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Recombination Dependence of the $O(\alpha_s^2)$ Three-Jet Cross Section in e^+e^- Annihilation

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Abstract. The $O(\alpha_s^2)$ QCD corrections to the thrust distribution for three-jet production in e^+e^- annihilation are found to depend on the algorithm for recombining four partons into three jets. Several recombination schemes are discussed and results are compared. Results of the recombination approach are compared also to recent direct calculations of three-jet thrust distributions. The importance of terms proportional to jet resolution parameters is stressed.

1 Introduction

It is well known that the study of jet production in high energy e^+e^- annihilation is one way to test the validity of perturbative QCD [1]. Therefore, in the past much effort has been devoted to the calculation of higher order QCD corrections to the e^+e^- annihilation total cross section [2] and to various differential cross sections [3–11]. At the level of perturbation theory up to $O(\alpha_s^2)$ e^+e^- annihilate into two-, three- and four-parton final states: $e^+e^- \to q\bar{q}$, $q\bar{q}g$, $q\bar{q}gg$, $q\bar{q}q\bar{q}$. Individually, the loop-corrected two-parton and three-parton diagrams are infrared and collinear divergent. These divergences cancel if the virtual corrections are combined with appropriately integrated real contributions. In the case of three-jet differential cross sections the divergences cancel if the $q\bar{q}g$ loop terms are taken together with the four-parton contributions where two unresolved partons (qg or $q\bar{q}$) are integrated over to produce one jet. This procedure yields finite jet cross sections which, however, depend on the resolution parameters. As resolution criteria for two partons two methods have been applied in the past. First there is the Sterman-Weinberg definition [12], where two partons are considered irresolvable if either parton has energy less than $\varepsilon\sqrt{q^2}/2$ ($\sqrt{q^2}$ being the total c.m. energy) or the angle between the two partons is less than δ. The second procedure for defining irresolvable partons is based on an invariant mass constraint. Here two partons are said to be unresolved if their invariant mass squared $(p_i+p_j)^2$ is less than cyq^2.

$O(\alpha_s^2)$ differential three-jet cross sections with (ε, δ)-resolution have been first calculated in [4] and with invariant mass resolution in [5]. These calculations were based on the most singular terms of the four-parton production cross section which are responsible for the infrared and mass singularities. In this work it was assumed that the non-singular pieces give small contributions proportional to ε, δ or y, respectively, which could be neglected. We can expect this for very small values of the resolution parameters. Therefore in some of the phenomenological analyses of e^+e^- annihilation data the resolution parameter was chosen rather small. So, for example, in many analyses based on the work of Sjöstrand [13], who incorporated the formulae of [5] into the string fragmentation model of the Lund group, the squared mass cut parameter was chosen to be $y=0.015$. But it had not been checked whether this value of y is small enough to make the subleading terms negligible in the total sum of the $O(\alpha_s^2)$ three-parton and four-parton contributions.

The magnitude of subleading contributions has been studied recently by the TASSO Collaboration [14] in connection with the analysis of their data towards a determination of α_s with the result that subleading terms are not negligible and must be included

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for a reliable determination of α_s*,**. The TASSO procedure is as follows. One starts from the FKSS formulae [4] for $d\sigma_{3\text{-jet}}$ based on leading four-parton contributions with (ε, δ) resolution. The (ε, δ)-values were chosen very small, i.e. $\varepsilon_0 = 0.01$ and $\delta_0 = 0.1$ (radians), with the understanding that for such (ε, δ)-values the subleading terms $O(\varepsilon_0, \delta_0)$ are really negligible. However, to separate the real three- and four-jet events one must go to larger (ε, δ)'s, i.e. $\varepsilon = 0.2$, $\delta = 40°$, for example. Furthermore, for smaller ε, δ, for example $\varepsilon = 0.1$ and $\delta = 0.5$, the higher order correction is large and negative [5], which indicates that we are already outside the range of perturbation theory. To obtain $d\sigma_{3\text{-jet}}$ with these cut values the four-parton terms in the phase space region between the smaller ε_0, δ_0 and the larger ε, δ constraints are calculated numerically and added to $d\sigma_{3\text{-jet}}$ obtained with the (ε_0, δ_0)-cut. In this calculation two of the partons must be combined if they are inside the (ε, δ)-constraint to obtain three jets. It was found out, however, that the way and how, which of the two partons are combined in one jet, is important and the amount of correction through nonleading terms depends on the recombination process. This possibility was already emphasized in early work by Gottschalk [16].

It is the purpose of this work to study this recombination dependence in more detail. We shall describe several possibilities for recombining two partons in one jet. The influence of the recombination on the three-jet cross section is studied in terms of the thrust distribution, as an example of a one-variable distribution. This influence exists for any variable and may be smaller or larger than in the thrust distribution. This must be found out then in separate calculations. Of course, one would be interested to know variables for which the recombination dependence is minimal.

In the meantime the three-jet cross section $d\sigma_{3\text{-jet}}$ was calculated with all subleading four-parton contributions included from the start [17]***. Also here it was found that the three-jet cross section depends on the way how the variables describing three jets were formed out of the momenta of the four partons. This result is similar to the recombination dependence described above. In these calculations the invariant mass resolution was applied to distinguish three and four jets. Therefore the three-jet thrust distribution cannot be compared directly with the results of the TASSO approach. We can make a comparison by adding also the four-jet thrust distribution in the two cases.

The outline of the paper is as follows: in Sect. 2 we describe the different procedures for combining four partons into three jets and present the results for the $O(\alpha_s^2)$ three-jet thrust distributions. The result for two of the procedures is then compared with the thrust distributions obtained in a direct calculation in [17]. In Sect. 3 we draw some final conclusions from the comparisons given in Sect. 2.

2 Thrust distribution in different recombination schemes

We consider the three-jet cross section $\sigma_{3\text{-jet}}(x_1, x_2; \varepsilon, \delta)$ which depends on the two scaled energies x_1 and x_2 ($x_i = 2E_i/\sqrt{q^2}$ with $i = 1, 2$ for quark and antiquark momenta as usual). In $O(\alpha_s^2)$ this cross section is obtained from

$$\sigma_{3\text{-jet}}(x_1, x_2; \varepsilon, \delta) = \sigma_{\text{virt}}^{(3)}(x_1, x_2) + \{\int_{3\text{-jet}} d\sigma^{(4)}\}(x_1, x_2; \varepsilon, \delta) \tag{2.1}$$

where $\sigma_{\text{virt}}^{(3)}(x_1, x_2)$ stands for the loop corrections with final parton state $q\bar{q}g$, which are infrared and mass singular. This divergence (negative) is compensated by the divergence (positive) in the second term in (2.1) which is the contribution of the four-parton diagrams evaluated over the three-jet kinematical region parametrized by the resolution parameters ε and δ. This three-jet region is defined as usual in the form that the four partons are considered as three jets if either one parton has a scaled energy $x_i < \varepsilon$ (soft parton) or has an angle $\theta_i < \delta$ with any other parton (collinear parton). For an illustration see Fig. 1a, b, where a

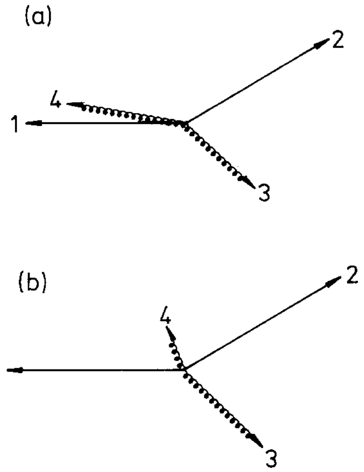

Fig. 1a, b. Recombination of four parton into three jets with a almost collinear gluon and b with soft gluon emitted under large angle as an example

* Actually in the TASSO paper only part of these corrections were used for determining α_s, although the complete corrections had been computed. For more details see [11] and [14]

** This was found also by R.Y. Zhu in his thesis [15] by comparing with the approach first used in [10] based on matrix elements of [3]

*** For work, where part of subleading terms were included, see Gottschalk and Shatz [14] soft gluon and a gluon collinear with a quark are considered. If two partons fulfil these conditions the contribution is considered as part of the two-jet cross section. For those terms in $\sigma^{(4)}$ which yield the infrared and collinear singularities, denoted as singular terms of $\sigma^{(4)}$, the integration within the ε, δ resolution has been done analytically and the results are reported in [4, 5] (FKSS-formulas). These results are correct only for very small ε, δ values, i.e. $\varepsilon \lesssim 0.01$, $\delta \lesssim 0.1$ [11, 14, 15]. For calculating the corrections in case ε and δ is much larger, which is needed for physical applications, we adopt the procedure already mentioned in Sect. 1. One chooses ε_0, $\delta_0 = 0.01$, 0.1 and calculates the last integral in (2.1) in the phase space region with recombination values lying between ε_0, δ_0 and ε, δ. The numerical computations are done with the iterative Monte Carlo routine VEGAS, which was necessary to obtain sufficient accuracy. The integration involves either the integration over an energy x_i with $\varepsilon_0 \leq x_i \leq \varepsilon$ or, if $x_i \geq \varepsilon$, integration over an angle θ_{ij} between two partons with $\delta_0 \leq \theta_{ij} \leq \delta$. This way a four-parton configuration is transformed into a three-jet configuration. However, the result for this averaging depends on the way the original four parton variables x_1, x_2, x_3, x_4 are related to the three-jet variables x_{I}, x_{II} and x_{III}. This produces the recombination dependence referred to in Sect. 1. The three jets are again considered as three massless partons and their variables are assumed to obey $x_{\mathrm{I}} + x_{\mathrm{II}} + x_{\mathrm{III}} = 2$. To study the influence of different definitions for the three-jet variables we considered several possibilities which we list in the following:

(1) *Sterman-Weinberg Recombination* (SW)

The three-jet variables x_{I}, x_{II}, x_{III} are given as twice the energies going into the q, $\bar{q}$ or g cone with opening angle δ, divided by the sum of the energies going into these cones. However, we must distinguish the case of a collinear parton (a) from the case of a soft parton (b):

(a) Suppose for example gluon 4 is produced so that its momentum with the momentum of quark 1 has an angle $\theta_{14} < \delta$ (see Fig. 1a). Then

$$\begin{aligned} x_{\mathrm{I}} &= x_1 + x_4 \\ x_{\mathrm{II}} &= x_2 \\ x_{\mathrm{III}} &= x_3. \end{aligned} \tag{2.2}$$

Since $x_1 + x_2 + x_3 + x_4 = 2$ we have also $x_{\mathrm{I}} + x_{\mathrm{II}} + x_{\mathrm{III}} = 2$.

(b) Gluon 4 is soft ($x_4 < \varepsilon$) and is emitted in such a way that the angles with all other partons θ_{i4} ($i = 1$, 2, 3) $> \delta$ (see Fig. 1b). Then

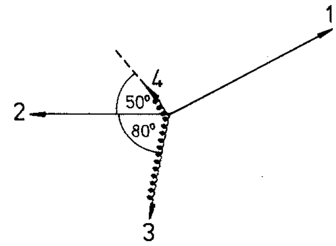

Fig. 2. Four-parton configuration with soft gluon emitted with large angle and scaled parton energies $x_1 = 0.85$, $x_2 = 0.60$, $x_3 = 0.50$ and $x_4 = 0.05$. All four momenta lie in one plane

$$\begin{aligned} x_{\mathrm{I}} &= \frac{2x_1}{x_1 + x_2 + x_3} = \frac{x_1}{1 - x_4/2} \\ x_{\mathrm{II}} &= \frac{2x_2}{x_1 + x_2 + x_3} = \frac{x_2}{1 - x_4/2} \\ x_{\mathrm{III}} &= \frac{2x_3}{x_1 + x_2 + x_3} = \frac{x_3}{1 - x_4/2}, \end{aligned} \tag{2.3}$$

i.e. the energy of the soft gluon is combined by rescaling.

An alternative to the scheme above would be to add x_4 randomly to x_1, x_2 and x_3 (scheme SW').

(2) *Minimal Mass Recombination* (MM)

In this scheme the recombination of two partons to one jet is controlled by the invariant mass of the two partons i and j: $y_{ij} = (p_i + p_j)^2/q^2$. Then those partons i, j are combined whose invariant mass is minimal. The corresponding jet four-momentum is equal to $p_i + p_j$. This means, that independent of whether one has the situation collinear or soft in (2.1), the three-jet variables are always given by formulas as in (2.2). This scheme was studied earlier by Zhu [15]. He considered in addition two possibilities. (i) The momentum scheme: the three-momentum vectors of the two partons are added and the resulting parton is again assumed massless. The energy conservation of the three resulting partons is reestablished by rescaling similarly to (2.3). (ii) The energy scheme: the energies of the partons, which are combined, are added up and are again interpreted as energies of massless partons. This scheme is identical to the scheme we are using here. It was found that the momentum and the energy scheme produce almost identical results, in agreement with [15]. Therefore we now restrict ourselves to the energy scheme.

We shall illustrate how the three-jet thrust changes in the various recombination or dressing schemes. The main difference appears in such configurations in which a soft gluon $x_4 < 0.2$ is emitted with a large angle $\theta_{4i} > 40°$. Such an event, assumed to lie in a plane, is shown in Fig. 2. This event has four-parton thrust $T=0.850$. In the SW and the SW′ scheme we obtain $T=0.872$ and $T=0.867$ respectively. In the MM scheme the thrust is $T=0.857$ (momentum scheme) and $T=0.850$ (energy scheme). The minimal mass occurs in combining parton 3 and 4. We see that the thrust in the MM scheme differs only little from the original four-parton thrust. In the two SW schemes the resulting thrust is larger than the parton thrust. Although the difference is small it has a strong effect on the thrust distribution which is steep in particular for the larger thrust values. Of course, similar to thrust also other jet variables change in the various schemes since the three-jet variables x_{I}, x_{II} and x_{III}, which enter into these variables, differ.

A further problem, which we encounter, is the separation of two-jet events from three- and four-jet configurations. Of course, four jets are separated equally in all schemes by demanding all $x_i>\varepsilon$ and $\theta_{ij}>\delta$ (i, $j=1, 2, 3, 4$) in the four-parton configurations. Then we remain with the two- and three-jet separation. To disentangle two jets it would be natural to eliminate all those four-parton terms where three partons are emitted in an angular cone of width δ, two partons are emitted with energies smaller than ε or two pairs of partons have angles less than δ. This procedure has been adopted in the SW, SW′ and MM scheme. Another possibility is to apply the two-jet criteria to the three-jet configurations already obtained via recombination of two partons from the four-parton sample. This procedure was applied only in the minimal mass scheme. After recombination of two partons with the smallest invariant mass the two-jet test with ε, δ constraint was applied to the three-jet events. We shall denote this procedure as MM′ which differs from MM just in the particular two-jet separation.

To study the differences originating from these various procedures we have calculated the $O(\alpha_s^2)$ corrections in these four schemes, SW, SW′, MM and MM′. We have determined the three-jet thrust distributions in these schemes as an example. Distributions in other variables should show similar variations. Since we are interested only in the $O(\alpha_s^2)$ corrections to $d\sigma/dT$ we write the full thrust distribution as

$$\frac{1}{\sigma_0}\frac{d\sigma_{3\text{-jet}}}{dT}=\frac{\alpha_s}{2\pi}A_1(T)+\left(\frac{\alpha_s}{2\pi}\right)^2 A_2(T) \tag{2.4}$$

and give the results for $A_2(T)$. The correctly normalized three-jet thrust distribution follows from (2.4) by dividing by $\sigma_{\mathrm{tot}}/\sigma_0$. σ_0 is the zeroth order cross section. $A_1(T)$ is the lowest order result with $\alpha_s/2\pi$ factored out. T is equal to max (x_{I}, x_{II}, x_{III}) with x_{I}, x_{II} and x_{III} as defined above in the different schemes. The parameters ε and δ are chosen as $\varepsilon=0.2$ and $\delta=40°$ whereas $\varepsilon_0=0.01$ and $\delta_0=0.1$.

Table 1. $O(\alpha_s)$ thrust distribution in terms of $A_1(T)$ as a function of thrust T averaged over bins of $\Delta T=0.025$. $A_1'(T)$ includes two-jet contributions. $A_2(T)$ is four-jet cross section for $\varepsilon=0.2$, $\delta=40°$

T	$A_1'(T)$	$A_1(T)$	$A_2(T)_{4\text{-jet}}$
0.675–0.700	2.024	2.024	76.4
0.700–0.725	4.601	4.601	138.8
0.725–0.750	7.530	7.530	187.6
0.750–0.775	11.06	11.06	244.0
0.775–0.800	15.55	15.55	310.4
0.800–0.825	21.53	21.53	378.4
0.825–0.850	29.91	29.91	409.2
0.850–0.875	42.31	42.31	438.8
0.875–0.900	62.01	62.01	380.4
0.900–0.925	96.59	81.67	308.8
0.925–0.950	167.9	102.2	98.4
	11.525	9.493	74.28

First we present in Table 1 the function $A_1(T)$ for thrust intervals of $\Delta T=0.025$ for T values ranging from 0.675 to 0.950. To see the influence of the two-jet elimination we give $A_1'(T)$, where the two-jet constraint with ε, δ is not applied and $A_1(T)$ with two jets taken out. Secondly, for later use we have given the genuine four-jet contribution to $A_2(T)$, i.e. those four parton contributions where all $x_i>\varepsilon$ and all $\theta_{ij}>\delta$ (i, $j=1, 2, 3, 4$). We see that $A_1(T)$ is changed in the last two T bins because of the two-jet subtraction*. The four-jet contribution to $A_2(T)$ has its maximum near $T=0.85$. The last line in Table 1 gives the integrated contribution up to $T=0.95$ for $A_1(T)$, $A_1'(T)$ and $A_2(T)$ respectively. In Table 2 we have collected our results for $A_2(T)$ for FKSS, SW, SW′, MM and MM′. The column FKSS is the old prediction based on the most singular four-parton terms**.

In FKSS several related approximations have been made. First, in the four-parton cross section only the most singular contributions were kept which were necessary to cancel the soft and collinear divergences with those of the virtual diagrams. These terms were integrated analytically with the further approximation that contributions of $O(\varepsilon, \delta)$ were neglected (except for one term proportional to ε, which could be calculated analytically. It was clear that this term did not yield all $O(\varepsilon, \delta)$ contributions). Second, all the terms from $q\bar{q}gg$ and $q\bar{q}q\bar{q}$ final states where either one quark is soft and at large angle or two quarks are collinear, obeying the resolution criteria, were ne-

* If one uses the invariant mass constraint with $y=0.05$ for separating two and three jets in lowest order the thrust distribution is given by $A_1'(T)$. This shows that results for large T are sensitive to the cut procedure already in lowest order

** The result for FKSS becomes positive for all T if the terms $O(\varepsilon)$, which had been calculated analytically in [4], are left out. The numbers for $A_2(T)$ in Table 2 are for this case: 1.356, 4.384, 10.85, 23.54, 46.83, 88.16, 161.1, 292.4, 540.9, 813.9, 1102.0

Table 2. $O(\alpha_s^2)$ thrust distribution in terms of $A_2(T)$ for three jets as a function of thrust with $\varepsilon=0.2$, $\delta=40°$ for various dressing schemes FKSS, SW, SW′, MM and MM′ as explained in the text

T	$A_2(T)$				
	FKSS	SW	SW′	MM	MM′
0.675–0.700	−13.45	−14.5	5.8	19.9	17.3
0.700–0.725	−29.52	− 2.9	21.1	36.9	52.0
0.725–0.750	−45.42	−15.9	52.8	87.9	82.4
0.750–0.775	−60.86	− 0.0	75.9	166.1	143.1
0.775–0.800	−75.30	80.9	114.5	218.8	224.1
0.800–0.825	−87.66	72.3	252.5	465.2	377.3
0.825–0.850	−95.99	138.8	362.3	575.2	646.2
0.850–0.875	−96.66	183.6	691.5	848.7	988.7
0.875–0.900	−83.54	686.6	693.0	854.6	1619.0
0.900–0.925	−50.22	751.7	721.3	555.5	1865.0
0.925–0.950	22.81	1070.0	310.6	785.2	2255.0
	−15.40	73.8	82.5	115.3	206.8

glected since they are less singular and are not needed to cancel infrared and collinear singularities. Comparing SW with FKSS we obtain the change caused by the non-singular terms. It amounts to a change of 20% for the 3-jet cross section integrated up to $T=0.95$ taking $\alpha_s=0.17$. It is obvious that it is not sufficient to include only the most singular parton terms as done in FKSS if one is interested in an accurate determination of α_s. The results for the other schemes are in the columns labelled SW′, MM and MM′. We see that these three schemes produce different $O(\alpha_s^2)$ contributions as compared to SW. If $A_2(T)$ is integrated up to $T=0.95$ (last line in Table 2) the change compared to SW leads to increases by 12%, 56% and 180%, respectively. In particular for the MM′ scheme the change in $A_2(T)$ is quite large. Also the MM scheme produces larger correction terms than SW. The numbers for $A_2(T)$ in Table 2 and for SW, SW′, MM and MM′ have Monte Carlo errors. In case of MM′ for instance they range from approximately 20% for the lowest T bins, to 6% for the highest T bins. To get an overview about the influence of the change caused by the non-singular contributions to $A_2(T)$ for the total thrust distribution we have plotted in Fig. 3 the sum of $O(\alpha_s)$ and $O(\alpha_s^2)$ contributions to $1/\sigma_{\text{tot}}(d\sigma/dT)$ for $\alpha_s=0.15$. The $O(\alpha_s^2)$ contribution is small and negative in the case of approximate FKSS result but turns into an essentially positive contribution for SW and even more positive for MM′. The other two schemes SW′ and MM lie between the SW and MM′ curve.

All the results presented so far have been computed for $\varepsilon_0=0.01$ and $\delta_0=0.1$. In order to test whether these ε_0, δ_0 are chosen small enough we have changed ε_0 and δ_0 by a factor 2. It was found that the change in the $O(\alpha_s^2)$ corrections were within the statistical Monte Carlo errors.

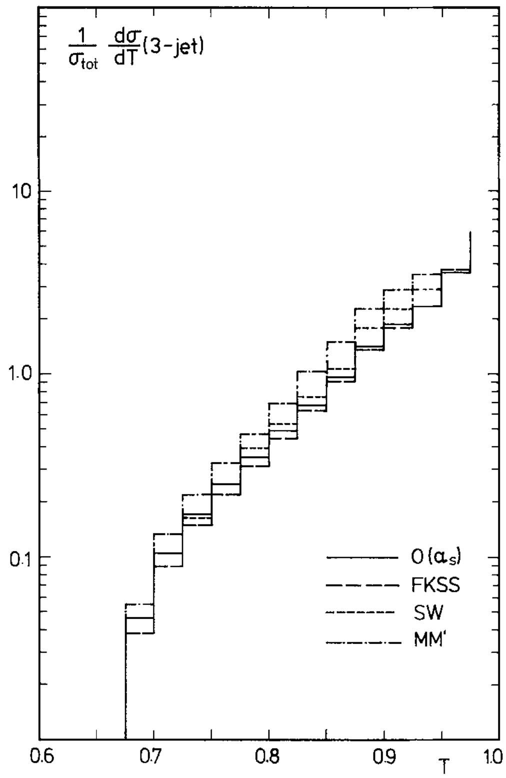

Fig. 3. Three-jet thrust distribution for FKSS, SW and MM′ parton dressing $O(\alpha_s^2)$ together with $O(\alpha_s)$ prediction for $\alpha_s=0.15$ and $\varepsilon=0.2$, $\delta=40°$

Last year a calculation of the three-jet cross section with all non-singular four parton contributions included from the start, has been completed [17]. This was achieved by partial fractioning of the four-parton cross section so that direct double pole terms could be avoided. In this calculation the separation of two, three and four jets was done only with squared invariant mass constraints. For example, when all $y_{ij}>y$, $(i, j=1, 2, 3, 4)$, the four parton contribution is in the four-jet class. If one $y_{ij}<y$ it is counted as three jet etc. The calculations have been performed for varying y in the range $0.01\leq y\leq 0.05$. For comparison we select $y=0.05$, which is of the same order of magnitude as $\varepsilon=0.2$, $\delta=40°$ ($\varepsilon^2\simeq\delta^2/4\simeq y$). Nevertheless the four-jet cross section for $y=0.05$ is not the same as for $\varepsilon=0.2$, $\delta=40°$. In the case of the invariant mass cut this cross section in terms of $A_2(T)$ integrated up to $T=0.95$ is equal to 13.56 whereas with the (ε, δ) cut according to Table 1 it is 78.28. Therefore we must add the three- and four-jet contributions to $A_2(T)$ to make a reasonable comparison. In Table 3 we collected the results for $A_2(T)_{3+4\text{-jet}}$ for two of our procedures SW and MM′ and confronted them with the results of [17], denoted by KL and KL′, based on the complete invariant mass cut calculation. In this approach, in which the non-singular four-par-

Table 3. $O(\alpha_s^2)$ thrust distribution in terms of $A_2(T)$ for the sum of three and four jets in SW and MM′ dressing compared to two complete three-jet calculations KL and KL′ of [17]

T	$A_2(T)_{3+4\text{-jet}}$			
	SW	MM′	KL	KL′
0.675–0.700	61.9	93.7	81.05	76.4
0.700–0.725	135.9	190.8	170.5	157.6
0.725–0.750	171.7	270.0	267.9	243.5
0.750–0.775	244.0	387.1	382.1	345.5
0.775–0.800	391.4	534.5	531.3	476.1
0.800–0.825	450.7	755.7	724.9	629.5
0.825–0.850	548.0	1055.0	963.4	805.5
0.850–0.875	622.4	1428.0	1270.0	1027.8
0.875–0.900	1067.0	1999.0	1788.0	1370.0
0.900–0.925	1061.0	2174.0	2701.0	1976.0
0.925–0.950	1168.0	2353.0	4712.0	2972.0
	148.0	281.0	339.8	252.0

ton terms are integrated over the total phase space, except the four-jet region, the result is not unique either. Again it depends how the three-jet variables are defined in terms of the original four-parton variables. If, for example, p_1, p_2, p_3, p_4 denote the four momenta of outgoing quark, antiquark, gluon 3, gluon 4, respectively, and gluon 3 is considered soft or collinear with the quark, in the KL scheme the three-jet variables were chosen $y_{134}=(p_1+p_3+p_4)^2/q^2=1-x_{\mathrm{II}}$, $y_{24}=(p_2+p_4)^2/q^2=1-x_{\mathrm{I}}$, whereas in KL′ the relations were $y_{134}=1-x_{\mathrm{II}}$, $y_{123}=(p_1+p_2+p_3)^2/q^2=1-x_{\mathrm{III}}$, always with the constraint $x_{\mathrm{I}}+x_{\mathrm{II}}+x_{\mathrm{III}}=2$. There is no a priori reason to prefer either of these two definitions. This non-uniqueness of three-jet variables in the degenerate quark-gluon region is similar to the non-uniqueness of the recombination scheme considered above. This problem that three-jet variables cannot uniquely be defined in terms of the four-parton momenta is quite general. In [17] these two particular choices for specific contributions of the $q\bar{q}gg$ final state were studied in detail. But many more possibilities are conceivable.

In the KL approach the main effect is a change of the two-jet kinematic region. Since two-jet contributions have been subtracted the thrust distribution changes if one goes from the KL to the KL′ approach. More details are found in [17]. From Table 3 we see that the $O(\alpha_s^2)$ corrections are smaller in the KL′ than in the KL scheme. The change is minor for $T\lesssim 0.8$ but it is appreciable if one approaches the two-jet region, $T\to 1$. We observe that the results for KL are very similar to the results of the minimal approach MM′ except for the last two T bins. As we have seen already in connection with the first order result $A_1(T)$ the last two T bins are very much influenced by the cut procedure. With the invariant mass cut criteria as in KL and KL′ the two-jet region is identical with $T\geq 0.95$ (if $y=0.05$) whereas with our ε, δ cuts the two-jet region extends down to $T\simeq 0.9$.

In connection with analysing e^+e^- annihilation data it is of interest to see what effect the various recombination procedures have if we want to determine the strong coupling constant α_s. It is clear that the approach SW will give larger couplings α_s than MM′ or KL. We have not confronted our results with complete hadronisation models as was done in the TASSO Collaboration work [14]. To get a rough idea on the change of α_s we adopted the following approach. In the TASSO work not the full SW scheme was used. Instead the analysis is based on the so-called extended FKSS approach, which contains only roughly 50% of the SW corrections if compared to FKSS*. As compared to the full SW scheme only part of the non-singular four-parton terms were included, in particular only those terms from $q\bar{q}gg$ and $q\bar{q}q\bar{q}$ final states, where one quark is soft and at large angle or two quarks are collinear with the (ε, δ)-constraint. In [11] one of us estimated the effect on α_s if the complete SW approach had been used. The result was that $\alpha_s=0.144$ at c.m. energy of 34.6 GeV follows from the TASSO analysis with independent jet fragmentation. With this value of α_s we have calculated $1/\sigma_{\mathrm{tot}}\, d\sigma/dT$ for three plus four jets using the results in Tables 1 and 3. This gives, for example, in the T interval $0.800\leqq T\leqq 0.875$ the result $1/\sigma_{\mathrm{tot}}\, d\sigma/dT=0.953$. We have chosen this T interval since it is outside the region where two-jet effects come in and where the cross section is still fairly large. Now we fit α_s in all the other approaches SW′, MM, MM′, KL and KL′ to the cross section data in this T-interval, i.e. $1/\sigma_{\mathrm{tot}}\, d\sigma/dT=0.953$. The results are: $\alpha_s=0.129$(SW′), 0.122(MM), 0.120(MM′), 0.123(KL) and 0.130(KL′). So SW′ and KL′ as one group and MM, MM′ and KL as the other group lead to two different α_s. The range of all values together with the SW value is from $\alpha_s=0.120$ to $\alpha_s=0.144$ with $\alpha_s=0.132$ as mean value. Of course, a fit to the thrust distribution up to $T=0.9$ may produce somewhat different numbers and it may also turn out that some of our schemes might give a better fit to the experimental thrust distribution than others. But this is not the topic of this paper.

Our results for the MM′ scheme are very similar to the results of Zhu [15] of the MARK J Collaboration. This is not surprising, since he applied the minimal mass recombination scheme (our MM′) in his study of second order corrections to the energy-energy correlations. This approach was also used earlier by Ali and Barreiro [10]. Zhu's approach was also followed in recent analyses of the MARK J Collabor-

* see first footnote on p. 544

ation whereas the approach of Ali and Barreiro was also applied, among others, in the TASSO analysis [14] for the energy-energy correlation. It is clear that all these analyses give values for α_s similar to the one we obtained for the MM′ scheme above.

3. Summary and Conclusions

In this paper we studied the dependence of the $O(\alpha_s^2)$ corrections to the three-jet cross section on the dressing, i.e. the method to combine non-singular four-parton configurations into three jets. As an example we calculated the thrust distribution applying Sterman-Weinberg (ε, δ)-cuts (SW). We investigated several schemes and found that the main effect arises from the different treatment of soft large angle partons. The SW scheme, in which the soft parton is averaged over, gives smaller $O(\alpha_s^2)$ corrections than the MM scheme, where the soft parton is recombined according to the minimal invariant mass. A recent $O(\alpha_s^2)$ calculation applying invariant mass ($y-$) cuts has shown that the thrust distribution depends on the choice of the three-jet variables. This is similar to the recombination dependence just described. Since no clear-cut theoretical reason can be given to select either of these schemes, $O(\alpha_s^2)$ predictions for three-jet cross sections are not unique. This leads to a systematic uncertainty in the determination of the strong coupling constant α_s.

Besides the dressing scheme dependence, however, there are other aspects that influence the determination of α_s from e^+e^- annihilation jet production. These are, for example, the choice of kinematical variables which are used for the comparison of data with theoretical model predictions [11] and in particular, as is well known, the way the fragmentation of the gluon is described, i.e. independent fragmentation versus string fragmentation [11, 14, 18].

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