CHIRAL PERTURBATION THEORY AND THE EVALUATION OF $1/N_c$-CORRECTIONS TO NONLEPTONIC DECAYS

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We present a discussion of the treatment of $1/N_c$-corrections in chiral perturbation theory for nonleptonic weak processes. We compare the approach with an explicit cutoff and dimensional regularization in chiral perturbation theory and present evidence that they are equivalent. The quadratic dependence on the cutoff can be subtracted consistently at each level of $1/N_c$. We discuss the identification of the explicit cutoff and μ_{QCD} and contrast our results with the recent calculation of Bardeen et al.

Low-energy hadronic interactions and purely hadronic weak decays have so far not been accurately computed within quantum chromodynamics (QCD). Several features observed in the strong interactions are, however, qualitatively explained. Current algebra, isospin and $SU(3)_V$ have a natural foundation within QCD. One notable exception is the $\Delta I = 1/2$ rule observed in kaon decays. The corresponding rule in baryon decay can be understood on the basis of the wavefunction of the baryon. There are at present three main approaches to describe nonleptonic decays: Lattice gauge theory calculations [1], QCD sum rules [2,3] and the $1/N_c$ expansion [4–7]. They all have in common that they use chiral symmetry. In the first approach it is used to reduce the physical decays to decays involving only two mesons. In the second approach it is used to describe the phenomenological part of the sum rule in a way that includes the chiral properties immediately [#1].

In the approach of refs. [4–7] chiral properties are used to extrapolate from the quark picture at some momentum scale to the meson picture. In this letter we would like to clarify the claim made in ref. [6] that their approach is different from the standard chiral perturbation theory approach [8,9]. We will also argue that it is very difficult, if not impossible, to identify the physical cutoff and the renormalization scale of QCD.

First we give a short description of the chiral lagrangian including the η' and we will argue that the η' does not change the $1/N_c$ behaviour of $K\to\pi\pi$ nonleptonic decays. Then we present the chiral lagrangian to second order in momentum squared/quark masses, only explicitly showing the terms relevant for the discussion of F_π, F_K and the matrix elements of the currents involved in the calculation of K decays and K^0–$\overline{K^0}$ mixing. We calculate the physical meson decay constants F_π and F_K in this approach paying special attention to regularizing and the treatment of divergences. We will show how the quadratic divergences arising can be consistently absorbed so that the cutoff approach and the dimensional regularization are equivalent. We will use a cutoff in euclidean space as a regulator so that all divergences can be seen explicitly. The same will then be done for the B parameter and the decay $K^+\to\pi^+\pi^0$ to show how the same principle can be applied there. Unfortunately the exact treatment will introduce extra free parameters at the next to leading order in $1/N_c$ so that no clear predic-

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[#1] This approach does not reproduce the observed $\Delta = 1/2$ enhancement. The upper limit in ref. [3] and the results of ref. [6] are in contradiction. More effort is needed to understand this discrepancy.

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tion can be made. We will not treat the $\Delta I=1/2$ case explicitly. There similar arguments can be made.

In the last part of this letter we will argue that the large difference in chiral corrections between the K^0–$\overline{K^0}$ and the K^+ decays makes it difficult to identify the cutoff used in the meson calculations with the renormalization scale of the operator product expansion of the effective weak lagrangian.

At the low-energy end we describe the hadronic interactions via the chiral lagrangian. Since we are also interested in the $1/N_c$ limit the lowest states are given by the π, K, η, η' pseudoscalar nonet.

In the large-N_c limit QCD is invariant under

$$SU(3)_L\times SU(3)_R\times U(1)_V\times U(1)_A \tag{1}$$

which is spontaneously broken to

$$SU(3)_V\times U(1)_V\,. \tag{2}$$

The lagrangian up to second order in momenta invariant under (1) and describing the nine Goldstone bosons is

$$\mathscr{L}_2=\tfrac{1}{8}f_8^2\,\mathrm{tr}\,\partial_\mu\Sigma\partial^\mu\Sigma^\dagger+\tfrac{1}{2}\partial_\mu\eta_1\,\partial^\mu\eta_1\,. \tag{3}$$

In (3) η_1 is the Goldstone boson associated with $U(1)_A$ and Σ is a 3×3 special unitary matrix parametrized via

$$\Sigma=\exp[(2\mathrm{i}/f_8)M] \tag{4}$$

and

$$M=\begin{pmatrix}\pi^0\sqrt{2}+\eta_8/\sqrt{6} & \pi^+ & K^+\\ \pi^- & -\pi^0/\sqrt{2}+\eta_8/\sqrt{6} & K^0\\ K^- & \overline{K^0} & -\sqrt{2/3}\,\eta_8\end{pmatrix}. \tag{5}$$

The fields transform under a chiral transformation $(L, R, e^{i\alpha})\in SU(3)_L\times SU(3)_R\times U(1)_A$ as

$$\Sigma\to L\Sigma R^\dagger,\quad \eta_1\to\eta_1+f_1\alpha\,. \tag{6}$$

The term in the QCD lagrangian that breaks (1) explicitly can to first order in the quark masses be described by

$$\mathscr{L}'_2=v\,\mathrm{tr}[m\Sigma\exp(\mathrm{i}\eta_1/f_1)+m^\dagger\Sigma^\dagger\exp(-\mathrm{i}\eta_1/f_1)] \tag{7}$$

with

$$m=\begin{pmatrix}m_u & & \\ & m_d & \\ & & m_s\end{pmatrix}. \tag{8}$$

Expanding (7) to second order in the meson fields leads to the U(1) problem because it implies three light mass eigenstates in the π^0, η_1, η_8 system [10]. Up to this order in momenta and quark masses all other terms invariant under (1) can be reduced to the ones in (3) and (7).

Including effects of $U(1)_A$ breaking via the anomaly the lagrangian to this order in momenta is given by

$$\mathscr{L}''_2=F_1+F_2\,\mathrm{tr}\,\partial_\mu\Sigma\partial^\mu\Sigma^\dagger+F_3\partial_\mu\eta_1\,\partial^\mu\eta_1+[F_4\,\mathrm{tr}\,m\Sigma\exp(\mathrm{i}\eta_1/f_1)+\mathrm{h.c.}]\,. \tag{9}$$

F_1, F_2, F_3 are arbitrary real functions of η_1/f_1 and F_4 a complex one. Parity invariance requires them to satisfy $F_i(x)=F_i(-x)^*$.

The η_1 has only a rather indirect effect on the processes considered in this letter. Apart from mixing it can be seen from (3) and (7) that there are no tree graph contributions involving the η_1. Loop contributions are also not present from (3) so they are suppressed by at least another factor of $1/N_c$ from explicit $U(1)_A$ breaking.

Fig. 1. One-loop Feynman diagrams for meson field renormalization and corrections to the decay constants.

Contributions from loops in (7) can also be described via fourth-order terms in momenta only involving Σ. This was the approach taken in ref. [9]. That term has no contribution to the processes considered here [7].

If we now try to go beyond lowest order in momenta/quark masses we have to add more terms to the lagrangian. Including terms with only a single flavour trace and removing all those that do not contribute to matrix elements with one or two mesons in currents derived from the lagrangian, leads to the following extra terms [#2]:

$$\mathcal{L}_4 = c_1\,\mathrm{tr}\,\partial_\mu\Sigma\partial^\mu\Sigma^\dagger(m^\dagger\Sigma^\dagger+\Sigma m) + c_2\,\mathrm{tr}(m\Sigma m\Sigma + m^\dagger\Sigma^\dagger m^\dagger\Sigma^\dagger)\,. \tag{10}$$

All these terms are leading in N_c so c_1, c_2, f_8^2 and v are all of order N_c in the large-N_c limit. The term proportional to c_2 does not play any role in the further analysis [7].

If we want to include the next order in $1/N_c$ there are two types of contributions:

(1) tree level contributions from terms in the lagrangian with more than one flavour trace, and

(2) loop contributions with one meson loop from $\mathcal{L}_2$.

The tree level term relevant for the present analysis is

$$\mathcal{L}'_4 = c_3\,\mathrm{tr}(\partial_\mu\Sigma\partial^\mu\Sigma^\dagger)\,\mathrm{tr}(m\Sigma+m^\dagger\Sigma^\dagger)\,. \tag{11}$$

The loop diagrams are subdominant in $1/N_c$ since loops are suppressed by factors $p^2/(4\pi f_8)^2$ [8] and hence suppressed by $1/N_c$. As an example let us derive F_π and F_K in terms of the left-handed current [#3] derived from the lagrangian (3), (10) and (11):

$$(L_\mu)_{ij} = -\mathrm{i}\tfrac{1}{4}f_8^2(\Sigma\partial_\mu\Sigma^\dagger)_{ij} - \mathrm{i}c_1(\Sigma\partial_\mu\Sigma^\dagger m^\dagger\Sigma^\dagger - \partial_\mu\Sigma m + m^\dagger\partial_\mu\Sigma^\dagger - \Sigma m\partial_\mu\Sigma\Sigma^\dagger)_{ij}$$
$$- 2\mathrm{i}c_3(\Sigma\partial_\mu\Sigma^\dagger)_{ij}\,\mathrm{tr}(m\Sigma+m^\dagger\Sigma^\dagger)\,. \tag{12}$$

We will work in the isospin limit $m_u = m_d = m$ in the remainder of this letter. Including the diagrams of fig. 1 with a cutoff Λ we get for the physical decay constants

$$F_\pi = f_8\{1 + (4c_1/f_8^2)2m + (8c_3/f_8^2)(2m+m_s) - (1/f_8^2)[2I_2(m_\pi^2) + I_2(m_K^2)]\}\,, \tag{13}$$

$$F_K = f_8\{1 + (4c_1/f_8^2)(m+m_s) + (8c_3/f_8^2)(2m+m_s) - (3/4f_8^2)[2I_2(m_K^2) + I_2(m_\pi^2) + I_2(m_8^2)]\}\,, \tag{14}$$

where

$$I_2(m_i^2) = (1/16\pi^2)[\Lambda^2 - m_i^2\log(1+\Lambda^2/m_i^2)]\,. \tag{15}$$

The cutoff dependence comes in several places in the following way [#4]:

(1) A possible Λ^4 contribution is forbidden due to chiral symmetry.

(2) The Λ^2 terms are by power counting and chiral symmetry of the form $\Lambda^2\times$tree level from $\mathcal{L}_2$.

(3) The $\log\Lambda^2$ terms can always be written as $\log\Lambda^2\times$tree level from $\mathcal{L}_4$ [8].

As a consequence we can always absorb the cutoff dependence. We can define renormalized quantities:

$$f_8^R = f_8 - 3\Lambda^2/16\pi^2 f_8,\quad c_1^R = c_1 + (v/16\pi^2 f_8^2)\tfrac{3}{2}\log(\Lambda^2/\mu^2),\quad c_3^R = c_3 + (v/16\pi^2 f_8^2)\tfrac{1}{2}\log(\Lambda^2/\mu^2)\,. \tag{16}$$

Using these we can rewrite (13) and (14) as

[#2] The term proportional to c_1 in (10) corresponds to the term used in refs. [6,7] via the lowest-order equation of motion.

[#3] This current corresponds to $(L_\mu)_{ij} = \bar{q}_j\gamma_\mu\frac{1}{2}(1-\gamma_5)q_i$.

[#4] Using a Λ cut-off and naive Feynman rules the results are not chirally invariant [11]. However, adding a term to the lagrangian of the form $-\mathrm{i}\delta^4(0)\log\det g_{ab}(\phi)$, where g_{ab} is defined in the lagrangian via $\mathcal{L} = \frac{1}{2}g_{ab}\partial_\mu\phi^a\partial^\mu\phi^b$ makes the results chirally invariant. This term corresponds to the difference between the naive measure and a chirally invariant one in the functional integral [12]. We have checked that this term does not contribute to any of the processes discussed in this letter.

$$F_\pi = f_8^R\{1+[4c_1^R/(f_8^R)^2]2m+[8c_3^R/(f_8^R)^2](2m+m_s)$$
$$+[1/(4\pi f_8^R)^2][m_K^2\log(\mu^2/m_K^2)+2m_\pi^2\log(\mu^2/m_\pi^2)]\}, \tag{17}$$
$$F_K = f_8^R\{1+[4c_1^R/(f_8^R)^2](m+m_s)+[8c_3^R/(f_8^R)^2](2m+m_s)$$
$$+[3/4(4\pi f_8^R)^2][2m_K^2\log(\mu^2/m_K^2)+m_\pi^2\log(\mu^2/m_\pi^2)+m_8^2\log(\mu^2/m_8^2)]\}. \tag{18}$$

Changing f_8 to f_8^R in the denominator in (17) and (18) is higher order in $1/N_c$ and thus allowed. We also used the Gell-Mann–Okubo relation

$$m_\pi^2 = 8vm/f_8^2, \quad m_K^2 = 4v(m+m_s)/f_8^2, \quad m_8^2 = \tfrac{4}{3}m_K^2 - \tfrac{1}{3}m_\pi^2. \tag{19}$$

The expressions on the right-hand side of(17) and (18) are μ independent. The dependence of the coefficients c_i^R on μ cancels the explicit μ dependence of logarithmic terms.

From this it is obvious that the approach with an explicit cutoff and the dimensional regularization approach are completely equivalent if all the relevant contributions are properly taken into account.

We will now proceed with the same analysis including the effective weak lagrangian. The traditional four-quark lagrangian transforms as $(8_L, 1_R)$ and $(27_L, 1_R)$ under $SU(3)_L\times SU(3)_R$ and at the quark level all terms have a current × current structure:

$$O_{ijkl} = a(\mu_{QCD})\bar{q}_j\gamma_\mu\tfrac{1}{2}(1-\gamma_5)q_i\bar{q}_l\gamma^\mu\tfrac{1}{2}(1-\gamma_5)q_k, \tag{20}$$

where $a(\mu_{QCD})$is a Wilson coefficient. The full expression for the weak operator including its dependence on $1/N_c$ can be found in ref. [5].

At the leading level in $1/N_c$ there is no strong interaction connection between the two currents in (20). The form of the current in QCD can be derived from a left-handed chiral transformation of the QCD lagrangian. Performing the same left chiral transformation on (3), (10) and (11) leads to the left current in (12) and so (19) can be replaced by

$$O_{ijkl} = (L_\mu)_{ij}(L^\mu)_{kl}. \tag{21}$$

If we now include $1/N_c$ corrections this one-to-one correspondence between (20) and (21) is not valid any-more. We then have to include in (21) all operators that have the correct $SU(3)_L\times SU(3)_R$ transformation properties up to the order in momentum that we are working:

$$O_{ijkl} = (L_\mu)_{ij}(L^\mu)_{kl} + \text{tree level from other operators} + \text{loops in (21)}. \tag{22}$$

Other operators also includes subleading $1/N_c$ corrections of the same form as (21) but with a different coefficient. Unfortunately these other operators introduce new free parameters at the next to leading order in $1/N_c$. This reflects the fact that the four-quark operator is not simply a product of bare currents, but is a composite operator whose overall scale is unknown. Only the leading $1/N_c$ term is fixed in the nonlinear sigma model. As an example the operator relevant for K^0–$\overline{K^0}$ mixing can be written as

$$O_{s\bar{d}s\bar{d}} = -(\tfrac{1}{16}f_8^4+g_1)(\Sigma\partial_\mu\Sigma^\dagger)_{s\bar{d}}(\Sigma\partial^\mu\Sigma^\dagger)_{s\bar{d}}$$
$$-(\tfrac{1}{2}c_1f_8^2+g_2)(\Sigma\partial_\mu\Sigma^\dagger)_{s\bar{d}}(\Sigma\partial^\mu\Sigma^\dagger m^\dagger\Sigma^\dagger-\partial^\mu\Sigma m+m^\dagger\partial^\mu\Sigma^\dagger-\Sigma m\partial^\mu\Sigma\Sigma^\dagger)_{s\bar{d}}$$
$$-(f_8^2c_3+g_3)(\Sigma\partial_\mu\Sigma^\dagger)_{s\bar{d}}(\Sigma\partial^\mu\Sigma^\dagger)_{s\bar{d}}\,\mathrm{tr}(m\Sigma+m^\dagger\Sigma^\dagger)-g_4(m^\dagger\Sigma^\dagger)_{s\bar{d}}(m^\dagger\Sigma^\dagger)_{s\bar{d}}+\dots. \tag{23}$$

In (23) the g_i are of order N_c and these contributions have not been discussed in ref. [6]. The leading term is of order N_c^2 and the g_i are the nonfactorizable contributions to the decay process. Nonleading factorizable contributions are embedded in the term proportional to c_3. The ... stand for other operators with the same transformation under $SU(3)_L\times SU(3)_R$. The complete list is rather long. The g_i in (23) allow to absorb the divergences in the loop contributions into renormalized g_i^R analogous to (15). In particular the quadratic divergence can be absorbed in g_1. Using the diagrams of fig. 2 we obtain

Fig. 2. One-loop Feynman diagrams relevant for K^0–$\overline{K^0}$ mixing. A square is an insertion of the weak operator and a circle a strong interaction vertex.

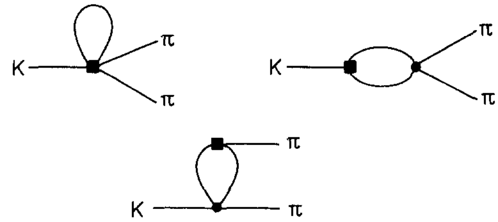

Fig. 3. One-loop Feynman diagrams relevant for $K^+ \to \pi^+\pi^0$.

$$\langle K^0 | O_{s\bar{d}s\bar{d}} | \overline{K^0} \rangle$$
$$= (\tfrac{1}{2} f_8^2 + 8 g_1 / f_8^2) m_K^2 + (8 c_1 + 16 g_2 / f_8^2)(m + m_s) m_K^2 + 16 (c_3 + g^3 / f_8^2)(2m + m_s) m_K^2 + (8 g_4 / f_8^2) m_s^2 + \ldots$$
$$- m_K^2 [\tfrac{5}{2} I_2(m_K^2) + \tfrac{3}{2} I_2(m_8^2) + I_2(m_\pi^2)] + m_K^4 I_3(m_K^2) + I_4(m_K^2) - \tfrac{3}{4} I_4(m_8^2) - \tfrac{1}{4} I_4(m_\pi^2) , \tag{24}$$

where

$$I_3(m_i^2) = (1/16\pi^2)[\Lambda^2/(\Lambda^2 + m_i^2) - \log(1 + \Lambda^2/m_i^2)], \quad I_4(m_i^2) = m_i^2 I_2(m_i^2) - \tfrac{1}{2}\Lambda^4/16\pi^2 . \tag{25}$$

A Λ^4 divergence would break chiral symmetry. As our results have to obey chiral symmetry (cf. footnote 4) there can be no Λ^4 term. Inspection of (24) shows indeed that they cancel. If we collect all quadratically divergent terms and the terms second order in momenta

$$\langle K^0 | O_{s\bar{d}s\bar{d}} | \overline{K^0} \rangle = (\tfrac{1}{2} f_8^2 + 8 g_1 / f_8^2) m_K^2 - (\Lambda^2/16\pi^2) 5 m_K^2 + \ldots , \tag{26}$$

where as required by chiral symmetry there is no $m_\pi^2 \Lambda^2$ term. Using (16) and

$$g_1^R = g_1 - (\Lambda^2 f_8^2 / 16\pi^2) \tfrac{1}{4} \tag{27}$$

this can be rewritten as

$$\langle K^0 | O_{s\bar{d}s\bar{d}} | \overline{K^0} \rangle = [\tfrac{1}{2} (f_8^R)^2 + 8 g_1^R / (f_8^R)^2] m_K^2 + \ldots . \tag{28}$$

Notice that g_1 is formally of order N_c and that the quadratic divergence is of the same order in N_c so that it can be subtracted consistently.

Let us now show that the same g_1^R also removes the quadratic divergence in K^+ decay. We only show the quadratically divergent terms and the terms second order in momenta. The relevant operators here are

$$O_{d\bar{s}u\bar{u}} \quad \text{and} \quad O_{u\bar{s}d\bar{u}} . \tag{29}$$

Including field renormalization and the diagrams in fig. 3 we obtain

$$\langle \pi^+\pi^0 | O_{d\bar{s}u\bar{u}} | K^+ \rangle = \langle \pi^+\pi^0 | O_{u\bar{s}d\bar{u}} | K^+ \rangle$$
$$= (i/2\sqrt{2}) [(\tfrac{1}{2} f_8 + 8 g_1 / f_8^3)(m_K^2 - m_\pi^2) - (\Lambda^2 / 16\pi^2 f_8) \tfrac{7}{2} (m_K^2 - m_\pi^2)] + \ldots . \tag{30}$$

We see here that the subtractions (16) and (27) take care of the quadratic divergence. Both in (24) and (30) the logarithmic divergences can be absorbed analogously to (18) using all possible terms in (23). This has to be the case since our results obey chiral symmetry (cf. footnote 4) and have hence to be of the form $\log \Lambda^2 \times p^4$-tree level in O_{ijkl}.

Since we can renormalize away all divergences there is no need to introduce a physical cutoff. However, even if one wishes to keep an explicit cutoff dependence the finite part of the extra terms still has to be included because they are contributions at the same order of $1/N_c$ as the divergences. The terms proportional to g_i are allowed in the nonlinear sigma model at the level of $1/N_c$ that we are considering. Finite parts of the g_i may be small or even vanish since the nonlinear sigma model is less restrictive than the full QCD theory. In ref. [6] these finite parts have been explicitly set to zero. The authors claim that these finite parts correspond to contributions either of higher resonances or of mesonic loops [13]. We cannot exclude this possibility, but we do not consider this proven. In at least one example these effects do not reproduce all next to leading $1/N_c$ terms. The η' mass cannot be generated in this way but has to be put in as an explicit subleading $1/N_c$ term [14]. So to recapitulate, there are two equivalent ways of going beyond leading $1/N_c$ in the chiral lagrangian:

(1) Using the bare parameters f_8, c_i, g_i, ... and an explicit cutoff Λ, or

(2) using the renormalized parameters f_8^R, c_i^R, g_i^R, ... at a specified renormalization point μ.

The use of dimensional regularization as in the standard chiral perturbation theory [9] corresponds to implicitly using f_8^R and g_1^R since the quadratic divergence is subtracted in the evaluation of integrals.

The above shows that it is already interpretationally difficult to identify the cutoff Λ with the renormalization scale in QCD, μ_{QCD}. In ref. [6] it is claimed that this identification is meaningful and provides a good numerical matching with the μ_{QCD} dependence. We will now look at this matching in a little bit more detail setting the bare g_i equal to zero. This corresponds to the method used in ref. [6]. As before we will only discuss the simpler case of the $(27_L, 1_R)$ operator because this operator is multiplicatively renormalized in QCD at the one loop level. The $(8_L, 1_R)$ operators do however mix among themselves so that a simple discussion is not possible. In the leading logarithm approximation the dependence of (20) on μ_{QCD} for the 27_L operator is

$$a(\mu_{QCD}) \propto [\alpha_s(\mu_{QCD}^2)]^{-3/11} . \tag{31}$$

The exponent in (31) is the leading N_c part of the anomalous dimension. This is subleading in $1/N_c$ since it vanishes for $N_c = \infty$.

There are two processes of interest here, K^0–$\overline{K^0}$ mixing and K^+ decay. They both have the same $a(\mu_{QCD})$ dependence. To cancel this both matrix elements should depend on μ_{QCD} like

$$\langle f | O_{ijkl} | i \rangle \propto [\alpha_s(\mu_{QCD}^2)]^{3/11} . \tag{32}$$

In ref. [6] it is claimed that this is exactly the behaviour produced by the dependence on the cutoff Λ. In particular for the processes in question the Λ dependence is given by (24) [6] [#5]

$$F_B = 1 - (1/16\pi^2 f_8^2)[4\Lambda^2 - \tfrac{20}{3} m_K^2 \log(\Lambda^2/m_K^2)] \tag{33}$$

for the K^0–$\overline{K^0}$ case and by

$$F_{3/2} = 1 - (1/16\pi^2 f_8^2)[4\Lambda^2 - \tfrac{1}{2} m_K^2 \log(\Lambda^2/m_K^2)] \tag{34}$$

for the K^+ decay case. Here we have set $m_\pi^2 = 0$. The Λ dependence in both cases is quite different due to the order of magnitude difference in the logarithmic terms. In ref. [6] it is claimed, however, that a good matching to (31) can be obtained for both cases. the Λ dependence in $F_{3/2}$ is essentially purely quadratic while in F_B the logarithmic terms significantly change the quadratic behaviour in the region 600–800 MeV.

We have plotted both these functions in fig. 4 together with the dependence on μ_{QCD} expected (32). We have matched that dependence to the functions F at the scale of 800 MeV. It is obvious from fig. 4 that no matching is achieved for both cases simultaneously.

One remark is in order here, the subleading logarithms produce a difference in μ_{QCD} dependence between the two cases. It is difficult, however, to imagine that the difference will be large enough to match the difference between (33) and (34).

Taking the limit where m_K^2 becomes very small the behaviour of F_B and $F_{3/2}$ becomes identical and purely quadratic. This is actually the limit in which one should check the matching, since the coefficients $a(\mu)$ are also calculated for massless quarks. We are then matching, however, a quadratic (Λ) and logarithmic (μ_{QCD}) dependence which is impossible over an extended region [#6]. In view of this we expect the numerical matching of the

[#5] The logarithmic dependence was calculated first in ref. [15] with the use of dimensional regularization.

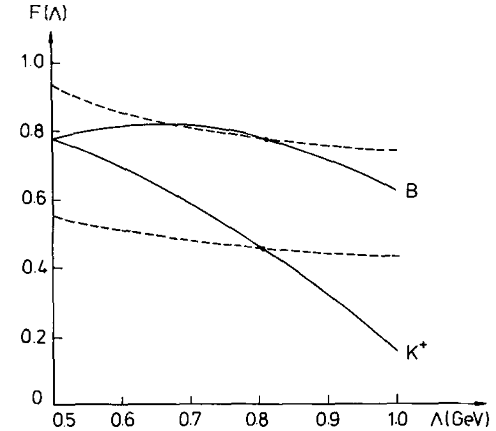

Fig. 4. A comparison of the Λ dependence for K^0–$\overline{K^0}$ mixing and $K^+ \to \pi^+\pi^0$ with the μ_{QCD} dependence. The full curves are the chiral behaviour (33) and (34) and the dotted curves are the expected QCD behaviour (32) normalized to the chiral curves at 800 MeV.

more complex $\Delta I = 1/2$ case obtained in ref. [6] to be of accidental nature. The fact that quadratic Λ dependence matches the logarithmic μ dependence at one point is not enough, in our opinion, to claim good matching. However, additional corrections (as proposed in ref. [6]) like vector meson effects, could improve this matching.

In conclusion, we have shown that the chiral perturbation theory approach of ref. [6] and the standard one [9] are equivalent, as long as one keeps the terms proportional to g_i. We have also pointed out some contributions at the same level in $1/N_c$ that were not discussed in ref. [6]. We have also argued that it is rather difficult both from the interpretational point of view, since the cutoff dependence can be consistently removed, and from the numerical point of view to identify the explicit cutoff in chiral perturbation theory and the renormalization scale in QCD.

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Note added. After submitting our paper, we received a preprint [16] where there is a different criticism of ref. [6].

#6 Notice that there is almost a factor of 2 difference in $F_{3/2}$ between 600 and 800 MeV.

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