USE OF FOUR-DIMENSIONAL SPIN METHODS IN THE CALCULATION OF RADIATIVE QCD CORRECTIONS

Gerhard A. SCHULER
Deutsches Elektronen-Synchrotron DESY, D-2000 Hamburg, Fed. Rep. Germany

Susumu SAKAKIBARA [1]
College of Science and Engineering, Iwaki Meisei University, Iwaki-shi, Fukushima-ken 970, Japan

Jürgen G. KÖRNER
Institut für Physik, Johannes Gutenberg-Universität, D-6500 Mainz, Fed. Rep. Germany and Deutsches Elektronen-Synchrotron DESY, D-2000 Hamburg, Fed. Rep. Germany

Received 16 April 1987

We demonstrate the applicability of four-dimensional spin methods to the calculation of higher-order QCD radiative corrections. These new techniques can lead to substantial simplifications of radiative correction calculations.

1. Introduction. With the completion of the high-energy accelerators TRISTAN, TEVATRON and SLC and the planned commissioning of LEP and HERA in the coming years one expects large data samples on jet production to become available in the next few years. The interpretation of these jet events and their polarization dependence within QCD requires the calculation of a large number of higher-order QCD corrections. Up to now the favoured method to calculate QCD radiative corrections is by means of dimensional regularization. Since the necessary calculations using dimensional regularization techniques are technically and algebraically quite involved [1–3] one would like to develop simpler methods to do the radiative corrections.

In the last few years new techniques have been developed using four-dimensional spin techniques to dramatically simplify tree diagram calculations in massless QCD and QED. Among these are the use of helicity methods [4], use of the two-component Weyl formalism [5] and the exploitation of supersymmetry relations [6]. It would be highly desirable to use these simple and compact tree level expressions along with their one-loop counterparts as input integrands in radiative correction calculations. Not only are the ensuing integrands shorter and easier to calculate, but also the structure of the integrands can be analysed and interpreted more easily in terms of their four-dimensional spin and helicity content. Also, when using helicity techniques, one can organize the singularity structure of cross section expressions quite efficiently. Once the radiative corrections have been done for the spin-averaged cross sections the radiative corrections to polarization type observables can be performed without much additional effort. Also the inclusion of parity-violating and polarization effects involving γ_5 or the antisymmetric tensor $\epsilon_{\alpha\beta\gamma\delta}$ is quite straightforward if one uses four-dimensional spin and helicity techniques.

Such a program can be realized within the dimensional reduction scheme proposed by Siegel [7]: Spin degrees of freedom are kept fixed in four dimensions whereas the momenta (and derivatives) are continued from four to n dimensions. This allows one to use the usual dimensional regularization techniques to regularize ultraviolet (UV) and infrared/mass

[1] On leave of absence from Institut für Physik, Johannes Gutenberg-Universität, D-6500 Mainz, Fed. Rep. Germany.

0370-2693/87/$ 03.50 © Elsevier Science Publishers B.V.
(North-Holland Physics Publishing Division)

(IR/M) divergencies in scalar integrals after the spin algebra has been done in four dimensions.

Although dimensional reduction has some formal inconsistencies [8,9], the scheme can be used as a working prescription to calculate Feynman diagrams when proper care is taken to circumvent the formal inconsistencies. This is not difficult in practise. In fact, the dimensional reduction scheme has been succesfully applied to the two-loop calculation of Ward identities [10], the axial anomaly [11], and anomalous dimensions [12,13].

In this paper we show how to apply dimensional reduction to the calculation of one-loop level radiative QCD (and QED) corrections to physical cross sections. To our knowledge this has not been discussed in the literature before. This requires the knowledge of the appropriate counterterms that result from the UV divergent wave function and vertex renormalization graphs. These global counter terms are identified and calculated. After addition of these counterterms the radiatively corrected cross sections calculated in dimensional reduction agree with the dimensional regularization result.

As an illustration we have recalculated the $O(\alpha_s^2)$ radiative QCD corrections to $e^+e^- \to q\bar{q}G$ within the dimensional reduction scheme including the appropriate counter terms. We have explicitly verified that the result agrees with the dimensional regularization result calculated in ref. [1]. We briefly comment on characteristic differences of the two schemes at intermediate steps of the calculation.

2. *Regularization by dimensional reduction.* The idea of dimensional reduction is to only continue to $n\neq 4$ dimensions coordinates x_μ and momenta p_μ, while leaving all other tensors and spinors in four dimensions. In particular, the gluon field G_μ and the Dirac spinors are left with four components and the algebra of γ-matrices is *unaltered.* Dimensional reduction is defined by decomposing four-dimensional space into the direct sum of n- and $(4-n)$-dimensional subspaces [#1]:

$$4=n\oplus(4-n). \tag{1}$$

This is realized by correspondingly splitting the n-dimensional metric tensor according to $(n\equiv 4-\epsilon)$:

$$g^{(4)}_{\mu\nu}\equiv g^{(n)}_{\mu\nu}+g^{(\epsilon)}_{\mu\nu},$$

$$g^{(4)}{}_\mu{}^\mu\equiv 4,\quad g^{(n)}{}_\mu{}^\mu=n,\quad g^{(\epsilon)}{}_\mu{}^\mu=\epsilon. \tag{2}$$

The orthogonality eq. (1) then reads

$$g^{(n)}{}_\mu{}^\alpha g^{(\epsilon)}{}_{\alpha\nu}=0. \tag{3}$$

Corresponding covariants as $p^{(4)}_\mu, p^{(n)}_\mu, p^{(\epsilon)}_\mu$ or $\gamma^{(4)}_\mu, \gamma^{(n)}_\mu, \gamma^{(\epsilon)}_\mu$ are introduced according to

$$\gamma^{(\epsilon)}_\mu := g^{(\epsilon)}_{\mu\nu}\gamma^{(4)\nu},\quad \text{etc.} \tag{4}$$

Dimensional reduction is then arrived at by postulating that the γ_μ (p_μ) of the lagrangian density and thereby of the Feynman rules obey

$$\gamma_\mu\equiv\gamma^{(4)}_\mu,$$

$$p_\mu\equiv p^{(n)}_\mu,\quad \text{i.e., } p^{(\epsilon)}_\mu=0. \tag{5}$$

Thus, the Dirac algebra remains in four dimensions yielding Feynman rules which are formally the same as in four dimensions. In particular, the gauge field propagator is $-ig^{(4)}_{\mu\nu}/k^2$. On the other hand, all momenta become n-dimensional allowing for (loop and phase-space) integrations in n dimensions. For example, symmetric integration transforms $k_\mu k_\nu$ momentum integrals into $g^{(n)}_{\mu\nu}$. One must, of course, carefully distinguish between the different metric tensors.

It is well known that dimensional reduction becomes inconsistent as soon as γ_5 comes into play. However, we will never have to calculate any parity odd trace. Instead we calculate p.v. cross sections via helicity amplitudes [14] [#2].

Corresponding to the decomposition eq. (1) we split the four-dimensional lagrangian density $\mathscr{L}$ into

$$\mathscr{L}=\mathscr{L}^{(n)}+\mathscr{L}^{(\epsilon)}, \tag{6}$$

[#1] This decoposition is based on the requirement that $\gamma\cdot p\gamma\cdot p=p^2$, where p_μ (γ_μ) is n-dimensional (four-dimensional).

[#2] It is also known that dimensional reduction can lead to computational ambiguities even without the presence of γ_5. It is the calculational algorithms used by algebraic computer programs that can originate these ambiguities [15,9]. However, when performing the Dirac algebra before doing the integrations, every manipulation is perfectly unique. In other cases one can easily modify the computer algorithms to prevent amgibuities. In addition the latter can show up first in two loops including 10 or more γ-matrices.

where $\mathscr{L}^{(n)}$ is the lagrangian density of *ordinary* dimensional regularization. Remembering $\partial_\mu^{(\epsilon)}=0$ we find that $\mathscr{L}^{(\epsilon)}$ contains ϵ-dimensional gluon fields $G_\mu^{(\epsilon)}$ besides the n-dimensional gluon fields $G_\mu^{(n)}$ $(G_\mu \equiv G_\mu^{(n)}+G_\mu^{(\epsilon)})$, the n-dimensional derivatives and the four-dimensional quark fields q. The ϵ-parts $\epsilon_\mu \equiv G_\mu^{(\epsilon)}$ of the (four-dimensional) gluon field G_μ are called ϵ-scalars (under the Lorentz group in n dimensions). $\mathscr{L}^{(\epsilon)}$ contains the ϵ-propagator and its couplings $G^{(n)}G^{(n)}\epsilon\epsilon$, 4ϵ, $G^{(n)}\epsilon\epsilon$ and $qq\epsilon$. Thus $\mathscr{L}^{(\epsilon)}$ describes the interactions of ϵ-scalars, which are not present in the usual procedure, and originates all differences between the ordinary dimensional regularization and dimensional reduction. Separating the ϵ-scalar contribution from $\mathscr{L}$ is only of "academic" interest as a comparison of the two methods, namely to determine minimal subtraction (MS) within dimensional reduction (see below). In practical computations with dimensional reduction we operate of course totally with $\mathscr{L}$ since otherwise the technical advantages of dimensional reduction would be lost.

We now adjust the dimensional reduction MS scheme in such a way that the UV poles *and* the finite terms they induce are the *same* as in usual dimensional regularization. Supposing that we only have an UV divergent amplitude we clearly get the same (finite) result in both methods. Once the dimensional reduction renormalization contributions are established up to one-loop, dimensional reduction can be used with great advantages for all one-loop calculations. In the case where UV and IR/M singularities are simultaneously present we handle the UV realm as mentioned above. Sinc ethe IR/M poles cancel between virtual and real diagrams all remaining differences between dimensional reduction and dimensional regularization drop out at the end [#3].

In order to determine the difference between dimensional regularization and dimensional reduction in the UV realm we calculate the UV divergencies that arise at one-loop order. The corresponding Feynman diagrams (including their correct weights) are shown in figs. 1a–1e, called qqG, 3G, Z_q, N_C-part

[#3] Up to collinear initial state singularities which, however, can be absorbed into the initial state parton distribution.

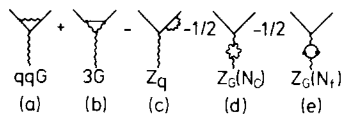

Fig. 1. One-loop diagrams for coupling constant renormalization.

and N_f-part of Z_G, respectively: Here N_C (N_f) denotes the number of colours (flavours) and $C_F \equiv (N_C^2-1)/(2N_C)$. To allow for a comparison with usual dimensional regularization we also need the *finite* contributions coing from the UV part of the n-dimensional integrals. The results are given in table 1 [#4]. In this table we have left out a common factor $-igt^A\alpha_s/4\pi\epsilon$ where t^A are the SU(3) colour matrices [#5]. The column denoted by "reduction" gives the respective UV poles and the finite parts which they induce within dimensional reduction. The third column gives the UV content of standard dimensional regularization. The last two columns originate from $\mathscr{L}^{(\epsilon)}$.

The entries of the "reduction" column can be seen to equal the sum of the last three columns (row by row). Therefore the last two columns account for the difference of dimensional reduction and dimensional regularization. Of course, this is nothing but the realization of eq. (1) to one-loop order. For reasons that will become clear below we have separated off the $\mathscr{L}^{(\epsilon)}$ contributions and refer to $\gamma_\mu^{(\epsilon)}$-terms as ϵ-operator contributions.

In standard dimensional regularization the MS renormalized amplitude is arrived at by subtracting the UV poles. They are given by the (true) UV poles of the "regularization" column. In order to match the results of dimensional reduction with those of usual dimensional regularization we have to subtract *both* the (true) UV poles of column three *and* the (total) contributions of the last two columns. Note that the contribution of this dimensional reduction counterterm (by counterterm we denote the *negative*

[#4] To complete the prescriptions eqs. (2)–(5) for computing with dimensional reduction we mention that Lorentz invariance only applies to the n-dimensional parts. We thus get additional quantities in a covariant expansion. E.g. besides $\gamma_\mu \equiv \gamma_\mu^{(4)}$ we also get $\gamma_\mu^{(n)}$ or $\gamma_\mu^{(\epsilon)}$, respectively. This is technically clear since symmetric integration on $k_\mu k_\nu$ also produces $g_{\mu\nu}^{(n)}$ terms which in turn transforms γ_μ into $\gamma_\mu^{(n)}$ via eq. (2).

[#5] We have also left out terms of order ϵ^2 since they can be neglected up to one-loop accuracy.

Table 1

	Colour/ flavour	Reduction	Regularization	ϵ-operator	ϵ-scalar
	$2C_F - N_C$	qqG $(1-\frac{1}{2}\epsilon)\gamma_\alpha^{(4)} + \gamma_\alpha^{(\epsilon)}$	$\mathrm{qqG}^{(n)}$ $(1-\epsilon)\gamma_\alpha^{(n)}$	$\mathrm{qq}\epsilon$ $2\gamma_\alpha^{(\epsilon)}$	$\mathrm{qqG}^{(n)}$ $\frac{1}{2}\epsilon\gamma_\alpha^{(n)}$
	N_C	3G $(3+\frac{1}{2}\epsilon)\gamma_\alpha^{(4)} - \gamma_\alpha^{(\epsilon)}$	$3\mathrm{G}^{(n)}$ $3\gamma_\alpha^{(n)}$	$\mathrm{G}\epsilon\epsilon$ $2\gamma_\alpha^{(\epsilon)}$	$3\mathrm{G}^{(n)}$ $\frac{1}{2}\epsilon\gamma_\alpha^{(n)}$
	$2C_F$	Z_q $-\gamma_\alpha^{(4)}$	Z_q $(-1+\frac{1}{2}\epsilon)\gamma_\alpha^{(n)}$	Z_q $-\gamma_\alpha^{(\epsilon)}$	Z_q $-\frac{1}{2}\epsilon\gamma_\alpha^{(n)}$
	N_C	Z_G $(\frac{5}{3}-\frac{1}{9}\epsilon)\gamma_\alpha^{(4)} + \frac{1}{3}\gamma_\alpha^{(\epsilon)}$	$Z_{G^{(n)}}$ $(\frac{5}{3}+\frac{1}{18}\epsilon)\gamma_\alpha^{(n)}$	Z_ϵ $2\gamma_\alpha^{(\epsilon)}$	$Z_{G^{(n)}}$ $-\frac{1}{6}\epsilon\gamma_\alpha^{(n)}$
	N_f	Z_G $(-\frac{2}{3}+\frac{1}{9}\epsilon)\gamma_\alpha^{(4)} - \frac{1}{3}\gamma_\alpha^{(\epsilon)}$	$Z_{G^{(n)}}$ $(-\frac{2}{3}+\frac{1}{9}\epsilon)\gamma_\alpha^{(n)}$	Z_ϵ $-\gamma_\alpha^{(\epsilon)}$	$Z_{G^{(n)}}$ 0

of the UV-terms which have to be subtracted in MS; thus counterterms are *added*) is most easily calculated by first peforming all contractions within the trace and then doing the trace.

Let us collect the results for the dimensional reduction counterterms as they can be used for practical calculations. It is most convenient to write them as shown in table 2 (again suppressing a factor $-\mathrm{i}gt^A\alpha_s/4\pi\epsilon$). We also quote the sum T_α^{ct} of the counterterms:

$$T_\alpha^{\mathrm{ct}} = -\mathrm{i}gt^A \frac{\alpha_{\mathrm{s,reg}}^{\mathrm{MS}}}{4\pi\epsilon} \{-2C_F\gamma_\alpha^{(\epsilon)} - N_C[(\tfrac{11}{3}-\tfrac{1}{6}\epsilon)\gamma_\alpha^{(4)} - \tfrac{5}{3}\gamma_\alpha^{(\epsilon)}] + \mathrm{N_f}(\tfrac{2}{3}\gamma_\alpha^{(4)} + \tfrac{1}{3}\gamma_\alpha^{(\epsilon)})\}. \tag{7}$$

We stress again that, when using the counterterm eq. (7), calculations within dimensional reduction and usual dimensional regularization give the same results in the UV realm. Therefore the coupling constant $\alpha_{\mathrm{s,reg}}^{\mathrm{MS}}$ of standard dimensional regularization has to be used in eq. (7). In the next section we present an explicit example how these counterterms are used within dimensional reduction.

Up to now we have presented a direct construction of the UV counterterms of dimensional reduction. To give an interpretation of these counterterms we derive them again in a more formal way. To this end we set up the minimal subtraction scheme (MS) for dimensional reduction. We first note that local gauge invariance is valid only for the n-dimensional gauge field $G_\mu^{(n)}$ ($G_\mu \equiv G_\mu^{(n)} + \epsilon_\mu$) since $\partial_\mu^{(\epsilon)} = 0$. Thus $G_\mu^{(n)}$ are the true gauge particles. Consequently, the Ward identities of gauge invariance only lead to the equality of the coupling constants for $G_\mu^{(n)}$ [#6]. The ϵ-scalar couplings are renormalized *independently* (coupling: $\mathrm{qq}\epsilon \neq \mathrm{qqG}^{(n)}$, $3\mathrm{G}^{(n)} \neq \mathrm{G}^{(n)}\mathrm{G}^{(n)}\epsilon$ but $3\mathrm{G}^{(n)} = \mathrm{qqG}^{(n)}$). Since our goal is only one-loop, we are not concerned with the renormalization of $\mathrm{G}^{(n)}\epsilon\epsilon$-couplings or any four-point coupling. We will therefore only consider the $\mathrm{qqG}^{(n)}$ and $\mathrm{qq}\epsilon$ couplings.

Since $G_\mu^{(n)}$ are the true gauge particles we split the

[#6] I.e. for $G_\mu^{(n)}$ there exists a universal coupling constant in all orders of perturbation theory which we denote by g_{red}.

Table 2

	qqG	3G	Z_q	Z_G	Z_G
	$N_C - 2C_F$ $(1+\frac{1}{2}\epsilon)\gamma_\alpha^{(4)} + \gamma_\alpha^{(\epsilon)}$	$-N_C$ $(3+\frac{1}{2}\epsilon)\gamma_\alpha^{(4)} - \gamma_\alpha^{(\epsilon)}$	$-2C_F$ $(-1-\frac{1}{2}\epsilon)\gamma_\alpha^{(4)}$	$-N_C$ $(\frac{5}{3}-\frac{1}{6}\epsilon)\gamma_\alpha^{(4)} + \frac{1}{3}\gamma_\alpha^{(\epsilon)}$	$-N_f$ $-\frac{2}{3}\gamma_\alpha^{(4)} - \frac{1}{3}\gamma_\alpha^{(\epsilon)}$

(four-dimensional) lagrangian $\mathscr{L}$ into three pieces [#7]: $\mathscr{L} \equiv \mathscr{L}^{\rm red} + \mathscr{L}^{\rm op} + \mathscr{L}^{\rm r}$. $\mathscr{L}^{\rm red}$ describes the interactions containing n-dimensional *external* fields $G_\mu^{(n)}$, $\mathscr{L}^{\rm op}$ describes the coupling of (external) ϵ-scalars and $\mathscr{L}^{\rm r}$ contains the rest. We define the coupling constant of dimensional reduction as $g_{\rm red}$, the coupling constant at e.g. the $qqG^{(n)}$ vertex. We also define g_ϵ to be the coupling constant of the $qq\epsilon$ vertex. Clearly, in lowest order, the two coupling constants are equal and there is no contribution coming from $\mathscr{L}^{\rm r}$.

However, at the one-loop level, the $qqG^{(n)}$ and $qq\epsilon$ vertices are renormalized *differently*. They receive contributions coming from insertion of loops containing the *full* (four-dimensional) gauge field propagator (and the fermion propagator). Defining MS renormalization as the scheme where just the UV-poles are subtracted, the contributions to $g_{\rm red}$ and g_ϵ correspond to the sum of the (true) UV poles of (column 3+column 5) and to the sum of column 4 of table 1, respectively. Here we see explicitly that the ϵ-scalar coupling is renormalized differently from the gauge particle coupling. We find (see also ref. [11]):

$$\alpha_{\rm s,red}^{\rm MS} = \mu^{-\epsilon} Z_\alpha \alpha_{\rm s,red}^{\rm B},$$

$$\alpha_{{\rm s},\epsilon}^{\rm MS} = \mu^{-\epsilon} Z_\epsilon \alpha_{{\rm s},\epsilon}^{\rm B}, \tag{8}$$

with

$$Z_\alpha = 1 + b\alpha_{\rm s,red}/2\pi\epsilon,$$

$$Z_\epsilon = 1 + b_\epsilon \alpha_{{\rm s},\epsilon}/2\pi\epsilon,$$

$$b = \tfrac{11}{3}N_{\rm C} - \tfrac{2}{3}N_{\rm f},$$

$$b_\epsilon = 2C_{\rm F} + 2N_{\rm C} - N_{\rm f}. \tag{9}$$

We now postulate that the two renormalized couplings are equal:

$$\alpha_{\rm s,red}^{\rm MS} \stackrel{!}{=} \alpha_{{\rm s},\epsilon}^{\rm MS}. \tag{10}$$

We then arrive at the MS counter term $T_\alpha^{\rm red}$ of dimensional reduction

$$T_\alpha^{\rm red} = -{\rm i}gt^A \frac{\alpha_{\rm s,red}^{\rm MS}}{4\pi\epsilon}\left(-b\gamma_\alpha^{(n)} - b_\epsilon \gamma_\alpha^{(\epsilon)}\right). \tag{11}$$

We observe that subtracting the UV poles [i.e., adding eq. (11)], no ϵ-operator contribution survives and in addition we subtract the same (true) UV poles as in standard dimensional regularization. However, in eq. (11) we use $\alpha_{\rm s,red}$ whereas in eq. (7) there is $\alpha_{\rm s,reg}$. The difference in the MS renormalization contributions to $g_{\rm red}$ and to the usual dimensional regularization coupling $g_{\rm reg}$ is just the last column of table 1. These are contributions containing *internal* ϵ-scalar lines. Their (finite) sum accounts for the difference between the two couplings $g_{\rm red}$ and $g_{\rm reg}$ to one-loop. Writing

$$\alpha_{\rm s,red} = \alpha_{\rm s,reg}(1 + k\alpha_{\rm s,reg}), \tag{12}$$

we find

$$k = N_{\rm C}/6\cdot 2\pi. \tag{13}$$

Adding this term to eq. (11), i.e., to the (true) UV poles of dimensional reduction, we again find the counter term eq. (7):

$$-{\rm i}g_{\rm red} t^A \gamma_\alpha^{(4)} + T_\alpha^{\rm red} = -{\rm i}g_{\rm reg} t^A \gamma_\alpha^{(4)} + T_\alpha^{\rm ct}. \tag{14}$$

3. An explicit example. Let us now, for purposes of illustration, turn to a specific example, namely the calculation of the $O(\alpha_s^2)$ corrections to $e^+e^- \to q\bar{q}G$ within dimensional reduction. This example is sufficiently complex to exhibit all the features that are necessary to calculate radiative corrections at the one-loop level in dimensional reduction.

We begin by determining the counterterm according to the contributions of table 1 [#8]. In terms of the Born term amplitude $M_{\mu\alpha}^{\rm B}$ ($p_{\rm q} = p_1$, $p_{\bar{\rm q}} = p_2$, $p_{\rm G} = p_3$) defined by [$s_{ij} = (p_i + p_j)^2$; μ = photon index, α = gluon index]

$$M_{\mu\alpha}^{\rm B} = -{\rm i}gt^A \bar{u}(p_1)\left(\gamma_\alpha \frac{\not{p}_1 + \not{p}_3}{s_{13}}\gamma_\mu - \gamma_\mu \frac{\not{p}_2 + \not{p}_3}{s_{23}}\gamma_\alpha\right) v(p_2), \tag{15}$$

one finds for the counterterm

$$M_{\mu\alpha}^{\rm ct} = M^{{\rm B}\mu'\alpha'}\{(A+B)g_{\mu\mu'}^{(4)} g_{\alpha\alpha'}^{(4)} + {\rm C}g_{\mu\mu'}^{(4)} g_{\alpha\alpha'}^{(\epsilon)} + {\rm D}g_{\mu\mu'}^{(\epsilon)} g_{\alpha\alpha'}^{(4)}\}, \tag{16}$$

[#7] In contrast to eq. (6).

[#8] As usual the MS renormalized loop contributions are given by the sum of the MS counterterms and the loop calculation where the UV and IR/M-poles are identified.

where

$$A = (\alpha_s/4\pi)(\tfrac{1}{3}N_f - \tfrac{11}{6}N_C)2/\epsilon,$$

$$B = (\alpha_s/4\pi)(\tfrac{1}{6}N_C),$$

$$C = (\alpha_s/4\pi)(\tfrac{1}{6}N_f - \tfrac{1}{6}N_C + N_C - C_F)2/\epsilon,$$

$$D = (\alpha_s/4\pi)(-C_F)2/\epsilon. \tag{17}$$

A represents the counterterm originating from $\gamma_\alpha^{(4)}$, B the ϵ-scalar and C the ϵ-operator contribution at the gluon vertex, and D is the ϵ-operator contribution at the photon vertex. Note that one has a counterterm for the electromagnetic (EM) vertex within dimensional reduction *even though* the Ward identities guarantee that he EM vertex is UV finite in dimensional regularization.

We then fold the counterterm amplitude $M_{\mu\alpha}^{\mathrm{ct}}$ with the Born term amplitude in order to obtain the counterterm that has to be added to the hadron tensor. We obtain for the trace of the hadron tensor ($x_i = 2p_i{\cdot}q/q^2$, $q = p_1 + p_2 + p_3$)

$$H_\mu^{\mathrm{ct}\,\mu} = 64\pi^2(\alpha_s/2\pi)^2 N_C C_F$$

$$\times \Bigg[(\tfrac{1}{3}N_f - \tfrac{11}{6}N_C)(2/\epsilon)B^4 + (\tfrac{1}{6}N_C)B^4$$

$$+ (\tfrac{1}{6}N_f - \tfrac{1}{6}N_C + N_C - C_F)\left(B^4 - \frac{2(1-x_3)}{(1-x_1)(1-x_2)}\right)$$

$$+(-C_F)B^4\Bigg], \tag{18}$$

where $B^4 = (x_1^2 + x_2^2)/(1-x_1)(1-x_2)$.

In order to display our normalization we write down the differential cross section for $e^+e^- \to q\bar{q}G$ for a quark with charge e_q:

$$\frac{d^2\sigma}{dx_1\,dx_2} = \frac{1}{64\pi^2}\frac{4\pi\alpha^2}{3q^2}\,e_q^2 H_\mu{}^\mu. \tag{19}$$

Let us now turn to the tree diagram contributions. For the dimensional regularization and dimensional reduction cases we obtain to $O(y^0)$ accuracy [where y denotes the invariant mass cutoff $s_{ij} \leqslant yq^2$, and $B^s = x_3^2/(1-x_1)(1-x_2)$]

$$H^{\mathrm{reg}}{}_\mu{}^\mu(\mathrm{tree}) = 64\pi^2(\alpha_s/2\pi)^2 N_C C_F$$

$$\times(1-\tfrac{1}{2}\epsilon)(B^4 - \tfrac{1}{2}\epsilon B^s)(a/\epsilon^2 + b/\epsilon + c),$$

$$H^{\mathrm{red}}{}_\mu{}^\mu(\mathrm{tree}) = 64\pi^2(\alpha_s/2\pi)^2 N_C C_F$$

$$\times\left(B^4(a/\epsilon^2 + b/\epsilon + c + d)\right.$$

$$\left. + e\,\frac{-2(1-x_3)}{(1-x_1)(1-x_2)}\right). \tag{20}$$

For the present discussion we are not interested in the explicit form of the contribution of the term $(a/\epsilon^2 + b/\epsilon + c)$ that multiplies the respective n-dimensional and four-dimensional Born terms. They can be read off from the corresponding expressions in ref. [1] if needed. Let us rather concentrate on the difference terms proportional to d and e. They survive after the singular terms in eq. (20) have been cancelled against the respective singular loop contributions. One finds

$$d = -C_F - \tfrac{1}{6}N_f, \tag{21}$$

$$e = -\tfrac{1}{6}N_f + \tfrac{1}{6}N_C. \tag{22}$$

The difference term proportional to d that multiplies the Born term B^4 is determined by the difference of n-dimensional and four-dimensional Altarelli–Parisi (AP) kernels. There is no difference in the AP kernels proportional to N_C which explains the absence of a N_C term in eq. (21).

The difference term proportional to e does not have the Born term structure as eq. (20) shows. This contribution arisese from the azimuthal dependence of the splitting functions $G \to GG$ and $G \to q\bar{q}$. The azimuthal dependence averages out for the n-dimensional (four-dimensional) contribution after n-dimensional (four-dimensional) azimuthal averaging. However, dimensional reduction prescribes n-dimensional azimuthal averaging of a four-dimensional matrix element which leads to the contribution involving e in eq. (20). This non-Born-term-like contribution can, however, be seen to exactly cancel the corresponding non-Born-term-like ϵ-operator contribution from the gluon self-energy contributions in eq. (18).

As a final step we have calculated the loop contributions within dimensional reduction (using four-dimensional matrix elements). Let us stress that one need not keep track of the n-dimensional metric tensor $g_{\mu\nu}^{(n)}$ resulting from symmetric integration if all manipulations related to the spin algebra (traces, contractions, etc.) are done *before* the n-dimensional integrations. Then, after adding up the counterterms eq. (18), the tree contributions eq. (20), and the loop contributions we obtained a finite result which is in complete agreement with the dimensional regularization result in ref. [1].

References

[1] R.K. Ellis, D.A. Ross and A.B. Terrano, Nucl. Phys. B 178 (1981) 421;
K. Fabricius, G. Kramer, G. Schierholz and I. Schmitt, Z. Phys. C 11 (1982) 315.
[2] J.G. Körner, G.A. Schuler, G. Kramer and B. Lampe, Z. Phys. C 32 (1986) 181.
[3] R.K. Ellis and J.G. Sexton, Nucl. Phys. B 269 (1986) 445;
B. Lampe, Desy preprint DESY 86-126 (1986).
[4] P. de Causmaecker, R. Gastmans, W. Troost and T.T. Wu, Phys. Lett. B 105 (1981) 215; Nucl. Phys. B 206 (1982) 53;
F.A. Berends, R. Kleiss, P. de Causmaecker, R. Gastmans and T.T. Wu, Phys. Lett. B 103 (1981) 124;
F.A. Berends, R. Kleiss, P. de Causmaecker, W. Troost and T.T. Wu, Nucl. Phys. B 206 (1982) 61.
[5] G.R. Farrar and F. Neri, Phys. Lett. B 130 (1983) 109;
A. Kersch and F. Scheck, Nucl. Phys. B 263 (1986) 475.
[6] S.J. Parke and T.R. Taylor, Phys. Lett. B 157 (1985) 81; Nucl. Phys. B 269 (1986) 410.
[7] W. Siegel, Phys. Lett. B 84 (1979) 193.
[8] W. Siegel, Phys. Lett. B 94 (1980) 37.
[9] L.V. Avdeev and A.A. Vladimirov, Nucl. Phys. B 219 (1983) 262.
[10] D.M. Capper, D.R.T. Jones and P. van Nieuwenhuizen, Nucl. Phys. B 167 (1980) 479.
[11] D.R.T. Jones and J.P. Leveille, Nucl. Phys. B 206 (1982) 473.
[12] G. Altarelli, G. Curci, G. Martinelli and S. Petrarca, Nucl. Phys. B 187 (1981) 461.
[13] M. Daniel and J.A. Penarocha, Nucl. Phys. B 236 (1984) 467.
[14] J.G. Körner, S. Sakakibara and G.A. Schuler, to be published.
[15] G.A. Schuler, thesis Mainz (1986), Internal Report DESY T-87-01 (February 1987).