Spin Analysis of the χ_b States

T. Skwarnicki, D. Antreasyan, D. Besset, J. K. Bienlein, E. D. Bloom, I. Brock. R. Cabenda, A. Cartacci, M. Cavalli-Sforza, R. Clare, G. Conforto, S. Cooper, R. Cowan, D. Coyne, C. Edwards, A. Engler, G. Folger, A. Fridman,[a] J. Gaiser, D. Gelphman, G. Godfrey, F. H. Heimlich, R. Hofstadter, J. Irion, Z. Jakubowski, S. Keh, H. Kilian, I. Kirkbride, T. Kloiber, W. Koch, A. C. König, K. Königsmann, R. W. Kraemer, R. Lee, S. Leffler, R. Lekebusch, A. M. Litke, W. Lockman, S. Lowe, B. Lurz, D. Marlow, W. Maschmann, T. Matsui, F. Messing, W. J. Metzger, B. Monteleoni, R. Nernst, B. Niczyporuk, G. Nowak, C. Peck, P. G. Pelfer, B. Pollock, F. C. Porter, D. Prindle, P. Ratoff, B. Renger, C. Rippich, M. Scheer, P. Schmitt, J. Schotanus, A. Schwarz, D. Sievers, K. Strauch, U. Strohbusch, J. Tompkins, H. J. Trost, R. T. Van de Walle, H. Vogel, U. Volland, K. Wacker, W. Walk, H. Wegener, D. Williams, and P. Zschorsch

(The Crystal Ball Collaboration)

California Institute of Technology, Pasedena, California 91125
Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213
Cracow Institute of Nuclear Physics, Cracow, Poland
Deutsches Elektronen Synchrotron DESY, Hamburg, Germany
Universität Erlangen-Nürnberg, Erlangen, Germany
Istituto Nazionale di Fisica Nucleare and University of Firenze, Firenze, Italy
I. Institut für Experimentalphysik, Universität Hamburg, Hamburg, Germany
Harvard University, Cambridge, Massachusetts 02138
University of Nijmegen and National Instituut voor Kernfysica en Hoge-Energiefysica–Nijmegen, Nijmegen, The Netherlands
Princeton University, Princeton, New Jersey 08544
Department of Physics, High Energy Physics Laboratory, and Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
Universität Würzburg, Würzburg, Germany

(Received 22 August 1986)

Angular correlations in the cascade reaction $e^+e^- \rightarrow \Upsilon(2S) \rightarrow \gamma\chi_b$, $\chi_b \rightarrow \gamma\Upsilon$, $\Upsilon \rightarrow \mu^+\mu^-$ or $\Upsilon \rightarrow e^+e^-$, have been used for the first time to investigate the spins of the χ_b states. The results support the χ_b spins predicted by the potential models of heavy quarkonia.

PACS numbers: 14.40.Gx, 13.40.Hq, 13.65.+i

The χ_b states have been detected[1] by observation of monochromatic lines in the inclusive photon spectrum from hadronic decays of the $\Upsilon(2S)$ resonance:

$$e^+e^- \rightarrow \Upsilon(2S) \rightarrow \gamma\chi_b, \ \ \chi_b \rightarrow \text{hadrons}. \qquad (1)$$

Assuming that these γ transitions proceed via electric dipole ($E1$) emission, one can infer the spin (J) of the three observed χ_b states from the relative partial widths of the $\Upsilon(2S)$ decays, which are predicted by nonrelativistic potential models[2] to be $\Gamma_{E1}(^3S_1 \rightarrow {}^3P_J) \propto (2J+1)E_\gamma^3$, where E_γ denotes the energy of the emitted photon. In fact, the measured branching ratios[1] of $\Upsilon(2S) \rightarrow \gamma\chi_b$ very much favor the theoretical prediction that the spins are 0, 1, 2, increasing with increasing χ_b mass. The spins cannot be determined from the inclusive spectrum with use of the angular distributions, because the resonance signals are observed on a very large background.

The two higher-mass χ_b states have also been observed[3,4] in the radiative cascade transitions between the $\Upsilon(2S)$ and Υ resonances:

$$e^+e^- \rightarrow \Upsilon(2S) \rightarrow \gamma\chi_b, \ \ \chi_b \rightarrow \gamma\Upsilon,$$
$$\Upsilon \rightarrow \mu^+\mu^- \text{ or } \Upsilon \rightarrow e^+e^-. \qquad (2)$$

Although the number of observed events is small, this channel has very low background. A study of the angular correlations in the cascade sample provides the possibility of measurement of χ_b spins directly, as has been done already for the charmonium system.[5-8] The results of such an analysis are presented below.

The data for this analysis were collected with the Crystal Ball detector at the e^+e^- storage ring DORIS-II at the Deutsches Elektronen-Synchrotron (DESY). An integrated luminosity of 63 pb^{-1} on the $\Upsilon(2S)$ resonance was accumulated, corresponding to $(200 \pm 16) \times 10^3$ $\Upsilon(2S)$ resonance decays.

The main part of the Crystal Ball detector[8] consists of a highly segmented spherical shell of NaI crystals. The good energy resolution for electromagnetically showering particles $\sigma(E)/E = (2.7\% \text{ GeV}^{1/4})/E^{1/4}$ plays the crucial role in this analysis, allowing separation of the events decaying via different χ_b states. The uniform acceptance over a large solid angle and good angular resolution for photons (1°–2°) make the Crystal Ball well suited to study angular correlations in the $\gamma\gamma l^+l^-$ channel.

Events with two photons and two nearly back-to-back muons or electrons have been selected by use of criteria very similar to those described in Ref. 4. The cuts have been slightly changed[9] to maximize the geometrical acceptance. The energy distribution of the low-energy photon corresponding to the radiative decay of the $\Upsilon(2S)$ in the final cascade sample is plotted in Fig. 1. The two peaks at 108 and 132 MeV, with widths consistent with our energy resolution, correspond to the two higher-mass χ_b states seen in the inclusive analyses of $\Upsilon(2S) \rightarrow \gamma\chi_b$ (we will call them χ_b^α and χ_b^β states). Transitions to the third χ_b state with expected photon energy about 164 MeV are not seen in this channel with the present experimental statistics.

Within the ranges indicated in Fig. 1 we obtain $N^\alpha=66$ events ($N_\mu^\alpha=34$ $\gamma\gamma\mu^+\mu^-$ and $N_e^\alpha=32$ $\gamma\gamma e^+e^-$) for the χ_b^α state and $N_\beta=71$ events ($N_\mu^\beta=33$ $\gamma\gamma\mu^+\mu^-$ and $N_e^\beta=38$ $\gamma\gamma e^+e^-$) for the χ_b^β state. The backgrounds are estimated from the fit displayed in Fig. 1. We find 2.9 ± 1.1 events in each χ_b sample coming from background processes (mainly from double radiative Bhabha scattering). Because of a low-energy tail in the NaI line shape we expect a feed-down from the χ_b^β resonance to the χ_b^α sample of 7.7 ± 0.9 events. The total background contribution is $(16 \pm 2)\%$ in the χ_b^α sample and $(4 \pm 2)\%$ in the χ_b^β sample.

Because the number of events is small, one-dimensional angular distributions cannot distinguish among different spin hypotheses. The full angular correlation in the cascade process is analyzed to extract the maximum information about the χ_b spins. The full angular distribution[5] $W(\Omega)$ is a function of six independent angles Ω (e.g., directions of the two photons and the direction of one of the final-state leptons, the two of which are exactly back-to-back in the Υ rest frame). $W(\Omega)$ depends on the χ_b spin (J), the transition multipoles, and the beam polarization (P). The DORIS-II beams are highly transversely polarized at the $\Upsilon(2S)$ energy. We obtain a value $P=(75 \pm 5)\%$ measuring the azimuthal angular distribution of the muons from the QED process $e^+e^- \rightarrow \mu^+\mu^-$. With unpolarized beams we would need twice the number of events to have the same sensitivity for the χ_b determination,[5] even though no additional phase information is gained from the polarization (see Ref. 9 of Skwarnicki[5]).

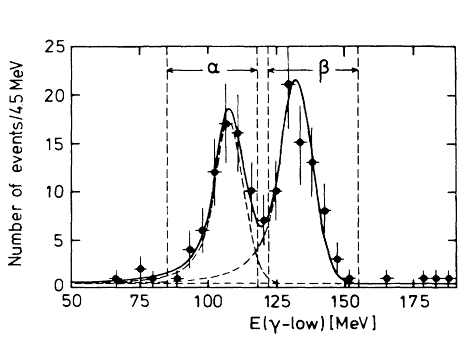

FIG. 1. Energy distribution of the low-energy photon in the sample of $\gamma\gamma\mu^+\mu^-$ and $\gamma\gamma e^+e^-$ events. The solid line shows the fit to the data of two monoenergetic γ lines (asymmetric NaI line shape) and flat background. The dashed lines indicate the cuts defining the data samples for the spin analysis.

In order to find out whether the angular distributions of the events are consistent with a hypothetical χ_b spin $J_{\rm hyp}$, we have used likelihood functions and likelihood ratios which result in test functions of the form

$$T(\Omega_1, \ldots, \Omega_N) = \sum_{i=1}^{N} l(\Omega_i). \qquad (3)$$

Here Ω_i denotes the measured values of all six independent angles in the ith event, N the number of events in the data sample, and $l(\Omega)$ one of the various functions of measured angles to be specified later. To get a confidence level for the given spin hypothesis, $J_{\rm hyp}$, one compares the observed value of the test function $T_{\rm obs}$ with the probability density distribution of T under this spin hypothesis (see e.g., Eadie *et al.*,[10] pp. 215–218). The Monte Carlo (MC) method can be used to obtain this probability density distribution. In principle, one generates a large number of MC experiments, with N events each (the same number of events as in the *real* data sample), according to the theoretical[5] angular distribution $W_{J_{\rm hyp}}(\Omega)$ convoluted with the detector acceptance and resolution. The distribution of T values calculated for the MC experiments gives an estimate of the true distribution of T for $J_{\rm hyp}$. The central-limit theorem (see e.g., Ref. 10, p. 40) predicts that distributions of T are Gaussian for large N. Applying the Gaussian approximation, we express our results in standard deviations, $\Delta = |T_{\rm obs} - \langle T\rangle_{J_{\rm hyp}}| / \sigma(T)_{J_{\rm hyp}}$. Here $\langle T\rangle_{J_{\rm hyp}}$ is the mean value and $\sigma^2(T)_{J_{\rm hyp}}$ is the variance of the distribution of T under the spin hypothesis $J_{\rm hyp}$, both obtained with the MC simulation.

In practice a somewhat modified method was used to calculate the hypothetical density distributions in order to save computer time. In the MC, the directions of the three successive two-body decays were generated *isotropically*. After the detector response was simulated, the MC events were selected by the same programs as used for the real data. Because of the Gaussian limit mentioned above, there was no need to group MC events into experiments of N events each, because all necessary parameters describing the distribution of T for $N=N_\mu+N_e$ events could be obtained from single-event expectation values of $l(\Omega)$ and $l(\Omega)^2$: $\langle T\rangle = N_\mu\langle l\rangle_\mu + N_e\langle l\rangle_e$, $\sigma^2(T) = N_\mu\sigma^2(l)_\mu + N_e\sigma^2(l)_e$, where $\sigma^2(l) = \langle l^2\rangle - \langle l\rangle^2$. A total number of about $n_\mu = 2\times10^4$ $\gamma\gamma\mu^+\mu^-$ and $n_e = 10^4$ $\gamma\gamma e^+e^-$ MC events passing all cuts were available. When the expectation values were calculated, $\langle l\rangle_{J_{\rm hyp}}$ and $\langle l^2\rangle_{J_{\rm hyp}}$, the isotropically generated MC events were given the weights $w_j = W_{J_{\rm hyp}}(\Omega_j)$, such that

$$\langle l\rangle_{J_{\rm hyp}} = \sum_{j=1}^{n} w_j l(\Omega_j) \left(\sum_{j=1}^{n} w_j\right)^{-1} \quad (n = n_\mu \text{ or } n_e),$$

and similarly for $\langle l^2 \rangle$. The reweighting method applied here allowed economies of computation, necessary to simulate properly the detector response.

The deviation of the true distributions of T from their Gaussian approximations was studied with a large MC sample of about 4×10^6 events grouped into 5×10^4 experiments for each $J_{\rm hyp}$. However, these MC events were subjected only to the simple geometrical cuts and not to the full detector response simulation. The deviations from the Gaussian distributions with parameters obtained from the same MC samples were small, at least in the region of ± 3 standard deviations. In addition to the Gaussian approximation, there are other systematic effects in our analysis. They arise from uncertainty in the beam polarization value, background in the data samples, and the limited number of the MC events. All four contributions to the systematic errors were added linearly and are expressed by the quotation of a range of corresponding confidence levels.

To test spin hypothesis J for a given line we use the log-likelihood function,

$$T = \ln \prod_{i=1}^{N} W_J(\Omega_i) = \sum_{i=1}^{N} \ln W_J(\Omega_i), \tag{4}$$

as the test function (Ref. 10, p. 271). Starting with $J=0$, we find that the observed value of the test function (4) disagrees with the mean value of this test function for the $J_{\rm hyp}=0$ MC simulation at 2.9 standard deviations for the χ_b^α sample and 5.2 standard deviations for the χ_b^β sample. The corresponding confidence level (two-sided probability) for spin 0 for the χ_b^α state is $(0.4^{+0.6}_{-0.4})\%$ and much less for the χ_b^β state (see Table I).

For higher spin values $(J>0)$ the angular distribution is not uniquely determined by the spin hypothesis, but also depends on the multipole structure of the cascade photon transitions. Both transitions have to be pure dipole for $J=0$ by angular momentum conservation; thus there is no ambiguity for the spin-0 case. Because of the limited experimental statistics, we do not leave multipolarities as free parameters in our analysis, but we adopt the prediction of the nonrelativistic quarkonium model that the electric dipole transitions dominate for any spin value. For χ_b states of spin 1 and 2, higher multipoles can contribute: magnetic quadrupole ($M2$) for spin 1 and up to electric octupole ($E3$) for spin 2. The single-quark-transition picture in the quarkonium model predicts negligible octupole transition rates (see Ref. 6, Sect. IV). In the nonrelativistic quark model the quadrupole amplitudes are also expected to be very small.[5,6] This prediction finds experimental support in the Crystal Ball results for charmonium,[8] which show that all cascade transitions are pure dipole, within small errors, except for a possible nonzero quadrupole amplitude in the radiative decay of the spin-2 χ_c state $[\Gamma_{M2}/\Gamma_{E1} = (12^{+52}_{-7})\%]$. Magnetic quadrupole transitions, being relativistic effects, should be even more suppressed in the Υ family; the scaling rule[5] $\Gamma_{M2}/\Gamma_{E1} \propto (1/M_Q)^2$, M_Q the quark mass, gives an additional suppression by an order of magnitude.

Even constraining multipoles, we cannot rule out any $J>0$ hypotheses at high confidence levels with use of the test function (4). In fact, our MC simulation shows that discrimination power among different spin values decreases rapidly with increasing spin $J_{\rm hyp}$ for a test function of the type (4). The most powerful test of a spin hypothesis J against another one J' is obtained by application of the likelihood-ratio test (Ref. 10, p. 224),

$$T = \ln \frac{\prod_{i=1}^{N} W_J(\Omega_i)}{\prod_{i=1}^{N} W_{J'}(\Omega_i)} = \sum_{i=1}^{N} \ln \frac{W_J(\Omega_i)}{W_{J'}(\Omega_i)}. \tag{5}$$

To limit the number of spin combinations, we assume the quarkonium model prediction, $J \le 2$. Having previously ruled out spin 0, only the two possibilities $J_{\rm hyp}=1,2$ remain. Results of tests using the test function (5) and $J=2$, $J'=1$ are presented in Table I. The data favor spin 2 for the χ_b^α state and spin 1 for the χ_b^β state, as predicted by the semirelativistic potential models of heavy quarkonia. The confidence levels for the reverse spin assignments (one-sided probabilities) are $(4^{+6}_{-1})\%$ for $J^\alpha=1$ and $(4^{+1}_{-2})\%$ for $J^\beta=2$.

TABLE I. Results of the spin tests.

Test function $T=\sum \ln t$	State	$J_{\rm hyp}$	Δ	Confidence level (%)
$t=W_{J=0}$	χ_b^α	0	2.9	$0.4^{+0.6}_{-0.4}$
	χ_b^β	0	5.2	<0.01
$t=W_{J=2}/W_{J=1}$	χ_b^α	1	1.8	4^{+6}_{-1}
		2	0.3	38^{+8}_{-4}
	χ_b^β	1	0.4	34^{+12}_{-4}
		2	1.7	4^{+1}_{-2}
$t=\begin{cases} W_{J=2}/W_{J=1} \text{ for } \alpha \\ W_{J=1}/W_{J=2} \text{ for } \beta \end{cases}$	$\chi_b^\alpha, \chi_b^\beta$	1, 2	2.5	$0.6^{+1.2}_{-0.3}$
		2, 1	0.5	32^{+11}_{-4}

Noting that in the quarkonium model the states cannot have the same spin, it makes sense to test the global spin assignment $J^{\alpha}=1$, $J^{\beta}=2$ against $J^{\alpha}=2$, $J^{\beta}=1$, with combined data from both χ_b samples. With the likelihood ratio test,

$$T=\ln\frac{\prod_{i=1}^{N^{\alpha}} W_{J=2}(\Omega_i)\prod_{j=1}^{N^{\beta}} W_{J=1}(\Omega_j)}{\prod_{i=1}^{N^{\alpha}} W_{J=1}(\Omega_i)\prod_{j=1}^{N^{\beta}} W_{J=2}(\Omega_j)} =\sum_{i=1}^{N^{\alpha}}\ln\frac{W_{J=2}(\Omega_i)}{W_{J=1}(\Omega_i)}+\sum_{j=1}^{N^{\beta}}\ln\frac{W_{J=1}(\Omega_j)}{W_{J=2}(\Omega_j)}, \quad (6)$$

we find (Table I) that the data agree very well with the expected spin assignment $J^{\alpha}=2$, $J^{\beta}=1$, and rule out the hypothesis $J^{\alpha}=1$, $J^{\beta}=2$ which has a confidence level of only $(0.6^{+1.2}_{-0.4})\%$. (This last test can be considered simply as quantifying the combination of the previous two independent line results with the added constraint from the quark model that the spins must be different.) It should be mentioned that not only does T_{obs} agree with $\langle T\rangle_{J_{\text{hyp}}}$ under the expected J_{hyp} for all test functions, but also the observed single-event distributions of $l(\Omega)$ are compatible with the MC simulation.

In summary, the analysis of angular correlations in the cascade process

$$e^+e^-\to\Upsilon(2S)\to\gamma\chi_b\to\gamma\gamma\Upsilon \to\gamma\gamma(e^+e^- \text{ or } \mu^+\mu^-)$$

allows us to rule out with high confidence a spin-0 assignment for the two higher-mass χ_b states. Assuming pure electric dipole photon transitions we can also *exclude* at 99.4% confidence level the global spin assignment $J=1$ for the highest mass χ_b state and $J=2$ for the next highest one. The data agree with the χ_b spins predicted by potential models of heavy quarkonia: spin 1 for $\chi_b(9891)$ and spin 2 for $\chi_b(9915)$.

We would like to thank the DESY and the Stanford Linear Accelerator Center directorates for their support. Special thanks go to the DORIS machine group and the experimental support groups at DESY. The Nijmegen group acknowledges the support of FOM–Nederlandse Organisatie voor Zuiver-Wetenschappelijk Onderzoek. The Erlangen, Hamburg, and Würzburg groups acknowledge financial support from the Bundesministerium für Forschung und Technologie and the Deutsche Forschungsgemeinschaft (Hamburg). This work was supported in part by the U. S. Department of Energy under Contracts No. DE-AC03-81ER4005 (California Institute of Technology), No. DE-AC02-76ER03066 (Carnegie-Mellon University), No. DE-AC02-76ER03064 (Harvard University), No. DE-AC02-76ER03072 (Princeton University), and No. DE-AC03-76SF00515 (Stanford Linear Accelerator Center), No. DE-AC03-76SF00326 (Stanford University), and by the National Science Foundation under Grants No. PHY75-22980 (California Institute of Technology), No. PHY81-07396 (High Energy Physics Laboratory), and No. PHY82-08761 (Princeton University).

[a]Permanent address: Départment de Physique des Particules Elémentaires, Centre d'Etudes Nucléaire de Saclay, Gif-sur-Yvette, France.

[1]C. Klopfenstein *et al.*, Phys. Rev. Lett. **51**, 160 (1983); P. Haas *et al.*, Phys. Rev. Lett. **52**, 799 (1984); R. Nernst *et al.* (Crystal Ball Collaboration), Phys. Rev. Lett. **54**, 2195 (1985); H. Albrecht *et al.*, (ARGUS Collaboration), Phys. Lett. **160B**, 331 (1985).

[2]For a review of potential models see, for example, W. Buchmüller, CERN Report No. CERN-TH3938/84, 1984, (unpublished), and in *Fundamental Interactions in Low-energy Systems, Proceedings of the Fourth Course of the International School of Physics of Exotic Atoms, Erice, Italy 1984*, edited by P. Dalpiaz, G. Fiorentini, and G. Torelli (Plenum, New York, 1985), p. 233.

[3]F. Pauss *et al.*, Phys. Lett. **130B**, 439 (1983).

[4]W. Walk *et al.* (Crystal Ball Collaboration), Phys. Rev. D **34**, 2611 (1986).

[5]L. S. Brown and R. N. Cahn, Phys. Rev. D **13**, 1195 (1976).

[6]G. Karl, S. Meshkov, and J. L. Rosner, Phys. Rev. D **13**, 1203 (1976).

[7]W. Tanenbaum *et al.*, Phys. Rev. D **17**, 1731 (1978).

[8]M. Oreglia *et al.*, Phys. Rev. D **25**, 2259 (1982); M. Oreglia, Ph.D. thesis, Stanford University, 1980, SLAC Report No. 236 (unpublished).

[9]The line energies and branching ratios obtained in this analysis are in excellent agreement with those of Ref. 4. For details of the cuts used in this analysis see T. Skwarnicki, DESY Internal Report No. DESY-F31-86-02 (1986), and Ph.D. thesis, Cracow Institute of Nuclear Physics, 1986 (unpublished).

[10]W. T. Eadie *et al.*, *Statistical Methods in Experimental Physics* (North-Holland, Amsterdam, 1971), p. 224.