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Upon the determination of heavy quark fragmentation functions in e^+e^- annihilation

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Abstract. Results on the fragmentation of heavy quarks from analyses of inclusive lepton production in e^+e^- annihilation are studied. The use of various fragmentation variables is closely examined and their differences resolved, providing a common basis for comparison of experimental results. The mean value of the fraction of available energy-momentum carried by the primary heavy hadron, defined as

$$z=\frac{(E+p_{\|})_{\text{hadron}}}{(E+p)_{\text{quark}}},$$

is determined to be $\langle z\rangle_c=0.67\pm0.02\pm0.02$ and $\langle z\rangle_b=0.83\pm0.01\pm0.02$ for an unknown mixture of charmed and bottom flavoured hadrons respectively. The corresponding values of the parameter ε_Q of the Peterson fragmentation function are

$$\varepsilon_c=0.06{}^{-0.01-0.01}_{+0.02+0.02} \text{ and } \varepsilon_b=0.006{}^{-0.001-0.002}_{+0.001+0.002}.$$

The ratio $\varepsilon_c/\varepsilon_b$ can be approximately related to M_b^2/M_c^2 giving a value of $10{}^{+4+5}_{-2-4}$, in agreement with an expectation of ~ 10. Measurements of the charged multiplicity of hadronic events containing heavy quark jets are investigated in terms of the mean value of z.

Introduction

A number of experiments in e^+e^- annihilation have now reported results on the fragmentation of heavy quarks using a variety of methods. Despite the wealth of information available, comparisons of the results are hindered by the many different formulations of the fragmentation variable. A complete evaluation of the available data cannot thus be made, until the differences between these formulations are unravelled. Such an attempt is presented in this paper with particular emphasis on the results arising from analyses of inclusive lepton production.

The Peterson fragmentation function

The fragmentation of heavy quarks into heavy hadrons is of both theoretical and experimental interest. An understanding of the underlying process provides knowledge of the inclusive hadron spectrum and its energy dependence in e^+e^- annihilation. The fragmentation of quarks (and gluons) into hadrons occurs at large distances where perturbative Quantum Chromodynamics (QCD) [1] no longer applies. To this effect, non-perturbative models, such as the independent jet [2] and the colour string models [3], are introduced to describe the hadronisation process. The development of the longitudinal fragmentation process in these models, is parametrised by a scaling function $f(z)$, where z is the fraction of available energy-momentum, $(E+p)$, carried by the primary hadron. The actual form of this function has been, and still is, the subject of theoretical and experimental endeavour. Originally it was assumed that $f(z)$ for heavy quarks, Q, was similar to that for light quarks, q, which fragment principally into pions and kaons, with a distribution of z which steeply falls as z increases [4]. However, kinematic considerations [5] for a heavy quark fragmenting into a hadron ($Q\bar{q}$ or Qqq) suggest that the momentum of the heavy quark is retained by the hadron containing Q, leading to a 'harder' distribution in z (i.e. peaked towards higher values of z) than for the light quarks, q. Pursuing these arguments and calculating the transition probability for the process $Q\rightarrow Q\bar{q}+q$, Peterson et al. developed the following fragmentation function [6]:

$$f(z) \propto \frac{1}{z\left[1-\frac{1}{z}-\frac{\varepsilon_Q}{(1-z)}\right]^2} \tag{1}$$

where z is defined as

$$z=\frac{(E+p_{\parallel})_{\text{hadron}}}{(E+p)_{\text{quark}}}; \tag{2}$$

$(E+p_{\parallel})_{\text{hadron}}$ is the sum of the energy and momentum component parallel to the fragmentation direction carried by the primary hadron. $(E+p)_{\text{quark}}$ is the energy-momentum of the quark after accounting for initial state radiation, gluon bremsstrahlung and photon radiation in the final state. The parameter ε_Q, is, for each heavy quark, Q, expected to have a value approximately equal to

$$\varepsilon_Q \approx \frac{M_q^2}{M_Q^2} \tag{3}$$

i.e. the ratio of the squares of the masses of the light and heavy quarks forming the primary (or leading) meson.

Although other forms of the fragmentation function have been proposed [2–3; 7], the Peterson function has been widely adopted in analyses determining the 'hardness' of heavy quark fragmentation functions; its biggest attraction being that it has only one free parameter, ε_Q, which is to be determined experimentally for each heavy quark, Q. The first experimental results to indicate a hard fragmentation for heavy quarks were those obtained from analyses of the momentum spectra of charmed hadrons [8].

Heavy quark fragmentation from charmed hadron production

In the continuum of e^+e^- annihilation, the heavy c quarks are plentifully produced and their fragmentation into the charmed particles, D, D^*, $D_s(F)$, $D_s^*(F^*)$, Λ_c, has been directly observed. In particular, much is known of the $c \to D^*$ process through the successful reconstruction of the hadronic decays of the D^*. The inclusive D^* cross section is, however, usually determined as a function of the fragmentation variable, x_E (4) or x_p (5) rather than z (2), as these variables, unlike z, are experimentally accessible on an event by event basis.

$$x_E=\frac{E_{\text{hadron}}}{E_{\text{beam}}} \tag{4}$$

$$x_p=\frac{p_{\text{hadron}}}{\sqrt{E_{\text{beam}}^2-m_{\text{hadron}}^2}}. \tag{5}$$

Clearly, x_E or x_p is not the same as z when the effects of initial state radiation and gluon bremsstrahlung are considered. (The effect of final state radiation is relatively insignificant.) These processes result in a quark energy which is less than the energy of the incoming beam of electrons or positrons, E_{beam}, and therefore, by definition, $x_E, x_p \leq z$. Despite this significant difference between the variables, the x_E or x_p distribution is, nonetheless, fitted directly to the Peterson form of the fragmentation function (with x_E, x_p replacing z in (1)) and the mean value of the variable x_E, x_p and equivalently $\varepsilon(x_E), \varepsilon(x_p)$ are quoted. Corrections therefore have to be applied to relate the measured distributions of $f(x)$ to the theoretical distributions of $f(z)$. This has been done in [8] which shows that of the energy of the primary c quark (after allowing for the effects of initial state radiation and gluon bremsstrahlung), the resulting D^* retains a fraction of $\langle z \rangle = 0.70 \pm 0.01 \pm 0.03$. It is also illustrated in [8] that the Peterson functional form does not provide an adequate description of the measured x_E spectrum, but is well suited to parametrising the underlying z spectrum.

Heavy quark fragmentation from inclusive lepton production

The fragmentation of b quarks into b flavoured mesons and baryons in the e^+e^- annihilation continuum is, on the other hand, far less well explored due to the small b quark cross section and the very inefficient reconstruction of the b flavoured hadrons. However, significant contributions towards an understanding of both the b and c fragmentation have been made from studies of inclusive lepton production in $c\bar{c}$ and $b\bar{b}$ events. Such processes are described by the fragmentation of the heavy quark into a heavy hadron which

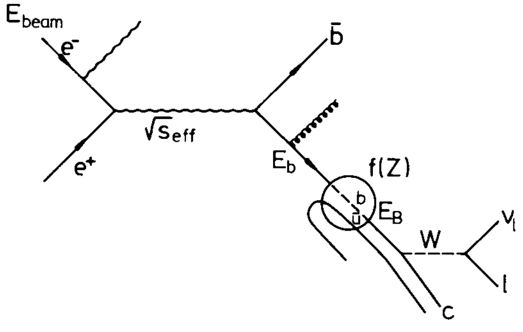

Fig. 1. The fragmentation and semileptonic decay of a b quark. A b quark is produced with energy E_b by the reaction $e^+e^- \to b\bar{b}(g)(g)$. The b fragments into a B meson of energy $E_B = zE_b$ with probability $f(z)$. The B meson subsequently decays semileptonically, $B \to l\nu_l X$, with the lepton momenta reflecting the z of the primary B

subsequently undergoes a semi-leptonic decay. This is depicted in Fig. 1 for a B meson. The lepton momentum, $p(l)$, being dependent upon the momentum of the parent hadron, contains information on the fragmentation of the heavy quark. The transverse component of the lepton momentum relative to the jet axis, $p_{\perp}(l)$ facilitates separation of the quark flavours. This separation may be aided by using an event shape variable. Typical analyses thus proceed by deducing the $p(l)$, $p_{\perp}(l)$ spectra of the quark flavours* according to the Peterson fragmentation function (for various values of ε_Q), and fitting these to the corresponding spectra of the data. Such statistical analyses do not however restrict the choice of fragmentation variable; although (2) is theoretically preferred, other variables in addition to (2), and (4), have been used in analyses of inclusive leptons:

$$z_E = \frac{E_{\text{hadron}}}{E_{\text{quark}}} \approx z \quad (6)$$

$$x_\gamma = \frac{2E_{\text{hadron}}}{\sqrt{s_{\text{effective}}}} \quad (7)$$

where $\sqrt{s_{\text{effective}}}$ is the energy of the virtual photon produced in the e^+e^- collision after accounting for initial state radiation (and therefore $x_E \leqq x_\gamma \leqq z$, z_E).

The existence of many fragmentation variables has, not surprisingly, led to confusion in the past**. There are however further complications. All fragmentation variables described so far are – in analyses of inclusive leptons – reconstructed from the Monte Carlo model used; i.e. once an event has been generated, the quantities E_{hadron}, E_{quark}, E_{beam} etc. are accessed and the fractions z, z_E, x_E are calculated. In one exception [11] however, the fraction, z is not reconstructed from the generated Monte Carlo event, but is taken directly from that allocated by the primordial fragmentation function. At this point it is necessary to distinguish between the two, as their values differ for reasons dependent upon the technicalities of Monte Carlo models; z(rec) and z(pri) thus refer to the reconstructed and primordial values of z respectively.

There is an inherent problem within independent jet models that stems from the creation of massive jets of hadrons from massless quarks, and leads to the non-conservation of energy and momentum during the process. To rectify this, certain technical modifications are applied in order to provide final states that are balanced in energy and momentum. These amendments consist of a rescaling in the particle momenta and energies after completion of the fragmentation process, leading to an inequality between the values of z(pri) and z(rec) which is particularly significant in events in which a hard gluon is emitted. In string models, fragmentation proceeds along the colour-flux lines connecting the partons rather than along the actual parton direction (as in independent jet models). By construction, energy and momentum are conserved locally at each step of the particle generation process. The recipe required to achieve this however, constrains the energy-momentum of the produced hadron, leading to a slight difference between the values of z(pri) and z(rec). In addition, the sum of the energy and momentum available for the formation of the primary hadron is not strictly interpreted as the energy-momentum of the primary quark (as in (2)), but rather as the energy-momentum component of the remaining, unfragmented system. The consequence of this different definition of 'available energy' in the string model, is that in some cases of hard gluon emission, the produced hadron can have a larger energy-momentum than the primary quark, leading to a value of z(rec), as defined by (2), which is greater than unity. Whatever the fragmentation model however, the value of z of the primary hadron given by the primordial fragmentation function is not, in general, identical to that which is reconstructed from the parton and hadron final state momenta and energies. The value of z(rec) for a given event, is generally larger than z(pri), with the magnitude of the difference depending on the respective strength of the gluon coupling constant (α_s) and upon the definition (and value) of the cut-off parameter used to resolve between partons. The resulting shift in the overall mean value of z, from $\langle z(\text{pri})\rangle$ to $\langle z(\text{rec})\rangle$ does not differ greatly between the two types of fragmentation model, after allowing for the different values of α_s required by the two models to describe the data.

The situation is thus somewhat confused. Experiments unfold the $p(l)$ distribution as a function of one of a number of fragmentation variables, z(pri), z(rec) or x_E, and quote the corresponding values of

* The $p(l)$, $p_{\perp}(l)$ distributions of the quark flavours are obtained from a Monte Carlo simulation model. The distributions are mainly governed by the mass of the quark. There are, however, other influences. For a given value of z, the $p(l)$ distribution depends slightly on the identity of the parent hadron. A primary D^*, for instance, decays into the two-body state D, π or D, γ. The lepton resulting from the subsequent weak decay of the D meson will thus have a softer momentum spectrum than had the D meson been formed in the primary hadronisation. The differences are, however, relatively small. The value of the strong coupling constant and the perturbative QCD cut-off parameter employed in the Monte Carlo simulation have a more significant effect. The uncertainty in their values is not, however, always considered as a source of systematic error

** Early fits of the D^* cross section as a function of x_E (4) to the Peterson function found good agreement for $\varepsilon(x_E)=0.25$. Unaware of the 'subtle' differences in fragmentation variables, some experiments wrongly interpreted this value of $\varepsilon(x_E)$ to be a valid representation of the charm fragmentation spectrum in z_E (6) [9] and in z (2) [10]

Table 1a, b. A compilation of the latest results on **a** charm **b** bottom quark fragmentation from inclusive lepton studies. The additional systematic error given in brackets in the final column refers to the uncertainty in extracting $\langle z(\text{rec})\rangle$ from $\langle x_E\rangle$ or $\langle z(\text{pri})\rangle$

a

Expt.	Ref.	l	MC Model	E_{cm} (GeV)	$\varepsilon_c(x_E)$	$\varepsilon_c(z(\text{pri}))$	$\varepsilon_c(z(\text{rec}))$	$\langle x_E\rangle_c$ (%)	$\langle z(\text{pri})\rangle_c$ (%)	$\langle z(\text{rec})\rangle_c$ (%)
MARK J	[11]	μ	FF+Ali	37	–	$(0.79\mp0.11\mp0.15)^2$	–	–	$46\pm3\pm3$	$51\pm3\pm3$ (±2)
TASSO	[12, 13]	μ	Lund	34.5	–	–	$0.006^{-0.005-0.005}_{+0.017+0.052}$	–	77^{+5+3}_{-7-11}	$83\pm9^{+5}_{-15}$
TASSO	[14]	e	FF+Ali	34.6	–	–	$0.19^{-0.13-0.08}_{+0.29+0.17}$	–	–	57^{+10+5}_{-9-6}
JADE	[15]	μ	Lund	34.6	–	–	$0.015^{-0.006-0.009}_{+0.009+0.018}$	–	–	$77\pm3\pm5$
MAC	[16]	μ	FF+Ali	29	–	–	–	–	–	17–67
DELCO	[17]	e	Lund	29	$0.14^{-0.05}_{+0.07}$	–	–	59 ± 4	–	76 ± 4 (±3)
TPC	[18]	μ	Lund	29	$0.14^{-0.07-0.05}_{+0.14+0.08}$	–	–	$60\pm6\pm4$	–	$76\pm6\pm4$ (±3)

b

Expt.	Ref.	l	MC Model	E_{cm} (GeV)	$\varepsilon_b(x_E)$	$\varepsilon_b(z(\text{pri}))$	$\varepsilon_b(z(\text{rec}))$	$\langle x_E\rangle_b$ (%)	$\langle z(\text{pri})\rangle_b$ (%)	$\langle z(\text{rec})\rangle_b$ (%)
MARK J	[11]	μ	FF+Ali	37	–	$(0.164\mp0.024\mp0.055)^2$	–	–	$74\pm2\pm5$	$78\pm2\pm5$ (±2)
TASSO	[12, 13]	μ	Lund	34.5	–	–	$0.0025^{-0.0025-0.0013}_{+0.029\ +0.011}$	–	85^{+10+2}_{-12-7}	87^{+13+2}_{-15-9}
TASSO	[14]	e	FF+Ali	34.6	–	–	$0.005^{-0.005-0.005}_{+0.022+0.020}$	–	–	84^{+15+15}_{-10-11}
JADE	[15]	μ	Lund	34.6	–	–	$0.0035^{-0.002-0.0025}_{+0.004+0.005}$	–	–	$86\pm4\pm5$
MAC	[16]	μ	FF+Ali	29	–	–	$0.008^{-0.008}_{+0.037}$	–	–	80 ± 10
DELCO	[17]	e	Lund	29	$0.033^{-0.017}_{+0.032}$	–	–	72 ± 5	–	83 ± 5 (±3)
TPC	[18]	μ	Lund	29	$0.011^{-0.007-0.007}_{+0.015+0.011}$	–	–	$80\pm5\pm5$	–	$93\pm5\pm5$ (±3)
TPC	[19]	e	Lund	29	$0.033^{-0.019-0.012}_{+0.037+0.019}$	–	–	$74\pm5\pm3$	–	$83\pm5\pm3$ (±3)
MARK II	[9]	μ	FF+Ali	29	–	–	$0.042^{-0.041-0.035}_{+0.218+0.120}$	–	–	$73\pm15\pm10$
MARK II	[9]	e	FF+Ali	29	–	–	$0.015^{-0.011-0.011}_{+0.022+0.022}$	–	–	$79\pm6\pm6$

ε, denoted here by $\varepsilon(z(\text{pri}))$, $\varepsilon(z(\text{rec}))$, $\varepsilon(x_E)$. (Although note that in [12] $\varepsilon_q(z(\text{rec}))$ is determined but $\langle z(\text{pri})\rangle$ is quoted!) In order to make a comparison between the various experimental results, the effects of the different definitions of the fragmentation variable need to be examined and accounted for. These effects are depicted in Fig. 2 which shows the mean values of z(pri), z(rec) and x_E as a function of the value of ε used in (1), for (a) $c \to l\nu_l X$ and (b) $b \to l\nu_l X$ events at $E_{\text{cm}} = 29$ GeV, as determined from the Lund Monte Carlo model employing a value for the strong coupling constant of $\alpha_s = 0.165$ and a value for the perturbative QCD cut-off parameter of, $y_{\min} = m_{ij}^2/s = 0.015$, where m_{ij} is the minimum invariant mass between resolvable partons i and j.

This figure (and others at different centre of mass energies) is used to extract $\langle z(\text{rec})\rangle$ from measurements employing other fragmentation variables. Where this is done a systematic error is included to allow for the uncertainty in relating $\langle z(\text{pri})\rangle$ and $\langle x_E\rangle$ to $\langle z(\text{rec})\rangle$. This uncertainty stems from the omission in publications of the values of α_s and $y_{\min}$ used in the various fitting procedures (this mainly affects the relation between $\langle x_E\rangle$ and $\langle z(\text{rec})\rangle$), and from the imprecise experimental determination of the value of α_s, knowledge of which is required to correct the results of $\langle x_E\rangle$ and $\langle z(\text{pri})\rangle$ to $\langle z(\text{rec})\rangle$. The systematic error was estimated by studying the effects of changes in the value of α_s and $y_{\min}$ of $\pm 20\%$ and a factor of three respectively. The effects of using different fragmentation models were also included in the systematic error.

The experimental results, together with the extracted values of $\langle z(\text{rec})\rangle_Q$, are summarised in Table 1. The latter are consistent with scaling between the different centre of mass energies, and are thus combined to give an overall mean value of $\langle z(\text{rec})\rangle_Q$ of

$$\langle z(\text{rec})\rangle_c = 0.67 \pm 0.02 \pm 0.02$$
$$\langle z(\text{rec})\rangle_b = 0.83 \pm 0.01 \pm 0.02.$$

The results correspond to the following values of ε_Q:

$$\varepsilon_c(z(\text{rec})) = 0.06 {}^{-0.01-0.01}_{+0.02+0.02}$$

$$\varepsilon_b(z(\text{rec})) = 0.006 {}^{-0.001-0.002}_{+0.001+0.002}.$$

The ratio $\varepsilon_c/\varepsilon_b$ can be interpreted (from (3)) as a measure of the ratio of the squares of the masses of the b and c quarks:

$$\frac{M_b^2}{M_c^2} = 10 {}^{+4+5}_{-2-4}.$$

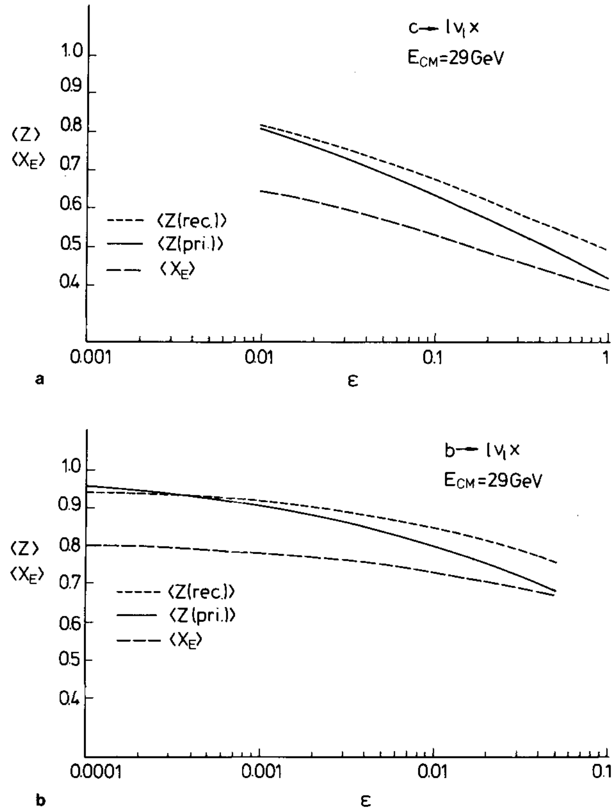

Fig. 2a, b. The relation of $\langle z(\text{pri})\rangle$ and $\langle x_E\rangle$ to $\langle z(\text{rec})\rangle$ as a function of ε_Q for **a** $c \to l\nu_l X$ and **b** $b \to l\nu_l X$ events at $E_{\text{cm}} = 29$ GeV; calculated using the Lund 5.2 Monte Carlo program with $\alpha_s = 0.165$ and $y_{\min} = m_{ij}^2/s = 0.015$, where m_{ij} is the invariant mass between partons i and j. In the Monte Carlo, the primary c and b quarks mostly form the lightest vector hadron state

This is in good agreement with an expectation of about 10 (using constituent quark masses of $M_c = 1.5$ GeV and $M_b = 4.8$ GeV).

Note that the fragmentation effect of shifting the primordial z values, z(pri), to higher values of z(rec) (as illustrated in Fig. 2), should be considered when generating Monte Carlo model events [20]. For example, in order to reproduce the above determined values of $\langle z(\text{rec})\rangle_Q$, the values of ε_Q required in the Lund Monte Carlo generation program (using the parameters given in Fig. 2), at various centre of mass energies, would be:

$E_{\text{cm}} = 29$ GeV: $\varepsilon_c(z(\text{pri})) = 0.10$, $\quad \varepsilon_b(z(\text{pri})) = 0.015$
$E_{\text{cm}} = 35$ GeV: $\varepsilon_c(z(\text{pri})) = 0.09$, $\quad \varepsilon_b(z(\text{pri})) = 0.012$
$E_{\text{cm}} = 45$ GeV: $\varepsilon_c(z(\text{pri})) = 0.08$, $\quad \varepsilon_b(z(\text{pri})) = 0.010$.

Thus, whilst z(rec) remains constant with energy, the value of z(pri) is seen to increase slightly with energy (as ε_Q decreases). This behaviour is a consequence of keeping the value of $y_{\min}$ constant with energy, resulting in a different value of the minimum invariant mass, m_{ij}, between the resolvable partons i, j at the various energy points.

Heavy quark fragmentation from charged multiplicity measurements

Experimental results on the mean charged multiplicity of multihadronic events containing heavy quarks jets, $\langle n \rangle_Q$, can be used to provide information on heavy quark fragmentation [21]. By distinguishing between the contribution to $\langle n \rangle_Q$ from the decay of the two primary hadrons (i.e. leading multiplicity, $\langle n_l \rangle_Q$) and from the remainder of the fragmentation process (i.e. non-leading multiplicity, $\langle n_{nl} \rangle$), the latter can be related to a corresponding average nonleading energy, $\langle E_{nl} \rangle$, by utilising the measured variation of the mean charged multiplicity in $e^+e^- \to q\bar{q}(g)(g)$ events, $\langle n \rangle$, as a function of centre of mass energy, E_{cm}★. The mean of the fragmentation variable, x_E, can then be obtained from the simple relation $\langle x_E \rangle = 1 - \langle E_{nl} \rangle / E_{cm}$.

The leading multiplicity in $b\bar{b}$ events is taken directly from [22] which measures $10.99 \pm 0.06 \pm 0.29$ for the multiplicity of $B\bar{B}$ events. In calculating the leading multiplicity in $c\bar{c}$ events, the procedure used in [23] is adopted. The D^*, D production ratio is taken as 3:1 [24], and, in accordance with isospin symmetry, the charged and neutral mesons are assumed to occur with equal probability. The branching ratio for the decay $D^{*+} \to D^0 \to \pi^+$ is taken as 50% [25] and the π^+ included in the leading multiplicity count. Then, using the measured D^+, D^0 charged multiplicities of $2.16 \pm 0.11 \pm 0.12$ and $2.47 \pm 0.10 \pm 0.09$ respectively [26], the total leading multiplicity of $c\bar{c}$ events obtained is $5.11 \pm 0.21 \pm 0.20$. (Multiplicity differences due to other c and b flavoured hadrons are not considered).

Recent results on the charged multiplicity of hadronic events containing heavy quark jets are summarised in Table 2. Included in the table are the non-leading multiplicity contributions ($\langle n_{nl} \rangle = \langle n \rangle_Q - \langle n_l \rangle_Q$), the corresponding non-leading energies, $\langle E_{nl} \rangle$, (corrected to allow for u, d, s production only in the hadronisation) and the resulting mean of the fragmentation variables x_E and z(rec). The value of $\langle z(\text{rec}) \rangle$ is obtained from $\langle x_E \rangle$ in the same manner as for the inclusive lepton analyses (Fig. 2).

It is seen from Table 2b that the results of the multiplicity of $b\bar{b}$ events give a very hard fragmentation for the b quark, with the mean value of z being $\langle z(\text{rec}) \rangle_b = 0.94^{+0.02+0.04}_{-0.02-0.04}$. This is in apparent disagreement with the determination from inclusive lepton production. The multiplicity results, however, suffer from the following systematic uncertainties. The analysis procedure used was to study hadronic events tagged by a lepton with high transverse momentum relative to the jet axis. The b contribution in this b enriched sample was determined from a fit to the p, $p_\perp$ spectra of inclusive leptons, performed in terms of the semi-leptonic branching ratios and fragmentation functions of heavy quarks. However, as demonstrated in [15], when such fits are performed in terms of the fragmentation variable x_E (4), the results of the semi-leptonic branching ratio measurements are highly affected by systematic uncertainties in the fragmentation process and QCD calculation – leading to an overestimation of the semi-leptonic branching ratio of the b quark★. In such cases, the actual b content of the b enriched sample is lower than that estimated from the fit. Consequently, the correction for the background contribution in the b enriched sample is underestimated, leading to an underestimated multiplicity measurement of b jets. This in turn results in an overestimation of the mean of the fragmentation variables x_E and z(rec)!

Such systematic errors are present in the determinations of $\langle n \rangle_b$ of [23; 29]. The corresponding effect in the determination of the flavour content in the c enriched sample [23] (tagged by a lepton with low transverse momentum relative to the jet axis) is an underestimation of the c content, leading to an overestimation in $\langle n \rangle_c$ and consequently an underestimation in $\langle x_E \rangle_c$ and $\langle z(\text{rec}) \rangle_c$. In [21], the flavour content in the c and b enriched regions was determined from a fit to the inclusive lepton sample employing the fragmentation variable z_E (6). However, ε_c was fixed to 0.25, which although considered at the time to be a good representation of the charm fragmentation in terms of x_E, is not a valid representation in z. This directly effects the determination of both the c and b semi-leptonic branching ratios.

★ The mean charged multiplicity in $e^+e^- \to q\bar{q}(g)(g)$ events, $\langle n \rangle$, arises from u, d, s, c and b production, depending on E_{cm}, whilst the non-leading multiplicity, $\langle n_{nl} \rangle$, is due to u, d and s fragmentation only. The relation between $\langle n_{nl} \rangle$ and $\langle E_{nl} \rangle$ is similar to that between $\langle n \rangle$ and E_{cm} only if multiplicity is independent of flavour content. This is, however, not the case. Corrections have to be applied in order to obtain the mean charged multipicity due to the light u, d, s quarks, $\langle n \rangle_q$, as a function of energy. Above the $b\bar{b}$ threshold, $\langle n \rangle_q$ is taken to be 90% of $\langle n \rangle$. (This is based on the measurements of [28; 31] which find $\langle n \rangle_q = 11.6$ and $\langle n \rangle = 12.87$ at 27.3 GeV.) In the energy region between the $c\bar{c}$ and $b\bar{b}$ thresholds, $\langle n \rangle_q$ is estimated to be 95% of $\langle n \rangle$. Below the $c\bar{c}$ threshold, $\langle n \rangle_q = \langle n \rangle$ is taken. These corrections have a significant effect on the determination of the non-leading energy

★ This is further supported by the observation that analyses using the variable x_E and varying both fragmentation parameters ε_c and ε_b of the Peterson function, give the highest values for the b semi-leptonic branching fraction (~25% higher than the world average) [15]

Table 2a, b. Results on **a** charm and **b** bottom fragmentation from charged multiplicity measurements. E_{cm}(eff) is the effective centre of mass energy after accounting for the effect of initial state radiation. The method column indicates the procedure used to identify the heavy quark jet (i.e. identification of D^* or inclusive lepton). The jet opposite to that containing the D^* or lepton is then studied (apart from [21] which determines the multiplicity of the entire event and then allows for the bias of the lepton). The systematic error given in brackets in the final column refers to the uncertainty in extracting $\langle z(\text{rec})\rangle$ from $\langle x_E\rangle$

a

Expt.	Ref.	E_{cm}(eff)	Method	$\langle n\rangle_c$	$\langle n_{nl}\rangle_c$	$\langle E_{nl}\rangle_c$	$\langle x_E\rangle_c$ (%)	$\langle z(\text{rec})\rangle_c$ (%)
TASSO	[27]	34.4 GeV	D^*	$15.0\mp1.0\mp0.6$	$9.9\mp1.0\mp0.6$	$20^{-3\;-2}_{+4\;+3}$	42^{+9+6}_{-12-9}	54^{+9+6}_{-12-9} (±3)
HRS	[28]	27.3 GeV	D^*	$13.2\mp0.4\mp0.5$	$8.1\mp0.4\mp0.5$	$14.4^{-1.6-2.0}_{+1.0+1.3}$	47^{+6+8}_{-3-5}	59^{+6+8}_{-3-5} (±3)
MARK II	[21]	29 GeV	l	$13.2\mp0.5\mp0.9$	$8.1\mp0.5\mp0.9$	$14.4^{-2.0-3.2}_{+1.3+3.0}$	50^{+7+11}_{-4-10}	64^{+7+11}_{-4-10} (±3)
TPC	[23]	29 GeV	l	$13.5\mp0.9\mp0.9$	$8.4\mp0.9\mp0.9$	$15.0^{-3.0-3.0}_{+3.2+3.2}$	48^{+11+11}_{-11-11}	61^{+11+11}_{-11-11} (±3)

b

Expt.	Ref.	E_{cm}(eff)	Method	$\langle n\rangle_b$	$\langle n_{nl}\rangle_b$	$\langle E_{nl}\rangle_b$	$\langle x_E\rangle_b$ (%)	$\langle z(\text{rec})\rangle_b$ (%)
MARK II	[21]	29 GeV	l	$16.1\;\mp0.5\;\mp1.0$	$5.1\mp0.5\mp1.0$	$5.7^{-0.8-1.8}_{+0.9+1.8}$	80^{+3+6}_{-3-6}	93^{+3+6}_{-3-6} (±3)
TPC	[23]	29 GeV	l	$16.7\;\mp1.0\;\mp1.0$	$5.7\mp1.0\mp1.0$	$6.7^{-1.7-1.7}_{+2.2+2.2}$	77^{+6+6}_{-8-8}	90^{+6+6}_{-8-8} (±3)
DELCO	[29]	29 GeV	l	$15.22\mp0.92\mp0.94$	$4.2\mp0.9\mp1.0$	$4.0^{-1.7-1.9}_{+1.7+1.8}$	86^{+6+7}_{-6-6}	100^{+6+7}_{-6-6}

For these reasons, in determining $\langle z(\text{rec})\rangle$, only the results of multiplicity measurements of heavy quark jets tagged by a D^* are used. This yields $\langle z(\text{rec})\rangle_c = 0.58^{+0.05+0.05}_{-0.03-0.04}$.

Summary and outlook

The use of various fragmentation variables in inclusive lepton production in e^+e^- annihilation has been investigated. Their differences have been resolved, providing a common basis for comparison of experimental results. This basis is the fraction of the sum of the energy and momentum of the primary quark (after allowing for the effects of initial state radiation and gluon bremsstrahlung) retained by the primary hadron. The value of this fraction, as reconstructed from the parton and hadron final state momenta and energies, z(rec), tends to be larger than that allocated by the primordial fragmentation function, z(pri). The effect is particularly significant in events in which a hard gluon is emitted. The overall combined mean value of $(z(\text{rec}))_Q$ and the corresponding value of $\varepsilon_Q(z(\text{rec}))$ is determined to be

$$\langle z(\text{rec})\rangle_c = 0.67\pm0.02\pm0.02,$$

$$\varepsilon_c(z(\text{rec})) = 0.06^{-0.01-0.01}_{+0.02+0.02}$$

$$\langle z(\text{rec})\rangle_b = 0.83\pm0.01\pm0.02,$$

$$\varepsilon_b(z(\text{rec})) = 0.006^{-0.001-0.002}_{+0.001+0.002}$$

for an unknown mixture of c and b flavoured hadrons respectively. These results lead to an estimate for the ratio of the squares of the masses of the b and c quarks of $\frac{M_b^2}{M_c^2} = 10^{+4+5}_{-2-4}$, in good agreement with an expectation of about 10.

In optimising the Lund Monte Carlo simulation model, the values of $\varepsilon_Q(z(\text{pri}))$ required by the primordial fragmentatioin functions to produce the above determined values of $\langle z(\text{rec})\rangle_Q$ depend on the value of α_s and the value of $y_{\min}$ used at a given centre of mass energy; e.g. using $\alpha_s = 0.165$, and $y_{\min} = 0.015$ the optimum values at PEP and PETRA energies are respectively:

$E_{cm} = 29$ GeV: $\varepsilon_c(z(\text{pri})) = 0.10$, $\varepsilon_b(z(\text{pri})) = 0.015$

$E_{cm} = 35$ GeV: $\varepsilon_c(z(\text{pri})) = 0.09$, $\varepsilon_b(z(\text{pri})) = 0.012$.

A further independent method of extracting information on the heavy quark fragmentation, arises from measurements of the charged multiplicity of hadronic events containing heavy quark jets. Hadronic events tagged by a D^* yield $\langle z(\text{rec})\rangle_c = 0.58\,{}^{+0.05}_{-0.03}\,{}^{+0.05}_{-0.04}$ which is consistent with the determination from inclusive lepton production. Charged multiplicity measurements of hadronic events tagged by an inclusive lepton yield $\langle z(\text{rec})\rangle_b = 0.94\,{}^{+0.02}_{-0.02}\,{}^{+0.04}_{-0.04}$. However, the present measurements suffer from significant systematic errors arising from the incorrect determination of the flavour content of the inclusive lepton sample.

The results of the charm fragmentation can be compared with the determination from D^* measurements which gives $\langle z(\text{rec})\rangle_c = 0.70 \pm 0.01 \pm 0.03$ [8]. In making such a comparison, it is noted that the value of $\langle z(\text{rec})\rangle_c$ resulting from inclusive lepton analyses and charged multiplicity measurements, is expected to be slightly softer than that arising from D^* measurements, as the former is an average of an unknown mixture of primary mesons and baryons. (Recall from (3) that a D_s (or D_s^*) meson, for instance, gives a larger value for ε_c, and therefore a softer fragmentation, than a D (or D^*) meson.)

Finally, despite the popular use of the Peterson form of $f(z)$ to determine the $\langle z\rangle$ of heavy hadrons, it is stressed that its derivation is relatively naive and it would indeed be surprising if it were to provide the ultimate description of the data. Within the limited statistics however, the distribution of the D^* cross sections is found to be in accordance with the Peterson fragmentation function, when fitted as a function of z rather than x_E [8]. The JADE collaboration [15] recently showed that the Peterson form also gives a good representation of both the charm and bottom fragmentation. A fit was performed to the p, $p_\perp$ spectra of inclusive muons by dividing the z(rec) region of the Lund Monte Carlo model into several intervals, and weighting these intervals without assuming any functional form for z(rec). The mean and *rms* values of z(rec) as determined from the 'free fit' are in good agreement with those determined from a fit to the Peterson function:

	free fit	Peterson
$\langle z(\text{rec})\rangle_c \pm \Delta z(\text{rec})_c =$	0.74 ± 0.20	0.77 ± 0.16
$\langle z(\text{rec})\rangle_b \pm \Delta z(\text{rec})_b =$	0.88 ± 0.07	$0.86 \pm 0.12.$

The statistical significance of this result is still limited however. More data are required to achieve any reasonable sensitivity to the detailed shape of the fragmentation function. However, with PETRA experiments having accumulated a further $\approx 90\ \text{pb}^{-1}$ of luminosity at $\sqrt{s} = 35$ GeV in 1986 (more than doubling the previous statistics at this energy), and PEP experiments totalling over $200\ \text{pb}^{-1}$, a more precise determination of the functional form of the fragmentation function, enabling more stringent tests of fragmentation models to be performed, is awaited.

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References

1. See for example E. Reya: Phys. Rep. 69 (1981) 195; A.H. Mueller: Phys. Rep. 73 (1981) 237; G. Altarelli: Phys. Rep. 81 (1982) 1; D.W. Duke, R.G. Roberts: Phys. Rep. 120 (1985) 276
2. R.D. Field, R.P. Feynman: Nucl. Phys. B136 (1978) 1; P. Hoyer et al.: Nucl. Phys. B161 (1979) 349; A. Ali et al.: Phys. Lett. 93B (1980) 155
3. B. Anderson et al.: Phys. Rep. 97 (1983) 33; Z. Phys. C – Particles and Fields 20 (1983) 317; T. Sjöstrand: Comp. Phys. Commun. 27 (1982) 243; 28 (1983) 229
4. L.M. Sehgal, P. Zerwas: Nucl. Phys. B108 (1976) 483; V. Barger et al.: Phys. Lett. 70B (1977) 51
5. Ya. I. Aziamov et al.: Leningrad preprint LNPI-222 (1976); M. Suzuki: Phys. Lett. 71B (1977) 139; J.D. Bjorken: Phys. Rev. D17 (1978) 171; D.M. Scott: Phys. Rev. D18 (1978) 210
6. C. Peterson et al.: Phys. Rev. D27 (1983) 105
7. M. Bowler: Z. Phys. C – Particles and Fields 11 (1981) 169
8. S. Bethke: Proceedings of the International Symposium on Production and Decay of Heavy Hadrons. K.R. Schubert, R. Waldi (Eds.) publ. DESY, p. 65. Heidelberg (1986); HD-PY 86/07 (1986); See also S. Bethke: Z. Phys. C – Particles and Fields 29 (1985) 175
9. N. Lockyer: Proceedings of the 11th SLAC Summer Institute on Particle Physics. P.M. McDonough (Ed.); See also MARK II Collab. M.E. Nelson et al.: Phys. Rev. Lett. 50 (1983) 1542
10. TASSO Collab. M. Althoff et al.: Phys. Lett. 138B (1984) 317
11. F.P. Poschmann: Ph.D. Thesis, University of Aachen, PITHA 85-21 (1985); See also MARK J Collab. B. Adeva et al.: Phys. Rev. Lett. 51 (1983) 443; In these analyses $\varepsilon(z(\text{pri}))$ is determined and $\langle z(\text{pri})\rangle_Q$ quoted, J. Schug: private communication
12. TASSO Collab. M. Althoff et al.: Z. Phys. C – Particles and Fields 22 (1984) 219; Here $\varepsilon(z(\text{rec}))$ is determined, but $\langle z(\text{pri})\rangle_Q$ quoted. The corresponding value of $\langle z(\text{rec})\rangle_Q$ is given in [13]
13. J.A. Thomas: Ph.D. Thesis, University of Oxford (1984); RAL HEP/T/114
14. TASSO Collab. M. Althoff et al.: Phys. Lett. 146B (1984) 443
15. JADE Collab. W. Bartel et al.: Z. Phys. C – Particles and Fields 33 (1987) 339
16. MAC Collab. B. Fernandez et al.: Phys. Rev. Lett. 50 (1983) 2054
17. DELCO Collab. T. Pal et al.: Phys. Rev. D33 (1986) 2708; See also DELCO Collab. D.E. Koop et al.: Phys. Rev. Lett. 52 (1984) 970. Here x_γ, i.e. (7), is used
18. TPC Collab. H. Aihara et al.: Phys. Rev. D31 (1985) 2719
19. TPC Collab. H. Aihara et al.: Z. Phys. C – Particles and Fields 27 (1985) 39
20. This is also the case in the $p\bar{p}$ Monte Carlo models of ISAJET and EUROJET which require different values of ε_Q to describe the data because of their different treatment of gluon bremsstrahlung. M.D. Negra: CERN EP/87-07 (1987)
21. MARK II Collab. P.C. Rowson et al.: Phys. Rev. Lett. 54 (1985) 2580. Here $\langle x_E\rangle_c = 60\,{}^{+9}_{-11}\,\%$ and $\langle x_E\rangle_b = 79\,{}^{+10}_{-5}\,\%$ are determined. In determining the non-leading energies, however, no

corrections were applied to allow for the absence of heavy quark production in the secondary hadronisation process

22. CLEO Collab. R. Giles et al.: Phys. Rev. D30 (1984) 2279
23. TPC Collab. H. Aihara et al.: Phys. Lett. 184B (1987) 299. The error given here is presumably statistical only; a systematic error of the same size as the statistical error is therefore quoted in Table 2
24. See for example JADE Collab. W. Bartel et al.: Phys. Lett. 161B (1985) 197; HRS Collab. E.H. Low et al.: PU-86-588 (1986); CLEO Collab. K. Han et al.: Phys. Rev. Lett. 55 (1985) 36
25. Particle Data Group: Phys. Lett. 170B (1986) 1, gives $BR(D^{*+} \rightarrow D^0 \pi^+) = (49 \pm 8)$ %
26. MARK II Collab. R.H. Schindler et al.: Phys. Rev. D24 (1981) 78
27. TASSO Collab. M. Althoff et al.: Phys. Lett. 135B (1984) 243
28. HRS Collab. P. Keston et al.: Phys. Lett. 161B (1985) 412. There is no mention here of any correction to $\langle n \rangle$ for the effects of initial state radiation. The effective centre of mass energy, E_{cm}(eff), is therefore taken to be 27.3 GeV [30]. The error on $\langle n \rangle$ quoted here is presumably statistical. Previous publications on the charged multiplicity of average jets give a systematic error of ± 0.6 from 19.6 pb^{-1} of data [30], and ± 0.3 from 185 pb^{-1} [31]. Since this publication is based on 111 pb^{-1}, a systematic error of ± 0.5 is estimated in Table 2
29. DELCO Collab. M. Sakuda et al.: Phys. Lett. 152B (1985) 399
30. HRS Collab. D. Bender et al.: Phys. Rev. D31 (1985) 1
31. HRS Collab. M. Derrick et al.: Phys. Rev. D34 (1986) 3304