 ±0.3	58.5 ±0.4
3-cluster	40.2 ±0.4	44.0 ±0.3	37.8 ±0.4
4-cluster	3.75 ±0.16	2.85 ±0.12	3.69 ±0.15
$\frac{\text{4-cluster}}{\text{3-cluster}}$	0.093 ± 0.004	0.065 ± 0.003	0.098 ± 0.004

** We optimized the parameters of this model requiring a good description of the measured multiplicity and momentum distributions of the charged particles. In particular, we use the QCD scale parameter $\Lambda=300$ MeV, the shower cutoff and virtual gluon mass $Q_g=700$ MeV and a maximum cluster-mass of 3.5 GeV for the 2-body decay into hadrons

* Charged particles are assumed to be pions and neutrals to be photons

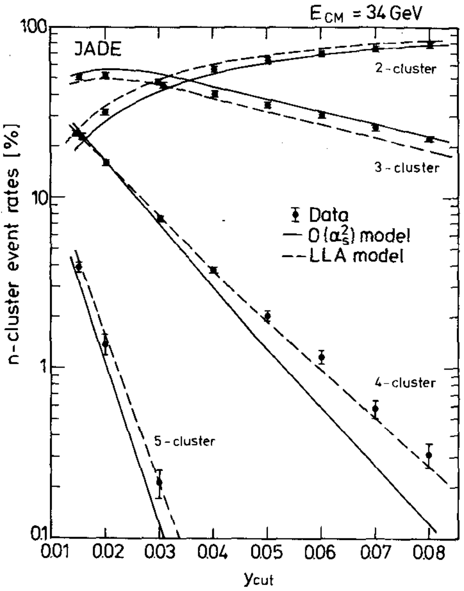

Fig. 2. *n*-cluster event rates of the data and model calculations at $E_{cm} = 34$ GeV, determined with the cluster algorithm described in the text, as a function of y_{cut}

correspond to the measured value. For the LLA model the 3-cluster rate is too low, but the rate of reconstructed 4-cluster events agrees with the data.

We checked that studies using different cluster algorithms or event classifications by topological variables like the sphericity-tensor [15] result in similar discrepancies between the data and the $O(\alpha_s^2)$ models.

Figure 2 shows the *n*-cluster event rates of the data and the models as a function of y_{cut}. The rates of the data are listed in Table 3. The rate of 4-cluster events predicted by the $O(\alpha_s^2)$ model falls below the data especially at high values of y_{cut}, whereas the LLA QCD model describes the data remarkably well.

At low y_{cut}-values, the reconstruction of clusters is dominated by fluctuations in the fragmentation process. At higher values of y_{cut}, the contribution of these fluctuations decreases. For instance, for the $O(\alpha_s^2)$ model the background of 2- and 3-parton events to the reconstructed 4-cluster events amounts to 66%, 42% and 25% at $y_{cut} = 0.02$, 0.04 and 0.06, respectively*. The small 4-cluster rate at high y_{cut} values, where the influence of 4-parton events is predominant, therefore does not seem to be caused by the treatment of the fragmentation, but indicates a deficiency of the 2nd order QCD predictions.

Table 3. *n*-cluster event rates of the data at $E_{cm} = 34$ GeV obtained with the cluster-algorithm described in the text (number in %)

y_{cut}	2-cluster	3-cluster	4-cluster	5-cluster
0.015	22.3 ± 0.4	50.2 ± 0.4	23.3 ± 0.4	3.88 ± 0.18
0.020	31.4 ± 0.4	51.3 ± 0.4	15.9 ± 0.4	1.37 ± 0.16
0.030	45.6 ± 0.4	46.7 ± 0.4	7.47 ± 0.24	0.21 ± 0.12
0.040	56.1 ± 0.4	40.2 ± 0.4	3.75 ± 0.16	–
0.050	63.4 ± 0.4	34.6 ± 0.4	2.02 ± 0.16	–
0.060	69.0 ± 0.4	29.8 ± 0.4	1.14 ± 0.10	–
0.070	74.1 ± 0.4	25.3 ± 0.4	0.57 ± 0.07	–
0.080	77.7 ± 0.4	22.0 ± 0.4	0.31 ± 0.05	–

Because of this deficiency, analyses which measure the strong coupling constant with no distinction between 3- and 4-jet events, will typically overestimate α_s, i.e. the production of 3-jet events in the model, in order to cancel partly the effects of missing 4-parton events. This is the reason why in Fig. 2 the 3-cluster rate of the $O(\alpha_s^2)$ model, with α_s determined from the energy-correlation asymmetry [6], is systematically above the data and matches the data only at high values of y_{cut}, where most of the generated 4-parton events are reconstructed as 3-cluster events and are not resolved separately any longer. As a consequence of the increased number of 3-cluster events in this model, the 4-cluster rate matches the data only at small y_{cut}-values, where 3-jet events, due to the dominant effects of fragmentation in this regime, have a high probability of being reconstructed as 4-cluster events.

These effects are seen directly if one determines α_s from the rate of 3-cluster events at y_{cut} values, where 3-cluster events are largely decoupled from the production of 4-cluster events. Thus the 3-cluster rate at $y_{cut} = 0.04$ leads to $\alpha_s = 0.152 \pm 0.004$, which is about 10% less than the value** of 0.165 ± 0.01 obtained previously in an analysis of the energy correlations [6]. Note that the rate of 4-cluster events decreases for smaller values of α_s thus increasing further the discrepancy between the data and the $O(\alpha_s^2)$ model.

Using the 2nd order QCD matrix elements of Gottschalk and Shatz [12, 13] together with the Lund fragmentation model, we obtain the value*** $\alpha_s = 0.134 \pm 0.003$ from the 3-cluster event rate at $y_{cut} = 0.04$. With this value of α_s, the model predicts a yet smaller rate of 4-cluster events than the model with the GKS matrix elements, and consequently also does not provide a satisfactory description of the data.

The LLA model, although it correctly describes the experimental rate of 4-cluster events, underestimates the 3-cluster rate and overestimates the rate

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* About 2/3 of this background is caused by c- and b-quark events

** The errors given are statistical only

*** A 10% decrease of α_s, compared to the value obtained using the GKS matrix elements, was expected by Gottschalk and Shatz [12] due to the approximations made by GKS

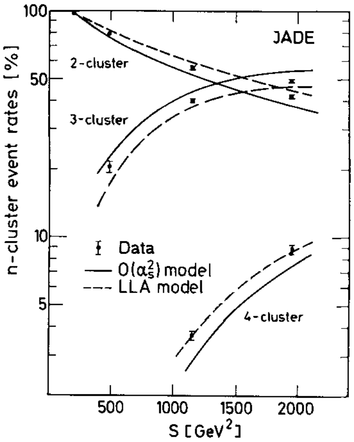

Fig. 3. n-cluster event rates of the data and model calculations as a function of $S=E_{\mathrm{cm}}^2$, determined with the cluster algorithm described in the text

of 2-cluster events. This presumably has to do with the fact that the leading log QCD calculations do not contain the full 1st or 2nd order QCD matrix elements.

In Fig. 3 the n-cluster event rates of the data and the model calculations are shown as a function of the center of mass energy between 14 GeV and 44 GeV. For the data, the numbers are also listed in Table 4. In order to provide a reasonable comparison of these rates at different center of mass energies, we require a constant minimum invariant two-cluster mass of $M_{kl}^{\mathrm{cut}}=6.8$ GeV in the cluster algorithm, which corresponds to $y_{\mathrm{cut}}=0.240$, 0.096, 0.040 and 0.024 at 14, 22, 34 and 44 GeV c.m. energy, respectively. The discrepancies described above between the data and both model calculations are visible in the whole energy range. The rate of 4-cluster events increases by more than a factor of two in going from 34 GeV to 44 GeV center of mass energy.

An important question is whether the observed discrepancies between the data and the models, as well as those between the two models, are due to uncertainties in the phenomenological treatment of the fragmentation, or whether they are due to differences in the QCD calculations used. The following tests are addressed to this question.

Table 4. n-cluster event rates of the data at different c.m. energies obtained with the cluster-algorithm described in the text (numbers in %)

$\langle E_{\mathrm{cm}}\rangle$	y_{cut}	2-cluster	3-cluster	4-cluster
14 GeV	0.240	98.5 ± 0.3	1.5 ± 0.3	–
22 GeV	0.096	79.4 ± 1.0	20.6 ± 1.0	–
34 GeV	0.040	56.1 ± 0.4	40.2 ± 0.4	3.75 ± 0.16
44 GeV	0.024	42.3 ± 0.6	48.4 ± 0.6	9.28 ± 0.36

A detailed study on the effects of changing model parameters was done for the Lund model. There is no way to increase the observable 4-cluster rate and to decrease the 3-cluster rate at the same time; e.g. a change in α_s alters both rates in the same direction. Only a flat or even softer fragmentation function for bottom-quarks, which is excluded by experimental measurements* [11], or an increase in σ_q of 50% will result in a higher observable 4-jet rate, but will destroy the overall agreement to the data in many other quantities, like p_t- and multiplicity distributions of the charged particles. As a further check we replaced the gaussian p_t-distribution in the fragmentation process of the model by an exponential function, which has a considerably larger tail at high p_t-values. The rate of 4-cluster events at $y_{\mathrm{cut}}>0.03$ turned out to be insensitive to this, however.

In addition, we performed a comparison between the $O(\alpha_s^2)$ and the LLA model on the level of quarks and gluons in order to investigate whether the observed differences in the cluster-multiplicities are already present at this stage and are thus due to differences in the QCD calculations implemented in these models. Such a comparison cannot be made directly, since the LLA model produces mostly multi-parton configurations with up to 10 quarks and gluons**, whereas the $O(\alpha_s^2)$ model generates only 2-, 3- and 4-parton events. Therefore those partons in the leading log cascade having a lower invariant pair-mass than a certain cutoff $y_{\min}$ (1), were com-

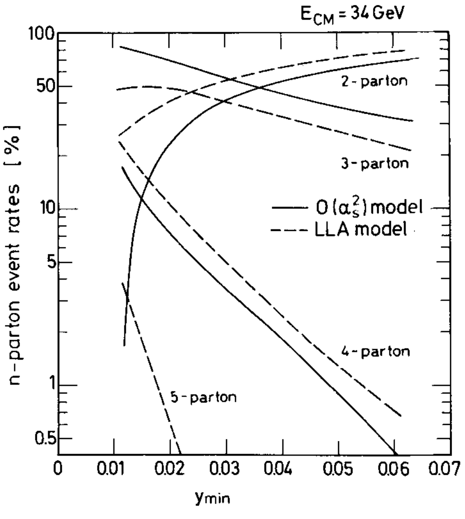

Fig. 4. n-parton event rates at $E_{\mathrm{cm}}=34$ GeV of the $O(\alpha_s^2)$ model and the LLA model as a function of $y_{\min}$

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* Note that the fragmentation function for bottom-quarks used in this analysis ($\varepsilon_b=0.018$) already corresponds to the softer limit allowed by the measurements

** The mean value is 5 partons per event at 34 GeV c.m. energy bined. This procedure is equivalent to a termination of the parton shower cascade at the given $y_{\min}$-cutoff. The remaining partons are then comparable to those of the $O(\alpha_s^2)$ model generator with the same value of $y_{\min}$.

In Fig. 4, the rates of n-parton events of the $O(\alpha_s^2)$ model and of the LLA model after the described recombination are shown as a function of $y_{\min}$ at $E_{\mathrm{cm}}=34$ GeV. The differences between both models are qualitatively the same as those observed for the reconstructed clusters after fragmentation. We conclude that the inability of both models to describe the experimentally observed rates of n-cluster events can be attributed to the parton dynamics of the models.

Studies of Acoplanarity Distributions

The acoplanarity [16] of an event is defined as

$$A=4\cdot\min\left(\frac{\sum|p_{i\perp}|}{\sum|\mathbf{p}_i|}\right)^2, \qquad (3)$$

where $p_{i\perp}$ is the momentum component of particle i perpendicular to the plane that minimizes the expression in brackets. This observable is especially sensitive to spherical, 4-jet like events, which predominantly have large values of A. In addition to the analysis of cluster-multiplicities presented in the previous section, the study of acoplanarity distributions provides a further test of QCD model predictions.

In Fig. 5 we show the differential A-distributions of the data and model calculations at 22 GeV, 34 GeV and 44 GeV center of mass energies. The $O(\alpha_s^2)$ model describes the data at 22 GeV, but at 34 and 44 GeV the theoretical predictions fall below the data especially at large values of A, whereas the LLA model provides a good description of the measured acoplanarity distributions in the whole energy range. In the following, we study the contributions of 2-, 3- and 4-parton events to the acoplanarity distributions in order to express the observed difference between the data and the $O(\alpha_s^2)$ model in terms of n-parton event rates.

Within the $O(\alpha_s^2)$ model, the relative rates of 2-, 3- and 4-parton events are fixed once the value of α_s has been chosen. The results of the cluster analysis showed however that this model does not correctly describe the ratio of the 4-jet and 3-jet event rates. Ignoring the predicted rates of n-parton events ($n=2, 3, 4$), we empirically determine those rates which, after fragmentation, provide the best description of the A-distribution of the data. This is done by fitting the fractions of n-parton events, leaving their differential distributions as predicted by the 2nd order QCD matrix elements. The results obtained at 34 GeV center of mass energy are shown in Table 5. In the first row, the original 3- and 4-parton event rates of the $O(\alpha_s^2)$ model, using the parameters $\Lambda_{\overline{MS}}=500$ MeV and $y_{\min}=0.015$ as described above, are shown. The results of a fit with R_4 as a free parameter are shown in the second row. R_4 is increased by 30% relative to the value given by the original model, but the fit is rather bad, as seen from the χ^2 value. In order to provide a good description of the data, a 2-dimensional fit was performed leaving both R_4 and R_3 free (R_2 is always given by the constraint $R_2+R_3+R_4=100$%). The result is listed in the 3rd row of Table 4, showing that a good fit is achieved if

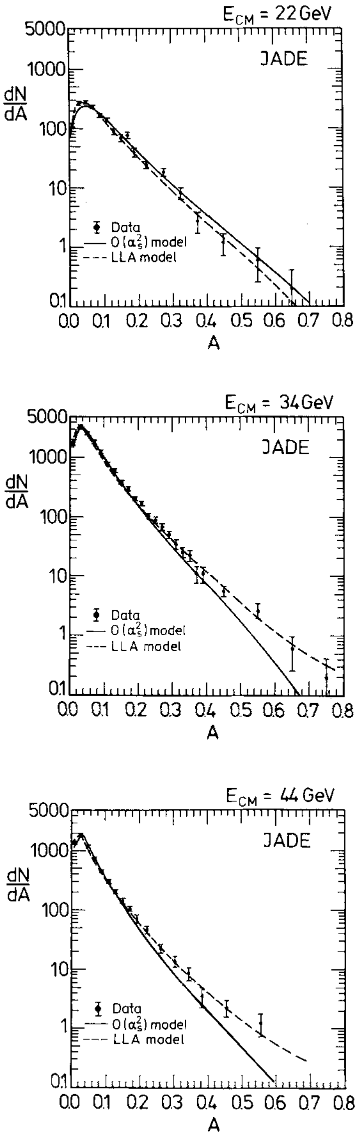

Fig. 5. Acoplanarity distributions of the data and the model calculations at 22, 34 and 44 GeV c.m. energies

R_4 is increased by a factor of 1.5 and R_3 is decreased by a factor of 0.8 with respect to the original numbers. Note that this result is not consistent with the QCD expectation, according to which both R_3 and R_4 increase (or decrease) simultaneously as a function of α_s.

The same fits of 3- and 4-parton event rates have been performed within the LLA model, defining the parton multiplicity of each model event as described in the previous chapter using $y_{\min}=0.015$. In the 4th row of Table 5 the original event rates of this model as well as the χ^2 of the A-distribution are shown. The results of a 2-dimensional fit of R_3 and R_4 are given in the 5th row. Within this model, the fitted 4-parton event rate is essentially identical to the original rate, but the fitted R_3 is increased relative to the original rate.

In summary, the trend of the models in the above fits is similar to that observed in the cluster analyses of the previous section. In comparison with the data, the $O(\alpha_s^2)$ model needs more 4-jet like and fewer 3-jet like events, and the LLA model needs more 3-jet like events, but already produces enough events with more than three jets.

An interesting question is whether the deficiency of spherical events observed for the $O(\alpha_s^2)$ model can be explained by the production of events with more than 4 jets. Note that these are expected from higher order perturbation theory and are not included in the $O(\alpha_s^2)$ model. Since such higher order contributions have not been calculated yet, we investigate this question by adding the events of the LLA model which have been classified as 5-parton events (R_5) to the event sample of the $O(\alpha_s^2)$ model. Leaving R_5 as a free parameter in a fit to the A-distribution yields $R_5=2.4\pm0.5\%$, as is shown in the 6th row of Table 5. This indicates that a relatively small rate of multi-parton events, which all have spherical event shapes, produces visible effects on the A-distribution. Fixing R_5 to 2% and repeating the 2-dimensional fit in R_3 and R_4 of the $O(\alpha_s^2)$ model, results in the numbers shown in the last row of Table 5. In this case the 'best fit' value of R_4 is unchanged compared to the original values shown in the 1st row, but R_3 still has to be decreased. Note that the inclusion of 5-parton events does not change the sum of the fitted 3- and 4-parton event rates, but, comparing the results shown in the 3rd and in the 7th row of Table 5, alters the relative ratio of R_4 to R_3. In summary, the fraction of 4-parton events in the $O(\alpha_s^2)$ model seems to be correct if the production of events with more than 4 partons is taken into account, but nevertheless the rate of 3-parton events is still too high.

The results of a similar analysis at center of mass energies around 44 GeV are presented in Table 6. The relative difference in R_3 and R_4 comparing the fitted and the original event rates of the $O(\alpha_s^2)$ model is somewhat bigger than at 34 GeV. The numbers in general demonstrate the same trends and lead to similar conclusions.

Up to now, the integrated rates of 3- and 4-parton events were varied and fitted in total, leaving their differential distributions as predicted. However the cluster analysis presented in the previous section indicates that it is specifically the hard, i.e. well pronounced 4-parton events which are missing in the $O(\alpha_s^2)$ model. In order to verify this conjecture we repeat the 2-dimensional fit in R_3 and R_4 at 34 GeV center of mass energy, using the $O(\alpha_s^2)$ model events as in the fits described above, but redefining the number of partons of each event by successively increasing the minimum required invariant parton pair mass $y_{\min}$ between any two partons. 4-parton events falling below this cut are then counted as 3-parton or even 2-parton events, and 3-parton events below the cut are counted as 2-parton events.

Figure 6 shows the ratio of the fitted 3- and 4-parton event rates to the corresponding numbers of the original model at 34 GeV, as a function of $y_{\min}$. The increase of the fitted R_4 with increasing $y_{\min}$ indicates that mainly the hard 4-parton events are

Table 5. Fit results of n-parton event rates R_n in the model calculations, obtained in the acoplanarity distribution at 34 GeV. The rates are given in % and correspond to $y_{\min}=0.015$ in the generator. For details see text

Row	Model	Fit in:	R_3	R_4	R_5	χ^2/d.f.
1	$O(\alpha_s^2)$	(orig.)	76.7±0.3	12.3±0.2	–	63/22
2	$O(\alpha_s^2)$	R_4	–	16.0±1.0	–	49/22
3	$O(\alpha_s^2)$	R_3, R_4	61.5±2.0	19.0±1.0	–	25.6/21
4	LLA	(orig.)	47.3±0.4	16.2±0.3	1.85±0.13	30/22
5	LLA	R_3, R_4	56.0±3.5	16.6±1.5	1.85 (fixed)	13.2/21
6	$O(\alpha_s^2)$	R_5	–	–	2.4 ±0.5	31.1/22
7	$O(\alpha_s^2)$	R_3, R_4	69.0±2.5	12.0±1.3	2.0 (fixed)	19.5/21

Table 6. Fit results of n-parton event rates in the model calculations, obtained in the acoplanarity distribution at 44 GeV. The rates are given in % and correspond to $y_{\min}=0.015$ in the generator. For details see text

Row	Model	Fit in:	R_3	R_4	R_5	χ^2/d.f.
1	$O(\alpha_s^2)$	(orig.)	74.3 ± 0.4	11.5 ± 0.3	–	62/16
2	$O(\alpha_s^2)$	R_4	–	15.5 ± 1.0	–	51/16
3	$O(\alpha_s^2)$	R_3, R_4	52.5 ± 3.0	20.3 ± 1.5	–	8.9/15
4	LLA	(orig.)	48.1 ± 0.4	14.1 ± 0.3	1.42 ± 0.10	30/16
5	LLA	R_3, R_4	54.0 ± 3.5	14.1 ± 1.2	1.42 (fixed)	11.9/15
6	$O(\alpha_s^2)$	R_5	–	–	2.0 ± 0.5	41.8/16
7	$O(\alpha_s^2)$	R_3, R_4	56.5 ± 3.0	16.4 ± 1.5	1.4 (fixed)	9.6/15

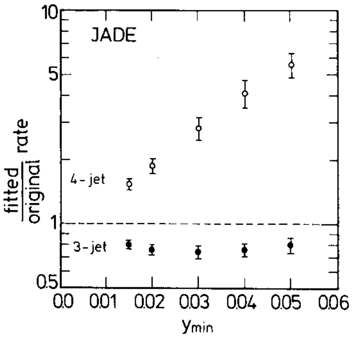

Fig. 6. The ratio of fitted to original 3- and 4-parton event rates as a function of the minimum invariant parton pair masses in the $O(\alpha_s^2)$ model, obtained by a 2-dimensional fit in A at $E_{\mathrm{cm}}=34$ GeV

missing. The simultaneously obtained ratio of the 3-parton event fractions, which is also shown in Fig. 6, does not depend on $y_{\min}$ within the errors, but needs to be decreased by approximately 20%.

In order to investigate to which extent the results obtained from the study of the A-distributions depend on uncertainties imposed by the fragmentation models, we performed similar systematic studies of the fragmentation parameters as described in the previous section. It turned out that the systematic uncertainties of the results presented in Tables 5 and 6 and in Fig. 6 are not bigger than the given statistical errors.

Summary and Discussion

Analyzing cluster multiplicities and acoplanarity distributions of hadronic final states, we find that $O(\alpha_s^2)$ models underestimate the rate of spherical and 4-jet like events at high energies. This deficiency is predominantly observed in the regions of well pronounced 4-jet events, and cannot be explained by adjusting the fragmentation parameters of the models. We cannot achieve a simultaneous description of the observed 3- and 4-jet event rates, which are both definitely predicted within the $O(\alpha_s^2)$ models as a function of the strong coupling constant α_s.

In addition we find that the rate of 3-jet events is overestimated in the $O(\alpha_s^2)$ models, if the value of α_s used is determined in analyses which do not discriminate between the contributions of 3- and 4-jet events. In such determinations of α_s, an increased production of 3-jet events, i.e. a larger value of α_s, partly compensates the effects of missing 4-jet events. In particular, the value of $\alpha_s=0.165\pm0.010$, which we previously determined in a study of the energy correlations at 34 GeV c.m. energy, using the 2nd order QCD matrix elements of GKS, decreases by 10% if it is adjusted to the observed rate of 3-cluster events in a region where the influence of 4-jet like events is small.

Events with more than 4 partons, which are expected from higher order QCD contributions, can partly compensate the deficiency of spherical, 4-jet like events in the $O(\alpha_s^2)$ model. However, since calculations in 3rd or even higher order perturbation theory are not available, the production rate of such higher order events as well as the magnitude of virtual higher order corrections to the 3- and 4-jet production are not known at all. Thus, no quantitative statements concerning an improved description of the data by higher order perturbative QCD calculations can be made.

The leading log QCD model, including effects of soft gluon interference, describes the observed number of spherical events, but underestimates the rate of 3-jet like events. The detailed reasons for this behaviour are not known at present. Such kind of models, however, provide an interesting alternative to the $O(\alpha_s^2)$ models especially at the higher energies of the forthcoming e^+e^- annihilation experiments, where the rates of 4- and multi-jet like events will be dominant.

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