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Determination of Semi-Muonic Branching Ratios and Fragmentation Functions of Heavy Quarks in $e^+ e^-$ Annihilation at $\langle\sqrt{s}\rangle = 34.6$ GeV

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Abstract. Muon inclusive multihadronic final states in e^+e^- annihilation at $\langle\sqrt{s}\rangle = 34.6$ GeV, collected by the JADE detector at PETRA, have been analysed in terms of the semi-muonic branching ratios and fragmentation functions of heavy quarks. Defining the fragmentation variable, ζ, as $z = \frac{(E+p_{\parallel})_{\text{hadron}}}{(E+p)_{\text{quark}}}$ we find $\langle z_c\rangle = (77 \pm 3 \pm 5)\%$ and $\langle z_b\rangle = (86 \pm 4 \pm 5)\%$. The corresponding semi-muonic branching ratios are measured to be $\mathrm{BR}(c \to \mu\nu_\mu X) = (7.8 \pm 1.5 \pm 2)\%$ and $\mathrm{BR}(b \to \mu\nu_\mu X) = (11.7 \pm 1.6 \pm 1.5)\%$. Defining ζ as $x_E = \frac{E_{\text{hadron}}}{E_{\text{beam}}}$ we observe an expected softening effect in the fragmentation due mainly to gluon emission, but find that this definition leads to semi-muonic

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branching ratio measurements which are strongly affected by uncertainties in the fragmentation process and the QCD calculation.

Introduction

In this paper we report on an analysis of muon-inclusive multihadronic events from which information on the fragmentation functions and measurements of the mean semi-muonic branching ratios of the heavy quarks are obtained. In particular, we examine the sensitivity of such measurements to the details of the model used to describe the production of multihadronic events. These studies allow us to quote results based on a method of analysis chosen to minimise such model-dependent systematic errors.

Theoretical Models and Experimental Methods

In the currently accepted theory of strong interactions, Quantum Chromodynamics (QCD) [1], a fundamental difficulty exists in describing the behaviour of quarks and gluons at large distance (or low energies). Here, perturbation theory is no longer applicable and non-perturbative models are introduced to describe the process of quark or gluon conversion into hadrons (termed hadronisation or fragmentation). The cross section of e^+e^- annihilation into quarks and gluons is thus calculated using second-order perturbative QCD while the subsequent transformation into hadrons at large distances is described by a phenomenological model. A commonly accepted mechanism for the latter process is that coloured quarks and gluons created at small distances fly apart stretching the colour lines of force (or string) between them. Through colour polarisation of the vacuum they then transform into jets of colourless hadrons with restricted transverse momentum with respect to the fragmentation direction* and with a flavour dependent distribution of longitudinal momentum. The transverse momentum distribution is parametrised by a gaussian distribution of width σ_q ($\sim$260 MeV), and the longitudinal momentum by a scaling function $f(\zeta)$ where ζ is the fraction of available momentum (or energy) carried by the hadron produced. Kinematic arguments [2] suggest that in the case of a fragmenting heavy quark (i.e. charm or bottom), a larger fraction of the available energy is carried by the hadron containing the heavy quark, than in the case of the light quarks which fragment principally into low momentum hadrons. This would lead to functions peaked towards high values of ζ (said to be 'hard') for charmed and bottom hadrons, produced solely from leading c and b quarks since $c\bar{c}$ and $b\bar{b}$ pair production in the fragmentation chain is heavily suppressed. The determination of the functional form of $f(\zeta)$ has been the subject of recent theoretical [3–8] and experimental [9–11] endeavour and a number of forms have been proposed. The Lund group [6] has proposed the form

$$f(\zeta)\propto\frac{1}{\zeta}(1-\zeta)^A \exp\left(-\frac{Bm_\perp^2}{\zeta}\right) \tag{1}$$

where $m_\perp$ is the transverse mass of the produced hadron and the parameters A and B are to be determined experimentally. In a previous analysis [12] the Lund functional form, together with other parameters of the model, were tuned to provide a good description of a number of particle distributions in multihadronic events. However, in analyses specific to the determination of the 'hardness' of the heavy quark fragmentation functions, the functional form adopted by experiments is that proposed earlier by Peterson et al. [8]:

$$f(\zeta)\propto\frac{1}{\zeta\left[1-\frac{1}{\zeta}-\frac{\varepsilon_q}{(1-\zeta)}\right]^2}. \tag{2}$$

Here, ε_q is the only free parameter which is to be determined experimentally for each heavy quark q. It is expected to be approximately proportional to the inverse of the square of the mass of the fragmenting heavy quark forming the leading meson.

Despite the universal adoption of the Peterson fragmentation function for c and b quarks, discrepancies do exist in the interpretation of the variable ζ. In fact, PEP and PETRA experiments between them have formulated no less than 5 definitions of ζ:

$$\zeta\equiv z=\frac{(E+p_\parallel)_{\text{hadron}}}{(E+p)_{\text{quark}}} \tag{3}$$

$$\zeta\equiv z_E=\frac{E_{\text{hadron}}}{E_{\text{quark}}}\approx z \tag{4}$$

$$\zeta\equiv x_E=\frac{E_{\text{hadron}}}{E_{\text{beam}}} \tag{5}$$

$$\zeta\equiv x_p=\frac{p_{\text{hadron}}}{\sqrt{E_{\text{beam}}^2-m_{\text{hadron}}^2}} \tag{6}$$

$$\zeta\equiv x_\gamma=\frac{2E_{\text{hadron}}}{\sqrt{s_{\text{effective}}}} \tag{7}$$

* In string models fragmentation proceeds along the colour-flux lines stretched between partons rather than along the parton direction

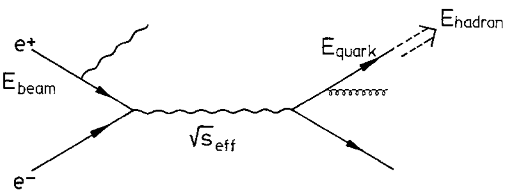

Fig. 1. The effect of initial state radiation and gluon emission leading to $E_q < \frac{1}{2} \cdot \sqrt{s_{\text{effective}}} < E_{\text{beam}}$

where E_{beam} is the beam energy, $\sqrt{s_{\text{effective}}}$ is the energy of the virtual photon produced in the $e^+ e^-$ collision after accounting for initial state radiation, and E_{quark} is the energy of the quark after further accounting for the emission of one or more gluons. $(E + p_{\|})_{\text{hadron}}$ is the energy and momentum component parallel to the fragmentation direction carried by the primary hadron. On examining these formulae it is evident that they will differ from each other when the effects of gluon emission and initial state radiation are considered. These effects, as depicted schematically in Fig. 1, lead to $E_{\text{quark}} \leqq \frac{1}{2} \cdot \sqrt{s_{\text{effective}}} \leqq E_{\text{beam}}$ and therefore, by definition, $x_E \leqq x_y \leqq z$.

The various definitions of ζ stem from the two methods employed in extracting information on the fragmenting heavy quark. The first results were obtained from analyses involving the reconstruction of charmed mesons through their hadronic decay. The reconstruction of the D^* meson using the $D^{*\pm} - D^0$ mass difference to tag the hadronic decay of the D^* has been particularly successful. In such analyses, the energy of the charmed meson is accurately determined and since the beam energy is a known quantity, the fraction x_E (5) or x_p (6) is readily accessible. The fraction z (3) or z_E (4), on the other hand, cannot be directly measured on an event by event basis since the charmed quark energy itself is unknown. Such a determination would entail the involved procedure of fitting to the hadron energy or momentum distribution of the data, the corresponding distribution as a function of z or z_E as derived from a QCD and fragmentation model. The determination of x as opposed to z is therefore preferred by such analyses in which the charmed meson is directly reconstructed.

More recently, information has been obtained from studies of inclusive leptons originating from the semi-leptonic weak decays of the heavy quarks. Here an indication of the fragmentation of the heavy quark lies in the momentum spectrum of the prompt lepton since this itself depends on the momentum spectrum of the parent hadron. The dependence of the muon momentum on ζ of the heavy hadron parent is illustrated in Fig. 2 for two definitions of ζ, namely z (3) and x_E (5). Figure 2 further shows that the softening of the muon momenta due to single gluon emission is only small. The lepton momentum spectra are thus relatively insensitive to the effects of first order QCD. The softening due to double gluon emission is, however, significantly larger.

To distinguish between the contributions from different quark flavours, a variable sensitive to the quark mass is used. A universal choice is the lepton momentum transverse to the thrust axis. Its mean value for c and b quarks is quite different, although this difference depends not only on the quark masses, but also on the extent of jet broadening due to gluon radiation. This effect is particularly significant for the transverse momentum spectrum of leptons from c decays. This

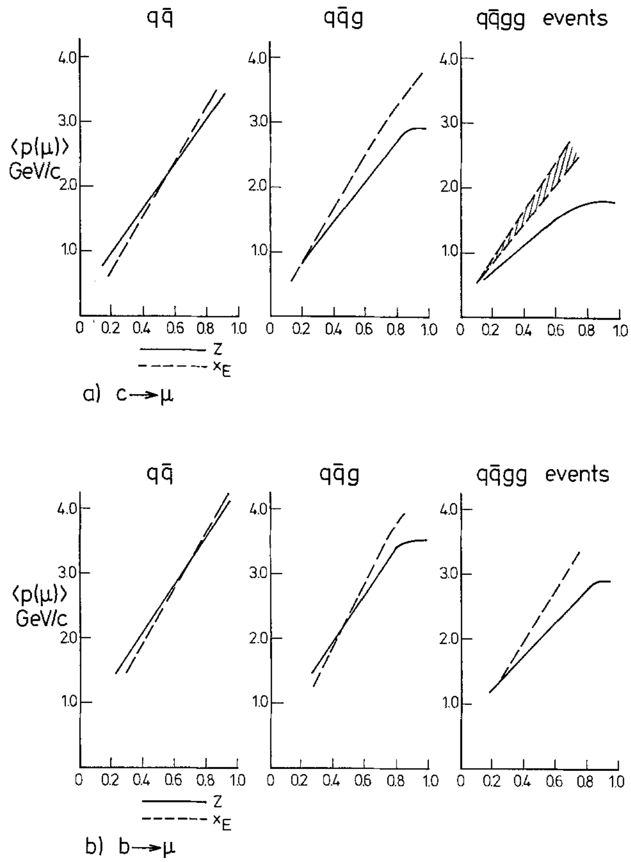

Fig. 2a, b. The mean muon momentum, $\langle p(\mu) \rangle$, from **a** $c \to \mu$ decay and **b** $b \to \mu$ decay as a function of z (solid line) and x_E (dashed line) of the heavy hadron parent, for two-, three- and four-parton events. The difference between the distributions with respect to z and x_E in the $q\bar{q}$ system is due to initial state photon radiation, whilst in the $q\bar{q}g(g)$ system there is a further effect due to the emission of one (two) gluon(s). Lund Monte Carlo prior to detector tracking. These events were generated using $\alpha_s = 0.165$ and $y_{\min} = \frac{m_{ij}^2}{s} = 0.015$ where m_{ij} is the invariant mass between partons i and j. With these values the total cross section is almost saturated with multi-parton events which comprise $\approx 85\%$ of the multihadronic events

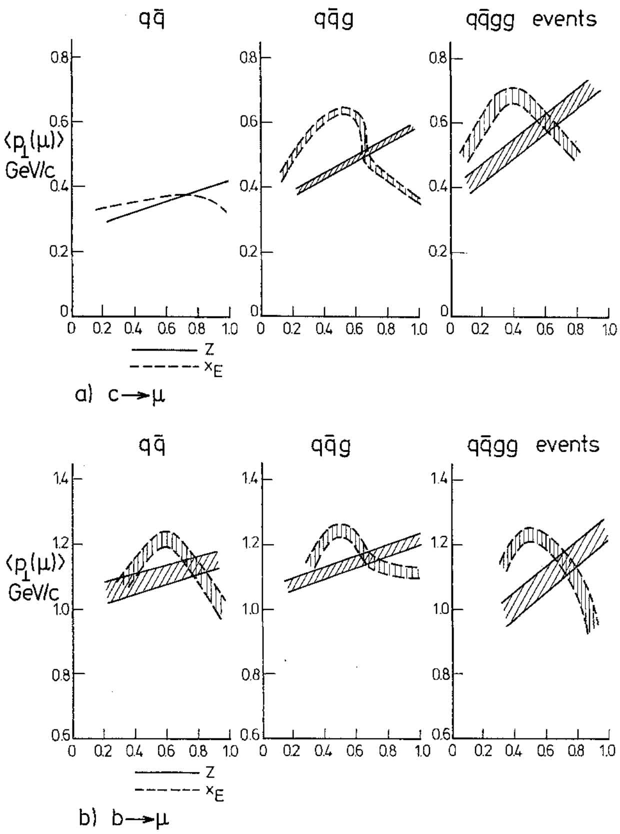

Fig. 3a, b. As Fig. 2, but plotting the average transverse muon momentum, $\langle p_\perp(\mu)\rangle$. The hatched areas encompass the spread in mean values. Note the different ordinate scales in **a** and **b**

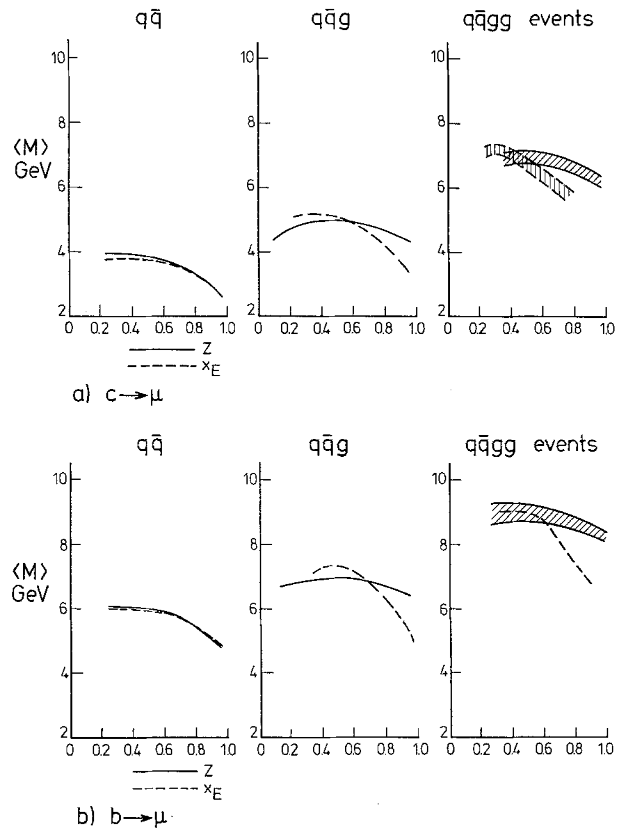

Fig. 4a, b. As Fig. 2, but plotting the average event transverse jet mass, $\langle M\rangle$. Note the different ordinate scales in **a** and **b**

is illustrated in Fig. 3a which shows the muon transverse momentum as a function of z and x_E for two-, three- and four-parton events. It is seen that the effect of gluon bremsstrahlung is a fairly uniform change in the mean value of $p_\perp(\mu)$ across the whole spectrum of z. The effect on the x_E distribution is however complicated by the migration of three- and four-parton events with hard gluon bremsstrahlung (and consequently higher $\langle p_\perp(\mu)\rangle$) from high bins of z into lower x_E bins. The lepton transverse momentum, with the inclusion of multi-parton events, thus results in a non-trivial distribution with respect to the fragmentation variable x_E. The variation of $\langle p_\perp(\mu)\rangle$ with z on the other hand, shows a steady increase with z attributed to phase space, and is less affected by any uncertainties in the QCD calculation and the fragmentation *ansatz*. The corresponding effect for leptons from b decays, as illustrated in Fig. 3b, is less dramatic since the extent of jet broadening due to gluon emission is less severe owing to the high mass of the b quark which already results in a broad b jet.

To achieve further separation between the quark flavours, a variable which indicates the topology of the event, such as thrust or sphericity, is sometimes used. In this respect, we have found the transverse jet mass variable, M [13–16], particularly fruitful. M is defined as

$$M=\frac{E_{\text{cm}}}{E_{\text{vis}}}\sum_n |p_\perp^{\text{out}}|, \tag{8}$$

where $|p_\perp^{\text{out}}|$ is the magnitude of the momentum component of the particle out of the event plane, and where the sum runs over all charged and neutral tracks. E_{vis} is the total visible energy of the event. The variation of $\langle M\rangle$ as a function of z and x_E is shown in Fig. 4 for two-, three- and four-parton events with leading c and b quarks. The x_E distribution is similarly affected by gluon emission.

Typical analyses proceed by deducing the prompt lepton momentum and transverse momentum (and sometimes thrust) distributions of the quark flavours as a function of ζ using a sample of simulated data (Monte Carlo). Then, by fitting to the corresponding distributions of the data, experiments are able to obtain information on the ζ distribution of the fragmenting heavy quark. Here, the choice of the definition of ζ is not restricted as all variables are *a priori* known in the Monte Carlo model. This allowed freedom in defining ζ (within the boundary of using a Monte Carlo model) is a major contributing factor to the existence of the many equations defining ζ. Although (3) is theoretically preferred, (4), (5) and (7) have also been used in analyses of inclusive leptons. These various definitions of ζ thus manifest themselves in different results for ε_q of the Peterson function, which in turn give different values for $\langle \zeta_q \rangle$ – a crucial factor to be accounted for before drawing conclusions and comparing results between experiments [10].

In addition to providing information on the heavy quark fragmentation functions, the production rate of prompt leptons yields valuable knowledge of the weak semi-leptonic branching ratios of the heavy quarks. The production mechanism of the prompt leptons is described by the spectator model [17]. The model predicts equal lifetimes and equal semi-leptonic branching ratios for charged and neutral charmed mesons, and likewise for charged and neutral bottom mesons; however, present data show unambiguously that the D^0 has a shorter lifetime [18] and a smaller semi-leptonic branching ratio [19; 52] than the $D^{\pm}$, indicating the existence of other, non-spectator, diagrams [20–21] in the hadronic decay system of D decays. The corresponding situation with B hadrons is however less well defined since charged and neutral B mesons have yet to be successfully isolated. An indication of the extent of the deviation from the spectator model in B decays can, nevertheless, still be obtained from measurements of the mean semi-leptonic branching ratios of all charged and neutral B hadrons. The spectator model with QCD and finite mass corrections applied predicts a mean branching ratio in the range 13–15% [22–24]. The world averages at the $\Upsilon(4S)$ resonance and in the continuum hover close to the lower limit [25]. More convincing experimental evidence is therefore required to test the validity of the spectator model in B decays.

Detection of Muon-Inclusive Multihadronic Events

The JADE detector [26] at PETRA has been used to collect a sample of high energy e^+e^- annihilation events with multihadronic final states containing muons. The data were collected at centre of mass energies in the range 33–37.5 GeV, with a mean value of 34.6 GeV, and correspond to an integrated luminosity of 69 pb^{-1}. A sample of 21,781 multihadronic events was selected by the standard JADE selection method [27], with the further requirement of the muon detection system being fully operative.

Muons were identified as penetrating tracks in the JADE muon filter, which is a segmented system with five layers of absorber and drift chambers covering a solid angle of 92% of 4π. A full description of the muon detection system and the procedure used for selecting prompt muons can be found in [28] and [29] respectively. A summary of the main selection criteria is, however, given here for convenience. For each event the muon selection programs extrapolated every charged track found by the jet chamber pattern recognition program [30] outwards into the muon filter as if it were a muon. All projected tracks which could be associated with a series of hits in the muon drift chambers, and which had a momentum greater than 1.8 GeV, were treated as candidate muon tracks. Further selection criteria were then imposed in order to enhance the proportion of prompt muon tracks relative to 'background' tracks, caused either by non-prompt muons from π and K decays or by 'punch-through' hadrons which either penetrated the layers of absorber without interaction or were produced as secondary particles in hadronic interactions in the absorber. This enhancement was achieved by requiring muon candidate tracks to traverse a minimum of 5.8 absorption lengths, to cross at least three chamber layers and to have an associated chamber hit in every

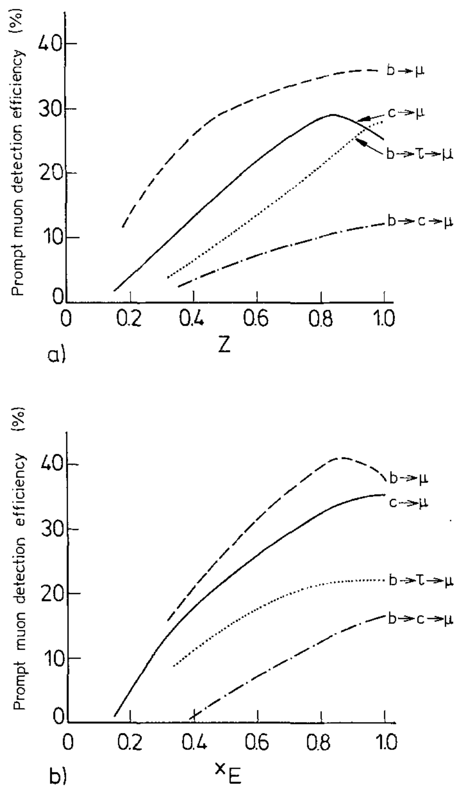

Fig. 5a, b. The prompt muon detection efficiency as a function of **a** z and **b** x_E of the heavy hadron parent for the processes $b \to \mu$ (dashed line), $c \to \mu$ (solid line), $b \to \tau \to \mu$ (dotted line) and $b \to c \to \mu$ (dashed-dotted line). The slight fall in detection efficiency at high values of ζ is due to the momentum cut of 9 GeV layer crossed. In addition an upper momentum cut of 9 GeV was imposed to reject hadron decays into muons. Such decays occasionally result in a kink in the inner detector track, which is not recognised by the pattern recognition software, and leads to an abnormally high reconstructed momentum. After applying all these selection criteria, a total of 959 muon candidate tracks were left which are attributed to the following physics processes.

Primary sources of inclusive muons:

1) $b \to c\mu^- \bar{\nu}_\mu$
2) $c \to s\mu^+ \nu_\mu$
3a) u, d, s, c, b: $\pi, K \to \mu\nu_\mu X$
3b) u, d, s, c, b: *hadron 'punch-through'*.

Secondary decays in b events:

4a) $b \to c W^-_{\text{virtual}}$; $c \to s\mu^+ \nu_\mu$
4b) $b \to c W^-_{\text{virtual}}$; $W^-_{\text{virtual}} \to \bar{c}s$; $\bar{c} \to s\mu^- \bar{\nu}_\mu$
5) $b \to c W^-_{\text{virtual}}$; $W^-_{\text{virtual}} \to \tau^- \bar{\nu}_\tau$; $\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$.

The Cabibbo suppressed decays, $b \to u\mu\nu_\mu$ and $c \to d\mu\nu_\mu$, are neglected.

The detection efficiency for prompt muons from the various sources is plotted in Fig. 5 as a function of (a) z and (b) x_E of the heavy hadron parent.

Data Analysis

The data were binned in $p(\mu)$, $p_\perp(\mu)$, M space and analysed in terms of the 5 processes listed above. In calculating the secondary decays in b events, the following estimates were taken:

$$\text{BR}(b \to c W^-_{\text{virtual}}) = 100\% \quad [31] \tag{9}$$

$$\text{BR}(W^-_{\text{virtual}} \to \bar{c}s) = 16\% \quad [21] \tag{10}$$

$$\frac{\text{BR}(b \to c\tau^- \bar{\nu}_\tau)}{\text{BR}(b \to c\mu^- \bar{\nu}_\mu)} = 26\% \quad [21] \tag{11}$$

$$\text{BR}(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau) = 17\% \quad [21;32]. \tag{12}$$

The shape of the uds background (i.e. process 3 above) spectra in $p(\mu)$, $p_\perp(\mu)$, M space was taken from the Monte Carlo model, which employed the Lund fragmentation function (1) for the light quarks with the parameters A and B set to 1.0 and 0.6 GeV^{-2} respectively. The uds background spectra are not however expected to depend significantly on the parameters of the fragmentation function. This was checked by plotting the $p(\mu)$, $p_\perp(\mu)$ and M spectra as a function of z of the first rank hadron i.e. the hadron containing the primary quark. No variation was seen apart from a slight increase ($\approx 20\%$) in the $p(\mu)$ spectrum between the lowest and highest of the z bins. The same was found to be true of the background spectra from c and b events. The background distributions from uds, c and b quarks were combined in a ratio predicted by the Lund Monte Carlo simulation, which assumes uds, c and b quark production in proportion to the square of the quark charges, after accounting for initial state radiation.

A maximum likelihood fit to the muon inclusive data sample was performed by weighting the $p(\mu)$, $p_\perp(\mu)$, M spectra of the c and b quarks with the Peterson fragmentation functions (by varying $\varepsilon_{c,b}$) and the semi-muonic branching ratios $\text{BR}(c, b \to \mu\nu_\mu X)$. The background fraction of the muon inclusive sample was not subtracted but was also left as a free parameter.

Two definitions for ζ were investigated, namely z and x_E. The results are summarised in Table 1a. An expected softening in the fragmentation due primarily to gluon emission is evident with $\langle x_E \rangle$ being in the order of 10–15% lower than $\langle z \rangle$. The results, however, also indicate that the branching ratios are dependent upon the choice of definition for ζ. These results having been derived from the same data sample were however expected to coincide. The errors quoted are statistical and include the effects of correlations between the parameters (Table 1b).

Table 1a. Branching ratios and fragmentation parameters for various choices of ζ. The mean value of ζ_q reflects the value of ε_q resulting from the fit. The χ^2/ν value is given as an indication of the goodness of the maximum likelihood fit

ζ	$\text{BR}(c \to \mu\nu_\mu X)\%$	$\text{BR}(b \to \mu\nu_\mu X)\%$	ε_c	ε_b	$\langle\zeta_c\rangle\%$	$\langle\zeta_b\rangle\%$	χ^2/ν
z	7.8 ± 1.5	11.7 ± 1.6	$0.015^{-0.006}_{+0.009}$	$0.0035^{-0.002}_{+0.004}$	77 ± 3	86 ± 4	$\frac{135}{120}$
x_E	$6.8^{+1.5}_{-1.4}$	$14.5^{+1.8}_{-1.7}$	$0.088^{-0.023}_{+0.032}$	$0.020^{-0.007}_{+0.012}$	64 ± 3	76 ± 3	$\frac{130}{120}$
x_E	7.9 ± 1.6	$12.9^{+1.7}_{-1.6}$	0.25 (fixed)	$0.015^{-0.006}_{+0.010}$	54 (fixed)	77 ± 3	$\frac{142}{121}$
x_E	7.8 (fixed)	11.7 (fixed)	$0.109^{-0.024}_{+0.033}$	$0.019^{-0.007}_{+0.012}$	62 ± 2	76 ± 3	$\frac{134}{122}$

Table 1b. Table of correlation coefficients between variables in fit using z

	ε_b	$BR(b \to \mu \nu_\mu X)$	ε_c	$BR(c \to \mu \nu_\mu X)$
$BR(b \to \mu \nu_\mu X)$	0.19	–	–	–
ε_c	0.04	−0.05	–	–
$BR(c \to \mu \nu_\mu X)$	0.05	−0.26	0.18	–
$N_{\text{background}}$	−0.06	0.00	−0.13	−0.85

A check was made to ensure that the difference in results for the branching ratios for different choices of ζ was neither due to an error in the normalisation of the fragmentation functions nor to a programming error. The analysis jobs were run on a set of Monte Carlo events with known heavy quark branching ratios and fragmentation parameters ε_q. Whilst the values of $\varepsilon_{c,b}$ differed between z and x_E, the results for the branching ratios were identical giving confidence in the analysis routines.

The relationship between z and x_E is determined mainly by the effects of gluon emission as calculated by QCD and implemented by the fragmentation *ansatz*. The effects of initial state radiation are, by comparison, small (at least for c quarks) and well described by QED. The difference in the results for the branching ratios suggest that the form of the relationship between these variables is not correctly represented in the Monte Carlo model. Consequently systematic errors arise in distributions of the heavy quark flavours due to our inability to correctly describe the fragmentation process and the multi-parton rate [33]. Whilst such systematic errors result in only a relatively small change in the mean values of $p_\perp(\mu)$ and M across the whole z spectrum, the effect on the x_E spectrum is more severe and complicated (as shown in Figs. 3 and 4). The branching ratio determination using x_E is therefore strongly affected. Fixing the parameter ε_c to 0.25 when fitting using x_E (in agreement with early D^* analyses) also greatly influences the results (third line in Table 1a). Moreover, the Peterson function has been shown not to be a good representation of the x_E spectrum of hadrons [11]. In view of these factors, the results using the z definition are clearly preferred. They are in excellent agreement with our previous result [16] in which a fit insensitive to the parameters of the fragmentation function of the heavy quarks was performed on essentially the same data sample.

The sensitivity of the results to the low $p(\mu)$, $p_\perp$, M regions, where the background is largest, was investigated by excluding them from the fit. No significant difference to the results was seen. The fraction of background given by the fit is $(47 \pm 6)\%$, which is in reasonable agreement with the Monte Carlo expectation of $(56 \pm 2)\%$ [34]. The results for the b quark proved insensitive to the fraction of background, whereas in the case of the c quark a strong negative correlation exists.

The background spectra of the c and b quarks were also obtained from a Monte Carlo employing the values of $\varepsilon_{c,b}$ determined from the fit. No difference in the results was observed, indicating that the background distributions are indeed relatively insensitive to the fragmentation function parameters. The total number of $b\bar{b}$ and $c\bar{c}$ events in the multihadronic sample was allowed to vary by 5%; likewise the number of charmed mesons per bottom event, as determined by (9) and (10) and the relative rate $\frac{b \to c \tau \nu_\tau}{b \to c \mu \nu_\mu}$ (11), was varied across a feasible range of values, and the small difference in the results incorporated in the systematic error.

Further contributions to the systematic error arise from the details of the fitting procedure; from uncertainties in the muon detection efficiency and in the background from $\tau^+\tau^-$ and two-photon events; and from uncertainties in the shapes of the $p(\mu)$, $p_\perp(\mu)$, M distributions of the quark flavours, due to errors in the reconstruction of the thrust axis and event planes and from a lack of understanding of the QCD effects.

Taking all these systematic effects into account, the results obtained are:

$$BR(b \to \mu \nu_\mu X) = (11.7 \pm 1.6 \pm 1.5)\%$$

$$BR(c \to \mu \nu_\mu X) = (7.8 \pm 1.5 \pm 2)\%$$

$$\varepsilon_c = 0.015 \begin{array}{l} -0.006 - 0.009 \\ +0.009 + 0.018 \end{array}$$

$$\langle z_c \rangle = (77 \pm 3 \pm 5)\%$$

$$\varepsilon_b = 0.0035 \begin{array}{l} -0.002 - 0.0025 \\ +0.004 + 0.005 \end{array}$$

$$\langle z_b \rangle = (86 \pm 4 \pm 5)\%.$$

The quantities $\langle z_{c,b} \rangle$ were also determined by dividing the z regions into several intervals and weighting these intervals without assuming any functional form. The results of the 'free fit' are compared with the direct fit to the Peterson function in Fig. 6 and are in good agreement. The mean and *rms* values of z as determined from the 'free fit' (fit to Peterson function) are:

$$\langle z_c \rangle \pm \Delta z_c = (74 \pm 20)\% \quad ((77 \pm 16)\%)$$

$$\langle z_b \rangle \pm \Delta z_b = (88 \pm 7)\% \quad ((86 \pm 12)\%).$$

The relationship

$$\sigma_{q\bar{q} \to \mu \nu_\mu X} = \frac{N_{q\bar{q} \to \mu \nu_\mu X}}{\mathscr{L} \mathscr{E}_{q\bar{q} \to \mu \nu_\mu X}(1 + \gamma_{q\bar{q}})}$$

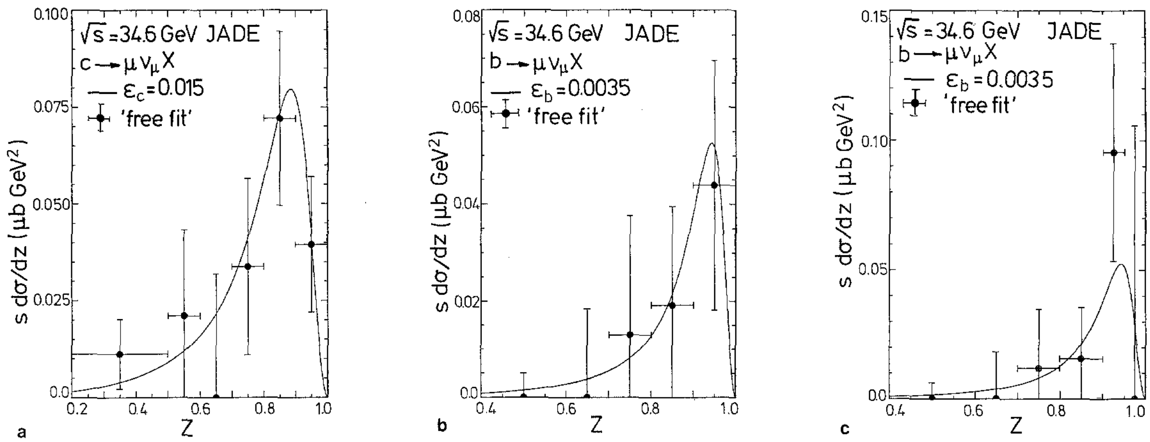

Fig. 6a–c. The **a** charm and **b** bottom quark fragmentation functions. The solid lines are the results of the fit to the Peterson fragmentation function. The points are the result of a 'free fit' assuming no functional form; **c** shows the results of a 'free fit' to the bottom quark fragmentation using smaller bins in the high z region. Contributions from $z > 0.95$ are not favoured. Note the different ordinate scales in **a–c**

where $\mathscr{L}$ is the total integrated luminosity, $N_{q\bar{q}\to\mu\nu_\mu X}$ is the number of $q\bar{q}\to\mu\nu_\mu X$ events detected and $\mathscr{E}_{q\bar{q}\to\mu\nu_\mu X}$ the corresponding overall detection efficiency and $(1+\gamma_{q\bar{q}})$ is the radiative correction factor, can be used to determine the following inclusive muon cross sections ($\langle\sqrt{s}\rangle = 34.6$ GeV):

$$\sigma(e^+e^- \to b\bar{b} \to \mu\nu_\mu X) = (6.3 \pm 0.9 \pm 0.8)\ \text{pb}$$
$$\sigma(e^+e^- \to c\bar{c} \to \mu\nu_\mu X) = (16.7 \pm 3.2 \pm 4.3)\ \text{pb}$$
$$\sigma(e^+e^- \to b\bar{b} \to c\bar{c}X \to \mu\nu_\mu X) = (5.0 \pm 1.0 \pm 1.3)\ \text{pb}$$
$$\sigma(e^+e^- \to b\bar{b} \to \tau\nu_\tau X \to \mu\nu_\mu X) = (0.28 \pm 0.04 \pm 0.04)\ \text{pb}.$$

The evaluation of these cross sections, unlike the semi-muonic branching ratio measurements, does not depend on the relative charm and bottom cross sections. They are in good agreement with theoretical expectations assuming the above determined branching ratios.

The composition of the muon-inclusive multihadronic sample in terms of the prompt muon and background processes is indicated in the $p_\perp(\mu)$ distribution of Fig. 7.

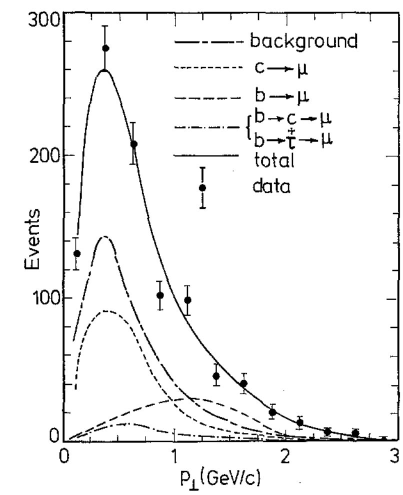

Fig. 7. The $p_\perp$ distribution with respect to the thrust axis of inclusive muons in the momentum range $1.8 < p(\mu) < 9$ GeV with predictions of the fit: $164\,b \to \mu$ events (long dashed line), $304\,c \to \mu$ events (short dashed line), $38\,b \to c \to \mu$ plus $4\,b \to \tau \to \mu$ events (dashed-dotted line), 449 background events (short-long dashed line) and 959 events in total (solid line)

Discussion and Comparison with Other Experiments

The results of this analysis are compared with those of other experiments in Tables 2 and 3 which list up-to-date figures. Our result of $\langle z_b\rangle = (86 \pm 4 \pm 5)\%$ compares with the world average from other experiments of $\langle z_b\rangle = (81 \pm 3)\%$, where the statistical and systematic errors have been added in quadrature. In calculating the world average, we have tried to compare 'like' with 'like' by using the results of $\langle z\rangle$ as determined by a fit to the Peterson function whenever applicable. In addition, results [42–44], which do not unfold the effects of initial state radiation and gluon bremsstrahlung, have been corrected by using the approximation $\langle z\rangle \approx \langle x_E\rangle + 11\%$. Similarly, our result of $\langle z_c\rangle = (77 \pm 3 \pm 5)\%$ compares with the world average of $\langle z_c\rangle = (60 \pm 3)\%$ from analyses of inclusive lep-

Table 2. A compilation of the latest results on heavy quark fragmentation. Updated result of [37] is preliminary

Expt.	Ref.	l	ζ	ε_c	$\langle\zeta_c\rangle$%	ε_b	$\langle\zeta_b\rangle$%
TASSO	[35]	μ	z	$0.006^{+0.017+0.052}_{-0.005-0.005}$	77^{+5+3}_{-7-11}	$0.0025^{+0.029+0.011}_{-0.0025-0.0013}$	85^{+10+2}_{-12-7}
TASSO	[36]	e	z	$0.19^{+0.29+0.17}_{-0.13-0.08}$	57^{+10+5}_{-9-6}	$0.005^{+0.022+0.020}_{-0.005+0.005}$	84^{+15+15}_{-10-11}
MARK J	[37]	μ	z	$(0.79\mp0.11\mp0.15)^2$	$46\pm3\pm3$	$(0.164\mp0.024\mp0.055)^2$	$74\pm2\pm5$
This expt.		μ	z	$0.015^{+0.009+0.018}_{-0.006-0.009}$	$77\pm3\pm5$	$0.0035^{+0.004+0.005}_{-0.002-0.0025}$	$86\pm4\pm5$
MAC	[38]	μ	z	–	17–67	$0.008^{+0.037}_{-0.008}$	80 ± 10
MARK II[a]	[40]	μ	z	–	–	$0.042^{+0.218+0.120}_{-0.041-0.035}$	$73\pm15\pm10$
MARK II[a]	[41]	e	z	–	–	$0.015^{+0.022+0.022}_{-0.011-0.011}$	$79\pm6\pm6$
DELCO	[42]	e	x_E	$0.14^{+0.07}_{-0.05}$	59 ± 4	$0.033^{+0.032}_{-0.017}$	72 ± 5
TPC	[43]	μ	x_E	$0.14^{+0.14+0.08}_{-0.07-0.05}$	$60\pm6\pm4$	$0.011^{+0.015+0.011}_{-0.007-0.007}$	$80\pm5\pm5$
TBC[b]	[44]	e	x_E	–	–	$0.033^{+0.037+0.019}_{-0.019-0.012}$	$74\pm5\pm3$

[a] ε_c fixed to 0.25 in fit [b] ε_c fixed to 0.24 ∓ 0.06 in fit

tons* and with the world average of $\langle z_c\rangle=(70.4\pm1.0\pm3.0)\%$ from D^* analyses [11].

Our result of $(11.7\pm1.6\pm1.5)\%$ for the semi-muonic branching ratio of the b quark agrees well with the world semi-leptonic** averages of $(11.8\pm0.5)\%$ at the $\Upsilon(4S)$ resonance and $(11.9\pm1.3)\%$ in the continuum. Results obtained using the x_E definition [42–44] are not included in this average. Pending re-analysis in terms of the fragmentation variable z, these results should be treated with some caution as they are likely, to some extent, to be subject to systematic effects similar to those observed in this analysis. We further omit the results of [40–41] as they fix ε_c to 0.25, which, although considered at the time to be a good representation of the charm fragmentation spectrum in x_E, is not a valid representation in z, which is the fragmentation variable used in their fit. With the inclusion of our result, the world average in the continuum becomes $(11.8\pm1.1)\%$, in excellent agreement with the $\Upsilon(4S)$ determination. This suggests that either b-flavoured hadrons in the continuum are of the same type and produced in the same proportion as on the $\Upsilon(4S)$ resonance (which by analogy with c quark results would seem improbable), or that all varieties of b hadrons have approximately similar branching ratios. Combining the continuum and $\Upsilon(4S)$ results gives an overall semi-leptonic branching ratio of $(11.8\pm0.5)\%$ which agrees with the predictions of the spectator model with QCD and non-spectator corrections [55].

Finally, our result of $(7.8\pm1.5\pm2)\%$ for the semi-muonic branching ratio of the c quark compares with a PEP and PETRA average of $(8.8\pm0.9)\%$ in which again results from [40–44] are not included. The inclusion of our result into the world average calculation gives $(8.6\pm0.9)\%$. This compares with an average on the $\psi(3{,}770)$ resonance of $(7.8\pm1.3)\%$ as determined by [50–51] which rely on charm cross section measurements for normalisation [56]; and an average of $(11.5\pm1.1)\%$ as determined by [19; 52] which tag D mesons and search for semi-leptonic decays in the recoiling system. The former result suggests that the D^+, D^0 mixture in the continuum is different from that at the $\psi(3{,}770)$ resonance (i.e. 56:44 respectively)

* The world average $\langle z_c\rangle=(60\pm3)\%$ from inclusive leptons is heavily weighted by the result of [37] which is of the order of five standard deviations away from the world average as determined from D^* analyses. Removing this result from the calculation gives a world average from inclusive leptons of $\langle z_c\rangle=(70\pm3)\%$, which is in excellent agreement with D^* measurements. Removing [37] from the corresponding world average calculation for b quarks gives $\langle z_b\rangle=(84\pm3)\%$

** We have assumed electron-muon universality

Table 3. A compilation of the latest results for the semi-leptonic branching ratio of heavy quarks. Updated results of [37–39; 46] are preliminary

Expt.	E_{CM} (GeV)	Ref.	l	ζ	BR$(c \to l \nu_l X)$%	BR$(b \to l \nu_l X)$%
PETRA:						
TASSO	35.5	[35]	μ	z	$8.2 \pm 1.2 {}^{+2}_{-1}$	$11.7 \pm 2.8 \pm 1$
TASSO	34.6	[36]	e	z	$9.2 \pm 2.2 \pm 4.0$	$11.1 \pm 3.4 \pm 4.0$
MARK J	35, 43	[37]	μ	z	$8.8 \pm 0.7 \pm 1.1$	$12.4 \pm 1.3 \pm 2.0$
CELLO	14, 22, 34	[45]	μ	–	$12.3 \pm 2.9 \pm 3.9$	$8.8 \pm 3.4 \pm 3.5$
CELLO	14, 22, 34	[45]	e	–	–	$14.1 \pm 5.8 \pm 3.0$
This expt.	34.6	–	μ	z	$7.8 \pm 1.5 \pm 2$	$11.7 \pm 1.6 \pm 1.5$
PEP:						
MAC[a]	29	[38]	μ	z	9 ± 3	$12.3 \pm 1.8 \left({}^{+1.7}_{-1.3} \pm 0.8\right)$
MAC[a]	29	[39]	e	z	8 ± 3	$11.3 \pm 1.9 \pm 3.0$
MARK II[b]	29	[40]	μ	z	$8.3 \pm 1.3 \pm 1.8$	$12.6 \pm 5.2 \pm 3.0$
MARK II[b]	29	[41]	e	z	$6.6 \pm 1.4 \pm 2.8$	$13.5 \pm 2.6 \pm 2.0$
DELCO	29	[42]	e	x_E	$11.6 {}^{+1.1}_{-0.9}$	$14.9 {}^{+2.2}_{-1.9}$
TPC	29	[43]	μ	x_E	$6.9 \pm 1.1 \pm 1.1$	$15.2 \pm 1.9 \pm 1.2$
TPC[c]	29	[44]	e	x_E	$9.1 \pm 0.9 \pm 1.3$	$11.0 \pm 1.8 \pm 1.0$
DORIS:						
ARGUS	10.58	[46]	e	–	–	$12.0 \pm 0.9 \pm 0.8$
CESR:						
CLEO	10.58	[47]	μ	–	–	$10.8 \pm 0.6 \pm 1.0$
CLEO	10.58	[47]	e	–	–	$12.0 \pm 0.7 \pm 0.5$
CUSB	10.58	[48]	μ	–	–	$11.2 \pm 0.9 \pm 1.0$
CUSB	10.58	[49]	e	–	–	$13.2 \pm 0.8 \pm 1.4$
SPEAR:						
DELCO	3.77	[50]	e	–	8.0 ± 1.5	–
LGW	3.77	[51]	e	–	7.2 ± 2.8	–
MARK II	3.77	[52]	e	–	10.0 ± 3.2	–
MARK III	3.77	[19]	e	–	$11.7 \pm 1.0 \pm 0.5$	–
LGW	3.9–7.4	[53]	e	–	8.2 ± 1.9	–
DORIS:						
DASP	3.99–4.08	[54]	e	–	8.0 ± 2.0	–
DASP	3.99–5.02	[54]	e	–	7.2 ± 2.0	–

[a] $\varepsilon_{c,b}$ fixed to 0.4 and $0.008 {}^{-0.008}_{+0.037}$ respectively. Fit however, performed to $p_\perp(\mu)$ distribution only, hence influence of ε_q values is relatively small

[b] ε_c fixed to 0.25 in fit; [c] ε_c fixed to 0.24 ∓ 0.06 in fit

– and/or other charmed mesons have different semi-leptonic branching ratios.

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