VERY SMALL AND RESOLUTION-INSENSITIVE $O(\alpha_s^2)$ CORRECTIONS TO A POLAR ASYMMETRY MEASURE OF ORIENTED THREE-JET EVENTS IN e^+e^- ANNIHILATION

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We calculate the $O(\alpha_s^2)$ polar angle distribution of the normal to the three-jet plane in e^+e^- annihilation into three jets. To $O(\alpha_s)$ the polar asymmetry parameter α, which determines the polar angle distribution, is a constant, $\alpha=-1/3$, over the whole phase space region. We find the $O(\alpha_s^2)$ corrections to $\alpha=-1/3$ to be very small, $O(10^{-3})$, and resolution insensitive. The $O(\alpha_s^2)$ corrections are also very small for an abelian version of QCD. We generated $\sim 68\,000$ two– and $O(\alpha_s)$ three-jet events by Monte Carlo at $\sqrt{q^2}=34$ GeV. We show that the polar asymmetry of the three-jet subsample can be determined quite accurately even when fragmentation effects are included. This demonstrates that present e^+e^- statistics should be sufficient to determine the polar asymmetry parameter experimentally. For scalar gluons the polar asymmetry deviates substantially from $\alpha=-1/3$ even at lowest order when measured on the Z_0 peak.

The measurement of the orientation of two-jet events in e^+e^- annihilation relative to the beam axis has been of crucial importance in establishing the spin-1/2 nature of quark-partons [1]. The corresponding measurement of the space orientation of three-jet events and a comparison with the QCD predictions is also very important for obvious reasons but has not been attempted so far due to the lack of statistics. Such measurements should be possible in the near future with higher statistics coming from the final analysis of the PETRA data, from PEP, TRISTAN and in particular from SLC and LEP running on the Z_0.

We point out in this letter that a determination of the polar asymmetry of the normal to the three-jet plane relative to the beam axis constitutes an important and feasible first measurement of the orientation of three-jet events. To lowest order $O(\alpha_s)$ in QCD the corresponding asymmetry parameter is a constant, $\alpha=-1/3$, over the whole region of phase space. In the following we will show that the $O(\alpha_s^2)$ corrections to α are very small and also resolution insensitive.

1. Generalities and notation. Consider the differential energy and polar angle distribution for the production of three massless partons in e^+e^- annihilation,

$$\frac{d\sigma}{dx_1\,dx_2\,d\cos\bar{\theta}}=\frac{1}{64\pi^2}\frac{4\pi\alpha^2}{3q^2}g_{11}(Q_f)[\tfrac{3}{8}(1+\cos^2\bar{\theta})H_{\bar{U}}+\tfrac{3}{4}\sin^2\bar{\theta}\,H_{\bar{L}}]\,. \tag{1}$$

The $x_i = 2E_i/\sqrt{q^2}$ are the scaled energies of the three partons ($i = 1, 2, 3$, for quark, antiquark and gluon, respectively). Eq. (1) gives the most general form of the polar angle distribution of a three-parton event. In the following we will be mainly concerned with the so-called transversity frame in which the event's z-axis is determined by the normal to the three-parton plane. Transversity frame quantities will be denoted by a bar in the following. Thus $\bar{\theta}$ is the polar angle of the normal to the three-parton plane with respect to the beam axis. The $H_{\bar{U}}$ and $H_{\bar{L}}$ are the transverse and longitudinal cross sections in the transversity frame. The $g_{11}(Q_f)$ specify the electroweak model dependence. For the standard model one has

$Q_f = \frac{2}{3}$, u, c, t:

$$g_{11} = \tfrac{4}{9} - \tfrac{4}{3}(-1 + 4\sin^2\theta_W)(1 - \tfrac{8}{3}\sin^2\theta_W)\mathrm{Re}\,\chi_Z$$
$$+ [(-1 + 4\sin^2\theta_W)^2 + 1][(1 - \tfrac{8}{3}\sin^2\theta_W)^2 + 1]\,|\chi_Z|^2 , \tag{2}$$

$Q_f = -\frac{1}{3}$, d, s, b:

$$g_{11} = \tfrac{1}{9} + \tfrac{2}{3}(-1 + 4\sin^2\theta_W)(-1 + \tfrac{4}{3}\sin^2\theta_W)\mathrm{Re}\,\chi_Z$$
$$+ [(-1 + 4\sin^2\theta_W)^2 + 1][(-1 + \tfrac{4}{3}\sin^2\theta_W)^2 + 1]\,|\chi_Z|^2 , \tag{3}$$

where

$$\chi_Z = gM_Z^2 q^2/(q^2 - M_Z^2 + \mathrm{i}M_Z\Gamma_Z) , \tag{4}$$

$$g = G_F/8\sqrt{2}\,\pi\alpha \approx 4.49 \times 10^{-5}\ \mathrm{GeV}^{-2} . \tag{5}$$

The longitudinal cross section $H_{\bar{L}}$ can be conveniently projected from the general hadron tensor $\mathscr{H}_{\mu\nu}$ by noting that

$$H_{\bar{L}} = \epsilon^{\mu}(0)\mathscr{H}_{\mu\nu}\epsilon^{*\nu}(0) , \tag{6}$$

with $[s_{ij} = (p_i + p_j)^2 = 2p_i p_j = y_{ij}\,q^2 = (1 - x_k)q^2]$

$$\epsilon_\mu(0) = \frac{2}{q^6\sqrt{X}}\,\epsilon_{\mu\alpha\beta\gamma}p_1^\alpha p_2^\beta p_3^\gamma . \tag{7}$$

Using standard ϵ-tensor identities one has

$$\epsilon_\mu(0)\,\epsilon_\nu^*(0) = -\hat{g}_{\mu\nu} + \frac{1}{Xq^2}(Z_1\hat{p}_{1\mu}\hat{p}_{1\nu} + Z_2\hat{p}_{2\mu}\hat{p}_{2\nu} + Z_3\hat{p}_{3\mu}\hat{p}_{3\nu}) , \tag{8}$$

where we have introduced the notation $(i, j, k = 1, 2, 3)$

$$X = (1 - x_1)(1 - x_2)(1 - x_3), \qquad Z_i = (1 - x_j)(1 - x_k) - (1 - x_i) ,$$
$$\hat{g}_{\mu\nu} = g_{\mu\nu} - q_\mu q_\nu/q^2, \qquad \hat{p}_{i\mu} = p_{i\mu} - q_\mu p_i q/q^2 . \tag{9}$$

Then, by expanding the hadron tensor along a standard set of gauge invariant covariants,

$$\mathscr{H}_{\mu\nu} = H_1\hat{g}_{\mu\nu} + H_2\hat{p}_{1\mu}\hat{p}_{1\nu}/q^2 + H_3\hat{p}_{2\mu}\hat{p}_{2\nu}/q^2 + H_4(\hat{p}_{1\mu}\hat{p}_{2\nu} + \hat{p}_{2\mu}\hat{p}_{1\nu})/q^2 , \tag{10}$$

we obtain $(H_{\bar{U}} + H_{\bar{L}} = -g_{\mu\nu}\mathscr{H}^{\mu\nu})$

$$H_{\bar{U}} - H_{\bar{L}} = -H_1 + \tfrac{1}{4}X_1^2 H_2 + \tfrac{1}{4}X_2^2 H_3 + \tfrac{1}{2}Z_3 H_4, \qquad H_{\bar{U}} + H_{\bar{L}} = -3H_1 + \tfrac{1}{4}X_1^2 H_2 + \tfrac{1}{4}X_2^2 H_3 + \tfrac{1}{2}Z_3 H_4. \tag{11}$$

For the sake of completeness we also list the relation of the transversity frame quantity $H_{\bar{U}} - H_{\bar{L}}$ to the three (helicity frame) longitudinal cross sections H_{L1}, H_{L2} and H_{L3} along quark, antiquark and gluon momenta [2,3] [‡1], which can be conveniently employed in the evaluation of higher-order QCD corrections. One has

$$H_{\bar{U}} - H_{\bar{L}} = -(H_U + H_L) - \frac{1}{2X}(x_1^2 Z_1 H_{L1} + x_2^2 Z_2 H_{L2} + x_3^2 Z_3 H_{L3}) , \tag{12}$$

where the longitudinal cross sections are given by

$$H_{Li} = \mathscr{H}_{\mu\nu} p_i^\mu p_i^\nu / E_i^2 , \tag{13}$$

and $H_U + H_L \equiv H_{\bar{U}} + H_{\bar{L}}$ is the total cross section.

The order $O(\alpha_s)$ Born term cross sections are given by (see, e.g., ref. [4])

$$H_1 = -32 N_c C_F \pi^2 \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}, \qquad H_2 = H_3 = -32 N_c C_F \pi^2 \frac{\alpha_s}{2\pi} \frac{4}{(1-x_1)(1-x_2)}, \qquad H_4 = 0 , \tag{14}$$

leading to ($N_c C_F = 4$)

$$H_{\bar{L}} = H_{\bar{U}} = 32\pi^2 N_c C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} . \tag{15}$$

The fact that $H_{\bar{L}} = H_{\bar{U}}$ to $O(\alpha_s)$ suggests that the polar angle distribution in eq. (1) can be rewritten as

$$d\sigma / d\cos\bar{\theta} \sim (1 - \tfrac{1}{3}\cos^2\bar{\theta})(H_{\bar{U}} + H_{\bar{L}}) - (\tfrac{1}{3} - \cos^2\bar{\theta})(H_{\bar{U}} - H_{\bar{L}}) . \tag{16}$$

From (16) one immediately reads off the $O(\alpha_s)$ value for the polar asymmetry, $\alpha = -1/3$, where

$$\alpha = (H_{\bar{U}} - 2H_{\bar{L}}) / (H_{\bar{U}} + 2H_{\bar{L}}) , \tag{17}$$

such that

$$d\sigma / d\cos\bar{\theta} \sim 1 + \alpha \cos^2\bar{\theta} . \tag{18}$$

In the following we shall calculate the $O(\alpha_s^2)$ corrections to $H_{\bar{U}} - H_{\bar{L}}$ and thereby to $\alpha = -1/3$. In order to make the α_s dependence explicit we define

$$H_{\bar{U}}^{(n)} \mp H_{\bar{L}}^{(n)} = \alpha_s^n \begin{Bmatrix} M^{(n)} \\ P^{(n)} \end{Bmatrix} . \tag{19}$$

Using $M^{(1)} = 0$ one has

$$\alpha = -\tfrac{1}{3} + \tfrac{8}{9}\alpha_s \frac{M^{(2)}}{P^{(1)}} \left[1 + \alpha_s \left(\frac{M^{(3)}}{M^{(2)}} - \frac{P^{(2)}}{P^{(1)}} \right) + \dots \right] . \tag{20}$$

Thus our task is to calculate the first nonvanishing contribution to $H_{\bar{U}} - H_{\bar{L}}$ which occurs at $O(\alpha_s^2)$. The calculation involves the evaluation of the $O(\alpha_s^2)$ loop and tree graph contributions.

Starting with the loop contributions one notes that the cross section difference $H_{\bar{U}} - H_{\bar{L}}$ is both ultraviolet (UV) and infrared/mass (IR/M) finite at $O(\alpha_s^2)$ since the UV and IR/M singularities must have the Born term structure and thus do not contribute to the cross section difference $H_{\bar{U}} - H_{\bar{L}}$. This means that $H_{\bar{U}} - H_{\bar{L}}$ need not be renormalized at $O(\alpha_s^2)$ and thus is renormalization scheme independent. A renormalization scheme dependence would only show up at the $O(\alpha_s^3)$ level. The above UV and IR/M structure was borne out in the explicit $O(\alpha_s^2)$ loop evaluation done in ref. [5].

The $O(\alpha_s^2)$ tree graph contributions to $H_{\bar{U}} - H_{\bar{L}}$ are also IR/M finite for the same reasons as described above.

[‡1] Note that refs. [2,3] contain a number of printing errors.

Up to now they have been evaluated only to $O(y^0)$ accuracy, i.e., the result is known for the leading and next-to-leading terms in the invariant mass cut-off parameter y $[(p_i+p_j)^2 \leqslant yq^2]$ [3,6]. Again the $O(y^0)$ contributions have the Born term structure and thus do not contribute to $H_{\bar{U}}-H_{\bar{L}}$. In particular this means that the cross section difference is a resolution insensitive $O(\alpha_s^2)$ measure of the orientation of three-jet events. Whether the higher y-contributions factor the Born term is not known at present.

Thus the $O(\alpha_s^2)$ contributions to $H_{\bar{U}}-H_{\bar{L}}$ are solely determined by the one-loop contributions at $O(y^0)$ accuracy.

2. *QCD results.* The results of the $O(\alpha_s^2)$ one-loop evaluation were given in terms of the invariants H_i in ref. [5]. Using eq. (11) one easily converts them to the desired $H_{\bar{U}}-H_{\bar{L}}$. One finds ($2p_ip_j=y_{ij}q^2$, $N_c=3$, $C_F=4/3$)

$$H_{\bar{U}}-H_{\bar{L}}=64\pi^2\left(\frac{\alpha_s}{2\pi}\right)^2 N_c C_F(1-X_3)\Bigg\{\tfrac{1}{2}N_c\left[\frac{x_1+x_2}{x_1x_2}+\frac{1}{x_1^2}\ln y_{23}+\frac{1}{x_2^2}\ln y_{13}\right]$$
$$+(C_F-\tfrac{1}{2}N_c)\Bigg[-\frac{(x_1-x_2)^2-2(x_1+x_2)}{x_1x_2x_3}+2\frac{2(1-x_1)(1-x_2)+x_3^2}{x_3^2(1-x_1)(1-x_2)}\ln y_{12}$$
$$+\frac{1-x_1+2x_2}{x_2^2(1-x_1)}\ln y_{13}+\frac{1-x_2+2x_1}{x_1^2(1-x_2)}\ln y_{23}+2\frac{r(y_{12},y_{13})}{(1-x_1)^2}+2\frac{r(y_{12},y_{23})}{(1-x_2)^2}\Bigg]\Bigg\}, \tag{21}$$

where $r(x,y)=\ln x\ln y-\ln x\ln(1-x)-\ln y\ln(1-y)-\mathscr{L}_2(x)-\mathscr{L}_2(y)+\frac{1}{6}\pi^2$, and where $\mathscr{L}_2(x)$ is the Spence function defined by

$$\mathscr{L}_2(x)=-\int_0^x dz\,\frac{\ln(1-z)}{z}. \tag{22}$$

In the terminology of refs. [2,5] the first and second square brackets contain the "QCD"- and "QED"-type contributions. The $O(\alpha_s^2)$ loop contributions to $H_{\bar{U}}-H_{\bar{L}}$ in (21) are remarkably simple compared to the $O(\alpha_s^2)$ total cros section result $H_{\bar{U}}+H_{\bar{L}}=H_U+H_L$ given, e.g., in refs. [2,5]. Note also that one encounters an overall factor $(1-x_3)$ in (21).

In order to be able to present our results in a once differential form we have calculated from (21) the differential thrust distribution, which we present in fig. 1. We have plotted the $O(\alpha_s^2)$ thrust distribution of $H_{\bar{U}}-H_{\bar{L}}$ together with the $O(\alpha_s)$ thrust distribution of the total cross section H_U+H_L. $H_{\bar{U}}-H_{\bar{L}}$ is very small, $O(10^{-3})$, compared to $H_U+H_L=H_{\bar{U}}+H_{\bar{L}}$ over the whole thrust range. We have checked that $H_{\bar{U}}-H_{\bar{L}}$ is very small over the entire (x_1, x_2) phase space region. We conclude that the $O(\alpha_s^2)$ corrections to the asymmetry parameter α are negligibly small and certainly not measurable for all practical purposes.

3. *"Abelian QCD" results.* In order to pinpoint the nonabelian nature of QCD a toy model was investigated in ref. [7] in which abelian gluons couple diagonally to quarks in the three-dimensional colour space. The strong coupling constant of this "abelian QCD" theory was then adjusted to give the same strength of the $O(\alpha_s)$ three-jet cross section. In such a theory the QCD-type contribution in eq. (21) due to the three-gluon coupling effects are switched off and one remains only with the QED-type contribution given by the second square bracket in eq. (21).

In such an "abelian QCD" the cross section difference $H_{\bar{U}}-H_{\bar{L}}$ is given by

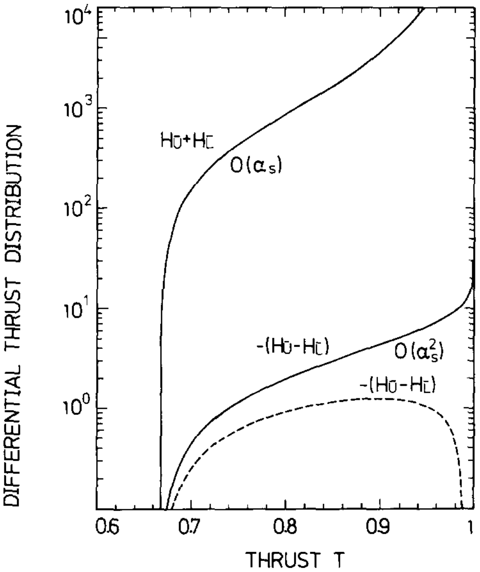

Fig. 1. $O(\alpha_s^2)$ thrust distributions of $H_{\bar{U}}-H_{\bar{L}}$ for QCD (full line) and "abelian QCD" (dashed line). Also shown is the $O(\alpha_s)$ thrust distribution for the total cross section $H_{\bar{U}}+H_{\bar{L}}=H_U+H_L$. We use $\alpha_s=0.16$.

$$H_{\bar{U}}-H_{\bar{L}}=192\pi^2\left(\frac{4}{3}\frac{\alpha_s}{2\pi}\right)^2(1-x_3)$$

$$\times\left\{-\frac{(x_1-x_2)^2-2(x_1+x_2)}{x_1x_2x_3}+2\frac{2(1-x_1)(1-x_2)+x_3^2}{x_3^2(1-x_1)(1-x_2)}\ln y_{12}+\frac{1-x_1+2x_2}{x_2^2(1-x_1)}\ln y_{13}\right.$$

$$\left.+\frac{1-x_2+2x_1}{x_1^2(1-x_2)}\ln y_{23}+2\frac{r(y_{12},y_{13})}{(1-x_1)^2}+2\frac{r(y_{12},y_{23})}{(1-x_2)^2}\right\}, \tag{23}$$

where $\alpha_s(\text{abelian})=\frac{4}{3}\alpha_s(\text{nonabelian})$ is the "abelian QCD" coupling constant adjusted to the three-jet cross section as described above.

The thrust distribution of $H_{\bar{U}}-H_{\bar{L}}$ corresponding to eq. (23) is plotted in fig. 1. The $O(\alpha_s^2)$ contribution can be seen to be even smaller than in the nonabelian case. Since both contributions are unmeasurably small one concludes that a polar asymmetry type measurement is not suited to pinpoint the nonabelian nature of QCD with its three-gluon coupling.

4. *Scalar gluon theory results.* In order to pinpoint the vector nature of the massless gauge boson in QCD the results of this theory have often been compared to the results of a corresponding theory with coloured scalar gluons. Although such a theory is not asymptotically free it is nevertheless renormalizable as is the "abelian QCD" theory.

The Born term Feynman diagrams in such a scalar gluon theory are given by (see, e.g., ref. [4])

$$T^V_\mu=\bar{g}_s\tfrac{1}{2}\lambda_i\bar{u}(p_1)(-\not{p}_3\gamma_\mu/s_{13}+\gamma_\mu\not{p}_3/s_{23})v(p_2),\qquad T^A_\mu=\bar{g}_s\,\tfrac{1}{2}\lambda_i\bar{u}(p_1)(-\not{p}_3\gamma_\mu\gamma_5/s_{13}+\gamma_\mu\gamma_5\not{p}_3/s_{23})v(p_2), \tag{24}$$

for the vector current and axial vector current contributions. $\bar{g}_s$ is the coupling constant of the scalar gluon theory. Note that the axial vector current contribution T^A_μ in eq. (24) is not conserved, $q^\mu T^A_\mu\neq 0$, due to the fact that the quarks that couple to the axial vector current vertex are not on-shell.

One finds again that $H_{\bar{U}}-H_{\bar{L}}=0$ for the VV part of the hadron tensor, leading to a value $\alpha=-1/3$ for the polar asymmetry parameter below the Z_0 peak, where the VV contribution dominates. However, for the AA

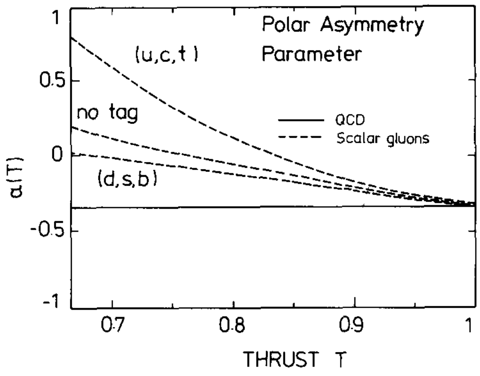

Fig. 2. Asymmetry parameter α as function of thrust. The $O(\alpha_s^2)$ correction to the $O(\alpha_s)$ result $\alpha=-1/3$ is too small to be visible. Also shown are lowest-order results for scalar gluon theory with massless quarks for the three cases (i) u,c,t, (ii) d.s.b and (iii) u+d, c+s, t+b (no tag).

contribution one finds $H_{\bar{U}}-H_L\neq 0$ and thereby $\alpha\neq -1/3$ already at the Born term level. Since the AA contribution is significant on the Z_0 peak this points to an interesting test of the spin property of the gluon.

Calculating the thrust distributions for $H_{\bar{U}}-H_L$ and $H_{\bar{U}}+H_L$ from the amplitudes (24) one finds

$$(H_{\bar{U}}-H_L)_{VV}=0, \qquad (H_{\bar{U}}-H_L)_{AA}=32\bar{g}_s^2(3T-2),$$

$$(H_{\bar{V}}+H_L)_{VV}=32\bar{g}_s^2\left(\frac{4-3T}{2(1-T)}(3T-2)-\ln(1-T)+(1-T)\ln(\tfrac{1}{2}T)+T\ln(2T-1)\right),$$

$$(H_{\bar{U}}+H_L)_{AA}=32\bar{g}_s^2\left(\frac{-6+7T}{2(1-T)}(3T-2)-\ln(1-T)+(1-T)\ln(T/2)+T\ln(2T-1)\right). \qquad (25)$$

In fig. 2 we have plotted the polar asymmetry parameter as a function of thrust for the three cases (i) u,c,t quarks, (ii) d,s,b quarks and (iii) the no tagging case (u+d), (s+c), (b+t). We have used the standard WS model values with $\sin^2\theta_W=1/4$ for the flavour couplings. One notes that the scalar gluon polar asymmetry values deviate substantially from $\alpha=-1/3$ for thrust values $T<0.9$, where the deviation from $\alpha=-1/3$ is biggest for tagged (u,c,t) quarks.

5. *Monte Carlo results.* In order to investigate whether the polar asymmetry parameter can be measured from present data we generated 67 794 hadronic events at $\sqrt{q^2}=34$ GeV using the $O(\alpha_s)$ Hoyer Monte Carlo [8]. This is the approximate number of hadron events accumulated at each of the PETRA detectors during PETRA's lifetime. Our three-jet criteria resulted in 11 509 three-jet events with thrust values below $T=0.9$ in the hadron sample. Using this three-jet subsample we measured the polar asymmetry parameter α using two different methods: (i) without fragmentation of the three partons, (ii) with fragmentation of the three originally produced partons and subsequent use of a standard jet algorithm to reconstruct the original three-parton plane. We then fitted the polar angle distribution of the normal to the hadron plane using the $1+\alpha\cos^2\bar{\theta}$ distribution of eq. (18).

The results of our fit are presented in table 1. The quality of the fit is generally quite good. In particular one finds that there is no degradation in the quality of the fit when fragmentation effects are included. The error on the accumulated fit $T\leqslant 0.9$ is quite small for both runs with and without fragmentation. We conclude that a data sample of $\sim 60\,000$ hadronic events should allow for an excellent measurement of the asymmetry parameter α.

In fig. 3 we show our fit to the accumulated three-jet sample with thrust cut-off $T\leqslant 0.9$, leading to $\alpha=-0.31\pm 0.02$ for both the fragmentation and no fragmentation cases. By comparing the two fits one notes that the misidentification of the original three-parton plane in the fragmentation run due to the inherent inef-

Table 1
Values of asymmetry parameter α from Monte Carlo runs with and without fragmentation. Number of events in the different thrust bins are determined from fragmentation run but are used for both runs.

Thrust bins (number of events)	α with fragmentation	α without fragmentation
$2/3<T\leqslant 0.75$ (1147)	-0.38 ± 0.07	-0.36 ± 0.08
$0.75<T\leqslant 0.80$ (1735)	-0.36 ± 0.06	-0.39 ± 0.06
$0.80<T\leqslant 0.85$ (3621)	-0.28 ± 0.04	-0.29 ± 0.04
$0.85<T\leqslant 0.90$ (5006)	-0.29 ± 0.04	-0.28 ± 0.04
$T\leqslant 0.90$ (11 509)	-0.31 ± 0.02	-0.31 ± 0.02

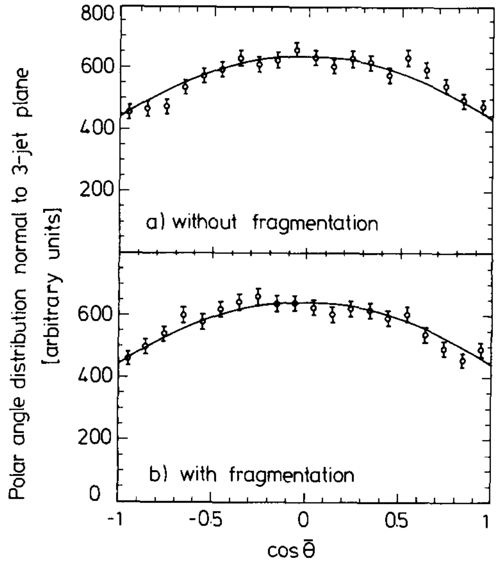

Fig. 3. Results of fit to angular distribution of the normal to the three-parton plane using $O(\alpha_s)$ Hoyer et al. MC data (11 509 three-jet events with $T\leqslant 0.9$). (a) without fragmentation, (b) with fragmentation. Both fits give $\alpha=0.31\pm0.02$.

ficiency of the jet algorithm is of a rather local nature which does not affect the global quality of the fit.

We have repeated the Monte Carlo run at the higher energy of $\sqrt{q^2}=93$ GeV corresponding to the Z_0 mass. The quality of the fragmentation fit showed no improvement relative to the 34 GeV run, which indicates again that fragmentation effects are not important in the Monte Carlo measurement of the polar asymmetry parameter at the lower energy of $\sqrt{q^2}=34$ GeV.

We finally did a high-statistics MC run for $O(\alpha_s^2)$ four-jet events with $0.001\leqslant y\leqslant 0.04$ in order to check the quality of the $O(y^0)$ approximation $(H_U-H_L)_{O(\alpha_s^2)}=0$ for the tree graph contributions. The parton plane was defined by the pair of partons with the large invariant mass. We found $(H_U-H_L)/(H_U+H_L)$ to be very small for these $O(\alpha_s^2)$ four-jet events and conclude that the $O(y^0)$ approximation used in this paper to calculate the $O(\alpha_s^2)$ contributions to H_U-H_L is reliable.

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References

[1] G. Hanson et al., Phys. Rev. Lett. 35 (1975) 1609.
[2] G. Kramer and B. Lampe, Phys. Scr. 28 (1983) 585.
[3] G. Kramer and B. Lampe, Commun. Math. Phys. 97 (1985) 257.
[4] J.G. Körner and D.H. Schiller, DESY preprint DESY 81-043 (1981).
[5] J.G. Körner and G.A. Schuler, Z. Phys. C 26 (1985) 559.
[6] G.A. Schuler and J.G. Körner, to be published.
[7] J.G. Körner, J. Willrodt and G. Schierholz, Nucl. Phys. B 185 (1981) 365.
[8] P. Hoyer, P. Osland, H.G. Sander, T.F. Walsh and P.M. Zerwas, Nucl. Phys. B 161 (1979) 349.