MEASURING THE γWW AND ZWW THREE GAUGE VERTEX WITH POLARIZED BEAMS

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The effects of beam polarization on $e^+e^- \to W^+W^- \to 4$ fermions are considered with special emphasis on the measurement of the three vector boson vertices. Transverse beam polarization will be crucial to separate anomalous γWW and ZWW couplings or even to detect anomalies.

After the discovery of the weak bosons the next task is a precise determination of their properties. While SLC/LEP I are designed to study the Z^0 in detail, a precision test of W properties beyond the abilities of hadronic colliders will be the main objective for studying e^-e^- collisions at a center of mass energy $\sqrt{s} = 190–200$ GeV at LEP II.

With an expected luminosity of 500 pb^{-1} yr^{-1} and a W-pair cross section of 15–20 pb (the precise value depending on $\sin^2\theta_W$ and details of radiative corrections) LEP II will produce of order 10^4 W pairs per experiment and year. Even if the 53% of these, which have both W's decay into $q\bar{q}$ pairs, are of limited use only due to jet and/or flavor identification problems, the 40% semi-leptonic events (i.e. one W decaying into leptons, the other one into a $q\bar{q}$ pair) and the remaining leptonic events should suffice to perform detailed investigations of the angular distributions of the decay products of the W's, thus allowing a measurement of the W's couplings to the other fields involved in the reaction [1] [‡1].

At tree level, within the standard model, $e^+e^- \to W^+W^-$ proceeds via photon-, Z- and neutrino-exchange, as shown in fig. 1. It is the interference between these three graphs and in particular the large gauge theory cancellations between them [2] which determine the properties of the reaction. By studying W-pair production, experiments at LEP II are thus uniquely suited to directly measure the nonabelian γWW and ZWW vertices entering in the first two graphs of fig. 1. The general strategy of this measurement, determination of all accessible angular distributions of the W decay products, was described in detail in ref. [1]. There we stated that a separate measurement of the γWW and the ZWW vertex is very difficult without beam polarization. The purpose of this paper is to quantify this statement and to indicate the extent to which beam polarization can improve the separation of the γWW and the ZWW vertex.

In order to determine these vertices, the experimental distributions should be compared with the ones derived from the most general phenomenological vertex, compatible with Lorentz invariance, that can contribute to $e^+e^- \to W^+W^-$ for on-shell W's. The Feynman rule for this vertex [1,3] is

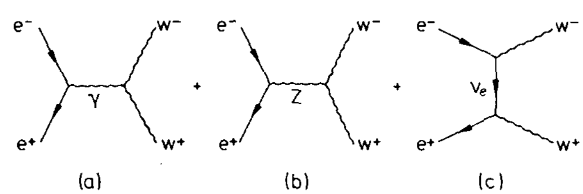

Fig. 1. The three Feynman graphs contributing to $e^+e^- \to W^+W^-$ at tree level.

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[‡1] A more complete reference list can be found in this paper.

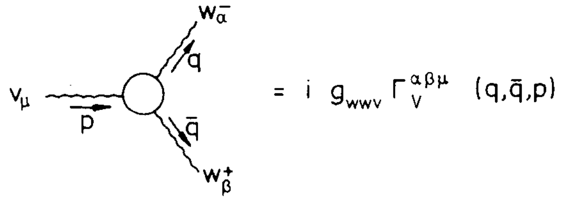

Fig. 2. Feynman rule for the general WWV vertex (V = γ or Z).

given in fig. 2 with

$$\Gamma_V^{\alpha\beta\mu}(q, \bar{q}, P)$$
$$= f_1^V (q-\bar{q})^\mu g^{\alpha\beta} - (f_2^V/m_W^2)(q-\bar{q})^\mu P^\alpha P^\beta$$
$$+ f_3^V (P^\alpha g^{\mu\beta} - P^\beta g^{\mu\alpha}) + i f_4^V (P^\alpha g^{\mu\beta} + P^\beta g^{\mu\alpha})$$
$$+ i f_5^V \epsilon^{\mu\alpha\beta\rho}(q-\bar{q})_\rho - f_6^V \epsilon^{\mu\alpha\beta\rho} P_\rho$$
$$- (f_7^V/m_W^2)(q-\bar{q})^\mu \epsilon^{\alpha\beta\rho\sigma} P_\rho (q-\bar{q})_\sigma, \qquad (1)$$

for V = γ, Z and $g_{WW\gamma} = -e$, $g_{WWZ} = -e \cot\theta_W$. f_1, f_2 and f_3 are frequently written in terms of κ and λ

$$f_1^V = 1 + \lambda_V p^2/2m_W^2, \quad f_2^V = \lambda_V,$$
$$f_3^V = 1 + \kappa_V + \lambda_V, \qquad (2)$$

which are directly related to the W magnetic dipole and electric quadrupole moment μ_W and Q_W

$$\mu_W^\gamma = e(1 + \kappa_V + \lambda_V)/2m_W,$$
$$Q_W^\gamma = -e(\kappa_V - \lambda_V)/m_W^2. \qquad (3)$$

Within the standard model all the form factors f_i^V vanish at tree level except for

$$f_1^V = 1, \quad f_3^V = 2, \quad V = \gamma, Z. \qquad (4)$$

In composite models of the weak bosons [4] at least some of them may be sizeable, however, and one should even be prepared to observe non-vanishing values of the *CP* violating couplings f_4, f_6 and f_7.

The form factors f_i enter linear in the amplitudes for $e^+e^- \to W^+W^-$. To be more specific consider the amplitudes $\mathcal{M}_{\sigma,\bar{\sigma};\lambda,\bar{\lambda}}$ for fixed e^- and e^+ helicities σ and $\bar{\sigma}$ and W^+ and W^- helicities λ and $\bar{\lambda}$ in the e^+e^- center of mass frame. Let (Θ, Φ) be the W^- polar and azimuthal angle with respect to the e^--beam, $\Delta\sigma = \sigma - \bar{\sigma}$, $\Delta\lambda = \lambda - \bar{\lambda}$, and $J_0 = \max(|\Delta\sigma|, |\Delta\lambda|)$. Separating some overall angular dependence in terms of the conventional *d*-functions [5],

$$\mathcal{M}_{\sigma,\bar{\sigma};\lambda,\bar{\lambda}}(\Theta, \Phi) = \sqrt{2}\, e^2\, \Delta\sigma (-1)^{\bar{\lambda}} d^{J_0}_{\Delta\sigma,\Delta\lambda}(\Theta)$$
$$\times \exp(i\,\Delta\sigma\Phi) \tilde{\mathcal{M}}_{\Delta\sigma,\lambda,\bar{\lambda}}(\Theta), \qquad (5)$$

very simple expressions are obtained for the matrix elements $\tilde{\mathcal{M}}$. For positive e^- helicity, i.e. in the scattering of right-handed electrons, the neutrino exchange graph in fig. 1 does not contribute and one finds [1] ($\beta = (1 - 4m_W^2/s)^{1/2}$)

$$\tilde{\mathcal{M}}_{+,\lambda,\bar{\lambda}}(\Theta) = -\beta\{A^\gamma_{\lambda\bar{\lambda}} - [s/(s - m_Z^2)] A^Z_{\lambda\bar{\lambda}}\}. \qquad (6a)$$

For left-handed electrons, on the other hand, neutrino-exchange interferes with photon- and Z-exchange

$$\tilde{\mathcal{M}}_{-,\lambda,\bar{\lambda}}(\Theta) = -\beta\{A^\gamma_{\lambda\bar{\lambda}}$$
$$+ [s/(s - m_Z^2)](1/2\sin^2\theta_W - 1) A^Z_{\lambda\bar{\lambda}}\}$$
$$+ (1/2\beta\sin^2\theta_W)$$
$$\times [B_{\lambda\bar{\lambda}} - (1 + \beta^2 - 2\beta\cos\Theta)^{-1} C_{\lambda\bar{\lambda}}]. \qquad (6b)$$

The dependence of the $A^V_{\lambda\bar{\lambda}}$ on the form factors f_i^V and the precise form of the $B_{\lambda\bar{\lambda}}$ and $C_{\lambda\bar{\lambda}}$ are given in table 1. Two different linear combinations f_R and f_L of the photon and Z form factors f^γ and f^Z enter in the right- and left-handed amplitudes (6a), (6b):

$$f_R = f - [s/(s - m_Z^2)] f^Z = f^\gamma - 1.32 f^Z, \qquad (7a)$$
$$f_L = f^\gamma + (1/2\sin^2\theta_W - 1)[s/(s - m_Z^2)] f^Z$$
$$= f^\gamma + 1.63 f^Z. \qquad (7b)$$

Table 1
The coefficients $A^V_{\lambda\bar{\lambda}}$ (V = γ, Z), $B_{\lambda\bar{\lambda}}$ and $C_{\lambda\bar{\lambda}}$ of eq. (6) for the general VWW vertex (eq. (1)). $\beta = (1 - 4m_W^2/s)^{1/2}$ and $\gamma = \sqrt{s}/2m_W$.

$\Delta\lambda$	$(\lambda\bar{\lambda})$	$A^V_{\lambda\bar{\lambda}}$	$B_{\lambda\bar{\lambda}}$	$C_{\lambda\bar{\lambda}}$
2	(+ −)	0	0	$2\sqrt{2}\beta$
−2	(− +)	0	0	$2\sqrt{2}\beta$
1	(+0)	$\gamma(f_3^V - i f_4^V + \beta f_5^V + i\beta^{-1} f_6^V)$	2γ	$2(1+\beta)/\gamma$
1	(0−)	$\gamma(f_3^V + i f_4^V + \beta f_5^V - i\beta^{-1} f_6^V)$	2γ	$2(1+\beta)/\gamma$
−1	(0+)	$\gamma(f_3^V + i f_4^V - \beta f_5^V + i\beta^{-1} f_6^V)$	2γ	$2(1-\beta)/\gamma$
−1	(−0)	$\gamma(f_3^V - i f_4^V - \beta f_5^V - i\beta^{-1} f_6^V)$	2γ	$2(1-\beta)/\gamma$
0	(+ +)	$f_1^V + i\beta^{-1} f_6^V + 4i\gamma^2\beta f_7^V$	1	$1/\gamma^2$
0	(− −)	$f_1^V - i\beta^{-1} f_6^V - 4i\gamma^2\beta f_7^V$	1	$1/\gamma^2$
0	(00)	$\gamma^2[2f_3^V + 4\gamma^2\beta^2 f_2^V - (1+\beta^2) f_1^V]$	$2\gamma^2$	$2/\gamma^2$

Here the numerical values hold for $\sqrt{s} = 190$ GeV, $m_Z = 93$ GeV and $\sin^2\theta_W = 0.223$. Hence, in order to distinguish WWγ from WWZ form factors, a separate measurement of the right- and left-handed amplitudes $\mathcal{M}_R$ and $\mathcal{M}_L$ of eqs. (6a), (6b) is required.

For the general case of arbitrary longitudinal polarization $P_L^{\pm}$ and transverse polarization $P_T^{\pm}$ of the $e^{\pm}$ beams (for details see e.g. ref. [6]) the differential cross section is generically given by

$$\begin{aligned} d\sigma \sim & (1 - P_L^-)(1 + P_L^+) |\mathcal{M}_L|^2 \\ & + (1 + P_L^-)(1 - P_L^+)|\mathcal{M}_R|^2 \\ & + 2 P_T^- P_T^+ [\mathrm{Re}(\mathcal{M}_L \mathcal{M}_R^*) \cos 2\Phi \\ & + \mathrm{Im}(\mathcal{M}_L \mathcal{M}_R^*) \sin 2\Phi]. \end{aligned} \tag{8}$$

Here summation over final-state helicities is implied and the azimuthal angle dependence of eq. (5) is shown explicitly.

For the standard model values of the form factors f^γ, f^Z (eq. (4)) one finds $|\mathcal{M}_R|^2/|\mathcal{M}_L|^2 \approx 10^{-2}$ over a large range of e^+e^- center of mass energies. This suppression of W^+W^- production from right-handed electrons is partially due to the absence of the neutrino t-channel and partially to the cancellation of f^γ and f^Z in eq. (7a). An important consequence is that, without beam polarization, only about 80 W pairs can be produced from right-handed electrons with an integrated luminosity of 500 pb^{-1} at $\sqrt{s} = 190$ GeV. Since in practice longitudinal polarization, if it can be achieved at all, will not be perfect, they will always be hidden in a background of a few thousand W pairs originating from the scattering of left-handed electrons. Due to the limited statistics $|\mathcal{M}_R|^2$ can hence only be determined within a factor of 2–3 and a variation of $\mathcal{M}_R$ due to anomalous couplings will thus be difficult to observe.

Beam polarization can be used in two different ways to increase the sensitivity to the right-handed combination f_R of photon and Z form factors. With longitudinal polarization [9] the "signal" from $|\mathcal{M}_R|^2$ can be increased depressing the "background" from $|\mathcal{M}_L|^2$ simultaneously or one can use transverse polarization to make $\mathcal{M}_R$ interfere with the much larger $\mathcal{M}_L$ which again enhances the signal (see eq. (8)). These two options will now be studied in some detail.

Transverse polarization introduces a 2Φ modulation of all the distributions that can be studied with unpolarized beams. (As before Φ is the angle between the e^- polarization direction and the $e^+e^- \to W^+W^-$ scattering plane.) Since in the standard model $|\mathcal{M}_L|$ is about 10 times larger than $|\mathcal{M}_R|$ (on average), eq. (8) shows that the 2Φ modulation of observables can at most be of order 10%. Actually, for most angular distributions of the W's and their decay products, cancellations between different W helicities tend to reduce the size of the modulation to the few percent level. Hence the mere observation of a sizeable 2Φ modulation will indicate the existence of anomalous couplings.

One example is given in fig. 3, where the distribution of the charged leptons in $W^- \to \ell^-\bar{\nu}$ (and $W^+ \to \ell^+\nu$) versus $2\Phi + 2\phi$ (and $2\Phi + 2\bar{\phi}$) is given for the standard model (solid line) and $\lambda_\gamma = 0.8$, $\lambda_Z = -0.8$ (dashed line). Here ϕ (and $\bar{\phi}$) are the azimuthal angles of the charged leptons around the W^--direction with respect to the scattering plane [‡2]. The anomalous coupling case is clearly distinguishable from the standard model.

The anomalous couplings and the angular distribution depicted in fig. 3 are just one particular choice. For a more systematic treatment the significance of deviations from the standard model was calculated in the κ_γ–κ_Z and the λ_γ–λ_Z planes. In order to obtain the 1σ to 6σ contour lines in these planes, signals were combined from the W differential cross section $d\sigma/d\cos\Theta$, and from all 1W inclusive distributions and all azimuthal angle correlations (around the W axis) of the decay fermions in $W^- \to f_1\bar{f}_2$ and $W^+ \to f_3\bar{f}_4$ (see ref. [1]). Only those correlations were considered which require charge identification of at most one of the final-state fermions.

Errors are based on a 5% systematic error of the luminosity measurement and on the statistical error of 4000 W pairs decaying "semileptonically",

‡2 A Monte Carlo program for $e^+e^- \to W^+W^- \to f_1\bar{f}_2f_3\bar{f}_4$ is obtainable from the author. It includes the anomalous couplings of eq. (1), finite W-width, beam polarization, and finite masses for the final state fermions.

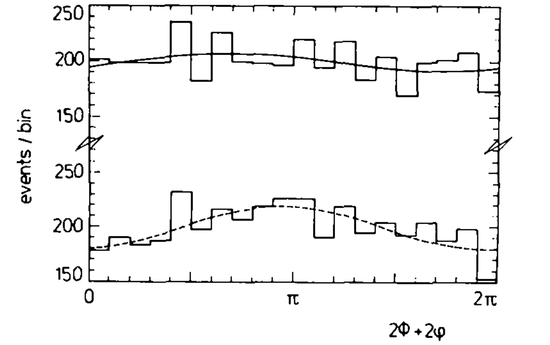

Fig. 3. Azimuthal angle correlation of the angle Φ between the scattering plane and the e^- polarization direction and of ϕ, the azimuthal angle of ℓ^- in $W^- \to \ell^- \bar{\nu}$ around the W^- direction. The curves are a fit of $a + b \cos x + c \sin x$ to the Monte Carlo generated histograms for the standard model (solid line) and anomalous couplings $\lambda_\gamma = 0.8$, $\lambda_Z = -0.8$ (dashed line). 100% transverse polarization was chosen.

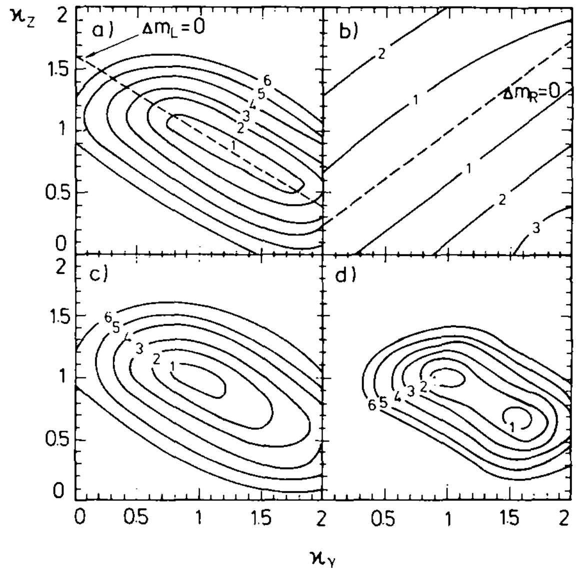

Fig. 4. Significance of deviations of κ_γ and κ_Z from their standard model values. 1σ to 6σ contour lines correspond to an ideal experiment with 4000 $W^+W^- \to \ell\nu q\bar{q}$ events (see text for details). The contours are for (a) no polarization; (b) 70% transverse polarization, 2ϕ modulation only; (c) 70% transverse polarization, all observables; (d) 50% longitudinal polarization of one beam. The dashed lines in (a) and (b) give the combinations of κ_γ and κ_Z for which $\mathcal{M}_L$ and $\mathcal{M}_R$, respectively, are as in the standard model.

e.g. $W^- \to e^-\bar{\nu}$, $W^+ \to q\bar{q}$. This number is the tree level expectation for e, μ, and τ combined, for an integrated luminosity of 500 pb^{-1} at $\sqrt{s} = 190$ GeV. Radiative corrections (in particular initial state radiation and the finite W-width) and experimental cuts may well reduce this number to 1600 events only [7]. However, since events with both W's decaying hadronically should be useful also, this reduction has not been taken into account.

The resulting contour lines for a deviation of either κ or λ from their standard model value (4) are given in figs. 4 and 5. Figs. 4a and 5a assume no beam polarization while figs. 4b and 5b are the 1σ to 5σ contour lines obtainable from the 2Φ modulation of the observables described before. For the latter a 70% transverse polarization of both beams was assumed, well below the 92.4% asymptotic level from the Sokolov–Ternov effect [8]. Figs. 4c and 5c combine the signals from Φ integrated observables (figs. 4a, 5a) and 2Φ modulation.

Obviously transverse polarization significantly improves the measurement of anomalous couplings along the $\Delta\mathcal{M}_L = 0$ line in figs. 4a, 5a. Along this line anomalous photon and Z couplings are such that only the right-handed amplitude $\mathcal{M}_R(\sim\tilde{\mathcal{M}}_+$ in eq. (6a)) differs from its standard model value i.e. $\kappa_L = \kappa_{GSW}$ and $\lambda_L = \lambda_{GSW} = 0$ in eq. (7b). Without polarization the deviation from the standard model $\Delta\mathcal{M}_R$ has nothing large to interfere with and hence a low sensitivity obtains. With transverse polarization $\Delta\mathcal{M}_R$ interferes with the large amplitude $\mathcal{M}_L$ to produce deviations from the standard model in the 2Φ modulation of observables (eq. (8)). For these modulations it is variations of $\mathcal{M}_L$ which produce no effect: $\Delta\mathcal{M}_L$ is multiplied by the small right-handed amplitude $\mathcal{M}_R$ and thus it cannot appreciably change the 2Φ modulations of observables. As a result the contour lines in figs. 4b, 5b are parallel to the $\Delta\mathcal{M}_R = 0$ line.

In terms of the right- and left-handed combinations of eq. (7), one can deduce rough experimental errors to be expected at LEP II from figs. 4 and 5. One finds $\Delta\kappa_R = {}^{+1.4}_{-0.37}$, $\Delta\lambda_R = {}^{+1.0}_{-0.37}$ and $\Delta\kappa_L$, $\Delta\lambda_L = \pm 0.15$ without beam polarization which for the right-handed combination can be improved to $\Delta\kappa_R = \pm 0.37$, $\Delta\lambda_R = {}^{+0.31}_{-0.25}$ with 70% transverse polarization. Correspondingly the sep-

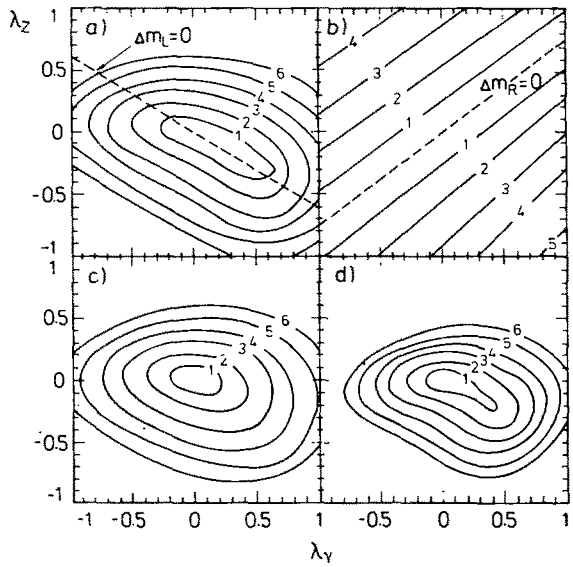

Fig. 5. Same as fig. 4 but for variations of λ_γ and λ_Z.

aration of photon and Z form factors of the W improves by a factor of 2–3. Hence, once anomalous couplings are discovered, transverse polarization will be very helpful to identify the source of the anomaly.

Longitudinal beam polarization is an alternative way of enhancing the effects of a deviation $\Delta\mathcal{M}_R$ from the standard model. For a comparison with transverse polarization, 50% longitudinal polarization of one of the beams was assumed. Combining the signals obtainable when running half of the time with left-handedly and right-handedly polarized beams each, the 1σ to 6σ contour lines in figs. 4d, 5d were obtained, which can be compared directly to figs. 4c, 5c. The usefulness of 70% transverse and 50% longitudinal polarization is comparable. However, while transverse polarization builds up automatically in storage rings, it is questionable whether longitudinal polarization can be achieved at all at LEP II.

In either case substantial degrees of polarization P_T, P_L are needed to significantly improve the detection capabilities of LEP II experiments for anomalous W-couplings. This is obvious for transverse polarization where the size of the effect (the 2Φ modulation) is proportional to P_T^2 (eq. (8)). With $P_T = 50\%$ for example, the contour lines in figs. 4b, 5b correspond to half the indicated sensitivity only, and hence a minor improvement of the measurement without beam polarization results. A transverse polarization of 50%–60% seems to be the minimal requirement.

Results similar to the ones presented for κ and λ can be obtained for the other anomalous couplings in eq. (1). When all the $f_i^V (i = 1, \ldots, 7;\ V = \gamma, Z)$ are treated as free parameters, a new problem arises, however: fitting 14 free parameters to a limited set of observables, frequently leaves open some discrete ambiguities. In many cases they can be resolved by the additional observation of 2Φ modulations due to transverse polarization.

It has been demonstrated that beam polarization can considerably improve the measurement of anomalous γWW and ZWW form factors, once they are discovered. Transverse beam polarization of 50% or more is required, in order to be useful. For particular values of the W form factors beam polarization may even be necessary to discover a deviation from the standard model.

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[1] K. Hagiwara, K. Hikasa, R.D. Peccei and D. Zeppenfeld, Probing the weak boson sector in $e^+e^- \to W^+W^-$, preprint DESY 86-058, to be published in Nucl. Phys. B.
[2] W. Alles, Ch. Boyer and A.J. Buras, Nucl. Phys. B 119 (1977) 125.
[3] K.J.F. Gaemers and G.J. Gounaris, Z. Phys. C 1 (1979) 259.
[4] L.F. Abott and E. Farhi, Phys. Lett. B 101 (1981) 69; H. Fritzsch and G. Mandelbaum, Phys. Lett. B 102 (1981) 319.
[5] Particle Data Group, M. Aguilar-Benitez et al., Phys. Lett. B 170 (1986) 1.
[6] K. Hagiwara and D. Zeppenfeld, Nucl. Phys. B 274 (1986) 1.
[7] A. Blondel, private communication.
[8] C. Bovet, B. Montague, M. Placidi and R. Rosmanith, Polarization and polarimeters at LEP, in: Physics at LEP, eds. J. Ellis and R.D. Peccei, Report CERN 86-02, Vol. 1 (CERN, Geneva) p. 58.
[9] C.L. Bilchak and J.D. Stroughair, Phys. Rev. D 30 (1984) 1881.