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Heavy Quark Production in Order α_s^3

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Abstract. We calculate one loop corrections to heavy quark production in proton antiproton collisions. We cancel ultraviolet and infrared singularities and give the cross sections on the partonic level.

1. Introduction

The calculation of strong corrections to parton parton scattering process was begun several years ago. Two groups [8, 12] calculated gluon corrections to the scattering of two non-identical quarks in order α_s^3. It was found that the inclusion of $O(\alpha_s^3)$ terms substantially modifies the $O(\alpha_s^2)$ tree level results.

At the energies of the present $p\bar{p}$ collider gluons cannot be neglected. At supercollider energies they will even play a pre-eminent role. This increases the number of diagrams to be considered in $O(\alpha_s^3)$. Calculating the heavy quark production means that we restrict ourselves to diagrams with a specific final state.

We are interested in charm production at the collider. Heavy quark production at the ISR-FNAL energies is not under the quantitative control of perturbative QCD. The reason is that a large fraction of the cross section at the small ISR energies is diffractive.

However, the present UA1/UA2 triggers are not sensitive to such a component. Thus it is reasonable to calculate QCD corrections to the diagrams in Fig. 1 in order to see whether perturbative QCD describes the "high $p_\perp$" data at the collider correctly [1]. The diagrams in Fig. 1 and their corrections are probably the dominant source of heavy flavour production. Weak production mechanisms such as $p\bar{p}\to Z\to c\bar{c}$ etc. have much smaller cross sections.

In our calculation all particles are massless. So "heavy" quarks means quarks that can be identified, such as c and b, whose masses are still small compared to the energy involved at the collider.

Furthermore we work in the Feynman gauge and regularize ultraviolet and infrared singularities by going to $n=4-2\,\varepsilon$ dimensions.

In this paper we fully work on the parton level. No folding with the distribution functions is undertaken. The aim is to prove that all infrared singularities cancel and to obtain all finite contributions for the cross sections of the parton processes $q\bar{q}\to Q\bar{Q}$ and $gg\to Q\bar{Q}$ in order α_s^3.

The paper is organized as follows: In Sect. 2 we revisit the tree level contributions and the virtual corrections. In Sect. 3 the real corrections are discussed and in Sect. 4 I present the results cancelling all singularities and going to $\varepsilon=0$. An Appendix is devoted to the calculation of a complicated phase space integral.

2. Born Graphs and Virtual Corrections

Consider the processes

$$q(p_1)+\bar{q}(p_2)\to Q(p_3)+\bar{Q}(p_4) \tag{2.1}$$

$$g(p_1)+g(p_2)\to Q(p_3)+\bar{Q}(p_4). \tag{2.2}$$

Define as usual $s=2p_1p_2=2p_3p_4$, $t=-2p_2p_4=-2p_1p_3$, $u=-2p_1p_4=-2p_2p_3$ with $s+t+u=0$. $q^2=s$ is the energy squared of the process (on the parton level). From t and u one can construct the scattering angle θ:

$$t_s=-t/s=\tfrac{1}{2}(1-\cos\theta) \tag{2.3}$$

$$u_s=-u/s=1-t_s. \tag{2.4}$$

The tree level contributions for the above processes have been calculated some time ago [2], virtual corrections to them have only just been published [3]

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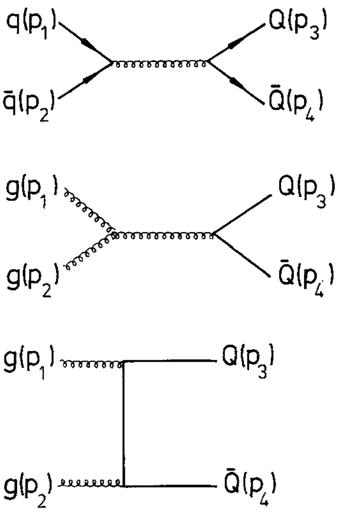

Fig. 1.

$$\frac{d\sigma^{\text{virtual}}_{q\bar{q}\to Q\bar{Q}}}{dt_s}=\left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^2\left\{N_4\, B^{(4)}_{q\bar{q}\to Q\bar{Q}}+\frac{\alpha_s(\mu^2)}{2\pi}\right.$$
$$\cdot\left[N_n\left(\frac{4\pi\mu^2}{s}\right)^{\varepsilon}\frac{\Gamma(1+\varepsilon)\,\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\right.$$
$$\cdot B^{(n)}_{q\bar{q}\to Q\bar{Q}}\left(C_F\left(-\frac{4}{\varepsilon^2}-\frac{6}{\varepsilon}-\frac{8}{\varepsilon}\ln\frac{t_s}{u_s}\right)\right.$$
$$\left.\left.\left.+\frac{N_c}{\varepsilon}(4\ln t_s-2\ln u_s)\right)+N_4\,F_{q\bar{q}\to Q\bar{Q}}\right]\right\} \tag{2.5}$$

where

$$F_{q\bar{q}\to Q\bar{Q}}=B^{(4)}_{q\bar{q}\to Q\bar{Q}}\left[-16C_F+\left(\frac{85}{9}+6\zeta_2+\frac{11}{3}\ln\frac{\mu^2}{s}\right)N_c\right.$$
$$\left.-\frac{4}{3}T_R\left(\frac{5}{3}+\ln\frac{\mu^2}{s}\right)\right]$$
$$-2C_F^2(t_s^2-u_s^2)(12\zeta_2+\ln^2 t_s+\ln^2 u_s)$$
$$+\tfrac{1}{2}N_c\,C_F(18\zeta_2+2\ln^2 t_s+\ln^2 u_s-2t_s\ln u_s+4u_s\ln t_s)$$
$$+4C_F^2(t_s\ln u_s-u_s\ln t_s). \tag{2.6}$$

The renormalization has been done here already. μ is the (arbitrary) mass parameter which has been introduced to keep the coupling constant dimensionless in n dimensions. $C_F=4/3$, $N_c=3$ are the invariants of SU_3. We have used the abbreviation $\zeta_2=\frac{\pi^2}{6}\approx 1.6449$.

$$B^{(n)}_{q\bar{q}\to Q\bar{Q}}=C_F(t_s^2+u_s^2-\varepsilon) \tag{2.7}$$

is the n-dimensional Born level contribution and

$$N_n=\frac{4\pi^3\mu^{2\varepsilon}}{N_c\, s\,\Gamma(1-\varepsilon)}\,t_s^{-\varepsilon}(1-t_s)^{-\varepsilon}\left(\frac{4\pi\mu^2}{s}\right)^{\varepsilon} \tag{2.8}$$

contains mainly the two particle phase space

$$\mathrm{PS}^{(2)}=\left(\frac{4\pi\mu^2}{s}\right)^{\varepsilon}\frac{1}{8\pi\Gamma(1-\varepsilon)}\,dt_s\,t_s^{-\varepsilon}(1-t_s)^{-\varepsilon} \tag{2.9}$$

$$\frac{d\sigma^{\text{virtual}}_{gg\to Q\bar{Q}}}{dt_s}=\left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^2\left\{N_4\,B^{(4)}_{gg\to Q\bar{Q}}+\frac{\alpha_s(\mu^2)}{2\pi}\,C_F\,N_c\right.$$
$$\cdot\left[N_n\left(\frac{4\pi\mu^2}{s}\right)^{\varepsilon}\frac{\Gamma(1+\varepsilon)\,\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\right.$$
$$\cdot\left(B^{(n)}_{gg\to Q\bar{Q}}\left(-C_F\left(\frac{2}{\varepsilon^2}+\frac{3}{\varepsilon}\right)-N_c\left(\frac{2}{\varepsilon^2}+\frac{11}{3\varepsilon}\right)+\frac{4T_R}{3\varepsilon}\right)\right.$$
$$\left.+f_2^{(4)}\frac{N_c}{\varepsilon}\left(2\left(C_F-\frac{N_c}{2}\right)\ln(t_s u_s)+N_c(u_s^2\ln t_s+t_s^2\ln u_s)\right)\right)$$
$$\left.\left.+N_4\left(F_{gg\to Q\bar{Q}}+(u_s\leftrightarrow t_s)\right)\right]\right\} \tag{2.10}$$

where

$$F_{gg\to Q\bar{Q}}$$
$$=B^{(4)}_{gg\to Q\bar{Q}}\left(-\frac{7}{2}C_F+\frac{11}{6}\ln\left(\frac{\mu^2}{s}\right)N_c-\frac{2}{3}\ln\left(\frac{\mu^2}{s}\right)T_R\right)$$
$$-N_c\left(C_F-\frac{N_c}{2}\right)f_2^{(4)}\ln u_s\ln t_s+N_c(N_c-C_F)\left(\frac{1}{4}-\frac{1}{2}f_1^{(4)}\right)$$
$$+6\zeta_2\left(-\frac{1}{4}N_c\left(C_F-\frac{N_c}{2}\right)+2\left(C_F-\frac{N_c}{2}\right)^2\left(1+\frac{3}{4}f_2^{(4)}\right)\right.$$
$$\left.+N_c^2\left(\frac{1}{8}f_2^{(4)}-\frac{1}{2}f_1^{(4)}\right)\right)$$
$$+\ln^2 t_s\left[N_c^2\left(u_s-\frac{1}{4}-\frac{1}{4t_s}\right)-2N_c\left(C_F-\frac{N_c}{2}\right)\left(\frac{t_s}{2u_s}+\frac{u_s}{4}\right)\right.$$
$$\left.+\left(C_F-\frac{N_c}{2}\right)^2\left(\frac{u_s}{t_s}+\frac{2}{u_s}\right)\right]$$
$$+\ln t_s\left[N_c^2\left(f_1^{(4)}-\frac{3}{4}t_s-\frac{5}{4}\frac{u_s}{t_s}-\frac{1}{4}\right)+\left(C_F-\frac{N_c}{2}\right)^2\right.$$
$$\left.\cdot\left(\frac{3}{t_s}-1\right)-N_c\left(C_F-\frac{N_c}{2}\right)\left(\frac{u_s}{2}+\frac{4}{u_s}+\frac{1}{t_s}\right)\right] \tag{2.11}$$

$$B^{(n)}_{gg\to Q\bar{Q}}=C_F f_2^{(n)}-N_c f_1^{(n)} \tag{2.12}$$

$$f_2^{(n)}=\left(\frac{t_s}{u_s}+\frac{u_s}{t_s}-\frac{\varepsilon}{u_s t_s}\right)(1-\varepsilon) \tag{2.13}$$

$$f_1^{(n)}=(t_s^2+u_s^2-\varepsilon)(1-\varepsilon). \tag{2.14}$$

3. Real Corrections

The infrared singularities present in (2.5) and (2.10) can be cancelled by contributions from the processes

$$q(p_1)+\bar{q}(p_2)\to Q(p_3)+\bar{Q}(p_4)+g(p_5) \tag{3.1}$$

$$g(p_1)+g(p_2)\to Q(p_3)+\bar{Q}(p_4)+g(p_5). \tag{3.2}$$

There is also a third and a fourth process producing heavy quarks, namely $qg\to Q\bar{Q}q$ and $\bar{q}g\to Q\bar{Q}\bar{q}$. These, however, give only finite contributions –up to initial state singularities which can be absorbed into the distribution functions. As they have been described in detail in [4], we will not consider them any further.

The singularities of (3.1) and (3.2) come from the collinear regions $\mathbf{p}_5 \parallel \mathbf{p}_i$, $i=1, 2, 3, 4$ and from the infrared region $|\mathbf{p}_5|\to 0$. Introducing an angle cut δ and an energy cut Δ, we will integrate over these regions analytically. Then we do the cancellations of singularities with (2.5) and (2.10). The result will depend on δ and Δ. Integrating numerically over the rest of three particle phase space and adding this to the analytical result will give a δ- and Δ-independent cross section.

In contrast to jet calculations the cuts δ, Δ here are only technical devices. So they may be choosen very small numerically ($\Delta \leqq 10^{-3}$, $\delta\leqq 10^{-4}$). Therefore in the analytical calculation terms of order δ or Δ may be neglected.

For the $2\to 3$ processes we use the following variables

$$s_{ij}:=2p_i\,p_j/2p_1\,p_2. \tag{3.3}$$

First we give the four-dimensional matrix elements squared for the processes (3.1) and (3.2) [5]:

$$\begin{aligned}|M|^2_{q\bar{q}\to Q\bar{Q}g} &= -\theta_N\, C_F\,\frac{s_{13}^2+s_{24}^2+s_{14}^2+s_{23}^2}{s_{12}\,s_{34}\,s_{15}\,s_{25}\,s_{35}\,s_{45}}\Big\{C_F\,[(s_{14}+s_{23})\\
&\cdot(s_{13}\,s_{24}+s_{12}\,s_{34}-s_{14}\,s_{23})+s_{14}(s_{13}\,s_{12}+s_{24}\,s_{34})\\
&+s_{23}(s_{13}\,s_{34}+s_{12}\,s_{24})]+\left(C_F-\frac{N_c}{2}\right)\\
&\cdot[(s_{13}+s_{24})(s_{13}\,s_{24}-s_{12}\,s_{34}-s_{14}\,s_{23})\\
&+2s_{12}\,s_{34}(s_{14}+s_{23})-2s_{14}\,s_{23}(s_{12}+s_{34})]\Big\}.\end{aligned} \tag{3.4}$$

($|M|^2_{qg\to Q\bar{Q}q}$ can be deduced from this by the interchange $(5\leftrightarrow -2)$.)

$$\begin{aligned}|M|^2_{gg\to Q\bar{Q}g} &= -\theta_N\\
&\frac{s_{13}\,s_{14}(s_{13}^2+s_{14}^2)+s_{23}\,s_{24}(s_{23}^2+s_{24}^2)+s_{35}\,s_{45}(s_{35}^2+s_{45}^2)}{8s_{13}\,s_{23}\,s_{35}\,s_{14}\,s_{24}\,s_{45}}\\
&\cdot\Big\{-2\left(C_F-\frac{N_c}{2}\right)^2 s_{12}+N_c\left(C_F-\frac{N_c}{2}\right)\\
&\cdot\left[s_{34}-\frac{1}{s_{12}}(s_{13}\,s_{24}+s_{14}\,s_{23})-\frac{1}{s_{25}}(s_{23}\,s_{45}+s_{35}\,s_{24})\right.\\
&\left.-\frac{1}{s_{15}}(s_{35}\,s_{14}+s_{13}\,s_{45})\right]\\
&+\frac{N_c^2}{2s_{12}}\left[\frac{s_{35}\,s_{45}}{s_{15}\,s_{25}}(s_{13}\,s_{24}+s_{14}\,s_{23})+\frac{s_{13}\,s_{14}}{s_{12}\,s_{15}}\right.\\
&\left.\cdot(s_{23}\,s_{45}+s_{35}\,s_{24})+\frac{s_{23}\,s_{24}}{s_{12}\,s_{25}}(s_{35}\,s_{14}+s_{13}\,s_{45})\right]\Big\}.\end{aligned} \tag{3.5}$$

The absolute normalization θ_N of (3.4) and (3.5) can be read off the cancellation of the singularities (see below). (3.4) and (3.5) can then be integrated numerically over the finite regions.

We define

$$\zeta:=\tfrac{1}{2}(1-\cos\sphericalangle(3,5)),\qquad \eta:=\tfrac{1}{2}(1-\cos\sphericalangle(1,5))$$

and x to be the fraction of energy carried away by the outgoing gluon. Then we can do the numerical integration with the following phase space

$$\begin{aligned}d\mathrm{PS}^{(3)}&=\frac{s}{(4\pi)^4}\,\frac{x(1-x)}{(1-x\zeta)^2}\,dx\,d\eta\,d\zeta\,d\phi\\
&\Delta\leqq x\leqq 1,\ \delta\leqq\eta,\ \zeta\leqq 1-\delta,\ 0\leqq\phi\leqq 2\pi.\end{aligned} \tag{3.6}$$

Here ϕ is the azimuthal angle between the 125-plane and the 345-plane.

The invariants s_{ij} can be expressed by the integration variables of the phase space (3.6):

$$s_{12}=1 \tag{3.7a}$$

$$\begin{aligned}s_{13}&=\frac{1-x}{1-x\zeta}\,(\eta(1-\zeta)+\zeta(1-\eta)\\
&\quad-2\sqrt{\eta(1-\eta)\zeta(1-\zeta)}\cos\phi)\end{aligned} \tag{3.7b}$$

$$s_{14}=1-x\eta-s_{13} \tag{3.7c}$$

$$s_{15}=x\eta \tag{3.7d}$$

$$\begin{aligned}s_{23}&=\frac{1-x}{1-x\zeta}\,(\eta\zeta+(1-\eta)(1-\zeta)\\
&\quad+2\sqrt{\eta(1-\eta)\zeta(1-\zeta)}\cos\phi)\end{aligned} \tag{3.7e}$$

$$s_{24}=1-x(1-\eta)-s_{23} \tag{3.7f}$$

$$s_{25}=x(1-\eta) \tag{3.7g}$$

$$s_{34}=1-x \tag{3.7h}$$

$$s_{35}=\frac{x(1-x)\zeta}{1-x\zeta} \tag{3.7i}$$

$$s_{45}=\frac{x(1-\zeta)}{1-x\zeta}. \tag{3.7j}$$

We now come to the collinear integrations. Because of the singularities we have to generalize (3.6) to n dimensions

$$dPS^{(3)} = \frac{s}{(4\pi)^4 \Gamma^2(1-\varepsilon)} \left(\frac{4\pi\mu^2}{s}\right)^{2\varepsilon} (1-x\zeta)^{-2+2\varepsilon} \cdot (x(1-x))^{1-2\varepsilon} (\eta(1-\eta)\zeta(1-\zeta))^{-\varepsilon} \, dx\, d\eta\, d\zeta\, d\phi. \quad (3.8)$$

Let us begin with the $q\bar{q} \to Q\bar{Q}g$-case and consider $\mathbf{p}_3 \parallel \mathbf{p}_5$, i.e. $\zeta < \delta$. We define effective particles with momenta $p_{\mathrm{I}} = p_1$, $p_{\mathrm{II}} = p_2$, $p_{\mathrm{III}} = p_3 + p_5$, $p_{\mathrm{IV}} = p_4$. This means one can identify s_{14} as the effective $2 \to 2$ variable u_s and $2p_1 p_2$ as the $2 \to 2$-energy s. This is physically intuitive, but can also be derived from (3.7) for $\zeta \to 0$. Furthermore one has

$$s_{13} = (1-x)t_s, \quad s_{15} = x t_s, \quad s_{23} = (1-x)u_s, \quad s_{24} = t_s, \quad s_{25} = x u_s, \quad s_{45} = x. \quad (3.9)$$

In the numerator one can put $s_{35} = 0$, in the denominator one must use $s_{35} = \zeta x(1-x)$. Inserting this into the matrix element (3.4) and keeping only the pole in ζ one finds

$$\lim_{\zeta \to 0} \zeta |M|^2_{q\bar{q} \to Q\bar{Q}g} = \theta_N C_F B^{(n)}_{q\bar{q} \to Q\bar{Q}} P^{(n)}_{qq}(1-x)/(x(1-x)). \quad (3.10)$$

At all stages only simple poles in $\zeta(s_{35})$ appear. This is the reason why one can use the simple approximation (3.9) to (3.7).

(3.10) shows that in the collinear limit the matrix element factorizes into a Born type expression (with variables the effective $2 \to 2$ variables) and the n-dimensional Altarelli-Parisi function (AP-function)

$$P^{(n)}_{qq}(v) = \frac{2v}{1-v} + (1-\varepsilon)(1-v). \quad (3.11)$$

The Born type expression can be left as it is. Only the AP-function must be integrated. The variable of the AP-functions will always be $v = 1-x$.

With the approximations (3.9) the three particle phase space (3.8) factorizes into an effective two particle phase space times some integrations to be carried out:

$$PS^{(3)}_{\mathrm{out}} = PS^{(2)} \left(\frac{4\pi\mu^2}{s}\right)^{\varepsilon} \frac{s}{16\pi^2 \Gamma(1-\varepsilon)} \cdot \int_0^{\delta} d\zeta\, \zeta^{-\varepsilon} \int_{\Delta}^{1} dx (x(1-x))^{1-2\varepsilon}. \quad (3.12)$$

The ϕ-integration is trivial for (3.10). The region $x < \Delta$ is excluded here, because it will be considered in connection with the infrared limit. The result of the integration is

$$C^{q\bar{q}}_{\mathrm{out}} = N_n \left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^3 \left(\frac{4\pi\mu^2}{s}\right)^{\varepsilon} B^{(n)}_{q\bar{q} \to Q\bar{Q}} 2\Gamma(1-\varepsilon) C_F \cdot \left[\frac{3}{2\varepsilon} + \frac{2}{\varepsilon} \ln\Delta + \frac{13}{2} - 4\zeta_2 - 2\ln^2\Delta - \frac{3}{2}\ln\delta - 2\ln\delta \ln\Delta\right]. \quad (3.13)$$

A factor of 2 has been added to account for the case $\mathbf{p}_4 \parallel \mathbf{p}_5$.

We now come to the case, where the outgoing gluon is collinear with one of the incoming quarks. For $\mathbf{p}_1 \parallel \mathbf{p}_5$ it is appropriate to identify

$$s = 2p_1 p_2 \cdot (1-x) \quad (3.14\text{a})$$

$$u_s = (1-\zeta)/(1-x\zeta) \quad (3.14\text{b})$$

as the effective $2 \to 2$ variables. Then for $\eta \to 0$ one finds ($s_{ij} := 2p_i p_j/s$)

$$s_{13} = \frac{t_s}{1-x}, \quad s_{14} = \frac{u_s}{1-x}, \quad s_{15} = \frac{1-x}{x \cdot \eta},$$
$$s_{23} = u_s, \quad s_{24} = t_s, \quad s_{25} = \frac{x}{1-x}, \quad (3.15)$$
$$s_{34} = 1, \quad s_{35} = \frac{x t_s}{1-x}, \quad s_{45} = \frac{x u_s}{1-x}.$$

A remark is in order: In the case under consideration we can define effective momenta $p_{\mathrm{I}} = p_1 + p_5$, $p_{\mathrm{II}} = p_2$, $p_{\mathrm{III}} = p_3$, $p_{\mathrm{IV}} = p_4$. Therefore the relation $s_{23} = u_s$ is physically intuitive. This leads to (3.14).

In the limit $\eta \to 0$ the matrix elements and the three particle phase space again factorize:

$$\lim_{\eta \to 0} \eta |M|^2_{q\bar{q} \to Q\bar{Q}g} = \theta_N C_F B^{(n)}_{q\bar{q} \to Q\bar{Q}} P^{(n)}_{qq}(1-x) \frac{1-x}{x} \quad (3.16)$$

$$PS^{(3)}_{\mathrm{in}} = PS^{(2)} \left(\frac{4\pi\mu^2}{s}\right)^{\varepsilon} \frac{s}{16\pi^2 \Gamma(1-\varepsilon)} \int_0^{\delta} d\eta\, \eta^{-\varepsilon} \int_{\Delta}^{1} \cdot dx\, x^{1-2\varepsilon} (1-x)^{-1+\varepsilon}. \quad (3.17)$$

There is again a factor of 2 for the case $\mathbf{p}_2 \parallel \mathbf{p}_5$. Doing the integrations one finds

$$C^{q\bar{q}}_{\mathrm{in}} = N_n \left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^3 \left(\frac{4\pi\mu^2}{s}\right)^{\varepsilon} B^{(n)}_{q\bar{q} \to Q\bar{Q}} 2\Gamma(1-\varepsilon) C_F \cdot \left[\frac{3}{2\varepsilon} + 2\frac{\ln\Delta}{\varepsilon} + \frac{11}{4} + 2\zeta_2 - 2\ln^2\Delta - \frac{3}{2}\ln\delta - 2\ln\delta \ln\Delta\right]. \quad (3.18)$$

The same analysis can be made for the collinear limits of the process (3.2). The analogues of (3.10), (3.13), (3.16) and (3.18) read

$$\lim_{\zeta \to 0} \zeta |M|^2_{gg \to Q\bar{Q}g} = \theta_N N_c C_F^2 B^{(n)}_{gg \to Q\bar{Q}} P^{(n)}_{qq}(1-x) \frac{1}{x(1-x)} \qquad (3.19)$$

$$\lim_{\eta \to 0} \eta |M|^2_{gg \to Q\bar{Q}g} = \theta_N N_c^2 C_F B^{(n)}_{gg \to Q\bar{Q}} P_{gg}(1-x) \frac{1-x}{x}. \qquad (3.20)$$

The AP-function

$$P_{gg}(v) = \frac{2}{v} + \frac{2}{1-v} - 4 + 2v(1-v) \qquad (3.21)$$

is independent of the space time dimension.

$$C^{gg}_{\text{out}} = N_n \left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^3 \left(\frac{4\pi\mu^2}{s}\right)^\varepsilon B^{(n)}_{gg \to Q\bar{Q}} 2\Gamma(1-\varepsilon) N_c C_F^2 \cdot \left[\frac{3}{2\varepsilon} + \frac{2}{\varepsilon} \ln \Delta + \frac{13}{2} - 4\zeta_2 - 2 \ln^2 \Delta - \frac{3}{2} \ln \delta - 2 \ln \delta \ln \Delta\right] \qquad (3.22)$$

$$C^{gg}_{\text{in}} = N_n \left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^3 \left(\frac{4\pi\mu^2}{s}\right)^\varepsilon B^{(n)}_{gg \to Q\bar{Q}} 2\Gamma(1-\varepsilon) N_c^2 C_F \cdot \left[\frac{11}{3\varepsilon} + \frac{2}{\varepsilon} \ln \Delta + \frac{2}{\varepsilon} \ln T - \frac{11}{3} \ln \delta - 2 \ln \Delta \ln \delta - 2 \ln T \ln \delta - 2 \ln^2 \Delta + \ln^2 T - 2\zeta_2 + \frac{67}{18}\right]. \qquad (3.23)$$

In the last formula a cut T has been introduced to avoid the limit of the two heavy quarks being collinear $(1-x \to 0)$. This is a physical cut which has to be put on experimentally anyhow.

One should note that the factorization properties of (3.10), (3.16), (3.19) and (3.20) could have been forecast from more general considerations. In the infrared (IR) limit we shall also find factorization (however without AP-functions, see below).

Before turning to the IR limit I want to discuss an additional type of collinear singularities. These must be absorbed into the distribution functions of the quarks and gluons. They are proportional to $\tilde{P}_{qq}$ for the $q\bar{q} \to Q\bar{Q}$ case and to $\tilde{P}_{gg}$ for the $gg \to Q\bar{Q}$ case (cf. [3] and the discussion at the end of Sect. 5). $\tilde{P}$ are AP-functions modified in such a way that charge conservation

$$\int_0^1 dv \tilde{P}_{qq}(v) = 0 \qquad (3.24)$$

and momentum conservation hold [6]:

$$\tilde{P}_{qq}(v) = 2\frac{v}{(1-v)_+} + 1 - v + \frac{3}{2}\delta(1-v) \qquad (3.25)$$

$$\tilde{P}_{gg}(v) = \frac{2}{v(1-v)_+} - 4 + 2v(1-v) + \left(\frac{11}{6} - \frac{2}{3}\frac{T_R}{N_c}\right)\delta(1-v). \qquad (3.26)$$

Here $(1-v)_+^{-1}$ is the regular version of $(1-v)^{-1}$ in the usual sense [6]. As we calculate an integrated cross section we can use (3.24) to prove that the $q\bar{q} \to Q\bar{Q}$ case gets no contribution from these considerations. This is even true if one absorbs certain finite higher order contributions of deep inelastic scattering into the distribution functions [7, 8].

For the $gg \to Q\bar{Q}$ case we have

$$\int_T^{1-\Delta} dv \tilde{P}_{gg}(v) = 2[-\ln T - \tfrac{11}{12} - T_R/(12 N_c) + \varepsilon(-\tfrac{1}{2}\ln^2 T + 2\zeta_2 - \tfrac{67}{36})]. \qquad (3.27)$$

A term proportional to (3.27) is indeed needed to get a finite answer namely

$$C^{gg}_{\text{sub}} = -N_n \left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^3 \left(\frac{4\pi\mu^2}{s}\right)^\varepsilon \cdot B^{(n)}_{gg \to Q\bar{Q}} 2\Gamma(1-\varepsilon) N_c^2 C_F \cdot \left(-\frac{1}{\varepsilon}\right) \int_\Delta^{1-T} dx \tilde{P}_{gg}(1-x). \qquad (3.28)$$

Now we come to the infrared limit $x < \Delta$. We cannot work any more with the phase space (3.8) in the infrared limit, because parton 5 being infrared defines no z-direction any more. Therefore we have chosen another description of three particle phase space [9]:

$$d\text{PS}^{(3)} = d\text{PS}^{(2)} \left(\frac{4\pi\mu^2}{s}\right)^\varepsilon \frac{s}{16\pi^2 \Gamma(1-\varepsilon)} \cdot (s_{34} s_{45} s_{35})^{-\varepsilon} ds_{34} ds_{35} (1-s_{45})^{-1-2\varepsilon} \cdot \delta(1 - s_{34} - s_{45} - s_{35}) \frac{1}{N_\varphi} \sin^{-2\varepsilon} \varphi \, d\varphi$$
$$0 < s_{34}, \ s_{35} < 1, \ 0 < \varphi < \pi. \qquad (3.29)$$

Here φ is the azimuthal angle of $\mathbf{p}_5$ with respect to $\mathbf{p}_3$. (We have chosen $\mathbf{p}_3$ to define the z-direction.) N_φ is the normalization of the φ-integration, $N_\varphi = \Gamma(\tfrac{1}{2} - \varepsilon)\Gamma(\tfrac{1}{2})/\Gamma(1-\varepsilon)$.

One can calculate s_{34}, s_{45} and s_{35} in terms of our old variables x and ζ. $s_{34}=1-x$, $s_{35}=x(1-x)\zeta/(1-x\zeta)$, $s_{45}=x(1-\zeta)/(1-x\zeta)$. In the small x limit one receives

$$PS_{IR}^{(3)}=PS^{(2)}\left(\frac{4\pi\mu^2}{s}\right)^{\varepsilon}\frac{s}{16\pi^2\Gamma(1-\varepsilon)}\int_0^{\Delta} dx\, x^{1-2\varepsilon} \cdot\int_0^1 d\zeta(\zeta(1-\zeta))^{-\varepsilon}\frac{1}{N_\varphi}\int_0^{\pi}\sin^{-2\varepsilon}\varphi\, d\varphi \tag{3.30}$$

η is no longer a simple quantitiy in this system

$$\eta=\zeta u_s+(1-\zeta)t_s+2\cos\varphi\sqrt{\zeta(1-\zeta)u_s t_s}. \tag{3.31}$$

Therefore expressions with $s_{15}\sim\eta$ in the dominator are not easy to integrate (see below).

It is simple to define an effective two particle phase space in the infrared limit, because parton 5 is not involved in the definition of effective partons: I$=1$, II$=2$, III$=3$, IV$=4$. So $t=-2p_{\mathrm{I}}p_{\mathrm{III}}=-2p_1p_3$ etc.

Inserting the small x approximations of the s_{ij} into the transition probabilities one gets their IR-limit. One finds at most poles of second order in x (despite the appearance of $s_{15}s_{25}s_{35}s_{45}\sim x^4$ in the denominator of (3.4)). In fact

$$\lim_{x\to 0} x^2|M|^2_{q\bar q\to Q\bar Q g} =\theta_N B^{(n)}_{q\bar q\to Q\bar Q}\left\{N_c\left(\frac{u_s}{\zeta(1-\eta)}+\frac{u_s}{(1-\zeta)\eta}\right)\right. +\left(C_F-\frac{N_c}{2}\right)\left(\frac{2}{\zeta}+\frac{2}{1-\zeta}+\frac{2}{\eta}+\frac{2}{1-\eta}+\frac{4u_s}{(1-\zeta)\eta}\right. \left.\left.+\frac{4u_s}{\zeta(1-\eta)}-\frac{4t_s}{(1-\zeta)(1-\eta)}-\frac{4t_s}{\eta\zeta}\right)\right\} \tag{3.32}$$

$$\begin{aligned}\lim_{x\to 0} x^2|M|^2_{gg\to Q\bar Q g} &=\theta_N C_F N_c\left\{N_c^2\left(\frac{1}{2}f_2^{(n)}-f_1^{(n)}\right)\left(\frac{1}{1-\eta}+\frac{1}{\eta}\right)\right.\\ &+2\left(C_F-\frac{N_c}{2}\right)\left[\left(C_F-\frac{N_c}{2}\right)f_2^{(n)}-N_c f_1^{(n)}\right]\\ &\cdot\left(\frac{1}{\zeta}+\frac{1}{1-\zeta}\right)+N_c\left[\left(C_F-\frac{N_c}{2}\right)u_s f_2^{(n)}+\frac{N_c}{2}t_s f_1^{(n)}\right]\\ &\cdot\left(\frac{1}{\zeta(1-\eta)}+\frac{1}{\eta(1-\zeta)}\right)\\ &+N_c\left[\left(C_F-\frac{N_c}{2}\right)t_s f_2^{(n)}+\frac{N_c}{2}u_s f_1^{(n)}\right]\\ &\left.\cdot\left(\frac{1}{\zeta\eta}+\frac{1}{(1-\zeta)(1-\eta)}\right)\right\}.\end{aligned} \tag{3.33}$$

Note that (3.32) and (3.33) also show factorization properties, however, without AP-functions. The factors in front of the Born expressions are always independent of ε. This is in accordance with general considerations [10].

Integrating (3.32) and (3.33) with the measure (3.30) one gets

$$\mathrm{IR}^{q\bar q}=N_n\left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^3\left(\frac{4\pi\mu^2}{s}\right)^{\varepsilon}B^{(n)}_{q\bar q\to Q\bar Q}\frac{\Delta^{-2\varepsilon}}{-2\varepsilon} \cdot\left\{2N_c J_- +\left(C_F-\frac{N_c}{2}\right)[4J_\zeta+4J_\eta+8J_- -8J_+]\right\} \tag{3.34}$$

$$\begin{aligned}\mathrm{IR}^{gg}&=N_n\left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^3\left(\frac{4\pi\mu^2}{s}\right)^{\varepsilon}\frac{\Delta^{-2\varepsilon}}{-2\varepsilon}C_F N_c\\ &\cdot\left\{2N_c J_\eta(\tfrac{1}{2}f_2^{(n)}-f_1^{(n)})\right.\\ &+4\left(C_F-\frac{N_c}{2}\right)\left[\left(C_F-\frac{N_c}{2}\right)f_2^{(n)}-N_c f_1^{(n)}\right]J_\zeta\\ &+2J_- N_c\left[\left(C_F-\frac{N_c}{2}\right)f_2^{(n)}+\frac{N_c}{2}\frac{t_s}{u_s}f_1^{(n)}\right]\\ &\left.+2J_+ N_c\left[\left(C_F-\frac{N_c}{2}\right)f_2^{(n)}+\frac{N_c}{2}\frac{u_s}{t_s}f_1^{(n)}\right]\right\}.\end{aligned} \tag{3.35}$$

Here we introduced

$$J_\zeta=\int_0^1 d\zeta\,\zeta^{-1-\varepsilon}(1-\zeta)^{-\varepsilon}=\frac{\Gamma(-\varepsilon)\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)} \tag{3.36}$$

$$J_\eta=\int_0^1 d\zeta\,\zeta^{-\varepsilon}(1-\zeta)^{-\varepsilon}\frac{1}{N_\varphi}\int_0^{\pi} d\varphi\,\sin^{-2\varepsilon}\varphi\,\frac{1}{\eta} \tag{3.37}$$

$$J_+=\int_0^1 d\zeta\,\zeta^{-\varepsilon}(1-\zeta)^{-\varepsilon}\frac{1}{N_\varphi}\int_0^{\pi} d\varphi\,\sin^{-2\varepsilon}\varphi\,\frac{t_s}{\eta\zeta} \tag{3.38}$$

$$J_-=\int_0^1 d\zeta\,\zeta^{-\varepsilon}(1-\zeta)^{-\varepsilon}\frac{1}{N_\varphi}\int_0^{\pi} d\varphi\,\sin^{-2\varepsilon}\varphi\,\frac{u_s}{(1-\eta)\zeta} =J_+(t_s\leftrightarrow u_s) \tag{3.39}$$

and have made use of symmetry properties of η and $1-\eta$ under exchanges $t_s\leftrightarrow u_s$ and $\zeta\leftrightarrow 1-\zeta$. We have devoted an appendix to the calculation of J_η and J_+. Here we only quote the results

$$J_\eta=-\frac{1}{\varepsilon}+\zeta_2\varepsilon \tag{3.40}$$

$$J_+=-\frac{2}{\varepsilon}+2\ln t_s+\varepsilon(2L_2(t_s)+2\ln t_s\ln u_s-\ln^2 t_s). \tag{3.41}$$

In (3.41) L_2 means the Spence function.

Finally we receive

$$\mathrm{IR}^{q\bar{q}}=N_n\left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^3\left(\frac{4\pi\mu^2}{s}\right)^\varepsilon B^{(n)}_{q\bar{q}\to Q\bar{Q}} \cdot\left\{\frac{4}{\varepsilon^2}C_F+\frac{1}{\varepsilon}[-8C_F\ln\Delta-4N_c\ln t_s +8C_F\ln t_s+2N_c\ln u_s-8C_F\ln u_s] +C_F\left[-4\zeta_2+8\ln^2\Delta-16\ln\Delta\ln\frac{t_s}{u_s}-4\ln^2 t_s +4\ln^2 u_s+8L_2(t_s)-8L_2(u_s)\right] +N_c[2\zeta_2+8\ln\Delta\ln t_s-4\ln\Delta\ln u_s+2\ln^2 t_s -\ln^2 u_s-2\ln t_s\ln u_s-4L_2(t_s)+2L_2(u_s)]\right\} \tag{3.42}$$

$$\mathrm{IR}^{gg}=N_n\left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^3\left(\frac{4\pi\mu^2}{s}\right)^\varepsilon C_F N_c\frac{\Delta^{-2\varepsilon}}{-2\varepsilon} \cdot\left\{\frac{1}{\varepsilon}\left[N_c^2\left(2f_2^{(n)}+4f_1^{(n)}-\frac{2f_1^{(n)}}{u_s t_s}\right) +4C_F N_c(f_1^{(n)}-f_2^{(n)})-4C_F^2 f_2^{(n)}\right]+\varepsilon\hat{G} +N_c^2\ln t_s(-2f_2^{(n)}-2f_1^{(n)}+2f_1^{(n)}/t_s) +N_c^2\ln u_s(-2f_2^{(n)}-2f_1^{(n)}+2f_1^{(n)}/u_s) +4f_2^{(n)}C_F N_c\ln(u_s t_s)\right\} \tag{3.43}$$

$$\hat{G}=\zeta_2(f_2^{(n)}N_c^2-2(f_2^{(n)}+f_1^{(n)})C_F N_c+2f_2^{(n)}C_F^2) +N_c^2\ln^2 t_s(f_1^{(n)}+f_2^{(n)}-f_1^{(n)}/t_s)-2N_c C_F f_2^{(n)}\ln^2 t_s +4N_c C_F f_2^{(n)}\ln t_s\ln u_s +N_c^2\ln t_s\ln u_s\left(\frac{f_1^{(n)}}{t_s u_s}-2f_2^{(n)}-2f_1^{(n)}\right) +4f_2^{(n)}N_c C_F L_2(t_s)+2N_c^2 L_2(t_s)(f_1^{(n)}/ t_s-f_2^{(n)}-f_1^{(n)})+(t_s\leftrightarrow u_s). \tag{3.44}$$

4. Results

In the sum (2.5)+(3.13)+(3.18)+(3.42) and (2.10)+(3.22)+(3.23)+(3.28)+(3.43) the singularities drop out. For $\varepsilon=0$ one gets

$$\frac{d\sigma^{\delta,\Delta}_{q\bar{q}\to Q\bar{Q}}}{dt_s}=N_4\left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^2\left\{B^{(4)}_{q\bar{q}\to Q\bar{Q}}+\frac{\alpha_s(\mu^2)}{2\pi}\right. \cdot\left\{B^{(4)}_{q\bar{q}\to Q\bar{Q}}\left[C_F\left(\frac{37}{2}-8\zeta_2-4\ln^2 t_s+4\ln^2 u_s -6\ln\delta-8\ln\delta\ln\Delta-16\ln\Delta\ln\frac{t_s}{u_s} +8L_2(t_s)-8L_2(u_s)\right)\right.\right. +N_c(2\zeta_2+2\ln^2 t_s-2\ln t_s\ln u_s-\ln^2 u_s +8\ln\Delta\ln t_s-4\ln\Delta\ln u_s-4L_2(t_s) \left.\left.\left.+2L_2(u_s)\right)\right]+F_{q\bar{q}\to Q\bar{Q}}\right\}\right\} \tag{4.1}$$

$$\frac{d\sigma^{\delta,\Delta}_{gg\to Q\bar{Q}}}{dt_s}=N_4\left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^2\left\{B^{(4)}_{gg\to Q\bar{Q}}+C_F N_c\frac{\alpha_s(\mu^2)}{2\pi}\right. \cdot\left[B^{(4)}_{gg\to Q\bar{Q}}\left(2N_c\zeta_2+\frac{13}{2}C_F-5C_F\zeta_2 -2(N_c+C_F)\ln\delta\ln\Delta-2N_c\ln\delta\ln T +\left(\frac{N_c}{3}-\frac{3C_F}{2}\right)\ln\delta\right)+F_{gg\to Q\bar{Q}}+\zeta_2 N_c\left(C_F-\frac{N_c}{2}\right)f_2^{(4)} -4N_c C_F f_2^{(4)}\ln\delta+N_c C_F f_2^{(4)}(\ln^2 t_s-2\ln t_s\ln u_s +4\ln t_s\ln\Delta)+N_c^2((f_1^{(4)}+\tfrac{1}{2}f_2^{(4)})\ln t_s\ln u_s+4f_1^{(4)}\ln\delta) +\ln^2 u_s\left(-f_1^{(4)}+u_s^2-\frac{u_s}{2t_s}\right)N_c^2 -\ln\Delta\ln t_s\left(4u_s^2+2\frac{t_s}{u_s}\right)N_c^2 +\ln t_s\left[N_c^2\left(-2u_s^2-\frac{2}{u_s}\right)+N_c C_F\left(\frac{4}{u_s t_s}-4\right)\right] +L_2(t_s)\left(-1+2u_s^2+\frac{1}{u_s}\right)N_c^2 \left.\left.-2f_2^{(4)}N_c C_F L_2(t_s)+(t_s\leftrightarrow u_s)\right]\right\}. \tag{4.2}$$

5. Conclusions

We have calculated QCD-corrections to partonic processes important for collider experiments. However, not all possible QCD-corrections were considered, because we restricted ourselves to a specific final state. For example, the corrections to the process $gg\to gg$ which is important for the full collider jet cross section have not been calculated, though in principle our method is also applicable to them.

We have given analytical expressions only for those regions of phase space where the collinear and infrared singularities lie. In those regions three particle kinematics effectively reduces to two particle kinematics, if the cuts δ and Δ are chosen small enough. In this limit our analytical expressions become exact. The integration over the rest of phase space can be done numerically. If, for example, one would be interested in jet cross sections, one would have to introduce physical jet cuts Δ_{jet} and δ_{jet} and add to (4.1) and (4.2) the results of a numerical integration of (3.4) and (3.5) in the regions $\Delta < x < \Delta_{\text{jet}}$ and $\delta < \zeta$, $1-\zeta$, η, $1-\eta < \delta_{\text{jet}}$.

The partonic cross sections calculated in this way have to be folded with quark and gluon distribution functions. (Decay functions are not needed, because we assume the heavy quarks to be directly measured.) This is to be done in a future publication.

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Appendix

Here we calculate J_η and J_+ as needed in the infrared limit. First one can do the φ integration [11]

$$\frac{1}{N_\varphi}\int_0^\pi \frac{d\varphi \sin^{-2\varepsilon}\varphi}{\zeta u_s+(1-\zeta)t_s-2\sqrt{\zeta(1-\zeta)t_s u_s}\cos\varphi}$$
$$=(1+2\varepsilon^2\zeta_2)\left(\frac{s_+}{2}\right)^{2\varepsilon} r_-^{-1-2\varepsilon}$$
$$-\frac{2\varepsilon^2}{r_-}\left[L_2\left(1-\frac{s_-}{s_+}\right)+\ln\frac{s_-}{s_+}\ln\left(1-\frac{s_-}{s_+}\right)\right]+O(\varepsilon^3) \quad \text{(A.1)}$$

where

$$r_+=\zeta u_s+(1-\zeta)t_s \quad \text{(A.2)}$$

$$r_-=|\zeta-t_s| \quad \text{(A.3)}$$

$$s_\pm=r_+\pm r_-. \quad \text{(A.4)}$$

So

$$\frac{s_+}{2}=\begin{cases}\zeta u_s & \zeta>t_s\\ t_s(1-\zeta) & t_s>\zeta.\end{cases} \quad \text{(A.5)}$$

Because of

$$1-\frac{s_-}{s_+}=2\frac{r_-}{s_+} \quad \text{(A.6)}$$

the singularity for $r_-\to 0$ is removed in the second term of (A.1). That term gives only a contribution, when a pole in ζ is present and only for $\zeta\to 0$. Because of

$$\lim_{\substack{\zeta\to 0\\ \zeta<t_s}}\frac{s_-}{s_+}=0 \quad \text{(A.7)}$$

one gets a ζ_2 from the Dilogarithm and a $-1/\varepsilon$ from the ζ-integration.

From the first term in (A.1) one typically encounters the following ζ-integral

$$K_\alpha=\int_0^1 d\zeta\,\zeta^{-\varepsilon-\alpha}(1-\zeta)^{-\varepsilon}\{\zeta^{2\varepsilon}u_s^{2\varepsilon}\,\theta(\zeta-t_s)(\zeta-t_s)^{-1-2\varepsilon}$$
$$+(1-\zeta)^{2\varepsilon}t_s^{2\varepsilon}\,\theta(t_s-\zeta)(t_s-\zeta)^{-1-2\varepsilon}\}. \quad \text{(A.8)}$$

One needs K_0 for J_η and K_1 for J_+. K_0 can be calculated by expanding $(\zeta-t_s)^{-1-2\varepsilon}$ around the appropriate point. K_1 can be reduced to K_0 (modulo some simpler integral) by partial fractioning

$$\frac{1}{\zeta(\zeta-t_s)}=\frac{1}{t_s}\left(\frac{1}{\zeta-t_s}-\frac{1}{\zeta}\right). \quad \text{(A.9)}$$

Here we only quote the result for K_0

$$K_0=2\frac{\Gamma(-2\varepsilon)\Gamma(1-\varepsilon)}{\Gamma(1-3\varepsilon)}+\varepsilon\zeta_2+O(\varepsilon^2). \quad \text{(A.10)}$$

The final formulae for J_η and J_+ can be found in the main text.

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