A CRITIQUE OF SHABALIN'S SELF PENGUIN APPROACH TO THE $\Delta I = 1/2$ RULE

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We discuss a recent suggestion of Shabalin for explaining the $\Delta I = 1/2$ rule by means of an enhanced off-shell $\bar{s}d$ self-energy contribution, arising from self Penguin graphs. We point out that this contribution, in a complete short-distance treatment, is actually part of an operator which vanishes by the QCD equations of motion, and as such cannot give rise to any physical effects. Furthermore, if these Penguin-like contributions were to account for the $\Delta I = 1/2$ enhancement, necessarily the *CP* violating ratio ϵ'/ϵ is large and most probably already exceeds the present experimental bounds.

In a recent paper Shabalin [1] has suggested a novel explanation for the $\Delta I = 1/2$ enhancement in nonleptonic decays which, at first sight, looks very intriguing. According to Shabalin [1] this enhancement can be traced to the $\bar{d}s$ self-energy contribution, which is in fact much larger than previously estimated if gluonic effects are included. Although Shabalin's observation about the $\bar{d}s$ self-energy is correct, this by itself does not suffice to explain the $\Delta I = 1/2$ enhancement. Indeed, the main purpose of this note is to show that, when a proper short-distance expansion of the weak hamiltonian is performed, Shabalin's enhanced self-energy term does not contribute to the $K \to 2\pi$ amplitude at all! Furthermore, if it were to be really the dominant contribution, then very likely the *CP* violating ratio ϵ'/ϵ would already exceed its present experimental bound.

Since the $\bar{d}s$ self-energy is a purely $\Delta I = 1/2$ contribution, the suggestion that it has something to do with the $\Delta I = 1/2$ rule is very natural and, indeed, this suggestion has a long history [2–4]. Unfortunately, when one tries to translate this qualitative idea into practice one finds that, after properly renormalizing the self-energy, the GIM mechanism [5] makes the magnitude of Σ_{ds} uninterestingly small. Claims in the literature of a large self-energy contribution arose either out of incomplete calculations, or from not having renormalized the self-energy properly.

An appropriate normalization condition to impose on Σ_{ds} is that it should vanish on the d and s quark mass shell [6,7], so that the physical self-energy takes the form:

$$\Sigma_{ds} = (\not{p} - m_d)\tilde{\Sigma}_{ds}(\not{p} - m_s) \ . \tag{1}$$

The above fixes the counterterms to be added to the original lagrangian to remove the divergent pieces in the calculation of the self-energy. Using this subtraction procedure Chia, in a recent paper [8], finds, in a four-quark model, the following approximate expression for $\tilde{\Sigma}_{ds}$:

$$\tilde{\Sigma}_{ds}(p) \simeq \frac{G_F \sin\theta_c \cos\theta_c}{4\sqrt{2}\,\pi^2} \frac{m_c^2 - m_u^2}{M_W^2} \times [(\not{p} + m_d)R + m_s L] \ , \tag{2}$$

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Fig. 1. Self Penguin diagrams.

where R and L are the usual chiral projectors $\frac{1}{2}(1+\gamma_5)$ and $\frac{1}{2}(1-\gamma_5)$. Even if one contemplates that in the $K\to 2\pi$ matrix element of Σ_{ds} the quarks are way off-shell, the GIM factor of $\Delta m^2/M_W^2$ suffices to make the self-energy contribution numerically unimportant [8].

Shabalin's observation [1] is that the inclusion of gluon corrections changes the estimate of $\tilde{\Sigma}_{ds}$ radically. As already happened in the case of the Penguin operators [9], the effect of including the gluons changes the GIM cancellation from power-like to logarithmic. What Shabalin calculates specifically are the graphs of fig. 1, which we shall call self Penguin diagrams since the gluons end up on either the d or s quark lines. He finds that these diagrams lead to a contribution for $\tilde{\Sigma}_{ds}$ of order

$$\tilde{\Sigma}_{ds} \sim (G_F/\pi^3) \sin\theta_c \cos\theta_c \alpha_s \ln(m_c^2/\mu^2) \not{p} R. \qquad (3)$$

In the above μ is a low-energy cutoff which typifies up to where one can trust the calculation. It replaces the more naive u quark mass. We have also retained in the above only the $\not{p}$ term, since only this term is important for the off-shell behaviour of Σ_{ds}. Clearly the expression in (3) is very much larger than that in (2), especially if one takes $\alpha_s=\alpha_s(\mu)$ and assumes, as Shabalin does [1], that one can trust the calculation down to μ's such that $\alpha_s(\mu)=1$!

Having obtained a potentially large self-energy, via the self Penguin enhancement, Shabalin [1] then tries to evaluate the $K\to 2\pi$ amplitude by computing the K to vacuum tadpole involving Σ_{ds} [10]. For this latter calculation he makes use of a hybrid quark–chiral model [11], which allows him to estimate the off-shell contributions of the d and s quarks in the kaon. Shabalin claims, as a result of this calculation, that the Σ_{ds} tadpole contributes 70% to the value of the $\Delta I=1/2$ amplitude. He argues, furthermore, that nontadpole contributions arising, also from Σ_{ds}, can account for the rest.

It is difficult to judge the reliability of these claims. However, there is a simple way to estimate how off-shell the self-energy contribution must be to give Shabalin's result. For this one just replaces all $\not{p}$ terms in Σ_{ds} by an effective mass parameter M. Then one is left with the matrix element of $\bar{d}s$ from K to vacuum, which is known. Using Shabalin's calculation of Σ_{ds} one can then estimate which M is needed to reproduce the $\Delta I=1/2$ enhancement, and this will give one an idea of how off-shell the self-energy must be. We have done this exercise and find that one needs an M of order 1 GeV, which is rather large. Furthermore, since $\Sigma_{ds}\sim G_F M^3$, it is clear that the final result is quite sensitive to the effective off-shellness. Thus, on this ground alone, Shabalin's claim is far from convincing. Not only most Σ_{ds} be enhanced but also the quarks in the kaon must be quite off-shell.

If Shabalin's claim were to be correct – something which we shall argue below is not the case – then there would be an immediate consequence for the CP violating parameter ϵ'/ϵ. If the matrix element of Σ_{ds} dominates the $K\to 2\pi$ amplitude, then necessarily the ratio ϵ'/ϵ has a large value. This observation is analogous to the one made sometime ago by Gilman and Wise [12] regarding the Shifman–Vainshtein–Zakharov [9] explanation of the $\Delta I=1/2$ rule by means of Penguin graphs. For graphs involving the Penguin operator, the ratio of imaginary to real parts is rather well determined. If these graphs are the dominant contributions (as Shabalin [1] argues for the self Penguins), then so is the ratio of the imaginary to real parts in the $K\to 2\pi$ amplitude, which is a measure of the CP violation.

The parameter ϵ' is essentially given by the ratio of the imaginary to real part of the amplitude A_0 for the transition $K^0\to 2\pi$, in which the two pions are in the $I=0$ state. More precisely, one has [12]

$$|\epsilon'| = \frac{1}{\sqrt{2}} \frac{\mathrm{Im}\, A_0}{\mathrm{Re}\, A_0} \left|\frac{A_2}{A_0}\right| \simeq 3.2\times 10^{-2} \frac{\mathrm{Im}\, A_0}{\mathrm{Re}\, A_0}, \qquad (4)$$

where the numerical value given in eq. (4) uses the experimental value for $|A_2/A_0|$, which is small as a result of the $\Delta I=1/2$ enhancement. If the dominant contribution to A_0 comes from the off-shell matrix element of Σ_{ds}, then $|\epsilon'|$ can be estimated readily, by trivially extending Shabalin's calculation to the six-quark case. The result is given, approximately, by

$$|\epsilon'| \simeq 3.2\times 10^{-2} \frac{\mathrm{Im}\, V_{dt} V_{ts}^*}{V_{du} V_{us}^*} \frac{\langle \Phi^{ct} \rangle}{\langle \Phi^{uc} \rangle}. \qquad (5)$$

Here $V_{qq'}$ is the qq′ element of the Kobayashi–Maskawa matrix and $\langle \Phi^{ct} \rangle$ and $\langle \Phi^{uc} \rangle$ are the average values of the functions [1]

$$\Phi^{ij}(p) = \int_0^1 dx \int_{m_i^2}^{m_j^2} \frac{dm^2}{p^2} \int_0^1 dy \ln\left(1 - \frac{p^2 yx(1-x)}{m^2}\right) , \quad (6)$$

which enter in the self Penguin diagrams, taken over the momentum range of interest in the $K \to 2\pi$ matrix element. (For Φ^{uc}, again, m_u should be replaced by some effective cutoff μ.) The precise value of the ratio $\langle \Phi^{ct} \rangle / \langle \Phi^{uc} \rangle$ depends on the value of m_t and of the cutoff μ, but not too much on the momentum range over which the averaging is performed, since the functions are quite p-independent. Using values that we consider reasonable ($m_t = 50$ GeV, $\mu = 0.14$ GeV) this ratio is of order 1.5, so that

$$|\epsilon'| \simeq 5 \times 10^{-2} \frac{\mathrm{Im}\, V_{dt} V_{ts}^*}{V_{du} V_{us}^*} . \quad (7)$$

One may use the measured value for the CP parameter ϵ to estimate the Kobayashi–Maskawa matrix elements, or directly obtain an expression for $|\epsilon'/\epsilon|$. Using the results given by Buras [13] for $m_t = 50$ GeV, this procedure yields

$$|\epsilon'/\epsilon| \simeq 1.5 \times 10^{-2} B^{-1} , \quad (8)$$

where $B = 1$ corresponds to the usual vacuum insertion approximation. This value is quite a bit larger than the most recent results of the Chicago–Saclay [14] and Yale–BNL [15] collaborations. Given the uncertainty in the parameters that go into the ratio of $\langle \Phi^{ct} \rangle$ to $\langle \Phi^{uc} \rangle$ and in the calculation of ϵ, this result is not fatal but argues strongly against Shabalin's explanation.

There is, however, a more direct way to arrive at this conclusion. In calculating the weak transition of a kaon into two pions one must calculate the matrix element of the effective hamiltonian

$$H_{eff} = \frac{e^2}{8 \sin^2 \theta_W} \int d^4x \, D_F^{\mu\nu}(x; M_W^2) \times T(J_\mu^+(x) J_\nu^-(0)) + \text{counterterms} \quad (9)$$

between these hadronic states. Here $D_F^{\mu\nu}$ is the W-boson propagator. The short-distance aspects of the problem can be taken care of by making use of the operator-product expansion for the product of the two currents, retaining only operators of dimension less than or equal to six [16]. In this way, H_{eff} is expressed as a sum of composite operators $O_i(\mu)$, multiplied by certain coefficient functions. For the piece of H_{eff} responsible for $\Delta S = 1$ transitions, in a four-quark context, one has

$$H_{eff} = \frac{G_F}{2\sqrt{2}} \sin \theta_c \cos \theta_c \sum_i C_i(\mu) O_i(\mu) . \quad (10)$$

Here μ is the normalization scale which specifies the subtraction point for the composite operators $O_i(\mu)$. The μ dependence of the coefficient functions follows directly from the renormalization group equations and it depends on the anomalous dimensions of the operators O_i. The difficult part of the problem, of course, is the calculation of the matrix element of the $O_i(\mu)$ operators between the hadronic states, which requires knowledge beyond perturbation theory. Although the matrix elements of H_{eff} are μ-independent, the idea is to choose μ in a way to facilitate the evaluation of the matrix elements of $O_i(\mu)$. That is, μ should be close to the typical scale of momenta relevant for the hadronic transition.

In general there are three classes of operators which enter in eq. (10): operators which are gauge invariant and do not vanish by the QCD equations of motion, operators which are gauge invariant and vanish by the QCD equations of motion and gauge variant operators. On physical grounds, it is clear that only the first class of operators should contribute to physical matrix elements [9,16]. Gauge variant operators are clearly unphysical. Operators which vanish by the equation of motion should also give no contribution, provided they do not mix, under renormalization, with operators which are gauge invariant and do not vanish. The more technical condition is that the renormalization matrix Z connecting these two kinds of operators be triangular [17]. That is, if O and X are operators which vanish and not vanish because of the equations of motion, respectively, then what is required is that under renormalization

$$\begin{pmatrix} X \\ O \end{pmatrix}_{ren} = \begin{pmatrix} Z_{XX} & Z_{XO} \\ 0 & Z_{OO} \end{pmatrix} \begin{pmatrix} X \\ O \end{pmatrix} . \quad (11)$$

If this is the case, then for physical matrix elements

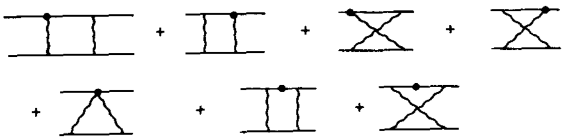

Fig. 2. Insertions of the Π^3 operator which show that this operator does not mix with the four-fermion operators.

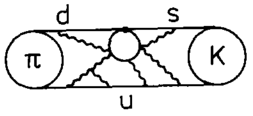

Fig. 3. Schematic "eye diagram" which is thought to be responsible for the $\Delta I = 1/2$ enhancement.

it suffices to just retain the X operator and calculate its anomalous dimension using Z_{XX} only.

The operator which is involved in Shabalin's [1] considerations is part of an operator which vanishes by the QCD equations of motions. The part of the self-energy contribution which could be important off-shell involved the structure, in momentum space, $\bar{d}\not{p}\not{p}\not{p}Ls$. This operator is part of the gauge invariant operator

$$\Pi^3 = \bar{d}\not{\Pi}\not{\Pi}\not{\Pi}Ls \,, \tag{12}$$

where, as usual $\Pi_\mu = p_\mu - gT^aA^a_\mu$. However, using the equation of motion for Π^3 acting on the quark field s, this operator is reduced to an operator with no momentum factors, since each $\not{\Pi}$ will be transmuted into a mass term. Thus it cannot give rise to important contributions off-shell! Of course, this statement is only true provided that one can show that Π^3 indeed does not mix with any four-quark operators of the Penguin operators. It is easy to show, by considering the graphs of fig. 2, that the insertion of the Π^3 operator in these box graphs never gives rise to a four-fermion operator. That is, although individually each of these graphs is logarithmically divergent, this divergence disappears in the sum of all the graphs. Hence $Z_{\Pi^3x} = 0$, where x stands for a four-fermion operator. We have not computed Z_{Π^3p}, with p being a quark–gluon Penguin operator, but we are sure that also this renormalization constant vanishes. Incidentally, $Z_{p\Pi^3}$ *does not* vanish. Indeed, it can be calculated precisely by the graphs of fig. 1 [18], the graphs which Shabalin calculated and which gave rise to his enhanced contribution.

The above demonstrates that the *short-distance* aspects of the $\bar{d}s$ self-energy are unimportant for the $K \to 2\pi$ transition amplitude. This does not mean that contributions which resemble self-energy contributions are not important for the $\Delta I = 1/2$ rule. It is likely that the $\Delta I = 1/2$ enhancement comes principally from terms in H_{eff} where the two u quarks in the four-fermion operators are contracted together and are festooned by myriads of soft gluons. However, these long-distance contributions (eye diagrams), shown schematically in fig. 3, are beyond a perturbative treatment. There have been various proposals in the literature [19–21] to try to account for various pieces of the long-distance corrections. In particular, the evaluation of the matrix elements of $O_i(\mu)$ is now being attempted by lattice techniques (for a recent careful discussion, see ref. [22]). A last point should be mentioned. Since in the diagrams of fig. 3 the loop involves only u quarks, effectively, the $\Delta I = 1/2$ enhancement of the real part of the $K \to 2\pi$ amplitude does not necessarily imply a similar enhancement for the imaginary part. That is, some of the operators entering in this loop have purely real coefficients and so cannot affect the imaginary part. Thus the enhancement we discussed earlier in connection with the Penguin operators and its implication for ϵ'/ϵ need not occur in real life.

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