

**$E_6$  BASED MECHANISM  
FOR THE GENERATION OF FERMION ELECTRIC DIPOLE MOMENTS:  
AN APPLICATION TO THE SOLAR NEUTRINO PUZZLE <sup>\*</sup>**

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We discuss the electric dipole moments (EDM) of fermions generated by  $CP$ -violating phases associated to the new Yukawa couplings involving heavy matter  $E_6$  fields predicted in the framework of superstring theories. While for neutron and electron it is not strictly necessary to resort to a superstring scenario to get a substantial EDM, in the neutrino case a sizeable EDM is a distinctive feature of the superstring. We thus focus on the neutrino EDM and discuss its relevance for the solution of the solar neutrino problem.

Superstring theory has emerged as a very promising theory for the ultimate description of nature [1]. However, its connection to the real world is at present still very aethereal. Thus, it is of foremost importance to extract as many phenomenological consequences as possible from superstring inspired models. There is clearly plenty of work done in this direction and much more is still to be done <sup>†1</sup>. In this paper we describe a mechanism that induces non-vanishing electric dipole moments (EDM) of leptons and quarks. The possibility of  $CP$ -violation – e.g. through non-zero EDM's – induced by superstrings has been already discussed in the literature [3]. Here we emphasize an alternative mechanism not found in the literature and focus, in particular, on the neutrino electric dipole moment (not discussed before) as a potential solution of the solar neutrino puzzle.

In the  $E_6$  based superstring models each generation of matter particles sits in a 27 representa-

tion. This 27 contains 12 new particles which are the following,

$$\nu^c(1, 1), H(1, 2), \bar{H}(1, 2), D(3, 1), \\ D^c(3^*, 1), N(1, 1),$$

where in parentheses we display their  $SU(3)_c \times SU(2)_L$  content. So, in particular, we see that there is an extra isosinglet colour triplet quark  $D$  and  $D^c$ .

The  $D$ -quark participates in the following trilinear Yukawa couplings to conventional standard model and supersymmetrical standard model particles as required by the most general  $E_6$  invariant superpotential,

$$\xi_1 D Q Q + \xi_2 D^c u^c d^c, \quad (1)$$

and

$$\lambda_1 D^c L Q + \lambda_2 D e^c u^c + \lambda_3 D \nu^c d^c, \quad (2)$$

where

$$L \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad \text{and} \quad Q \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L.$$

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<sup>†1</sup> See ref. [2] for a review.

Now, the coexistence of eqs. (1) and (2) would lead to an almost instantaneous proton decay. It has been therefore argued that either  $\xi_1, \xi_2 \neq 0$  or  $\lambda_1, \lambda_2, \lambda_3 \neq 0$  because of topological reasons or because of a discrete symmetry [2].

Since we will be mainly concerned with lepton EDM's we shall assume  $\xi_1 = \xi_2 = 0$  and consider a *CP*-violating mechanism involving the Yukawa couplings in eq. (2). Nonetheless, it is worth noticing that one can easily accomodate an EDM for the neutron, interestingly close to the present bound, using the Yukawa couplings in eq. (1). We shall not give the explicit result here [4].

As to the couplings in eq. (2) they produce a contribution to both EDM of leptons *and* quarks as is easily seen from figs. 1 and 2. The necessary complex phases to get a non-zero effect are provided by the Yukawa couplings at each vertex (the coupling constants are independent) and by the mass matrices of the relevant squarks or sleptons that propagate in the loops.

Let us make these points explicit by calculating the EDM of the neutrino.

The neutrino EDM involves the diagram 1a with a *D* quark and a  $\tilde{d}^c$  squark propagating in the loop. Now, since EDM implies helicity flip the amplitude necessarily involves L-R squark mixing. The corresponding L-R mass matrix has off-diagonal entries which are in general complex. Explicitly,

$$\begin{pmatrix} m_{LL}^2 & m_{LR}^2 \\ m_{RL}^2 & m_{RR}^2 \end{pmatrix}, \tag{3}$$

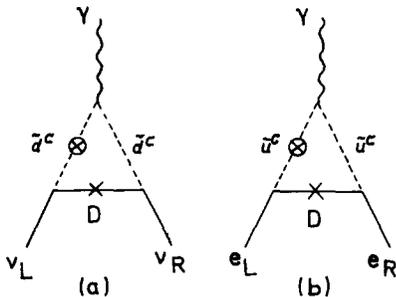


Fig. 1. Feynman diagrams that contribute to the EDM of (a) the neutrino and (b) the electron. The diagrams where the photon is attached to the *D*-quark are not shown.

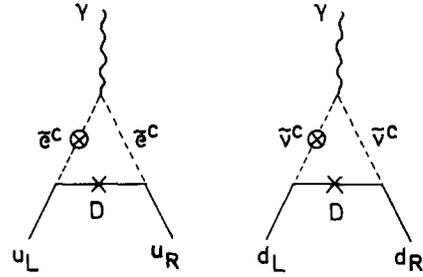


Fig. 2. Feynman diagrams that contribute to the EDM of quarks. The diagrams where the photon is attached to the *D*-quark are not shown.

where  $m_{LR}^2 = A m_f m_0$  and  $m_{RL}^2 = A^* m_f m_0 = m_{LR}^{2*}$ . The parameter *A* is a complex parameter that enters the game after soft SUSY breaking through trilinear scalar couplings [5].

To be specific let us particularize to the case at hand (diagram 1a). To go to a  $\tilde{d}$ -squark mass eigenstate basis from a weak  $\tilde{d}$ -squark basis we perform a rotation *R* given by

$$R_{ab} = \begin{pmatrix} e^{-i\alpha} \cos \phi & -e^{-i\alpha} \sin \phi \\ e^{i\alpha} \sin \phi & e^{i\alpha} \cos \phi \end{pmatrix}, \tag{4}$$

such that

$$(\tilde{d}_{\text{weak}})_a = \sum_b R_{ab} (\tilde{d}_{\text{mass}})_b, \tag{5}$$

and where the phase  $\alpha$  is chosen such that the two physical masses  $m_{a=1,2}$  are real. Then the relevant pieces in the lagrangian read

$$L_Y = \sum_b [\lambda_1 R_{1b} (\tilde{d}_{\text{mass}})_b \bar{D}_R \nu_L + \lambda_3 R_{2b} (\tilde{d}_{\text{mass}})_b d_b \bar{D}_L \nu_R] + \text{h.c.}, \tag{6}$$

and is *CP*-violating because of the phase  $\alpha$  in *R* and the phases of  $\lambda_1$  and  $\lambda_3$ .

From now on it is straightforward to compute the contribution from eq. (6) to the neutrino EDM.

The electric dipole moment is defined as

$$\langle f | J_\mu | f \rangle_{\text{EDM}} = ie D_f (q^2) \bar{u}(p') \sigma_{\mu\nu} q^\nu \gamma_5 u(p), \tag{7}$$

with  $q = p' - p$ . Evaluation of diagram 1a and the

related ones gives for  $|D_\nu(0)|$ ,

$$|D_\nu(0)| = \frac{1}{8\pi^2} \sum_{a=1,2} \text{Im}(\lambda_1 \lambda_3^* R_{1a} R_{2a}^*) \times \frac{m_D}{m_a^2 - m_D^2} \left( 1 - \frac{m_a^2}{m_a^2 - m_D^2} \log \frac{m_a^2}{m_D^2} \right). \quad (8)$$

Introducing  $R_{1a}$  and  $R_{2a}$  from eq. (4) and defining the complex phase  $\beta$

$$\lambda_1 \lambda_3^* = |\lambda_1 \lambda_3| e^{i\beta},$$

we finally get,

$$|D_\nu(0)| = \frac{|\lambda_1 \lambda_3|}{16\pi^2} \sin 2\phi \sin \delta \times \sum_{a=1,2} (-1)^a \frac{m_D}{m_a^2 - m_D^2} \times \left( 1 - \frac{m_a^2}{m_a^2 - m_D^2} \log \frac{m_a^2}{m_D^2} \right), \quad (9)$$

where  $\delta = \beta - 2\alpha$ .

The electron EDM is calculated along the same steps from diagram 1b. One obtains

$$|D_e(0)| = \frac{|\lambda_1 \lambda_2|}{16\pi^2} \sin 2\chi \sin \delta' \times \sum_{a=1,2} (-1)^a \frac{m_D}{m_a^2 - m_D^2} \times \left[ \frac{3}{2} + \frac{m_a^2 + 2m_D^2}{m_a^2 - m_D^2} \times \left( 1 - \frac{m_a^2}{m_a^2 - m_D^2} \log \frac{m_a^2}{m_D^2} \right) \right], \quad (10)$$

where  $\chi$  is the angle that diagonalises the  $\bar{u}$ -squark mass matrix,  $\delta' = \gamma - 2\chi$  ( $\gamma$  is the phase of  $\lambda_1 \lambda_2^*$ ) and  $m_{1,2}$  are the masses of  $\bar{u}$ -squarks.

Obviously, similar formulas can be derived for the u and d EDM's.

Actually, one can get even larger contributions than the ones implied by eqs. (9) and (10) if we recall that the Yukawa interactions in eq. (2) are in general flavour non-diagonal and the couplings  $\lambda_{1,2,3}$  connect different generations (they carry

family indices). These intergenerational Yukawas allow for the propagation in the loop of particles in generations other than the first. As a consequence, the mixing angles  $\phi$  and  $\chi$  in eqs. (9) and (10) are larger (recall that the off-diagonal mass terms are proportional to  $m_t$ ) by big factors.

The experimental bounds on  $D_e$  and  $D_n$  are [6]

$$D_e < 3 \times 10^{-24} e \text{ cm}, \quad (11)$$

$$D_n < 2.3 \times 10^{-25} e \text{ cm}.$$

As an illustration, take

$$m_1 = 65 \text{ GeV}, \quad m_2 = 60 \text{ GeV}, \quad m_D = 50 \text{ GeV}, \quad (12)$$

then, eq. (10) gives

$$D_e = 0.4 \times 10^{-22} |\lambda_1 \lambda_2|_{11} \sin \delta' e \text{ cm}, \quad (13)$$

if u circulates in fig. 1a, or,

$$D_e = 3.2 \times 10^{-19} |\lambda_1 \lambda_2|_{13} \sin \delta' e \text{ cm}, \quad (14)$$

for t circulating in the loop (we took  $m_t = 40$  GeV). (The subindices 11 and 13 in eqs. (13) and (14) remind us of the generations involved.) Analogous results follow for  $D_n$ .

Let us now turn to the EDM of the neutrino. The experimental upper limits on the neutrino EDM are furnished by astrophysical arguments. These limits are derived from the neutrino luminosity estimate of white dwarfs [7]. According to these estimates the EDM of the neutrino cannot exceed  $\sim 10^{-21} e \text{ cm}$ .

Very recently an explanation for the solar neutrino problem has been proposed. The problem is well known [8]. The flux of neutrinos reaching the earth from the sun is roughly a factor of 3 smaller than expected theoretically. It has been pointed out [9] that a magnetic moment of the neutrino on the order of  $\mu_\nu \sim 10^{-11} - 10^{-10} \mu_B$  would suffice for the magnetic field of the solar convective zone to flip the helicity of the emitted neutrinos by the amount necessary to explain the decrease in the detected flux (for the right-handed neutrinos are sterile). Furthermore, Okun has indicated that also a non-zero EDM of the neutrino would lead to the same consequences [10]. Therefore, both mechanisms would contribute independently to the de-

pletion of the flux of solar neutrinos. To support this interpretation is the fact that, allegedly, the flux of neutrinos from the sun attains its maximum value when the solar activity is less (and the magnetic field is correspondingly weaker) and vice versa. This flux variation proceeds over a period of eleven years. Thus, the above interpretation would explain this periodic modulation of the detected flux linked to the solar activity cycle.

The real question is how to generate a large magnetic moment and/or EDM for the neutrino. Okun has pointed out that the EDM should be  $O(10^{-22}-10^{-21}e \text{ cm})$  [10]. The question of a  $\mu_\nu$  generated via a superstring mechanism has already been discussed before [11]. So let us focus on the EDM. A major issue is how to simultaneously understand a neutron and electron EDM bound by eq. (11) and an EDM for the neutrino which should be 3–4 orders of magnitude larger.

In ref. [11] we showed that a choice of masses (the choice is, of course, not unique and serves only to illustrate the point)

$$\begin{aligned} m_{LL} &= (120 \text{ GeV})^2, & m_{RR} &= (110 \text{ GeV})^2, \\ m_{LR} &= (3 \text{ GeV})^2, \end{aligned} \quad (15)$$

and an inter-family mixing with the third generation could supply the right amount of  $\nu_\nu$  (through the same diagrams as in fig. 1a; see ref. [11] for details) to explain the anticorrelation of the neutrino flux data with the solar activity. Since for  $D_\nu = \mu_\nu$  the frequencies of magnetic and electric precessions coincide, it is clear that the same choices as eq. (15) above would do. Hence an EDM of the order of  $10^{-22}-10^{-21}e \text{ cm}$  can be obtained by our mechanism.

The second problem is to make this compatible with the bounds on  $D_e$  and  $D_n$ . The EDM of the electron and the quarks  $\underline{u}$  and  $\underline{d}$ , is proportional to the couplings  $\lambda_1$  and  $\lambda_2$  and the neutrino EDM depends on the couplings  $\lambda_1$  and  $\lambda_3$ . But the key point is that due to the very peculiar Aharanov–Bohm type of  $E_6$  breaking [12], these different Yukawa couplings need not be interrelated by  $E_6$  Clebsch–Gordan coefficients [13]. Therefore, one may perfectly well envisage the situation where  $\lambda_2 \ll \lambda_3$  (keeping, of course,  $\lambda_3$  small in order to avoid dangerous FCNC, as dis-

cussed in ref. [11]) such that  $D_e$  and  $D_n$  do not exceed the experimental limits and nevertheless  $D_\nu$  is sufficiently large for the above helicity flip effect to take place.

Although we have discussed the EDM for the electron and neutron we have made special emphasis on the EDM of the neutrino. The reason is threefold. First, it has not been discussed before (to our knowledge) in the context of superstring phenomenology. Second, in contrast to what happens for the electron and neutron, the proposed mechanism for the neutrino is distinctive of superstring inspired models. Indeed, whereas an EDM close to the experimental bounds can be easily obtained for the electron and neutron in SUGRA models [14] this is not true for the neutrino where the EDM turns out to be proportional to the neutrino mass (the helicity flip is unavoidably associated to the external legs). Thus, only for the neutrino a substantial EDM is a unique property of superstring inspired models. Finally, an EDM for the neutrino of  $O(10^{-22}-10^{-21}e \text{ cm})$  provides a supplementary source of helicity flip (in addition to the one due to a neutrino magnetic moment) necessary for solving the solar neutrino puzzle.

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