CALCULATION OF THE CONDENSATE $g_s\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$ ON THE LATTICE

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We calculate the condensate $g_s\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$ using Kogut–Susskind quarks in lattice-regularized quenched QCD. We obtain $g_s\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle/\langle\bar{q}q\rangle\approx 1.1\ \mathrm{GeV}^2$ at $\mu=1$ GeV.

1. Introduction. In this letter we shall present the first results of a calculation of the condensate $g_s\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$ on the lattice. This condensate is one of the parameters which determine the low-energy properties of QCD in the framework of QCD sum rules [1]. In contrast to lattice calculations, QCD sum rules do not determine hadronic properties in terms of one fundamental mass scale only, but they relate hadronic masses, couplings, etc. to a number of phenomenological parameters, which are vacuum expectation values of local spin-zero operators (condensates). These condensates must be determined phenomenologically. For some condensates Shifman, Vainshtein and Zakharov also gave estimates from a dilute instanton gas calculation.

One of the least known condensates is

$$g_s\langle\bar{q}\sigma_{\mu\nu}\tfrac{1}{2}\lambda^a G^a_{\mu\nu}q\rangle\ , \tag{1}$$

where q is a light quark field and $G^a_{\mu\nu}$ is the gluon field strength. This condensate is important in two areas of QCD sum rules. It gives large contributions to sum rules for mesons composed of one light and one heavy quark, as it appears there in the combination

$$m_H\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle\ , \tag{2}$$

i.e. multiplied by the (large) heavy quark mass m_H [#1]. The other important area is in sum rules for baryonic currents [2]. The two-point function of two local interpolating baryon fields ψ has the form

$$\int d^4x\,\exp(iqx)\langle 0|T(\psi_\alpha(x)\bar{\psi}_\beta(0))|0\rangle$$

$$=\not{q}_{\alpha\beta}F_1(q^2)+\delta_{\alpha\beta}F_2(q^2)\ , \tag{3}$$

where $F_2(q^2)$ has the form

$$F_2(q^2)=A\langle\bar{q}q\rangle q^2\ln(-q^2/\mu^2)$$

$$+Bg_s\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle\ln(-q^2/\mu^2)\ldots\ . \tag{4}$$

Here $\langle\bar{q}q\rangle$ is the chiral symmetry breaking quark condensate. The knowledge of the condensate $g_s\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$, appearing in the second term, is particularly important for the calculation of the πN sigma term.

The first attempt to determine $g_s\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$ was made in ref. [3], where it was claimed that

$$m_0^2:=g_s\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle/\langle\bar{q}q\rangle=0.5\text{–}1.0\ \mathrm{GeV}^2\ . \tag{5}$$

[1] Supported by BMFT.

[#1] This was first mentioned to us by V.I. Zakharov.

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In refs. [4,5] phenomenological determinations were attempted, which yielded $m_0^2 \approx 0.8$ GeV^2 and $m_0^2 \approx 0.2$ GeV^2, respectively.

Since the knowledge of $g_s \langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle$ is essential not only to improve the accuracy of QCD sum rule predictions, but is even indispensible for an accurate determination of the πN sigma term by QCD sum rules (whose value is still controversial [6,7]), a calculation of $g_s \langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle$ on the lattice is truly important.

2. *Method.* For the calculation we shall use Kogut–Susskind [8] quarks, whose action is

$$S_F = \sum_x \left(\tfrac{1}{2} \sum_\mu [\bar{\chi}_x (-1)^{x_1 + \ldots + x_{\mu-1}} U_{x,\mu} \chi_{x+\mu} - \text{h.c.}] + ma\bar{\chi}_x \chi_x \right) , \tag{6}$$

where $\bar{\chi}, \chi$ are single-component Grassmann fields. The latter translate into physical four-component, four-flavoured Dirac fields, which reside on elementary hypercubes, by [9]

$$\psi_\mu^f(x) = \tfrac{1}{8} \sum \Gamma_\alpha^{\mu f} (U_{x,1})^{\alpha_1} (U_{x+\alpha_1,2})^{\alpha_2} (U_{x+\alpha_1+\alpha_2,3})^{\alpha_3} \times (U_{x+\alpha_1+\alpha_2+\alpha_3,4})^{\alpha_4} \chi_{x+\alpha} ,$$

$$\alpha = (\alpha_1, \alpha_2, \alpha_3, \alpha_4) , \tag{7}$$

where the sum runs over the 16 corners of the hypercube and μ, f are Dirac and flavour indices, respectively. The action (6) preserves an explicit continuous chiral symmetry (for $m=0$) which was shown to be broken spontaneously in ref. [10]. We shall use antiperiodic boundary conditions for the fermion fields. The importance of this choice is discussed in ref. [11].

In the notation of eq. (7) the condensate can be written as

$$g_s a^5 \langle \bar{q}\sigma_{\mu\nu} \tfrac{1}{2}\lambda^a G_{\mu\nu}^a q \rangle = \tfrac{1}{4} \bar{\psi}(x) U_{x,\mu} U_{x+\mu,\nu} \sigma_{\mu\nu} \psi(x+\mu+\nu) \rangle . \tag{8}$$

On the RHS it is summed over f, which is corrected by the factor in front. Like the ordinary chiral condensate this expectation value is zero (for $m=0$) to all orders in perturbation theory. To calculate eq. (8) we need to compute quark propagators

$$\bar{\chi}_{x+\alpha}^A \chi_{x+\alpha}^B \tag{9}$$

(A, B are colour indices), in total 48 rows (for SU(3)) of the inverse of the fermion matrix for each gauge field configuration. Since the RHS of eq. (8) involves products of up to ten link matrices, which give rise to large fluctuations, it is favourable to write

$$\psi_\mu^f(x) = \tfrac{1}{8} \sum_\alpha \Gamma_\alpha^{\mu f} \chi_{x+\alpha} , \tag{10}$$

and insert in (8) only those two link matrices which make the expression gauge invariant. This definition has the same continuum limit.

The gluon interaction is described by the Wilson action

$$S_G = \beta \sum_p [1 - \tfrac{1}{3} \operatorname{Tr} \operatorname{Re} U(\partial p)] , \tag{11}$$

and we impose periodic boundary conditions on the gluon fields. All our calculations are done in the quenched approximation, where the contribution of virtual quark loops is neglected.

3. *Results and discussion.* The size of our lattice is 8^4. So far we have computed the condensate on five independent gauge field configurations at $\beta = 5.7$. We have performed this calculation at masses $ma = 0.02$, 0.035 and 0.05. To invert the fermion matrix we have used the conjugate gradient algorithm. We stop the iterative procedure once [12] $r_i^2 \leqslant 10^{-8}$. We also have computed

$$a^3 \langle \bar{q}q \rangle = \tfrac{1}{4} \langle \bar{\psi}(x) \psi(x) \rangle , \tag{12}$$

which we will use to set our scale. On each hypercube one can compute 6 condensates $\bar{\psi}(x) U_{x,\mu} U_{x+\mu,\nu} \times \sigma_{\mu\nu} \psi^{x+\mu+\nu)}$ due to reflection symmetry and 16 condensates $\bar{\psi}(x)\psi(x)$. The results of our calculation are shown in table 1 and fig. 1, where we have averaged over the 6 and 16 possibilities, respectively. We find that the mass dependence of both condensates is to a very good approximation linear for each of the five gauge field configurations. We therefore have extrapolated $\langle \bar{q}q \rangle$ and $g_s \langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle$ to $m=0$ individually for every configuration and taken the average at the end. The result is

$$a^3 \langle \bar{q}q \rangle = 0.064 \pm 0.08 , \tag{13}$$

and

$$g_s a^5 \langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle = 0.183 \pm 0.028 . \tag{14}$$

Table 1
The condensates $a^3\langle\bar{q}q\rangle$ and $g_s a^5\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$ for the individual gauge field configurations at the various quark masses.

Configuration	ma	$a^3\langle\bar{q}q\rangle$	$g_s a^5\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$
1	0.020	0.082±0.007	0.167±0.015
	0.035	0.101±0.006	0.188±0.017
	0.050	0.117±0.006	0.216±0.022
2	0.020	0.066±0.002	0.189±0.031
	0.035	0.088±0.002	0.240±0.036
	0.050	0.107±0.002	0.282±0.041
3	0.020	0.111±0.010	0.323±0.060
	0.035	0.132±0.011	0.377±0.064
	0.050	0.150±0.011	0.419±0.068
4	0.020	0.083±0.006	0.201±0.025
	0.035	0.098±0.006	0.229±0.029
	0.050	0.112±0.006	0.253±0.031
5	0.020	0.099±0.007	0.271±0.042
	0.035	0.119±0.007	0.312±0.047
	0.050	0.136±0.008	0.348±0.050

Though $\langle\bar{q}q\rangle$ and $g_s\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$ fluctuate considerably from one configuration to the other, we notice that the ratio of the two is relatively stable, which indicates that this quantity is less affected by fluctuations of the small eigenvalues of the fermion matrix.

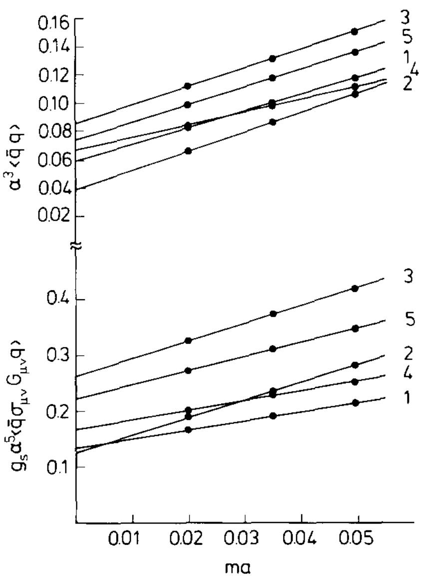

Fig. 1. The condensates $g_s a^5\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$ and $a^3\langle qq\rangle$ for configurations 1–5 versus the quark mass. Errors are not shown.

The result (13) is in agreement with previous calculations [10] (notice the difference in normalization though). If we want to use (13) to set the scale, we have to form the quantity

$$\langle\bar{q}q\rangle|_{\mu^2}=\left(\frac{\alpha_s(\mu^2)}{\alpha_s(\pi^2/a^2)}\right)^{-4/11}\langle\bar{q}q\rangle$$

$$\equiv\left(\frac{\alpha_s(\mu^2)}{\alpha_s(\pi^2/a^2)}\right)^{-4/11}\langle\bar{q}q\rangle|_{\pi^2a^2}\,, \tag{15}$$

where

$$\alpha_s(\mu^2)=\frac{2\pi}{\frac{11}{2}\ln(\mu^2/\Lambda^2_{\mathrm{mom}})+\frac{51}{11}\ln\ln(\mu^2/\Lambda^2_{\mathrm{mom}})}\,, \tag{16}$$

which is known "experimentally" [7]. Assuming the two-loop formula for the lattice spacing,

$$a=(83.5/\Lambda_{\mathrm{mom}})(\tfrac{8}{33}\pi^2\beta)^{51/121}\exp(-\tfrac{4}{33}\pi^2\beta)\,, \tag{17}$$

we obtain the "experimental" value of [13]

$$\langle\bar{q}q\rangle|_{1\,\mathrm{GeV}^2}=(225\ \mathrm{MeV})^3 \tag{18}$$

for $\Lambda_{\mathrm{mom}}\approx 160$ MeV. This scale parameter is $\approx 20\%$ lower than the value we obtained by fitting the mass of the ρ [12]. Notice also that $\langle\bar{q}q\rangle|_{1\,\mathrm{GeV}^2}$ and $\bar{\psi}\psi\rangle_{\mathrm{inv}}$ of ref. [10] are not identical; hence the difference in Λ_{mom}, though the Monte Carlo "data" agree within errors.

Similarly, for the condensate $g_s\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$ we obtain

$$g_s\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle|_{\mu^2}=\left(\frac{\alpha_s(\mu^2)}{\alpha_s(\pi^2/a^2)}\right)^{2/33}g_s\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$$

$$\equiv\left(\frac{\alpha_s(\mu^2)}{\alpha_s(\pi^2/a^2)}\right)^{2/33}g_s\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle|_{\pi^2/a^2}\,. \tag{19}$$

For the above choice of scale parameter this leads to

$$g_s\langle\bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle|_{1\,\mathrm{GeV}^2}=0.0126\pm0.0020\ \mathrm{GeV}^5\,, \tag{20}$$

and

$$m_0^2|_{1\,\mathrm{GeV}^2}=1.1\pm0.1\ \mathrm{GeV}^2\,. \tag{21}$$

Note that the two-loop formula (17) served only to compute Λ_{mom} which enters α_s. We estimate that the observed deviation from (17) at $\beta=5.7$ will change our results by less than the statistical errors.

This result supports the higher values of $g_s\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$ obtained from QCD sum rules (see above). If one whishes one can increase the statistics to reduce the errors. This would require however a major commitment of computer time, because on each configuration one has to compute (at least) 48 rows of the inverse of the fermion matrix.

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