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Charmed Baryon Lifetime Differences

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Abstract. We predict the quantitative lifetime differences of weakly decaying charmed baryons which arise due to preasymptotic bound state effects. These predictions are to a large extent model independent. We also give and compare detailed quantitative estimates obtained in a nonrelativistic quark model and in a bag model. Finally, we show how the various nonspectator effects can be separated and determined phenomenologically, once the lifetime of charmed baryons are measured with sufficient accuracy.

I. Introduction

The differing lifetimes of weakly decaying charmed particles have attracted considerable attention because of the possibility to study strong interaction effects governed by short *and* long distance dynamics. In the past, most efforts have been devoted to the lifetimes of charmed mesons, specifically to the ratio of the D^+ and D^0 lifetimes, which experimentally amounts to $\tau(D^+)/\tau(D^0)=2.40\pm0.16$ [1]. On the theory side, two main mechanisms have been suggested to account for the observed lifetime difference, light quark interference [2] and W-exchange (annihilation in the case of the D_s^+ (former F^+)) [3]. The interference mechanism is based on the fact that the final state resulting from D^+ decay contains two $\bar{d}$ quarks, one being the light constituent of the D^+, the other one emerging from the decay of the charm quark $c\to su\bar{d}$. The interference of these identical quark flavors due to the Pauli principle lengthens the lifetime of the D^+ with respect to the D^0. On the other hand, the W-exchange process $c\bar{u}\to s\bar{d}$ is only operative in D^0 decay shortening the lifetime of the D^0 with respect to the D^+. In contrast to the main spectator decay both of the above effects are controlled by the D wave functions, a quantity which is not very well known, at least not from first QCD principles. This makes quantitative estimates of the lifetime differences difficult and uncertain.

Straightforward quark model estimates [4] indicate a sizeable interference effect but give a totally negligible effect from W-exchange, the latter process being suppressed not only by the D-meson wave function but also by a small short-distance factor and most importantly by helicity conservation. From interference alone one then typically arrives at $\tau(D^+)/\tau(D^0)\sim1.3$ and, thus, fails to explain the experimental lifetime ratio quantitatively. Keeping only the leading contribution in the expansion of the nonleptonic decay rates in powers of $1/N_c$, N_c being the number of color degrees of freedom, improves [5] the theoretical lifetime ratio to $\tau(D^+)/\tau(D^0)\sim1.6$. This procedure [5, 6] is suggested by theoretical consistency arguments and by an analysis [5, 7] of nonleptonic two-body decays. As far as the W-exchange contribution is concerned it has been argued [3] that the helicity suppression (and partly also the color suppression) may be lifted by soft gluon radiation. Unfortunately a quantitative estimate [4] of the gluon enhancement is even more difficult than estimates of the corresponding plain valence quark processes leaving the lifetime question numerically unsolved. Some earlier calculations claiming a sizeable or even dominant decay rate via W-exchange used very large D wave functions corresponding to a decay constant $f_D\sim500$–700 MeV which is now excluded by the recent

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MARK III bound [8], $f_D < 340$ MeV. Fortunately, MARK III has also measured the branching ratios of about 15 exclusive D decay modes [9]. These data indicate that the nonleptonic two-body decays (including resonances) actually constitute a large fraction of the nonleptonic D decays. A detailed theoretical analysis of two body decays in the framework of quark models [7] supplemented by $1/N_c$ expansion [5] and QCD sum rules [10] has lead to an internally consistent and quantitatively fair description of the experimental results. Important for the present discussion are the implications of this work [11] on the D-meson lifetime question: the observed $D^+ - D^0$ lifetime different is dominantly due to quark interference enforced by about 20% W-exchange contribution. Needless to say, it would be very interesting to have further tests of this conclusion. There are two possible places, the D_s^+ and the charmed baryon system. In this paper we analyse the latter.

We are considering baryons with one charmed quark and two light quarks (u, d, or s). Four of these baryons decay weakly: the Λ_c^+ with the quark content (cdu), the Ξ_c^+ (former A^+) and the Ξ_c^0 (former A^0) with the quark contents (csu) and (csd), respectively, and the Ω_c^0 (former T^0) with the quark content (css). Experimentally not very much is known about these states. In particular, only two lifetimes are measured:

$$\tau(\Lambda_c^+) = \left(1.9 \begin{matrix} +0.5 \\ -0.3 \end{matrix}\right) \times 10^{-13}\ \text{s [1]}$$
$$\tau(\Xi_c^+) = \left(4.8 \begin{matrix} +2.9 \\ -1.8 \end{matrix}\right) \times 10^{-13}\ \text{s [12].} \quad (1)$$

Although the experimental errors are still large the above results indicate that the lifetimes of Λ_c^+ and Ξ_c^+ differ as expected if preasymptotic effects are important. The mechanisms responsible for the latter are again quark interference and eventually W-exchange. Annihilation into a virtual W is not possible because of the absence of antiquarks in baryon bound state. The essential difference to the meson case, however, lies in the fact that, W-exchange among valence quarks of baryons is neither helicity nor color suppressed and should therefore, lead to much more pronounced effects. More specifically, the relevance of W-exchange does not depend on soft gluon enhancement, the crucial question in charmed meson decays. Here, soft gluon radiation is a mere correction to the pure valence quark process. In this sense, the preasymptotic effects in charmed baryon decays are cleaner and calculable more reliably then in the case of charmed mesons. Another advantage is the possibility to study the influence of interference and W-exchange on the lifetimes of four states. Because of the different quark structure of these states, interference and $W-$exchange contribute in different combinations and with different strengths. Thus it is relatively easy to disentangle and determine the individual contributions, once sufficiently accurate data become available. The importance of such information for the understanding of the D and D_s lifetime differences is obvious. Although experimentally a very difficult task, it would be a pity not to exploit the favorable theoretical circumstances offered by the weak decays of charmed baryons.

In this paper we shall present detailed theoretical expectations on the Λ_c^+, $\Xi_c^{+,0}$ and Ω_c^0 lifetimes. The case of the Λ_c^+ has been treated earlier in the literature [6, 13]. Here, we extend these investigations to the other charmed baryons and develop the qualitative lifetime pattern in an essentially model independent way. We also give quantitative estimates using nonrelativistic and relativistic quark models and discuss the uncertainties. Furthermore we describe how to isolate the W-exchange and interference effects and determine their size phenomenologically, once the lifetimes are measured. Some results of our investigations have already been presented at the Heidelberg Workshop [14] and the Berkeley Conference [11]. Here we give a full account of our work.

The paper is organized as follows: in Sect. II we give the basics and develop the formalism. In Sect. III we present and discuss the qualitative predictions on the lifetime difference. These results are complemented in Sect. IV by quantitative estimates. Section V then gives a brief summary.

II. Preasymptotic Effects in Inclusive Decays

The starting point of our work is the effective weak Hamiltonian provided by the standard $SU(3) \times SU(2) \times U(1)$ model of electroweak and strong interactions. It follows from the Wilson short distance expansion of the product of weak currents and takes into account the QCD corrections to the bare effective Hamiltonian. In the usual form, the nonleptonic piece reads [15]

$$\mathscr{H}_{\text{eff}} = \sqrt{2} G_F U_{\bar{q}_3 c} U^*_{\bar{q}_1 q_2} [c_- \mathcal{O}_- + c_+ \mathcal{O}_+] \quad (2)$$

where $\mathcal{O}_\pm$ are local 4-quark operators

$$\mathcal{O}_\pm = (\bar{q}_{1L} \gamma_\mu q_{2L})(\bar{q}_{3L} \gamma^\mu c_L) \pm (\bar{q}_{3L} \gamma_\mu q_{2L})(\bar{q}_{1L} \gamma^\mu c_L) \quad (3)$$

with $\bar{q}_L \gamma_\mu q_L = \frac{1}{2} \bar{q} \gamma_\mu (1-\gamma_5) q$, and $U_{\bar{q}_\alpha q_\beta}$ are elements of the Kobayashi-Maskawa mixing matrix. The coefficients $c_\pm$ are the QCD corrected Wilson coefficients in leading logarithmic approximation given by

$$c_\pm(\mu^2) \cong \left(\frac{\alpha_s(\mu^2)}{\alpha_s(m_w^2)}\right)^{\frac{d_\pm}{2b}}. \quad (4)$$

Here, $\alpha_s(Q^2)=4\pi/b \ln(Q^2/\Lambda^2)$ is the running QCD coupling constant with b depending on the number of colors (N_c) and on the number of effective flavors (n_f)

$$b=\tfrac{1}{3}(11N_c-2n_f). \qquad (5)$$

Finally, the quantities $d_- = -2d_+ = 8$ are proportional to the anomalous dimensions of the operators $\mathcal{O}_-$ and $\mathcal{O}_+$, which are calculated by inserting the operators in all relevant irreducible Green's functions. The dependence of $c_\pm$ on the subtraction point μ according to (4) should cancel with the corresponding μ-dependence of the matrix element of the operators $\mathcal{O}_\pm$ in order to yield μ-independent physical amplitudes. Unfortunately, $\langle \mathcal{O}_\pm \rangle(\mu)$ cannot be calculated from first principles of QCD at present, because of the lack of knowledge of the true QCD wave functions. Thus, our results will depend on μ. The usual choice is $\mu \simeq O(m_c)$.

Using (2) the general expression of the total hadronic decay rate of a charmed baryon B is given by

$$\Gamma^{\text{had}}(B)=\frac{1}{M_B}\,\text{Im}\,\langle B|\,i\int d^4x\,\text{T}(\mathcal{H}_{\text{eff}}(x)\,\mathcal{H}^\dagger_{\text{eff}}(0))\,|B\rangle. \qquad (6)$$

In order to estimate the absorptive part of the matrix element on the rhs of (6) one invokes quark-hadron duality. This means that above some threshold energy, one represents the physical intermediate states in (6) by quarks instead of physical hadrons. The quark-hadron duality has been proven to work for a variety of processes in the framework of QCD sum rules [16]. Moreover, the duality holds even at surprisingly low scales, of order m_ρ^2. Therefore one can expect duality to work also in charmed baryon and meson decays, where the typical scale is of order ~ 1.5 GeV (the charm quark mass). Asymptotically, i.e. for sufficiently heavy quarks, the duality picture implies equal total decay rates and lifetimes for all weakly decaying particles carrying the heavy flavor in question. This cannot really be expected in the charm case due to the moderate value of the charm quark mass. Indeed the experimental lifetimes of charmed mesons and baryons are different indicating clearly the importance of preasymptotic effects. These effects can readily be estimated following the approach of [6].

The calculation is particularly straightforward in coordinate space. Performing the Wick contractions in the T-product of (6) one obtains

i) a term with the light quarks q_1, q_2 and q_3 contracted (Fig. 1a): it corresponds to the c-quark decay contribution,

ii) three terms with only two contractions and one normal-ordered product of uncontracted fields (Fig. 1b, c, d): they correspond to W-exchange (Fig. 1b) and quark interference (Fig. 1c, d) contributions.

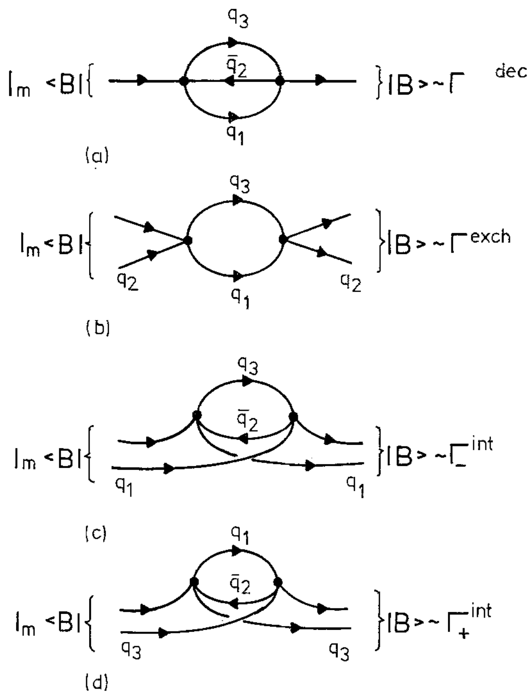

Fig. 1a–d. Contributions to nonleptonic decay rates of charmed baryons arising from the operator $(\bar{q}_1 q_2)(\bar{q}_3 c)$. **a** c-quark decay, **b** W-exchange, **c** destructive interference, **d** constructive interference

The normal-ordered product: $q_\alpha \bar{q}_\alpha$: $|B\rangle$ ($\alpha=2$, 1, 3 in Fig. 1b, c, d respectively) vanishes if the wave function of the charmed baryon does not carry the flavor q_α. In that case, the baryon decays only through the "spectator mechanism", i.e.

$$c \to q_1\,\bar{q}_2\,q_3. \qquad (7\text{a})$$

On the other hand if the quark flavors q_1 and/or q_3 are present in the charmed baryon wave function they interfere with the corresponding flavors in the final state of (7) (Fig. 1c, d). The interference effects are in principle different for q_1 and q_3 quarks. Trivially, no interference is associated with the quark flavor q_2 since there is no valence $\bar{q}_2$-antiquark in a baryon. However, in the crossing process

$$c\,q_2 \to q_1\,q_3, \qquad (7\text{b})$$

depicted in Fig. 1b, the q_2 flavor, if present in the baryon wave function, plays an active role, giving rise to baryon decay via W-exchange.

The explicit evaluation of the rate (6) has been performed in coordinate space, using the technique of [17]. One encounters two types of integrals

$$I=\int d^4x\,e^{ip\cdot x}\,\frac{1}{(x^2)^n},$$

$$I_{\mu\nu}=\int d^4x\, e^{ip\cdot x}\frac{x_\mu x_\nu}{(x^2)^n}.$$

Both integrals can be computed from the general formula

$$\int d^4x\, e^{ip\cdot x}\frac{1}{(x^2)^n}=i(-)^n\frac{\pi^2}{2^{2n-4}\Gamma(n)\Gamma(n-1)}(p^2)^{n-2}\cdot\ln(-p^2-i\eta). \quad (9)$$

Taking the imaginary part of the integral one readily obtains the following general expressions for the c-quark decay, W-exchange and interference contributions to the nonleptonic decay rates;

$$\Gamma^{\mathrm{dec}}(B)=\frac{G_F^2 m_c^5}{192\pi^3}\xi(2c_+^2+c_-^2)\frac{1}{2M_B}\langle B|\bar{c}(1-\gamma_5)c|B\rangle$$

$$\Gamma^{\mathrm{exch}}(B)_{q_2}=\frac{G_F^2 m_c^2}{4\pi}\xi\frac{1}{2M_B}\langle B|(c_+^2+c_-^2)(\bar{c}\Gamma_\mu c\bar{q}_2\Gamma^\mu q_2)+(c_+^2-c_-^2)(\bar{c}^i\Gamma_\mu c^j\bar{q}_2^j\Gamma^\mu q_2^i)|B\rangle$$

$$\Gamma_-^{\mathrm{int}}(B)_{q_1}=-\frac{G_F^2 m_c^2}{16\pi}\xi\frac{1}{2M_B}\langle B|(c_++c_-)^2(\bar{c}\Gamma_\mu c\bar{q}_1\Gamma^\mu q_1+\tfrac{2}{3}\bar{c}\gamma_\mu\gamma_5 c\bar{q}_1\Gamma^\mu q_1)+(5c_+^2+c_-^2-6c_+c_-)(\bar{c}^i\Gamma_\mu c^j\bar{q}_1^j\Gamma^\mu q_1^i+\tfrac{2}{3}\bar{c}^i\gamma_\mu\gamma_5 c^j\bar{q}_1^j\Gamma^\mu q_1^i)|B\rangle$$

$$\Gamma_+^{\mathrm{int}}(B)_{q_3}=-\frac{G_F^2 m_c^2}{16\pi}\xi\frac{1}{2M_B}\langle B|(c_+-c_-)^2(\bar{c}\Gamma_\mu c\bar{q}_3\Gamma^\mu q_3+\tfrac{2}{3}\bar{c}\gamma_\mu\gamma_5 c\bar{q}_3\Gamma^\mu q_3)+(5c_+^2+c_-^2+6c_+c_-)(\bar{c}^i\Gamma_\mu c^j\bar{q}_3^j\Gamma^\mu q_3^i+\tfrac{2}{3}\bar{c}^i\gamma_\mu\gamma_5 c^j\bar{q}_3^j\Gamma^\mu q_3^i)|B\rangle. \quad (10)$$

In the above, $\Gamma_\mu\equiv\gamma_\mu(1-\gamma_5)$, i,j are color indices and the subscripts q_α denote the uncontracted quark fields according to Figs. 1b, c, d. The factor ξ is given by

$$\xi=|U_{\bar{q}_3c}|^2|U_{\bar{q}_1q_2}|^2. \quad (11)$$

As already mentioned one generally obtains two different interference contributions, a destructive one (Γ_-^{int} in (10)) and a constructive one (Γ_+^{int} in (10)). This is quickly demonstrated. Since the baryon wave function is antisymmetric in color the second matrix elements in the expressions for Γ^{exch} and Γ^{int} reduce to the first ones with a minus sign yielding

$$\Gamma^{\mathrm{exch}}(B)_{q_2}=\frac{G_F^2 m_c^2}{2\pi}\xi c_-^2\mathscr{M}_{q_2}(B)$$

$$\Gamma_-^{\mathrm{int}}(B)_{q_1}=-\frac{G_F^2 m_c^2}{4\pi}\xi c_+(2c_--c_+)\mathscr{M}_{q_1}(B) \quad (12)$$

$$\Gamma_+^{\mathrm{int}}(B)_{q_3}=\frac{G_F^2 m_c^2}{4\pi}\xi c_+(2c_-+c_+)\mathscr{M}_{q_3}(B)$$

where

$$\mathscr{M}_{q_\alpha}(B)=\frac{1}{2M_B}\langle B|\bar{c}\Gamma_\mu c\bar{q}_\alpha\Gamma^\mu q_\alpha|B\rangle,\quad \alpha=2 \quad (13\mathrm{a})$$

$$\mathscr{M}_{q_\alpha}(B)=\frac{1}{2M_B}\langle B|\bar{c}\Gamma_\mu c\bar{q}_\alpha\Gamma^\mu q_\alpha+\tfrac{2}{3}\bar{c}\gamma_\mu\gamma_5 c\bar{q}_\alpha\Gamma^\mu q_\alpha|B\rangle\quad \alpha=1,3. \quad (13\mathrm{b})$$

The subscript q_α in $\mathscr{M}_{q_\alpha}$ again denotes the quark flavor matching a light quark in the baryon wave function. The second term in (13b) vanishes except in the case of Ω_c^0. The matrix elements $\mathscr{M}_{q_\alpha}(B)$ measure the strength of the "nonspectator" effects and are to be evaluated in ones favorite quark model. Before we do that, we want to discuss more generally the relations obtained.

It is clear from (10) and (12) that the quark decay rates Γ^{dec} scale like m_c^5 whereas the nonspectator effects represented by Γ^{exch} and Γ^{int}, scale like m_c^2. Therefore, numerical calculations are quite sensitive to the specific value of m_c used. As a reasonable choice, we take $m_c\sim 1.6$ GeV corresponding to the effective value of the charmed quark mass in the baryon bound state. Furthermore, preasymptotic effects in charmed baryon and meson decays are closely related. Most interesting is the relation between the exchange diagram in Λ_c^+ decay (Fig. 1b) and the interference diagram in D^+-decay. Basically, the interference diagram in D^+ decay is the exchange diagram Fig. 1b with the d-quarks crossed. The only differences occur in the short distance factors and the wave functions. Since meson and baryon wave functions are similar in order of magnitude one sees that large exchange contributions to baryon decays imply large interference effects in meson decays and vice versa. Finally, the ratios of interference to exchange contributions are given by

$$\frac{\Gamma_-^{\mathrm{int}}(B)_{q_1}}{\Gamma^{\mathrm{exch}}(B)_{q_2}}=-\frac{c_+(2c_--c_+)}{2c_-^2}\frac{\mathscr{M}_{q_1}(B)}{\mathscr{M}_{q_2}(B)}$$
$$\frac{\Gamma_+^{\mathrm{int}}(B)_{q_3}}{\Gamma^{\mathrm{exch}}(B)_{q_2}}=\frac{c_+(2c_-+c_+)}{2c_-^2}\frac{\mathscr{M}_{q_3}(B)}{\mathscr{M}_{q_2}(B)} \quad (14)$$

while

$$\left|\frac{\Gamma_+^{\mathrm{int}}(B)_{q_3}}{\Gamma_-^{\mathrm{int}}(B)_{q_1}}\right|=\frac{2c_-+c_+}{2c_--c_+}\frac{\mathscr{M}_{q_3}(B)}{\mathscr{M}_{q_1}(B)}. \quad (15)$$

These ratios are free of the uncertainties due to m_c and depend only little on the specific quark model used in the calculation of $\mathscr{M}_{q_\alpha}(B)$. Modulo $SU(3)$ breaking, $\mathscr{M}_{q_i}/\mathscr{M}_{q_j}=1$ in a nonrelativistic model, except one of the baryons is the Ω_c^0. As discussed later, Ω_c^0 contains an additional numerical factor which arises from spin statistics later. Also, the uncertainties in the short distance QCD factors due to the uncertain value of the scale μ in the coefficients $c_\pm(\mu)$, (4) are relatively moderate. Furthermore, the signs in (14) are unambiguously fixed, since $c_- > c_+ > 0$ for any μ. For the same reason $|\Gamma_+^{\text{int}}|$ is always larger than $|\Gamma_-^{\text{int}}|$, as can be seen from (15). These important facts enable us to predict the qualitative lifetime pattern of charmed baryons with high confidence.

III. Qualitative Lifetime Pattern of Charmed Baryons

It is sufficient to consider only the Cabibbo favored decays. In this case, one has $q_1 = u$, $q_2 = d$, $q_3 = s$ in the equations of the last section and

$$\xi = |U_{\bar{u}d}|^2\, |U_{\bar{s}c}|^2 \approx \cos^4\theta_c \tag{16}$$

in (10) and (12), specifically. The nonleptonic decay rates for $\Lambda_c^+(c\,ud)$, $\Xi_c^+(cus)$, $\Xi_c^0(cds)$ and $\Omega_c^0(css)$ are then given by

$$\begin{aligned}
\Gamma_{NL}(\Lambda_c^+) &= \Gamma^{\text{dec}}(\Lambda_c^+) + \Gamma^{\text{exch}}(\Lambda_c^+)_d + \Gamma_-^{\text{int}}(\Lambda_c^+)_u\\
\Gamma_{NL}(\Xi_c^+) &= \Gamma^{\text{dec}}(\Xi_c^+) + \Gamma_-^{\text{int}}(\Xi_c^+)_u + \Gamma_+^{\text{int}}(\Xi_c^+)_s\\
\Gamma_{NL}(\Xi_c^0) &= \Gamma^{\text{dec}}(\Xi_c^0) + \Gamma^{\text{exch}}(\Xi_c^0)_d + \Gamma_+^{\text{int}}(\Xi_c^0)_s\\
\Gamma_{NL}(\Omega_c^0) &= \Gamma^{\text{dec}}(\Omega_c^0) + \Gamma_+^{\text{int}}(\Omega_c^0)_s.
\end{aligned} \tag{17}$$

From the ratios given in (14) and using $c_+ \sim 0.74$, $c_- \sim 1.8$ (corresponding to $\Lambda_{\text{QCD}} \sim 250$ MeV, $N_c = 3$, $n_f = 4$ in (4)) one estimates, further,

$$\Gamma^{\text{exch}} : \Gamma_+^{\text{int}} : \Gamma_-^{\text{int}} \simeq 1 : 0.5 : -0.3 \tag{18}$$

modulo $SU(3)$ breaking (and a spin factor in the case of Ω_c^0). Consequently, we expect the uniform lifetimes due to the *c-quark decay mechanism* (Fig. 1a)

$$\tau(\Xi_c^0) \simeq \tau(\Lambda_c^+) \simeq \tau(\Omega_c^0) \simeq \tau(\Xi_c^+) \tag{19}$$

to be altered by the *W-exchange contributions* (Fig. 1b) to

$$\tau(\Xi_c^0) \simeq \tau(\Lambda_c^+) < \tau(\Omega_c^0) \simeq \tau(\Xi_c^+) \tag{20}$$

and further modified by the *light quark interference* effects (Fig. 1c, d) to

$$\tau(\Xi_c^0) < \tau(\Lambda_c^+) < \tau(\Xi_c^+). \tag{21}$$

The Ω_c^0 lifetime needs some extra considerations for the following reason. Because of the presence of two s-quarks in the Ω_c^0 (as opposed to one s-quark in the Ξ_c^+ and Ξ_c^0) which interfere with the s-quark produced when the c-quark decays, one quite generally expects $\Gamma_+^{\text{int}}(\Omega_c^0) > \Gamma_+^{\text{int}}(\Xi_c^+ \text{ or } \Xi_c^0)$. In fact, a nonrelativistic quark model gives

$$\Gamma_+^{\text{int}}(\Omega_c^0)_s = \tfrac{10}{3}\,\Gamma_+^{\text{int}}(\Xi_c^{+,0})_s \tag{22}$$

where the numerical factor arises simply from the spin wave function as shown later. Using again (18) one thus finds in total

$$\tau(\Omega_c^0) \simeq \tau(\Xi_c^0) < \tau(\Lambda_c^+) < \tau(\Xi_c^+). \tag{23}$$

It is also interesting to note that without short-distance QCD corrections, that is for $c_+ = c_- = 1$, one would have $\Gamma^{\text{exch}} : \Gamma_+^{\text{int}} : \Gamma_-^{\text{int}} = 1 : 3/2 : -1/2$ and

$$\tau(\Omega_c^0) < \tau(\Xi_c^0) < \tau(\Xi_c^+) < \tau(\Lambda_c^+). \tag{24}$$

Thus we see that the lifetime differences of the weakly decaying charmed baryons are characteristic for both the preasymptotic effects due to W-exchange and light quark interferences, *and* the hard gluon modifications of the effective weak Hamiltonian. The lifetime pattern exhibited in (23) is largely independent of details of the baryonic wave functions. Therefore, a fairly accurate lifetime measurement would allow a practically model-independent test of the dynamics of weak decays at preasymptotic scales. Such a test would certainly help to resolve the uncertainties in the explanation of the lifetime differences of the charmed mesons (D^+, D^0, D_s^+).

The remaining questions then concern the size of the lifetime differences of charmed baryons and absolute lifetime predictions. Obviously, the answers will depend on the quark model used to calculate the baryonic wave functions and on the choice of other parameters such as m_c, μ, etc. Our quantitative estimates are put together in the next section.

IV. Absolute and Relative Lifetime Estimates in Quark Models

In (10) we have expressed the various contributions to the non-leptonic decay rates in terms of the matrix elements $\mathscr{M}_{q_\alpha}(B)$ defined in (13) and the matrix element

$$m(B) = \frac{1}{2M_B}\langle B|\,\bar{c}(1-\gamma_5)\,c\,|B\rangle. \tag{25}$$

First we proceed to calculate the above matrix elements in a bag model*. Although calculated in the bag, the results are generally valid for any quark model with large (u) and small (v) components of the quark-wave functions. The standard procedure leads to

$$\begin{aligned}
&m(B) = \int d^3r\,(u_c^2 - v_c^2) \quad \text{for all baryons}\\
&\mathscr{M}_{q_\alpha}(B) = a_{q_\alpha} + b_{q_\alpha}, \quad \text{for } B = \Lambda_c^+, \Xi_c^+, \Xi_c^0\\
&\mathscr{M}_s(\Omega_c^0) = \tfrac{2}{9}(15a_s + 7b_s + 16c_s).
\end{aligned} \tag{26}$$

* A similar calculation has been performed earlier in order to estimate the interference effect in D^+ decay [18]

Here *a*, *b*, and *c* are the following bag integrals

$$a_q=\int d^3r(u_c^2(r)\,u_q^2(r)+v_c^2(r)\,v_q^2(r))$$
$$b_q=\int d^3r(u_c^2(r)\,v_q^2(r)+v_c^2(r)\,u_q^2(r)) \quad (27)$$
$$c_q=\int d^3r(u_c(r)\,u_q(r)\,v_c(r)\,v_q(r))$$

where $u_q(r)$ and $v_q(r)$ denote the large and small components of a particular radial quark wave function, respectively.

In the nonrelativistic (NR) limit, the small components vanish and (27) reduces to

$$a_q\to\int d^3r\,u_c^2(r)\,u_q^2(r)\equiv|\psi_{cq}(0)|^2$$
$$b_q\to 0,\qquad c_q\to 0, \quad (28)$$

where $\psi(0)$ is the baryon wave function at the origin. The matrix elements (26) now become rather simple:

$$\mathcal{m}B)=1 \quad \text{for all baryons}$$
$$\mathcal{M}_q(B)=|\psi_{cq}(0)|^2 \quad \text{for } B=\Lambda_c^+,\Xi_c^+,\Xi_c^0 \quad (29)$$
$$\mathcal{M}_s(\Omega_c^0)=\tfrac{10}{3}|\psi_{cs}(0)|^2.$$

The first relations in (10) and (29) constitute the basis of the well-known spectator model of weak decays of heavy-flavored hadrons: every reference to hadronic bound states has disappeared. The last relation in (29) explains (22).

Using the above results we are now in a position to give and compare quantitative predictions of relativistic and nonrelativistic quark models. From the above formulas one would naively expect the relativistic description to lead to larger matrix elements $\mathcal{M}_{q_\alpha}(B)$ then the nonrelativistic one. However, it is well-known from the physics of nonleptonic hyperon decays [19] that nonrelativistic approach (potential models, oscillator models) gives larger matrix elements than relativistic bag models. This inconsistency is presently not resolved. Bag-model wave functions describe correctly the parity-violating amplitudes A^{pv} of hyperons, however, fail in the case of the parity-conserving amplitudes B^{pc} which come out too small by a factor 2. The discrepancy with experiment is believed to be explained by contributions to B^{pc} coming from $1/2^+$ baryon resonances. On the other hand, the larger nonrelativistic wave functions lead to too big A^{pv} but correct B^{pc}. Here, agreement with experiment is achieved by introducing $1/2^-$ and $3/2^+$ resonance poles. At present, there are no compelling theoretical arguments in favor of any of these two schemes.

In the following, we shall adopt a bag model [20] that leads to a reasonable description of hyperon masses and decays [20, 21], keeping in mind that it may underestimate the preasymptotic effects in charmed baryon decays. The relevant bag parameters are $m_{u,d}=0$, $m_s=0.279$ GeV, $m_c=1.551$ GeV, and the bag radii: $R(\Lambda_c^+)=4.63\ \text{GeV}^{-1}$, $R(\Xi_c^+)=R(\Xi_c^0)=4.58\ \text{GeV}^{-1}$ and $R(\Omega_c^0)=5.02\ \text{GeV}^{-1}$. These yield the following values for the integrals in (26) and (27):

Table 1. Contributions to the nonleptonic decay rates [in units of 10^{-12} GeV] of charmed baryons and lifetimes in a relativistic bag model. The results are obtained with $m_c=1.6$ GeV, $c_+=0.74$, $c_-=1.8$ and the bag parameters given in the text

	Γ^{dec}	Γ^{exch}	Γ_-^{int}	Γ_+^{int}	Γ_{NL}	Γ^{tot}	τ (s)
Λ_c^+	0.98	0.55	−0.18	0	1.35	1.8	3.7×10^{-13}
Ξ_c^+	0.98	0	−0.19	0.36	1.15	1.5	4.4×10^{-13}
Ξ_c^0	0.98	0.57	0	0.36	1.91	2.3	2.8×10^{-13}
Ω_c^0	0.98	0	0	0.92	1.90	2.3	2.8×10^{-13}

$$\mathcal{m}(B)=0.94 \quad (30)$$

$$a_{u,d}(\Lambda_c^+)=2.61\times10^{-3}\ \text{GeV}^3$$
$$b_{u,d}(\Lambda_c^+)=0.47\times10^{-3}\ \text{GeV}^3$$
$$a_{u,d}(\Xi_c^{+,0})=2.70\times10^{-3}\ \text{GeV}^3$$
$$b_{u,d}(\Xi_c^{+,0})=0.48\times10^{-3}\ \text{GeV}^3$$
$$a_s(\Xi_c^{+,0})=3.64\times10^{-3}\ \text{GeV}^3 \quad (31)$$
$$b_s(\Xi_c^{+,0})=0.38\times10^{-3}\ \text{GeV}^3$$
$$a_s(\Omega_c^0)=2.85\times10^{-3}\ \text{GeV}^3$$
$$b_s(\Omega_c^0)=0.27\times10^{-3}\ \text{GeV}^3$$
$$c_s(\Omega_c^0)=0.10\times10^{-3}\ \text{GeV}^3.$$

For the matrix elements of (26), one then obtains numerically

$$\mathcal{M}_{u,d}(\Lambda_c^+)=3.08\times10^{-3}\ \text{GeV}^3$$
$$\mathcal{M}_{u,d}(\Xi_c^{+,0})=3.18\times10^{-3}\ \text{GeV}^3 \quad (32)$$
$$\mathcal{M}_s(\Xi_c^{+,0})=4.02\times10^{-3}\ \text{GeV}^3$$
$$\mathcal{M}_s(\Omega_c^0)=10.28\times10^{-3}\ \text{GeV}^3.$$

Note the relatively large flavor symmetry violation in $\mathcal{M}_s(\Xi_c^+)$. The enhancement of $\mathcal{M}_s(\Omega_c^0)$ is mainly due to the presence of two valence *s*-quarks as pointed out earlier. The numerical results on the nonleptonic decay rates calculated from (10) or (12) are summarized in Table 1. Also shown are the total decay rates and the corresponding absolute lifetimes. The latter estimates include a nominal semileptonic width $\Gamma_{SL}\simeq 0.4\times10^{-12}$ GeV which corresponds to a $2\times15\%$ semileptonic branching ratio from the spectator mechanism alone. For the lifetime ratios one then finds

$$\tau(\Omega_c^0):\tau(\Xi_c^0):\tau(\Lambda_c^+):\tau(\Xi_c^+)\simeq 0.75:0.75:1:1.2. \quad (33)$$

In the nonrelativistic case (29), the matrix elements $\mathcal{M}_q(B)$ are directly determined by the baryon wave functions

$$|\psi_{cq}(0)|^2\sim\langle B|\,\delta^3(\mathbf{r}_c-(\mathbf{r}_q)\,|B\rangle \quad (34)$$

Table 2. Contributions to the nonleptonic decay rates [in units of 10^{-12} GeV] and lifetimes in a nonrelativistic quark model. The results are obtained with $m_c = 1.6$ GeV, $c_+ = 0.74$, $c_- = 1.8$ and $|\psi(0)|^2 \simeq 10^{-2}$ GeV^3

	Γ^{dec}	Γ^{exch}	Γ_-^{int}	Γ_+^{int}	Γ_{NL}	Γ^{tot}	τ (s)
Λ_c^+	1.04	1.98	−0.65	0	2.37	2.8	2.3×10^{-13}
Ξ_c^+	1.04	0	−0.65	0.98	1.37	1.8	3.7×10^{-13}
Ξ_c^0	1.04	1.98	0	0.98	4.00	4.4	1.5×10^{-13}
Ω_c^0	1.04	0	0	3.26	4.30	4.7	1.4×10^{-13}

which essentially measure the overlap of the charmed quark with one of the light constituent quarks. A reasonable estimate of $|\psi_{cq}(0)|^2$ can be obtained from the hyperfine splitting in the charmed baryon system. For example, [22]

$$\Delta M = M_{\Sigma_c^+} - M_{\Lambda_c^+} = \frac{16}{9} \pi \alpha_s \frac{m_c - m_u}{m_u^2 m_c} |\psi_{cu}^{\Lambda_c}(0)|^2, \tag{35}$$

yields

$$|\psi_{cu}^{\Lambda_c}(0)|^2 \simeq 10^{-2}\ \text{GeV}^3, \tag{36}$$

if $\Delta M \sim 170$ MeV, $m_c = 1.8$ GeV, $m_u = 340$ MeV and $\alpha_s(m_c^2) = 0.4$ is used. The uncertainty entering through the effective QCD coupling constant α_s and the quadratic dependence on the constituent mass m_u can be reduced at the cost of introducing the D-meson wave function (Cortes and Sanchez-Guillen, [13])

$$\frac{|\psi^{\Lambda_c}(0)|^2}{|\psi^D(0)|^2} = \frac{2m_u}{m_c - m_u} \frac{M_{\Sigma_c^+} - M_{\Lambda_c^+}}{m_{D^*} - m_D}. \tag{37}$$

Using $|\psi^D(0)|^2 = 1/12 m_D f_D^2$ and, for example, the experimental upper limit [8], $f_D < 340$ MeV one reproduces the value (36), while the QCD sum rule estimate [23] $f_D = 170$ MeV gives $|\psi^{\Lambda_c}(0)|^2 = 2.5 \times 10^{-3}$ GeV^3. Table 2 displays the numerical results on the various widths and lifetimes using (36) for all baryons thereby neglecting flavor symmetry violation effects*. The corresponding lifetime ratios are

$$\tau(\Omega_c^0) : \tau(\Xi_c^0) : \tau(\Lambda_c^+) : \tau(\Xi_c^+) \simeq 0.6 : 0.6 : 1 : 1.6. \tag{38}$$

As expected and exemplified by (28) and (38) the uncertainty in the quantitative predictions on the relative lifetimes is considerable. This is even more true for the absolute lifetime estimates listed in Tables 1 and 2, which are at most reliable within a factor 1.5–2. Some reassurance comes from the rough agreement of our inclusive results with the lifetime ratios emerging from semileptonic decays and nonleptonic two-body decays alone [24].

$$\tau(\Omega_c^0) : \tau(\Xi_c^0) : \tau(\Lambda_c^+) : \tau(\Xi_c^+) \simeq 0.7 : 0.7 : 1 : 2. \tag{39}$$

* The small difference in Γ^{dec} obtained in the nonrelativistic model as compared to the bag model (see Table 1) is due to the negative contribution of the small components of the bag wave functions as can be seen from (26–29)

V. Summary and Conclusions

We have studied the lifetimes of the weakly decaying charmed baryons Λ_c^+, Ξ_c^+, Ξ_c^0 and Ω_c^0 using the effective weak Hamiltonian provided by the standard gauge model of electroweak and strong interactions, and baryon wave functions as described by typical quark models of QCD bound states. We have found an interesting pattern of lifetime differences of the four baryons which arises from light quark interferences in the c-quark decay process and from decays via W-exchange.

The qualitative features of this lifetime pattern depend only on the properties of the effective Hamiltonian, in particular, the QCD modifications, and on the color, flavor and spin structure of the baryonic bound states. Hence, the hierarchy of lifetimes presented in this paper is a very reliable prediction and, given the necessary data, allows a clear test of our correct understanding of the decay dynamics. The two existing lifetime measurements shown in (1) are consistent with our expectation and indicate the presence of W-exchange in Λ_c^+ decay. If W-exchange were absent or negligible one should observe $\tau(\Xi_c^+)/\tau(\Lambda_c^+) \leqq 1$!

The quantitative estimates shown in this paper are supposed to provide some rough idea of the size of the lifetime differences one can expect. Some of these estimates are subject to considerable uncertainties as suggested from the experience in similar quark model calculations and as illustrated by the numerical differences of our nonrelativistic quark model and bag model results. The deviations of the lifetime ratios from unity are essentially proportional to $|\psi(0)|^2/m_c^3$ where $\psi(0)$ denotes the magnitude of the baryon wave function at the origin. This clearly reveals the pre-asymptotic nature ($|\psi(0)|^2/m_c^3 \to 0$ for $m_c \to \infty$) and the bound state (or nonspectator) origin of the lifetime differences, and indicates the main source of quantitative uncertainties. For example, a 100% error in $|\psi(0)|^2/m_c^3$ typically translates into a 30% error in the lifetime ratios as one can easily convince oneself using the formulas and numerical tables provided. Keeping the above in mind, it is encouraging to note that the ratio

$$\frac{\tau(\Xi_c^+)}{\tau(\Lambda_c^+)} \simeq 1.6 \tag{40}$$

obtained in the nonrelativistic quark model with $|\psi(0)|^2 \simeq 10^{-2}$ GeV^3 and $m_c = 1.6$ GeV agrees with the measured one,

$$\frac{\tau(\Xi_c^+)}{\tau(\Lambda_c^+)} \simeq 2.5 {+1.7 \atop -1.0} \tag{41}$$

within the, unfortunately large, experimental errors. It is also obvious that the theoretical uncertainty in the absolute lifetimes is considerably bigger than in lifetime ratios mainly due to an extra factor m_c^5 coming from the spectator decay rates which set the scale. We estimate the computed lifetimes to be reliable within a factor 1.5 to 2. Interestingly, the same quark model which leads to (40) also predicts

$$\tau(\Xi_c^+) \simeq 3.7 \times 10^{-13}\ \mathrm{s}$$

and (42)

$$\tau(\Lambda_c^+) \simeq 2.3 \times 10^{-13}\ \mathrm{s}$$

in agreement with the present data quoted in (1).

As mentioned in the introduction, the agreement between quark model estimates (similar to the ones considered in this paper) and experiment on the lifetime ratio and semileptonic branching ratios of charmed mesons improves remarkably if only the leading terms in the $1/N_c$ expansion of the nonleptonic contributions are kept [5, 6]. It is an interesting question what happens in the charmed baryon case. However, because of the difficulties in applying $1/N_c$ expansion methods to baryons [25] we have left this question open. Another point ([26] and Cheng in [13]) not considered here are soft gluon effects suggested to enhance the helicity suppressed W-exchange (annihilation) process in $D^0 (D_s^+)$ decays [3]. This possibility is extremely difficult to evaluate theoretically in any reliable way. Furthermore, we repeat that in baryon decays W-exchange is not helicity suppressed. Hence, soft gluon effects play the role of (perhaps large) corrections to the corresponding valence quark processes.

To conclude, we stress that W-exchange reveals itself most clearly in the smallness of the Λ_c^+ and Ξ_c^0 lifetimes as compared to the Ξ_c^+ lifetime, while the interference effects are most noticeable in a surprisingly small lifetime of the Ω_c^0. The latter is shifted to the lower end of the lifetime hierarchy due to the presence of the s-quark constituents which strongly interfere with the s-quark produced in the c-quark decay. In fact, given accurate lifetime measurements, one could separate and determine the preasymptotic effects together with the spectator decay rate on purely phenomenological grounds as shown below.

$$\begin{aligned}
\Gamma_+^{\mathrm{int}} &= \tfrac{3}{4}\left[\tau^{-1}(\Omega_c^0) - \tau^{-1}(\Xi_c^+) + \tau^{-1}(\Lambda_c^+) - \tau^{-1}(\Xi_c^0)\right] \\
\Gamma_-^{\mathrm{int}} &= \tfrac{3}{4}\left[\tau^{-1}(\Omega_c^0) - \tau^{-1}(\Xi_c^+)\right] + \tfrac{7}{4}\left[\tau^{-1}(\Lambda_c^+) - \tau^{-1}(\Xi_c^0)\right] \\
\Gamma^{\mathrm{exch}} &= \tfrac{3}{4}\left[\tau^{-1}(\Omega_c^0) - \tau^{-1}(\Xi_c^0)\right] + \tfrac{7}{4}\left[\tau^{-1}(\Lambda_c^+) - \tau^{-1}(\Xi_c^+)\right] \\
\Gamma_{\mathrm{spectator}}^{\mathrm{tot}} &= \tfrac{5}{2}\left[\tau^{-1}(\Xi_c^+) + \tau^{-1}(\Xi_c^0) - \tau^{-1}(\Lambda_c^+)\right] - \tfrac{3}{2}\tau^{-1}(\Omega_c^0)
\end{aligned} \tag{43}$$

where $\Gamma_{\mathrm{spectator}}^{\mathrm{tot}} = \Gamma_{SL} + \Gamma^{\mathrm{dec}}$ and $SU(3)$ breaking effects are neglected. The above relations follow directly from the most general of our considerations condensed in (17) and (22). Clearly, such an analysis requires enormous experimental efforts but it does not appear unfeasible in principle. At any rate, it would put the theoretical understanding of short-distance dynamics (see (14) and (15)) and QCD bound state properties (see (26) and (29)) to very stringent tests.

While preparing this paper for publication we have become aware of a recent paper by M.A. Shifman and M.B. Voloshin, preprint ITEP 86-83, who arrived at similar conclusion.

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