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HEATING AFTER HIGHER DIMENSIONAL INFLATION

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Within the context of a class of higher dimensional models of gravity, we investigate the heating of the universe following an inflationary phase. High temperatures, typically on the order of 10^{17} GeV can be achieved. This allows for a subsequent production of baryon asymmetry and, if existing, superheavy cosmic strings.

1. Introduction

In higher dimensional models of inflation [1], the time evolution of the internal space can play the role of an inflaton scalar field φ in the reduced effective four-dimensional theory. Due to its gravitational origin, the potential for φ becomes exponentially flat for large φ. Sufficient inflation is obtained without extreme fine tuning of parameters. High dimensional gravity leads to a violation of the four-dimensional equivalence principle due to the presence of additional couplings of φ to gravity. As a consequence [1], the potential $W(\varphi)$ which determines the time evolution of the inflaton is different from the potential $V(\varphi)$ which determines the Hubble parameter H during the inflationary phase. One finds that $V(\varphi)$ vanishes exponentially for large φ. During inflation H is several orders of magnitude smaller than the inverse characteristic length scale of the internal space L^{-1}. This leads to acceptable values for the density fluctuations $\Delta\rho/\rho$. Typically, $\Delta\rho/\rho \approx 10^{-4}$–$10^{-5}$ on galactic scales can be obtained. The fluctuations decrease for larger length scales (they are about a factor 10 smaller on the present horizon scale). This picture for density fluctuations has been confirmed by independent calculations of Pollock [2].

In this paper we concentrate on the question of entropy production after the inflationary period. We calculate the interactions of the inflaton field with other (gauge nonsinglet) particles. We arrive at the remarkable conclusion that the

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temperature of the universe following inflation and subsequent φ decay is unusually large, typically on the order of 10^{17} GeV. We expect this result to be valid for a wide class of higher dimensional models. This confirms an earlier rough estimate [1] based on the large mass term of the φ field in the ground state. Pollock has reached at a similar conclusion using constraints from $\Delta\rho/\rho$ as an input [3].

Heating of the universe is due to the decay of the coherent field φ when it approaches its ground state value. This is best described in a four-dimensional language. We therefore perform in sect. 2 the dimensional reduction of the higher dimensional action, including the most general gravitational invariants with up to four derivatives. The effective four-dimensional Newton constant depends on the volume of the internal space and therefore on φ. This is not a very convenient formulation since the relevant physics depends on ratios of length scales such as H/M_{P}, L^{-1}/M_{P} etc. A formulation with a constant value of M_{P} is obtained by an appropriate Weyl scaling of the four-dimensional metric. This is described in sect. 3. For the discussion of both sects. 2 and 3 the detailed geometry of the internal space is not used explicitly. We present not only the terms needed for a discussion of heating, but also those relevant for the inflationary period and the calculation of $\Delta\rho/\rho$ [1].

In sects. 4 and 5 we proceed to a detailed calculation of the post-inflation heating temperature for the model of ref. [1] with the ground state $\mathscr{M}^4\times S^D$. An explicit calculation of cubic and quartic interactions of the inflaton with nonsinglet scalars can be found in the appendix. We find a maximal heating efficiency, so that almost all of the potential energy stored in the inflaton (geometry of internal space) is converted into heat after the inflationary period. In the conclusion we compare our results with those obtained from the more standard four-dimensional inflationary scenarios. In higher dimensional theories a high heating temperature is compatible with very small interactions during the inflationary phase, due to the predicted exponential behaviour of the coupling strength. We also argue that the existence of an inflationary solution is a crucial criterion for the selection of the "true" ground state of a higher dimensional theory.

2. Dimensional reduction: The coupled system of a scalar singlet and gravitation

In the next three sections we perform the dimensional reduction for the model of ref. [1], expanding on a "ground state" $\mathscr{M}^4\times S^D$. We start with the action*

$$S=-\frac{1}{\mathring{V}_D}\int \mathrm{d}^d\hat{x}\,\hat{g}^{1/2}\left(\alpha\hat{R}^2+\beta\hat{R}_{\hat{\mu}\hat{\nu}}\hat{R}^{\hat{\mu}\hat{\nu}}+\gamma\hat{R}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}\hat{R}^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}+\delta\hat{R}+\varepsilon\right). \qquad (1)$$

This is the most general form of an approximation including up to four derivatives

* Our conventions are specified in ref. [1].

for the effective action of d-dimensional gravity. The isometry group $SO(D+1)$ of the sphere S^D will appear as a gauge symmetry in the reduced four-dimensional action. We only keep the singlets under $SO(D+1)$ for the purpose of this section*. Since non-singlets appear at least quadratic in the effective action, we are guaranteed that every solution of the field equations for singlets in the reduced four-dimensional theory corresponds exactly to a solution of the higher dimensional field equations. (The truncation to $SO(D+1)$ singlets is "consistent" in the sense of ref. [5]). There is a one to one correspondence between cosmologies based on the reduced four-dimensional action and the solutions of the higher dimensional field equations discussed in ref. [1].

The most general ansatz for the vielbein $\hat{e}_{\hat{\mu}}^{\hat{m}}(x, y)$ consistent with $SO(D+1)$ symmetry is

$$\begin{aligned} \hat{e}_{\mu}^{m} &= \tilde{e}_{\mu}^{m}(x), \\ \hat{e}_{\alpha}^{m} &= 0, \\ \hat{e}_{\mu}^{a} &= 0, \\ \hat{e}_{\alpha}^{a} &= \mathring{e}_{\alpha}^{a}(y) l(x). \end{aligned} \tag{2}$$

Here $\mathring{e}_{\alpha}^{a}(y)$ is the internal vielbein corresponding to the ground state manifold. For a sphere S^D the function $l(x)$ can be interpreted as an x dependent ratio of the radius $L(x)$ over the constant ground state radius L_0. The volume of the internal space is proportional to $l(x)^D$. A variation of $l(x)$ only changes the volume, but not the shape of the internal space and is therefore a singlet with respect to the isometry group $SO(D+1)$.The only other singlet excitation is the four-dimensional gravitational field described by the vielbein $\tilde{e}_{\mu}^{m}(x)$.

Actually, the ansatz (2) holds for the volume degree of freedom plus gravitation for arbitrary internal space (there may, however, be additional scalar singlets). We perform dimensional reduction for this system for general ground states and use the special properties of S^D only at a late stage in sect. 4. This permits an easy use of parts of our results for more realistic theories. To obtain the effective four-dimensional action we insert the ansatz (2) into the action (1) and integrate over the internal coordinates. The curvature tensor corresponding to the ansatz (2) is calculated easily:

$$\hat{R}_{mnpq} = \tilde{R}_{mnpq},$$

$$\hat{R}_{manb} = -\eta_{ab}\tilde{e}_{m}^{\mu}\tilde{e}_{n}^{\nu}\left\{ l^{-2} l_{;\mu} l_{;\nu} + \left(l^{-1} l_{;\nu} \right)_{;\mu} \right\}, \tag{3}$$

$$\hat{R}_{abcd} = \mathring{R}_{abcd} l^{-2} - \left(\eta_{ac}\eta_{bd} - \eta_{ad}\eta_{bc} \right) \tilde{g}^{\mu\nu} l^{-2} l_{;\mu} l_{;\nu}. \tag{4}$$

* Dimensional reduction for the full theory, including all infinitely many nonsinglet excitations, has been carried out at the linearized level in ref. [4].

Here $\tilde{R}_{mnpq}(x)$ and $\mathring{R}_{abcd}(y)$ are the four dimensional and internal curvature tensors respectively, calculated from the vielbeins $\tilde{e}^m_\mu(x)$ and $\mathring{e}^a_\alpha(y)$. The semicolon denotes four dimensional covariant derivatives. All other components of $\hat{R}_{\hat{m}\hat{n}\hat{p}\hat{q}}$ vanish (except index permutations in eq. (3)). We notice that all derivative terms of $l(x)$ appear in the combination $l^{-1}l_{;\mu}$ and introduce the variable

$$s(x) = \ln l(x). \tag{5}$$

The effective d-dimensional action (1) for these singlet degrees of freedom yields for an arbitrary internal space

$$S = \int d^4x \int_{K^D} d^D y\, \mathring{e}(y)\, \frac{1}{\mathring{V}_D} \mathscr{L}, \tag{6}$$

$$\mathscr{L} = \mathscr{L}_G + \mathscr{L}_{Kin} - \tilde{e}\tilde{V} + \tilde{\mathscr{L}}_{R2} + \tilde{\mathscr{L}}_{KR} + \tilde{\mathscr{L}}_{K4}\,, \tag{7}$$

$$\mathscr{L}_G = -\tilde{e}\exp(Ds)\left(\delta + 2\alpha\mathring{R}\exp - 2s\right)\tilde{R}\,, \tag{8}$$

$$\mathscr{L}_{Kin} = \tilde{e}\exp(Ds)\Big\{\left[D(D+1)\delta + 2(D(D+1)\alpha + D\beta + 2\gamma)\cdot\mathring{R}\exp(-2s)\right]s_{;\mu}s^{\mu}_{;} + \left[2D\delta + (4D\alpha + 2\beta)\mathring{R}\exp(-2s)\right]s_{;}{}^{\mu}{}_{\mu}\Big\}\,, \tag{9}$$

$$\tilde{V} = \exp(Ds)\Big\{\left(\alpha\mathring{R}^2 + \beta\mathring{R}_{ab}\mathring{R}^{ab} + \gamma\mathring{R}_{abcd}\mathring{R}^{abcd}\right)\exp(-4s) + \delta\mathring{R}\exp(-2s) + \varepsilon\Big\}\,, \tag{10}$$

$$\tilde{\mathscr{L}}_{R2} = -\tilde{e}\exp(Ds)\left(\alpha\tilde{R}^2 + \beta\tilde{R}_{mn}\tilde{R}^{mn} + \gamma\tilde{R}_{mnpq}\tilde{R}^{mnpq}\right), \tag{11}$$

$$\tilde{\mathscr{L}}_{KR} = \tilde{e}\exp(Ds)\Big\{2D(D+1)\alpha\tilde{R}s_{;\mu}s^{\mu}_{;} + 2D\beta\tilde{R}^{\mu\nu}s_{;\mu}s_{;\nu} + 4D\alpha\tilde{R}s_{;}{}^{\mu}{}_{\mu} + 2D\beta\tilde{R}^{\mu\nu}s_{;\nu\mu}\Big\}\,, \tag{12}$$

$$\begin{aligned}\tilde{\mathscr{L}}_{K4} = -\tilde{e}\exp(Ds)\Big\{&\left[D^2(D+1)^2\alpha + D^2(D+1)\beta + 2D(D+1)\gamma\right]s_{;\mu}s^{\mu}_{;}s_{;\nu}s^{\nu}_{;}\\ &+\left[4D^2(D+1)\alpha + 2D^2\beta\right]s_{;\mu}s^{\mu}_{;}s_{;}{}^{\nu}{}_{\nu} + \left[4D^2\alpha + D^2\beta\right]s_{;}{}^{\mu}{}_{\mu}s_{;}{}^{\nu}{}_{\nu}\\ &+\left[2D^2\beta + 8D\gamma\right]s_{;\mu}s_{;\nu}s^{\mu\nu}_{;} + \left[D^2\beta + 4D\gamma\right]s_{;\mu\nu}s^{\mu\nu}_{;}\Big\}\,.\end{aligned} \tag{13}$$

Here all four-dimensional index manipulations are carried out by multiplication with $\tilde{e}^m_\mu(x)$ or its inverse $\tilde{e}^\mu_m(x)$, e.g. $\tilde{R}_{mn} = \eta^{pq}\tilde{R}_{mpnq}$, $\tilde{R}^{\mu\nu} = \tilde{e}^{m\mu}\tilde{e}^{n\nu}\tilde{R}_{mn}$, $\tilde{R} = \eta^{nm}\tilde{R}_{mn}$, $s^\mu_{;} = \tilde{g}^{\mu\rho}s_{;\rho} = \tilde{e}^\mu_m\tilde{e}^{m\rho}s_{;\rho}$, $\tilde{e} = \det\tilde{e}^m_\mu$ etc. (We also have defined $\mathring{R}_{ab} = \eta^{cd}\mathring{R}_{cadb}$, $\mathring{R} = \eta^{ab}\mathring{R}_{ab}$, $\mathring{e} = \det\mathring{e}^a_\alpha$, $\mathring{V}_D = \int d^D y\,\mathring{e}$, $\hat{g}^{1/2} = \mathring{e}\tilde{e}l^D(x)$.) If $\mathring{R}_{abcd}$ does not depend on the internal coordinates, the integration over K^D is trivial and $\mathscr{L}$ is the effective four-dimensional Lagrange density for the coupled system $s(x)$, $\tilde{e}^m_\mu(x)$★.

★ For $\mathring{R}_{abcd}$ depending on y one has to take "mean values" over internal space for quantities like $\mathring{R}_{ab}\mathring{R}^{ab}$ appearing in $\mathscr{L}$.

3. Weyl scaling

The field equations derived from (6) involve up to four derivatives. The degrees of freedom $s(x)$ and $\tilde{e}_\mu^m(x)$ are coupled in a complicated way. Essentially, this theory describes some kind of generalized Brans-Dicke theory with Brans-Dicke scalar $\sim s(x)$. In particular, we note that the terms involving only two derivatives of $\tilde{e}_\mu^m$ are s-dependent and that the $s_{;\mu}s^{;\mu}$ terms depend on the four-dimensional curvature. The appearance of an s-dependent Newton "constant" makes comparison with standard cosmology somewhat cumbersome. We rather want to decouple the kinetic terms for the graviton and for s at least for those terms which involve only two derivatives. This is done by an appropriate Weyl scaling of the vielbein, resulting in a constant coefficient of the curvature scalar in the Einstein-Hilbert piece $\mathscr{L}_G$ of the action. This is always possible for a region of s where the coefficient of R in (8) does not change sign. We note that for $\alpha\mathring{R} < 0$, $\delta > 0$ the effective Newton constant is positive only in the range

$$s > s_c\,,$$
$$\exp(-2s_c) = -\delta/2\alpha\mathring{R}\,. \tag{14}$$

Within this range* of s we rescale the vielbein

$$\tilde{e}_\mu^m(x) = w(x)\, e_\mu^m(x) \tag{15}$$

with $w(x)$ chosen so that the coefficient of the curvature scalar R built from $e_\mu^m(x)$ is constant. (Since the Weyl scaling (15) becomes singular at $s \to s_c$, we expect the four-dimensional action to have singularities for $s \to s_c$, corresponding to a "coordinate singularity" in field space.) The quantities appearing in (8)–(13) are to be replaced as follows:

$$\tilde{e} = w^4 e\,,$$
$$\tilde{g}_{\mu\nu} = w^2 g_{\mu\nu}\,,$$
$$\tilde{g}^{\mu\nu} = w^{-2} g^{\mu\nu}\,,$$
$$\begin{aligned}\tilde{R}_{mnpq} = w^{-2}\Big\{ & R_{mnpq} - (\eta_{mp}\eta_{nq} - \eta_{mq}\eta_{np})(\ln w)_{;\mu}(\ln w)_{;}^{\mu} \\ & + \big(e_m^\mu e_q^\nu \eta_{np} - e_m^\mu e_p^\nu \eta_{nq} - e_n^\mu e_q^\nu \eta_{mp} + e_n^\mu e_p^\nu \eta_{mq}\big) \\ & \times \big((\ln w)_{;\nu\mu} - (\ln w)_{;\nu}(\ln w)_{;\mu}\big)\Big\}\,,\end{aligned}$$
$$\begin{aligned}\tilde{R}_{mn} = w^{-2}\Big\{ & R_{mn} - \eta_{mn}(\ln w)_{;\;\mu}^{\;\mu} - 2\eta_{mn}(\ln w)_{;\mu}(\ln w)_{;}^{\mu} \\ & + 2e_m^\mu e_n^\nu \big[(\ln w)_{;\nu}(\ln w)_{;\mu} - (\ln w)_{;\nu\mu}\big]\Big\}\,,\end{aligned}$$
$$\tilde{R} = w^{-2}\big\{ R - 6(\ln w)_{;\;\mu}^{\;\mu} - 6(\ln w)_{;\mu}(\ln w)_{;}^{\mu}\big\}\,. \tag{16}$$

* For $\alpha\mathring{R} > 0$ this range extends to all values of s.

In eq. (16) and the following, covariant derivatives are formed with the rescaled metric $g_{\mu\nu}$. We therefore have to replace in (9), (12) and (13)

$$s_{;\mu} \to s_{;\mu},$$

$$s_{;\nu\mu} \to s_{;\nu\mu} - (\ln w)_{;\nu} s_{;\mu} - (\ln w)_{;\mu} s_{;\nu} + g_{\mu\nu} (\ln w)_{;\rho} s_{;}^{\rho},$$

$$s_{;\mu}{}^{\mu} \to w^{-2} \left(s_{;\mu}{}^{\mu} + 2(\ln w)_{;\mu} s_{;}^{\mu} \right). \tag{17}$$

Choosing the scale factor

$$w(x) = \exp(-\tfrac{1}{2} D s) \left(\frac{\delta + 2\alpha \mathring{R} \exp(-2s)}{\delta + 2\alpha \mathring{R}} \right)^{-1/2}, \tag{18}$$

one obtains in the rescaled variables:

$$\mathscr{L}_{\mathrm{G}} + \mathscr{L}_{\mathrm{Kin}} = -e(\delta + 2\alpha \mathring{R}) R + \tfrac{1}{2} e f^2(s) s_{;\mu} s_{;}^{\mu} + \text{total divergence}. \tag{19}$$

The Planck mass M_{P} can be identified

$$\frac{M_{\mathrm{P}}^2}{16\pi} = \delta + 2\alpha \mathring{R} \tag{20}$$

and $f^2(s)$ is given by

$$f^2(s) = \frac{M_{\mathrm{P}}^2}{16\pi} \left(1 + \frac{2\alpha \mathring{R}}{\delta} \exp(-2s) \right)^{-2} g^2(s), \tag{21}$$

$$g^2(s) = D^2 + 2D + 4\left(D^2 \alpha + 2\beta + 2\gamma \right) \frac{\mathring{R}}{\delta} \exp(-2s)$$
$$+ 4\alpha \left[(D^2 - 2D + 12)\alpha + 4\beta + 4\gamma \right] \frac{\mathring{R}^2}{\delta^2} \exp(-4s). \tag{22}$$

After Weyl scaling, the scalar potential reads ($\tilde{e}\tilde{V} = eV$)

$$V(s) = \left(\frac{M_{\mathrm{P}}^2}{16\pi} \right)^2 \left(\delta + 2\alpha \mathring{R} \exp(-2s) \right)^{-2} \exp(-Ds)$$
$$\times \left\{ \left(\alpha \mathring{R}^2 + \beta \mathring{R}_{ab} \mathring{R}^{ab} + \gamma \mathring{R}_{abcd} \mathring{R}^{abcd} \right) \exp(-4s) + \delta \mathring{R} \exp(-2s) + \varepsilon \right\}. \tag{23}$$

This potential vanishes exponentially for large positive s. The exponential behaviour is very general and reflects the gravitational origin of the scalar: Typically, the potential is a rational function of l, but the kinetic terms, as usual in gravitation, involve $l^{-1}l_{;\mu}$, suggesting the choice of the new variable s. The factor $\exp(-Ds)$ corresponds to the inverse of the volume of internal space. Of course, we still need to perform an appropriate rescaling of the scalar field in order to obtain the usual normalization of the part of the kinetic term which involves only two derivatives. In general this will not change the exponential behaviour of the scalar potential.

For the higher derivative terms, we obtain after Weyl scaling

$$\tilde{\mathscr{L}}_{\mathrm{R2}}+\tilde{\mathscr{L}}_{\mathrm{KR}}+\tilde{\mathscr{L}}_{\mathrm{K4}}=\mathscr{L}_{\mathrm{R2}}+\mathscr{L}_{\mathrm{KR}}+\mathscr{L}_{\mathrm{s2}}+\mathscr{L}_{\mathrm{s3}}+\mathscr{L}_{\mathrm{s4}}+\text{total divergence}\,; \tag{24}$$

$$\mathscr{L}_{\mathrm{R2}}=-e\exp(Ds)\left(\alpha R^2+\beta R_{mn}R^{mn}+\gamma R_{mnpq}R^{mnpq}\right); \tag{25}$$

$$\begin{aligned}\mathscr{L}_{\mathrm{KR}}={}&e\exp(Ds)\left(\delta+2\alpha\mathring{R}\exp(-2s)\right)^{-2}\left[a_1(s)Rs_{;\mu}s^{\mu}_{;}+a_2(s)R^{\mu\nu}s_{;\mu}s_{;\nu}\right]\\&+e\exp(Ds)\left(\delta+2\alpha\mathring{R}\exp(-2s)\right)^{-1}\left[b_1(s)s^{\mu}_{;}R_{;\mu}+b_2(s)s_{;\nu}R^{\mu\nu}{}_{;\mu}\right];\end{aligned} \tag{26}$$

$$\begin{aligned}a_1(s)={}&\left[(3D^2+2D)\alpha+D^2\beta+D^2\gamma\right]\delta^2\\&+\left[12D(D-2)\alpha+2(2D^2-3D+2)\beta+4D(D-2)\gamma\right]\delta\alpha\mathring{R}\exp(-2s)\\&+\left[4(3D^2-14D+12)\alpha+4(D^2-3D+6)\beta+4(D-2)^2\gamma\right]\alpha^2\mathring{R}^2\exp(-4s)\\a_2(s)={}&\left[D(D+2)\beta+2D^2\gamma\right]\delta^2+\left[4D^2\beta+8D^2\gamma\right]\delta\alpha\mathring{R}\exp(-2s)\\&+\left[4(D^2-2D-4)\beta+8(D^2-4)\gamma\right]\alpha^2\mathring{R}^2\exp(-4s),\\b_1(s)={}&(2D\alpha+D\beta)\delta+\left[4(D-6)\alpha+2(D-2)\beta\right]\alpha\mathring{R}\exp(-2s)\\b_2(s)={}&4D\gamma\delta+\left[8(D-2)\gamma-8\beta\right]\alpha\mathring{R}\exp(-2s);\end{aligned} \tag{27}$$

$$\mathscr{L}_{\mathrm{s2}}=-e\exp(Ds)\left(c_1(s)s_{;\mu}{}^{\mu}s_{;\nu}{}^{\nu}+c_2(s)s_{;\mu\nu}s^{\mu\nu}_{;}\right), \tag{28}$$

$$\begin{aligned}c_1(s)&=D^2(\alpha+2\beta+\gamma)-2D(6\alpha+3\beta+2\gamma)x_s+4(9\alpha+2\beta+\gamma)x_s^2,\\c_2(s)&=2D(D+2)\gamma-8D\gamma x_s+4(\beta+2\gamma)x_s^2,\\x_s&=\frac{2\alpha\mathring{R}\exp(-2s)}{\delta+2\alpha\mathring{R}\exp(-2s)}.\end{aligned} \tag{29}$$

The terms $\mathscr{L}_{s3(4)}$ contain cubic or quartic products of derivative terms of s like $s_{;\mu\nu}s^{\mu}_{;}s^{\nu}_{;}$ or $s_{;\mu}s^{\mu}_{;}s_{;\nu}s^{\nu}_{;}$. Their explicit form will not be needed for our purpose. Indeed, after rescaling of s to a scalar field φ with dimension of mass, the terms $\mathscr{L}_{s3}$ and $\mathscr{L}_{s4}$ correspond to dimension seven or eight operators. If all typical energies E are much smaller than M_{P}, their contribution is suppressed by three or four powers of E/M_{P}. Even for energy scales in the vicinity of the Planck mass, the contributions of $\mathscr{L}_{s3}$ and $\mathscr{L}_{s4}$ can be neglected whenever s is evolving slowly. This is the case for the inflationary solutions of ref. [1] for which an approximation for the action quadratic in the time derivatives of s is appropriate. Neglecting $\mathscr{L}_{s3}$ and $\mathscr{L}_{s4}$, we collect the various terms of the effective four-dimensional action:

$$\begin{aligned} S^{(4)} = -\int \mathrm{d}^4x\, g^{1/2}\Bigg\{ & \frac{M_{\text{P}}^2}{16\pi}R + \exp(Ds)\left(\alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu} + \gamma R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right) \\ & + V(s) - \tfrac{1}{2}\Bigg[f^2(s) g^{\mu\nu} + \frac{2\exp Ds}{\left(\delta + 2\alpha \mathring{R}\exp(-2s)\right)^2} \\ & \qquad \times \left(a_1(s) R g^{\mu\nu} + a_2(s) R^{\mu\nu}\right)\Bigg] s_{;\mu}s_{;\nu} \\ & - \frac{\exp(Ds)}{\left(\delta + 2\alpha\mathring{R}\exp(-2s)\right)}\left[b_1(s)R^{\mu}_{;} + b_2(s)R^{\nu\mu}{}_{;\nu}\right]s_{;\mu} \\ & + \exp(Ds)\left[c_1(s)s_{;\mu}^{\ \mu}s_{;\nu}^{\ \nu} + c_2(s)s_{;\mu\nu}s_{;}^{\mu\nu}\right]\Bigg\}. \end{aligned} \tag{30}$$

The functions $V(s)$, $f^2(s)$, $a_i(s)$, $b_i(s)$ and $c_i(s)$ are given in (23), (21), (27) and (29). We observe that even after Weyl scaling the kinetic term for s is still rather complicated. We do not expect to achieve much more decoupling by a simple rescaling of s. Fortunately, the effective four-dimensional action simplifies considerably for some of the situations of interest.

4. The Friedmann universe

For our present universe and for most of its evolution since the big bang all relevant length scales are much larger than the Planck length M_{P}^{-1}. We therefore can neglect all higher derivative terms and the action (30) is well approximated by

$$S_{\text{F}} = -\int \mathrm{d}^4x\, g^{1/2}\left\{\frac{M_{\text{P}}^2}{16\pi}R - \tfrac{1}{2}f^2(s)s_{;\mu}s^{\mu}_{;} + V(s)\right\}. \tag{31}$$

Stability of Minkowski spacetime (and of the Friedmann universe) requires positive kinetic energy for the scalar field

$$f^2(s) > 0. \tag{32}$$

This allows to normalize the kinetic term in the standard way be introducing a rescaled scalar field φ

$$f^2(s) s_{;\mu} s^{\mu}_{;} = \varphi_{;\mu}\varphi^{\mu}_{;},$$

$$\varphi = \int_0^s \mathrm{d}s' f(s'). \tag{33}$$

By definition we have

$$\frac{\partial V}{\partial \varphi}(\varphi = 0) = \frac{\partial V}{\partial s}(s = 0) = 0. \tag{34}$$

The second requirement of stability is a positive (or zero) mass term for φ

$$\mu_\varphi^2 = \frac{\partial^2 V}{\partial \varphi^2}(\varphi = 0) = f^{-2}(0)\frac{\partial^2 V}{\partial s^2}(s = 0) \geqslant 0. \tag{35}$$

Finally, a vanishing cosmological constant needs

$$V(\varphi = 0) = V(s = 0) = 0. \tag{36}$$

If the requirements (32), (35) and (36) are fulfilled the scalar field will settle at its minimum at a very early stage of cosmology. If enough entropy is created to heat the universe (this will be discussed in the next section) the subsequent evolution of the universe is given by the standard hot big bang model. To a very good approximation we can completely neglect the scalar field when its mass term μ_φ^2 is large. (Corrections to the Friedmann universe are suppressed by powers of T/μ_φ.)

To be more quantitative we turn now to a specific model [1, 6] with internal space forming a D-dimensional sphere S^D. Condition (36) needs a fine tuning of the higher dimensional cosmological constant

$$\varepsilon = \tfrac{1}{4}D(D-1)\delta^2/\zeta,$$

$$\zeta = D(D-1)\alpha + (D-1)\beta + 2\gamma. \tag{37}$$

The ground state radius and M_{P} are

$$L_0^2 = \frac{2\zeta}{\delta}, \qquad \zeta > 0,$$

$$\frac{M_{\mathrm{P}}^2}{16\pi} = \frac{\chi}{\zeta}\delta, \qquad \chi = (D-1)\beta + 2\gamma > 0 \tag{38}$$

with

$$\mathring{R}_{abcd} = -\frac{\delta}{2\zeta}\left(\eta_{ac}\eta_{bd} - \eta_{ad}\eta_{bc}\right),$$

$$\mathring{R} = -D(D-1)\delta/2\zeta. \tag{39}$$

One finds for the scalar potential (23)

$$V(s) = \left(\frac{M_{\mathrm{P}}^2}{16\pi}\right)^2 \frac{D(D-1)}{4\zeta} \exp(-Ds)(1-\exp(-2s))^2(1+\sigma\exp(-2s))^{-2},$$

$$\sigma = -\frac{D(D-1)\alpha}{\zeta}. \tag{40}$$

For the kinetic term one obtains

$$f^2(0) = \frac{M_{\mathrm{P}}^2}{16\pi} F_0,$$

$$F_0 = \chi^{-2}\left\{12\zeta^2 + 4(D-6)\zeta\chi + (D^2 - 6D + 12)\chi^2 - 4D(D-3)\chi\gamma\right\}. \tag{41}$$

For $D > 3$, the kinetic energy is positive provided

$$\gamma < \frac{12\zeta^2 + 4(D-6)\zeta\chi + (D^2 - 6D + 12)\chi^2}{4D(D-3)\chi} \tag{42}$$

(the upper bound is positive). We may expand the scalar potential in powers of φ

$$V(\varphi) = \tfrac{1}{2}\mu_\varphi^2\varphi^2 + \tfrac{1}{6}\nu_\varphi\varphi^3 + \tfrac{1}{24}\lambda_\varphi\varphi^4 + \mathrm{O}\left(\varphi^5/M_{\mathrm{P}}\right) \tag{43}$$

and one finds

$$\mu_\varphi^2 = \frac{D(D-1)}{(1+\sigma)^2\zeta F_0}\frac{M_{\mathrm{P}}^2}{8\pi},$$

$$\nu_\varphi = \frac{3}{2}\frac{D(D-1)}{(1+\sigma)^3\zeta F_0^{3/2}}\frac{M_{\mathrm{P}}}{\sqrt{\pi}}\left\{(\tilde{A} - D)(1+\sigma) - 2\right\}. \tag{44}$$

($\tilde{A}$ is defined in the appendix (A.14).) The ratio $\mu_\varphi^2/\nu_\varphi \approx F_0^{1/2}M_{\mathrm{P}}/12D\sqrt{\pi}$ gives roughly the range of φ for which the polynomial expansion is a valid approximation. We note that μ_φ^2 is positive ($F_0 > 0$) and is very roughly of the order M_{P}^2. There is no reflection symmetry $\varphi \to -\varphi$.

Conditions (32) and (35) assure stability only for the coupled system of singlet and gravitation in the low momentum range. The complete stability discussion for the model under consideration has been carried out in ref. [4]. In our context, where we use this model only as a prototype imitating cosmology for more realistic models, we only require stability for the singlet (42) together with $\zeta > 0$, $\chi > 0$ (38). This assures a realistic cosmology for late times if nonsinglet modes are not excited.

5. Heating of the universe

We have seen that the effective action reduces to the Einstein-Hilbert action at low momenta. In addition there are the massless gauge fields of $SO(D+1)$. In a more realistic model there would also be fermions with mass much smaller than M_P. If early cosmology provides the required initial conditions one will end with the standard hot big bang cosmology. Assume that an inflationary period – which occurs for a reasonable choice of parameters in our model – ends at t_1 with a scale factor a exponentially big compared to the inverse Hubble parameter so that the a^{-2} term in the cosmological equations can be neglected compared to $H^2 = \dot{a}^2/a^2$ until today ($k = 0$ cosmology). This solves the horizon and flatness problem [7]. We also assume that inflation is responsible for homogeneity and isotropy up to effects of small density perturbations $\Delta\rho/\rho$ whose spectrum is calculable in our model [1, 3]. The transition from the inflationary phase to the Friedmann universe must be such as to produce enough entropy and furthermore, the universe must be heated to a sufficiently high temperature T_0 in order to subsequently create the observed baryon asymmetry [8]. If cosmic strings [9] are responsible for galaxy formation, they should be produced after inflation (or near the end of the inflationary phase). In grand unified models with symmetries like SU(5) monopoles are produced [10] by the breaking to $SU(3) \times SU(2) \times U(1)$. This symmetry breaking should occur before or during the inflationary phase so that monopoles are sufficiently diluted. Also, the heating of the universe should not produce strong density fluctuations.

We define T_0 to be the temperature to which the universe is heated after inflation, T_B the temperature at which baryons are produced, T_S the temperature characterizing the phase transition producing strings and T_G the temperature at which a grand unified symmetry like SU(5) (or another symmetry whose breaking leads to monopoles) would be restored. Realistic cosmology requires

$$T_0 > T_B\,, \tag{45}$$

$$T_0 > T_S\,, \tag{46}$$

$$T_0 < T_G\,. \tag{47}$$

If the last condition is violated, the grand unified symmetry will (again) be broken after inflation thus producing an unacceptable monopole abundance. We note, however, that in higher dimensional models the topology of the internal space may not be consistent with SU(5) symmetry even if the higher dimensional symmetry is of the grand unified type. In this case there is no danger of restoration of SU(5) and condition (47) can be dropped. Consistency of the four dimensional description allows T_0 to be on the order of the compactification scale M_c, but it should not be much higher. (Only a finite number of low mass modes from the infinite tower of four-dimensional fields should be in thermodynamic equilibrium.)

Estimates of T_B, T_S and T_G depend on details of the model and its symmetry breaking. If cosmic strings associated with the breaking of a local symmetry are to play a role in galaxy formation, T_S should be $\approx 4 \cdot 10^{16}$ GeV. Sufficient baryon asymmetry is produced in many models if the universe cools down from such high temperatures. On the other hand, T_G should be safely above T_S so as to avoid monopole production after inflation.

A possible scenario could have a compactification scale $M_c \approx 10^{17} - 10^{18}$ GeV at which the higher dimensional symmetry breaks to a four dimensional $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_G$ symmetry. The topology of the internal space may not be compatible with SU(5) symmetry so that there is no restriction from T_G even if the universe is heated to a temperature T_0 of the order of M_c. The abelian symmetry $U(1)_G$ could play the role of a generation group, in which case the scale M_1 of its spontaneous breaking should be a factor 10–20 smaller than M_c in order to produce a realistic fermion mass spectrum [11]. Spontaneous breaking of a U(1) group produces strings and the string tension would be of the right order of magnitude for the strings to be relevant for galaxy formation ($T_S \approx M_1 \approx \frac{1}{10} - \frac{1}{20} M_c$). Finally, the baryon asymmetry may be produced at $T_B \lesssim T_S$. The crucial point for this type of scenario is a heating temperature T_0 around or somewhat smaller than the compactification scale M_c.

Our present model with $SO(D+1)$ symmetry is not a realistic one (it has no chiral fermions) and we therefore did not attempt to calculate explicitly the scales of symmetry breaking. (Although in the present model for $D = 9$ the grand unified symmetry SO(10) would indeed be broken for a large range in parameter space.) We believe, however, that high heating temperatures T_0 around or somewhat smaller than the compactification scale are possible for a wide class of higher dimensional inflationary scenarios [1, 3] – in sharp contrast with most inflationary models discussed so far. The reason is simply that the scalar potential does not have a polynomial form – the flat exponential tail responsible for inflation does not imply a small mass term at the origin of φ. We found indeed a scalar mass on the order of the compactification scale (44). Also, the cubic and quartic couplings at the origin are not very small. In the remainder of this section we wish to demonstrate, using the (calculable) couplings of φ to nonsinglet fields, that T_0 is not very different from M_c for the scalar potential (40) of our model. We will neglect for this discussion all higher derivative terms and use (31). We do not expect that the inclusion of higher derivative terms will significantly change the qualitative results. We have the following picture for heating after the inflationary phase [12]: Once the scalar field has moved outside the flat tail of the potential it follows a damped oscillation around the minimum at $\varphi = 0$. (The motion towards $\varphi = 0$ could even be over-damped.) Its total energy density E, composed from potential and kinetic energy, determines the Hubble constant

$$H^2 = \frac{8\pi}{3M_P^2} E\,,$$

$$E = V(\varphi) + \tfrac{1}{2}\dot{\varphi}^2\,. \tag{48}$$

As long as interactions with other particles can be neglected, gravitational damping leads to a decrease in energy and this in turn lowers H

$$\dot{E} \approx -3H\dot{\varphi}^2\,. \tag{49}$$

Assume now that φ has cubic couplings to particles whose mass is smaller than its own mass μ_φ. Suppose that it decays into these particles with a decay rate Γ. As long as the lifetime $\tau = \Gamma^{-1}$ of φ is longer than the Hubble time H^{-1} this effect can be neglected for the motion of φ. However, once Γ and H become comparable the decay of φ induces a damping force comparable to the gravitational damping (49). The energy of the coherent motion of φ is converted into kinetic energy of its decay products and the entropy therefore increases. The scalar singlet φ itself has no renormalizable couplings to the "massless" gauge bosons and chiral fermions. (Unrenormalizable terms $\sim \varphi F_{\mu\nu}F^{\mu\nu}$ etc., however, are possible.) Cubic couplings to non-singlet heavy scalar fields χ or heavy non-singlet fermions χ_F are expected. If φ decays into χ-particles these in turn will decay into the massless gauge bosons, quarks and leptons thereby establishing thermodynamic equilibrium. Once E is essentially converted into radiation energy the motion of φ becomes irrelevant and further evolution of the universe corresponds to a radiation dominated Friedmann universe.

We note that higher dimensional models lead to an infinite variety of χ particles. They have cubic (and higher order) couplings to φ. The requirement that there be some χ particles with masses smaller than μ_φ is fulfilled in our model for a wide range of parameters [4]. There is, in fact, no reason why φ should be the lightest one amongst the particles with masses of the order M_c★.

The temperature T_0 may be roughly estimated by assuming that all the φ field energy $E(t_2)$ is converted into radiation energy once Γ and $H(t_2)$ become equal at

★ Even if φ would be the lightest among the heavy particles, entropy may still be created through quartic interactions – if the mass gap between χ particles and φ is not too large – or by interactions of φ with massless particles involving intermediate (virtual) χ particles or other nonrenormalizable interactions.

time t_2.

$$N^* T_0^4 = E(t_2) = \frac{3H^2(t_2) M_P^2}{8\pi} = \frac{3\Gamma^2 M_P^2}{8\pi},$$

$$T_0 = \left(\frac{90}{8\pi^3 N}\right)^{1/4} M_P^{1/2} \Gamma^{1/2} \approx \tfrac{1}{4} M_P^{1/2} \Gamma^{1/2}. \tag{50}$$

(Here $N^* = \frac{1}{30}\pi^2 N$, where N denotes the number of effective degrees of freedom in equilibrium at T_0. N is of the order 10^2 for very early cosmology.) The partial width for φ decaying into two scalars χ_i (neglecting the mass of χ_i) is

$$\Gamma_{\chi_i} \approx \frac{1}{16\pi} \frac{\nu_{\chi_i}^2}{\mu_\varphi}. \tag{51}$$

Here μ_φ is given by (44) and we have calculated a typical cubic coupling ν_χ (as well as the quartic coupling λ_χ) in the appendix. We note that χ_i may belong to a representation of fairly high dimension (54 for the example in the appendix) which multiplies the previous decay rate formula. Also, φ may couple to several boson and fermion representations (for which the decay rates are similar since the typical Yukawa couplings of φ to heavy fermions are of the order of the gauge coupling). A rough estimate of Γ (also accounting for the mass of χ particles) is

$$\Gamma \approx \frac{\nu_\chi^2}{\mu_\varphi} \qquad \text{or} \qquad h^2 \mu_\varphi \tag{52}$$

with ν_χ a typical cubic scalar coupling and h a typical Yukawa coupling. In principle, both ν_χ and μ_φ depend on the expectation value of φ. For our scenario however, this effect is not very strong and we evaluate the quantities at the minimum at $\varphi = 0$. (Correction terms to μ_φ^2 are of the order $\nu_\varphi \varphi$ and for ν_χ of the order $\lambda_\chi \varphi$.)

We present in table 1 the relevant mass scales for the case $D = 9$ with two sets of values of the parameters. The first set $\gamma = -0.9\beta$, $\alpha = \beta > 0$ leads to an inflationary scenario [1]. For these values the "ground state" $\mathscr{M}^4 \times S^9$ is classically unstable [4] (only the singlet sector is stable as required for realistic cosmological equations once non-singlet excitations are neglected). In particular the field S_1 discussed in the appendix has both negative μ_χ^2 and negative kinetic term. (Also the absolute value of μ_χ is larger than μ_φ.) We have nevertheless calculated the cubic coupling ν_χ to get a feeling about the orders of magnitude involved. More generally, we find that there is a conflict between parameter values required for sufficient inflation and those that are needed for low momentum stability of the ground state. We may

TABLE 1

$D=9$	$\gamma=-0.9\beta$, $\alpha=\beta$, $\beta>0$	$\gamma=5\beta$, $\alpha=0$, $\beta>0$
L_0^{-1}	$4\beta^{-1/2}\cdot 10^{17}$ GeV	$2\beta^{-1/2}\cdot 10^{17}$ GeV
$\lvert\mu_\varphi\rvert$	$1.2\ L_0^{-1}$	$10\ L_0^{-1}$
$\lvert\mu_\chi\rvert$	$3.2\ L_0^{-1}$	$3\ L_0^{-1}$
ν_χ^2	$3.9\beta^{-1}\ L_0^{-2}$	$10\beta^{-1}\ L_0^{-2}$
Γ	$3.2\beta^{-1}\ L_0^{-1}$ (*)	$\beta^{-1}\ L_0^{-1}$
T_0	$1.3\beta^{-3/4}\cdot 10^{18}$ GeV (*)	$3.6\beta^{-3/4}\cdot 10^{17}$ GeV
T_M^*	$2.8\beta^{-1/4}\cdot 10^{17}$ GeV	$2.2\beta^{-1/4}\cdot 10^{17}$ GeV

*For realistic cases with $\mu_\varphi > 2\mu_\chi$ we would expect this value to be lowered by a factor $\mu_\varphi/2\mu_\chi \approx 1/6$.

interpret this to mean that for the "inflationary parameters" the SO(10) symmetry must be spontaneously broken, but we will not pursue the question of the true ground state for these parameters in our toy model. A second set of parameters $\alpha=0$, $\gamma=5\beta>0$ was chosen so that $\mu_\chi < \frac{1}{2}\mu_\varphi$. For these values all modes considered in ref. [4] are stable at low momenta. Both parameter sets give similar values for Γ and T^0. We conclude that typical values of Γ are of order $L_0^{-1} \approx$ a few 10^{17} GeV and typical heating temperatures are also a few times 10^{17} GeV.

We are confident that these results hold qualitatively for a wide range of parameters in our toy model. They may be characteristic for many other higher dimensional models. If the cubic and quartic couplings are not small the typical value of T_0 is of order L_0^{-1}. On the other hand the overall coupling strength of dimensionless cubic couplings is of the order of the gauge coupling g, dimensionless quartic couplings are $\sim g^2$ and cubic scalar couplings $\sim gL_0^{-1}$. (In our model $g^2 \sim \beta^{-1}$.) For realistic models g^2 cannot be too small ($g^2 \approx \frac{1}{4}$) and both L_0^{-1} and T_0 are therefore in a typical range 10^{17}–10^{18} GeV.

One may obtain an independent upper bound on the heating temperature if one assumes that after the end of the inflationary period the Hubble parameter monotonically decreases. In this case the radiation energy density cannot exceed $(3/8\pi)H^2(t_1)M_P^2$ where $H(t_1)$ is the Hubble parameter at the end of the inflationary period. This gives the bound

$$T_0 = \eta T_M\,,$$
$$T_M \approx \tfrac{1}{4}M_P^{1/2}H(t_1)^{1/2}\,,$$
$$\eta \leqslant 1 \tag{53}$$

(the parameter η may be called heating efficiency). During the inflationary period

H is given [1] by the potential $V(s)$ (eq. (40))

$$H^2 = \frac{8\pi}{3}\frac{V}{M_P^2} \tag{54}$$

and one has

$$T_M \leqslant (N^*)^{-1/4} V_M^{1/4} = T_M^* \tag{55}$$

with V_M the maximum of V in the region relevant for inflation. This maximum is at z_M with

$$D\sigma z_M^2 + z_M(D + 4 - D\sigma + 4\sigma) - D = 0\,,$$

$$z = \exp(-2s)\,. \tag{56}$$

The maximum temperature does not depend strongly on the specific form of V

$$T_M^* \approx 6\beta^{-1/4}\hat{V}_M^{1/4} \cdot 10^{17}\ \text{GeV}\,,$$

$$V_M = \left(\frac{M_P^2}{16\pi}\right)^2 \beta^{-1}\hat{V}_M\,. \tag{57}$$

For the two sets of parameter values discussed above we find $T_M^* = 2.8\beta^{-1/4} \cdot 10^{17}$ GeV and $T_M^* = 2.2\beta^{-1/4} \cdot 10^{17}$ GeV respectively. Comparison with T_0 (table 1) shows that the heating efficiency η is near its maximal value $\eta = 1$. The maximum temperature T_M (53) was also estimated for this model by Pollock [3], who estimated $H(t_1)$ from the requirement that density fluctuations $\Delta\rho/\rho$ are in an acceptable range. He finds typical values $T_M \approx$ a few times 10^{17} GeV. A rather consistent overall picture arises: The heating temperature is a few times 10^{17} GeV. The heating efficiency is maximal and heating takes place very fast ($\Gamma \gtrsim H(t_1)$). The temperature is high enough so that a subsequent phase transition at $T_S \approx 4.10^{16}$ GeV could produce superheavy cosmic strings.

Since $T_0 \approx L_0^{-1}$ no unacceptable monopole number needs to be produced. There seem to be no particular difficulties to produce a baryon asymmetry. We expect that a similar picture can be realized in a more general class of higher dimensional models.

6. Conclusions

Let us compare our picture for heating after inflation in higher dimensional theories with the standard four-dimensional inflationary scenarios. We note, first of all, that the characteristic scale in the transition is the compactification scale L_0^{-1}, typically on the order of a few times 10^{17} GeV. This is two orders of magnitude higher than a standard grand unification scale. Presumably, this still is sufficiently below the Planck mass so that a classical description in terms of a few invariants of the effective action can be trusted. The mass term of the inflaton, which determines the frequency of its oscillations around the ground state value, is of the order of the compactification scale.

After inflation is over, the heating of the universe critically depends on the couplings of the inflaton field to other matter fields. In the standard four-dimensional inflationary scenarios these couplings can be freely chosen and the discussion is restricted, in general, to renormalizable polynomial interactions. For polynomial interactions the strength of these couplings essentially is the same during the periods of inflation and subsequent heating. This usually leads to the following dilemma: The couplings must be very small during inflation so as not to disturb the flatness of the inflaton potential and thereby ensure that inflation will last long enough. On the other hand, small couplings during the heating period lead to comparatively low heating temperatures, certainly much too low for the production of cosmic strings relevant for galaxy formation, and often problematic for a creation of baryon asymmetry.

In higher dimensional models all these couplings are calculable. They turn out to be relatively large during the heating period. The mass term for the inflaton field typically is of the order L_0^{-2}. A high heating temperature of the order of the compactification scale is predicted. Nevertheless, the dilemma noted above does not occur in higher dimensional inflation. Indeed, the strength of all interactions decreases strongly if the internal length scale becomes larger than the compactification length L_0 (which is relevant for the ground state and for the heating period). This is related to the fact that before Weyl scaling, the four-dimensional Newton constant is proportional to the inverse volume of the internal space (compare (A.10)). It is one of the characteristic features of our higher-dimensional inflationary scenario that the internal radius is significantly larger than L_0 during the inflationary phase. The couplings of the inflaton field to other fields are therefore predicted to be very small during inflation. This may be equivalently expressed by saying that higher dimensional models do not predict polynomial interactions, but rather a specific exponential behaviour for the strength of the interactions.

There is another notable observation in our particular model which may have more general significance. We found that the parameter range (in α, β, γ) required for sufficient inflation did not overlap with the range for classical stability of the ground state. Although the singlet sector is stable for a range of parameters compatible with inflation, so that the model is well suited for a study of cosmological solutions, the appearance of unstable non-singlet modes (besides other failures) excludes it from being considered as a fully realistic model. Taking into account the non-singlet modes the cosmology of the action (1) with a choice of parameters compatible with inflation would asymptotically not approach the state $\mathscr{M}^4 \times S^D$ but some other state with a different symmetry – one which we do not know.

This brings us to the question of selection criteria for the "true" ground state of higher dimensional models. The field equations obtained from the effective action of a d-dimensional field theory (or a string theory) may admit many solutions with (approximate*) Poincaré symmetry P_E, $E \leqslant d$. They may be interpreted as "compactifications" with a flat E dimensional space-time embedded in a d dimensional space-time. Many of these solutions may be classically stable and possible quantum mechanical tunnelling rates to other solutions may vanish or be small compared to the inverse age of the universe. All such solutions can be considered as candidates for ground states. (It is even conceivable that there is a continuous spectrum of such solutions, depending on initial conditions for the relevant set of differential equations – compare the discussion of non-compact internal spaces in refs. [13, 11]. This would imitate the free continuous parameters α, β, γ in our model becoming dynamical quantities.) For every stable candidate ground state there would be an associated Friedmann universe approaching it asymptotically if the initial conditions for the matter density are appropriately set. (We discard here possible complications from stable massless scalar modes. The conditions for being an "attractor" universe in the words of Maeda [14] are then essentially equivalent to classical stability of the asymptotic flat solution as required in ref. [1].)

Will the universe evolve to one of these candidate ground states, and if so, to which one? It is essentially a cosmological question if the required initial value for the matter density being very near the critical density is generated. Early cosmology must create on effective E dimensional universe where the curvature for the $E-1$ spacelike coordinates is very small compared to the Hubble parameter H^2 – otherwise this solution would recollapse in a time of the order H^{-1} and never become flat. A large amount of entropy (with $\rho \approx M_P^2 H^2$) should also be created. This is exactly what inflation and subsequent heating of the universe are supposed to achieve.

Inflation in the sense of a fast (exponential) expansion of some of the spacelike dimensions compared to the other (internal) ones may be a relatively frequent phenomenon – in the sense that the field equations admit many solutions of this type. Although a special choice of parameters (initial values) may be necessary, no extreme fine tuning is needed to obtain sufficient inflation. (In our example one needs $\gamma \approx -\beta$ to within a 10% range.) Our example suggests, however, that not every inflationary solution finally ends in an asymptotic approach to a stable ground state solution. If inflation does not last forever (this may anyhow be excluded by instabilities of de Sitter space) the Hubble parameter must finally end its almost constant behaviour. If there is no classically stable solution with H decreasing to zero, to which the universe can make a transition after inflation, it is plausible that the universe will recollapse and after a short time all characteristic scales will again be of order M_P.

We can now formulate a criterion for the ground state: The ground state should be a stable solution with (approximate) Poincaré symmetry P_E for the field equa-

* This includes the case of (anti)-de Sitter symmetry if the cosmological constant is sufficiently small.

tions derived from the effective action. In addition there must exist an associated inflationary solution ending through transition to a universe which approaches asymptotically this ground state. We may call this criterion "evolutionary selection". If there is no inflation, none of the space dimensions will ever grow big. If there is no transition to a stable asymptotic solution the large characteristic length scales created by inflation presumably recollapse. Combining the requirements of inflation and of a stable asymptotic state with approximate Poincaré symmetry may be very restrictive – and perhaps a more or less unique universe can be singled out.

There are of course many other solutions without asymptotic approximate Poincaré symmetry. If all their characteristic length and time scales are near the Planck length we would hardly call them "ground state" because of the lack of static properties (approximate invariance under time translations) and spatially extended homogeneous structure, which are both required to give meaning to the distinction between a local excitation and the ground state itself. (Otherwise all excitations could equally well be called "ground states".) Imagine that the universe was originally characterized by such "random solutions" with length scales of order M_P^{-1}. (Classical description may not make sense at this stage and it may be more appropriate to speak of a soup of quantum bubbles.) If a region of the universe (by chance or, if preferred, by tunnelling from "nothing" [15]) fullfils the initial conditions of an inflationary solution, some space dimensions will expand exponentially. At the end of inflation they may recollapse into the quantum bubble soup – no interesting (classical) structures would emerge with such a short lived "excursion" from the random state. As a possible alternative, the large space dimensions could survive (and even further expand) by the approach to one of the ground state candidates. This would create a long living universe capable to produce structure and eventually intelligence. For this picture it is not important if our universe evolved from a small region of the original soup (to the extent that the concept "small" makes sense at the beginning of the universe...) or if it covers the whole universe. Also, our universe could be an extended object in a higher dimensional world [16] (described by a non-compact internal space) or rather have the more traditional topological structure $\mathbb{R}^E \times$ compact internal space.

Entropy production (heating) is a crucial ingredient in a possible transition to a Friedmann universe. We conclude from our investigations that this is probably not a very severe additional restriction – at least not in four dimensions.

Appendix

MASS AND COUPLINGS OF NON-SINGLET SCALARS

In addition to the scalar singlet φ our model contains infinitely many scalars in nontrivial representations of the gauge symmetry $SO(D+1)$. The sign of their mass and kinetic term decides on classical stability of the ground state. Their couplings to the scalar singlet φ are important for their possible production by decay (or fast oscillations) of φ and therefore for the heating of the universe at the end of the inflationary phase. We denote such a non-singlet scalar field by χ. All couplings must be at least quadratic in χ due to SO($D+1$) symmetry. Possible couplings are $\varphi\chi^2$, $\varphi^2\chi^2$ etc...

For a ground state with given symmetry there is an elegant way of calculating simultaneously the masses of non-singlet fields χ and all couplings to singlets of the form $\varphi^N\chi^2$. We only need to compute the contributions to the effective action which are quadratic in χ. We evaluate them for arbitrary configurations of fields which are singlets with respect to the symmetry of the ground state. Neglecting higher derivatives of χ and assuming χ to be a complex field, the effective action has the form

$$S_\chi^{(2)} = \varepsilon_\chi \int \mathrm{d}^4x\, g^{1/2} \left\{ D^\mu\chi^* D_\mu\chi - M_\chi^2 \chi^*\chi \right\}. \tag{A.1}$$

Here we have scaled χ so that $\varepsilon_\chi = \pm 1$. For negative ε_χ the kinetic energy is negative indicating classical instability. The "mass term" M_χ^2 is a functional depending on arbitrary background values of the singlet fields φ, $g_{\mu\nu}$ etc. It contains all information about interactions with singlets which are quadratic in χ and do not involve derivatives of χ.

In order to extract the mass term for χ and its cubic and quartic couplings to φ we evaluate M_χ^2 for constant φ and vanishing (four dimensional) curvature ($R_{\mu\nu\rho\sigma} = 0$), thus reducing M_χ^2 to a single function of φ. For the ground-state at $\varphi = 0$ the mass term $\mu_\chi^2|\chi|^2$ and the cubic and quartic couplings $\nu_\chi\varphi|\chi^2|$, $\frac{1}{2}\lambda_\chi\varphi^2|\chi|^2$ are given by

$$\mu_\chi^2 = M_\chi^2(\varphi = 0), \tag{A.2}$$

$$\nu_\chi = \frac{\mathrm{d}M_\chi^2}{\mathrm{d}\varphi}(\varphi = 0), \tag{A.3}$$

$$\lambda_\chi = \frac{\mathrm{d}^2 M_\chi^2}{\mathrm{d}\varphi^2}(\varphi = 0). \tag{A.4}$$

(This procedure can be generalized for the analysis of stability of the inflationary phase and a calculation of effective interactions during this phase. We would have to evaluate M_χ^2 for a curved background and expand around nonzero φ.)

The sign of ε_χ and the field equations can be read off from the work of ref. [4]. As an illustration we look at the scalar in the symmetric second rank tensor representation of SO($D+1$) corresponding to $S_1(1=2)$ in ref. [4]. Its field equations for flat four-dimensional space and given constant radius L of the internal space are ($\varepsilon_\chi = -\text{sign}(a_2)$)

$$\hat{g}^{\mu\nu}\partial_\mu\partial_\nu\chi - \frac{a_3}{a_2}\chi = 0, \tag{A.5}$$

$$L^{-2} = L_0^{-2}\exp(-2s) = \frac{\delta}{2\zeta}\exp(-2s), \tag{A.6}$$

$$a_2 = -\left(c_4 + c_5\exp(-2s)\right)\frac{\delta}{2\zeta},$$

$$a_3 = \left(c_1 + c_2\exp(-2s) + c_3\exp(-4s)\right)\frac{\delta^2}{4\zeta^2}; \tag{A.7}$$

$$c_1 = -\tfrac{1}{2}D(D-1)\zeta,$$

$$c_2 = (D^2 - D + 4)\zeta,$$

$$c_3 = \left(-\tfrac{1}{2}D^4 + D^3 - \frac{9}{2}D^2 + 4D\right)\alpha$$

$$+\left(-\tfrac{1}{2}D^3 + 3D^2 - \tfrac{1}{2}D + 6\right)\beta + (11D^2 + 9D - 12)\gamma,$$

$$c_4 = \zeta,$$

$$c_5 = -D(D-1)\alpha + 2(D+1)\beta + 2(6D+1)\gamma,$$

$$c_1 + c_2 + c_3 = \left(\tfrac{3}{2}D^2 + \frac{9}{2}D + 2\right)\beta + 4(3D^2 + 2D - 1)\gamma,$$

$$c_4 + c_5 = (3D+1)(\beta + 4\gamma). \tag{A.8}$$

In eq. (A.5) the metric is the higher dimensional metric $\hat{g}_{\mu\nu}$ and we have to correct this by the Weyl scaling (18). In terms of the dimensionless variable s one finds

$$M_\chi^2(s) = -w^2(s)\frac{a_3(s)}{a_2(s)}. \tag{A.9}$$

We note that $M_\chi^2(s)$ has the typical exponential dependence on s and vanishes for large s like

$$\lim_{s\to\infty} M_\chi^2(s) \to -\tfrac{1}{2}D(D-1)(1+\sigma)\exp(-Ds)L_0^{-2}. \tag{A.10}$$

This indicates that all couplings of s to nonsinglet fields are exponentially suppressed during the inflationary phase of ref. [1]. Radiative corrections from loops involving nonsinglet fields will therefore not spoil the exponential flatness of the potential $W(s)$.

For vanishing four-dimensional curvature we can use relation (33) for $\mathrm{d}\varphi/\mathrm{d}s = f(s)$ and it is now straightforward to calculate the mass μ_χ^2 and the couplings ν_χ, λ_χ:

$$\mu_\chi^2 = \frac{c_1+c_2+c_3}{c_4+c_5}\frac{\delta}{2\zeta}, \tag{A.11}$$

$$\nu_\chi = (c_4+c_5)^{-1}\left\{\left(-D+\frac{2\sigma}{1+\sigma}\right)(c_1+c_2+c_3) + 2\left[\frac{(c_1+c_2+c_3)c_5}{c_4+c_5} - c_2 - 2c_3\right]\right\} f^{-1}(0)\frac{\delta}{2\zeta}, \tag{A.12}$$

$$\begin{aligned}\lambda_\chi = {} & (\tilde{A}-D)f^{-1}(0)\nu_\chi \\ & + f^{-2}(0)\frac{\delta}{2\zeta}\left\{-\frac{4\sigma}{(1+\sigma)^2}\frac{(c_1+c_2+c_3)}{(c_4+c_5)}\right. \\ & + 2\frac{(D+2/(1+\sigma))c_2 + (2D+4(2+\sigma)/(1+\sigma))c_3}{c_4+c_5} \\ & - \frac{8c_5(c_2+2c_3)}{(c_4+c_5)^2} - 2\left(D+\frac{2}{1+\sigma}\right)\frac{c_5(c_1+c_2+c_3)}{(c_4+c_5)^2} \\ & \left. + \frac{8c_5^2(c_1+c_2+c_3)}{(c_4+c_5)^3}\right\},\end{aligned} \tag{A.13}$$

$$\tilde{A} = -2\{2D(D-1)(D-6)\alpha^2 + D(D-1)(D-2)\alpha\beta + 2D\alpha\gamma + 2(D-1)\beta^2 + 2(D+1)\beta\gamma + 4\gamma^2\}$$
$$\times\left\{12D(D-1)\alpha^2 + 4D(D-1)\alpha\beta - 2D(2D-7)\alpha\gamma + (D-1)(D-2)\beta^2 + 4\beta\gamma - \frac{4(D-4)}{D-1}\gamma^2\right\}^{-1}. \qquad (A.14)$$

To get a feeling for typical orders of magnitude, we take $D=9$ and $\alpha=0$. One obtains

$$\mu_\chi^2 = \frac{41\beta + 260\gamma}{7(\beta+4\gamma)} L_0^{-2}, \qquad (A.15)$$

$$\nu_\chi = -\frac{(553 - 82\tilde{c})\beta + (3376 - 520\tilde{c})\gamma}{7(\beta+4\gamma)} f^{-1}(0) L_0^{-2}, \qquad (A.16)$$

$$\lambda_\chi = \left\{(1784 - 1638\tilde{c} + 328\tilde{c}^2)\beta + (13316 - 8864\tilde{c} + 2080\tilde{c}^2)\gamma + 8[(553 - 82\tilde{c})\beta + (3376 - 520\tilde{c})\gamma] \times \frac{120\beta^2 + 18\beta\gamma - 7\gamma^2}{112\beta^2 + 8\beta\gamma - 5\gamma^2}\right\} \frac{f^{-2}(0) L_0^{-2}}{7(\beta+4\gamma)} \qquad (A.17)$$

with

$$\tilde{c} = \frac{c_5}{c_4 + c_5} = \frac{5}{14}\,\frac{2\beta + 11\gamma}{\beta + 4\gamma}, \qquad (A.18)$$

$$f^{-2}(0) L_0^{-2} = (1392\beta - 84\gamma)^{-1}. \qquad (A.19)$$

One finds for two characteristic cases the following order of magnitude for the quartic coupling

$$|\gamma| \ll |\beta|: \qquad \lambda_\chi \approx \frac{1}{2\beta},$$

$$|\gamma| \gtrsim |\beta|: \qquad \lambda_\chi \approx \frac{3}{4\beta}. \qquad (A.20)$$

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