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Radiative corrections with two Higgs doublets at LEP/SLC and HERA

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Abstract. Formulae for the radiative corrections to $e^+e^- \to f\bar{f}$ and $ep \to eX, \nu_e X'$ are given for two Higgs doublets in $SU(2) \times U(1)$. The magnitude of deviations from the minimal model is discussed for the $M_W - M_Z$ mass correlation, the e^+e^- forward-backward and polarization asymmetries and $\sigma(e^+e^- \to \text{hadrons})$ at LEP/SLC, and for $\sigma(\text{NC})/\sigma(\text{CC})$, charge and polarization asymmetries in deep inelastic ep scattering at HERA. Precision experiments can restrict the mass splitting between neutral and charged Higgs bosons to $\lesssim 100$ GeV. In the supersymmetric Higgs model the additional corrections remain unobservably small.

1 Introduction

In the standard $SU(2) \times U(1)$ model of the electroweak interaction a single neutral scalar boson is the only remnant of the spontaneous symmetry breaking mechanism. The minimal Higgs structure yields the identity

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1.$$

This experimentally confirmed relation [1], however, does not allow to conclude that the Higgs sector is minimal: $\rho = 1$ remains valid e.g. for any number of additional Higgs doublets.

The strongest motivation for extending the Higgs sector may come from supersymmetry. The supersymmetric version of the standard model requires two $SU(2)$ doublets to give masses to up and down quarks [2]. But also non-supersymmetric arguments advocate two Higgs doublets, such as the discussion of CP violation [3] and the Peccei–Quinn mechanism to solve the strong CP problem [4].

The minimal extension of the standard model has two scalar doublets Φ_1, Φ_2 in an otherwise conventional $SU(2) \times U(1)$ gauge theory. 3 of their 8 degrees of freedom are absorbed in forming the longitudinal polarization states of $W^\pm$, Z, and 5 remain as physical particles. Their spectrum consists of a pair of charged bosons $\phi^\pm$, two neutral scalars H_0, H_1, and a pseudoscalar H_2 (this terminology describes their behaviour on the interaction with fermions). These physical states are obtained by diagonalizing the mass matrix derived from the Higgs potential $V(\Phi_1, \Phi_2)$.

The original Higgs fields in the Lagrangian

$$\mathscr{L}_H = |D_\mu \Phi_1|^2 + |D_\mu \Phi_2|^2 - V(\Phi_1, \Phi_2) \tag{1.1}$$

are written as

$$\Phi_j = \begin{pmatrix} \phi_j^+ \\ (v_j + \eta_j + i\chi_j)/\sqrt{2} \end{pmatrix} \tag{1.2}$$

with vacuum expectation values v_1, v_2. By appropriate choice of the Higgs phases v_1 and v_2 may be taken to be real and positive [2]. The mass eigenstates can be obtained from the unmixed components in (1.2) by

$$\begin{aligned} \phi^\pm &= -\phi_1^\pm \sin\beta + \phi_2^\pm \cos\beta \\ H_2 &= -\chi_1 \sin\beta + \chi_2 \cos\beta \\ H_0 &= \eta_1 \cos\alpha + \eta_2 \sin\alpha \\ H_1 &= -\eta_1 \sin\alpha + \eta_2 \cos\alpha \end{aligned} \tag{1.3}$$

and the unphysical Goldstone fields are

$$\begin{aligned} \psi^\pm &= \phi_1^\pm \cos\beta + \phi_2^\pm \sin\beta \\ \chi &= \chi_1 \cos\beta + \chi_2 \sin\beta. \end{aligned} \tag{1.4}$$

The mixing angle β is related to v_1 and v_2:

$$\tan\beta = v_2/v_1, \tag{1.5}$$

whereas α depends on all the quadratic and quartic parameters of the potential in a rather involved way.

The situation $v_1 \gg v_2 (v_1 \ll v_2)$ leads to Yukawa couplings to the $I_3 = _{(}\pm_{)}\frac{1}{2}$ fermions enhanced by a factor $v_1/v_2 (v_2/v_1)$ compared to the minimal model.* Limits on v_2/v_1 from the leptonic sector are rather weak, [5, 6] ($\sim 0(10^2)$); more stringent limits have been

* If Φ_2 couples to $+\frac{1}{2}$ and Φ_1 to $-\frac{1}{2}$ fermions

derived from CP violation in K, D, and B mesons [7]:

$$\left(\frac{v_1}{v_2}\right)^2 \leqq \frac{2M_{\phi^+}}{m_t}. \tag{1.6}$$

In a non-supersymmetric two-Higgs model the mixing angles α, β and all the physical boson masses M_{ϕ^+}, M_0, M_1, M_2 are independent quantities which are not fixed by the theory. In the minimal supersymmetric model these parameters are severely constraint [2]:

$$\begin{aligned} M_{\phi^+}^2 &= M_W^2 + M_2^2 \\ M_{0,1}^2 &= \tfrac{1}{2}(M_Z^2 + M_2^2 \\ &\quad \pm \sqrt{(M_Z^2+M_2^2)^2 - 4M_Z^2 M_2^2 \cos^2 2\beta}) \\ \tan(2\alpha) &= \tan(2\beta)\cdot\frac{M_2^2+M_Z^2}{M_2^2-M_Z^2}. \end{aligned} \tag{1.7}$$

In such a model one of the neutral scalars is always lighter than the Z boson, whereas $M_{\phi^+} \geqq M_W$. From present e^+e^- experiments an experimental limit $M_{\phi^+} \gtrsim 18\,\text{GeV}$ [8] has been derived.

If the masses of the Higgs bosons are of the weak boson mass scale or heavier there is little chance to produce them in the e^+e^- and ep colliders of the next future. Indirect effects, however, can be present from virtual Higgs bosons in the radiative corrections to the standard fermionic processes $e^+e^- \to f\bar{f}$, $ep \to e(v)X$, as well as in the low-q^2 reactions μ decay and ν scattering. Calculations of 2-doublet 1-loop corrections (with $v_1 \approx v_2$) in the $M_W - M_Z$ correlation and the ρ parameter have been performed in [9, 10].

In an earlier paper we have studied the effects of a second Higgs doublet in the 1-loop corrections to $e^+e^- \to \mu^+\mu^-(\tau^+\tau^-)$ for the case of enhanced Yukawa couplings [6]. In this limit the mixing in the neutral scalar sector is very transparent (if the quartic couplings in V are all of the same order of magnitude): H_0 behaves like the single standard Higgs, whereas H_1, H_2 and $\phi^\pm$ appear as additional particles with enhanced Yukawa couplings to the leptons. It was found that charge and polarization asymmetries are rather insensitive to v_2/v_1 up to $v_2/v_1 = 0(10^2)$. For the purpose of investigating 1-loop effects of 2-doublet models in the asymmetries it is therefore allowed to neglect vertex corrections and treat the additional Higgs contributions only in the gauge bosons' diagonal and mixing propagators. This covers practically the whole range of $0 < \beta < \pi/2$; in particular the situation $v_1 \approx v_2$ is of special interest for supersymmetric 2-Higgs models.

In this paper we calculate the renormalized gauge boson 2-point functions in the on-shell scheme for general mixing in the scalar sector and discuss the radiative corrections in $e^+e^+ \to \mu^+\mu^-$, $e^+e^- \to$ hadrons at LEP/SLC and in deep inelastic ep scattering at HERA. Mass splitting between the neutral and charged Higgs bosons can give quite remarkable contributions to cross sections and asymmetries. In particular, if the neutral Higgses have masses around the Z mass scale (or below) precision measurements can tightly restrict the mass of the charged Higgs to $\lesssim 200\,\text{GeV}$. In the supersymmetric version big mass splittings are not allowed; therefore large deviations from the minimal model are not expected.

In Sect. 2 we list the formulae for the renormalized diagonal and non-diagonal self energies in the on-shell scheme. Section 3 contains the discussion of the $M_W - M_Z$ interdependence which enters the results for the e^+e^- and ep reactions presented in Sects. 4 and 5.

2 Renormalized vector boson 2-point functions

The propagators for the vector boson fields are decomposed as

$$\Delta_{\mu\nu}^j(k) = \left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\right)\Delta_T^j + \frac{k_\mu k_\nu}{k^2}\Delta_L^j, \quad j = \gamma, \gamma Z, Z, W. \tag{2.1}$$

Since we do not consider processes with heavy fermions it is sufficient to deal with the transverse parts Δ_T^j only. They are related to the corresponding 1-loop self energies Σ^j by

$$\begin{aligned} \Delta_T^j &= \frac{i}{k^2 - M_j^2}\left[1 - \frac{\Sigma^j(k^2)}{k^2 - M_j^2}\right], \quad j = \gamma, Z, W \\ \Delta_T^{\gamma Z} &= -\frac{i}{k^2 - M_Z^2}\,\frac{\Sigma^{\gamma Z}(k^2)}{k^2}. \end{aligned} \tag{2.2}$$

The renormalization is performed in the on-shell scheme for the 2-doublet model as described in [6]. This means that the loop calculations are done with the physical mass eigenstates of gauge and Higgs bosons, and the counter terms are fixed by the conditions that the input parameters

$$M_W, M_Z, M_0, M_1, M_2, M_{\phi^+} \tag{2.3}$$

are the physical masses, and α becomes the electromagnetic fine structure constant in the Thomson limit.

For the calculation of gauge boson self energies we need only the kinetic part of the Higgs Lagrangian (1.1) and not the details of the Higgs potential. Only the mixing angles α, β from (1.3) enter the results as input parameters in addition to (2.3).

The renormalized Σ^j, which are finite by the method of [6], can be split up into

$$\Sigma^j = \hat{\Sigma}^j + \tilde{\Sigma}^j \tag{2.4}$$

where $\hat{\Sigma}^j$ denotes the standard self energy including a single standard Higgs [11]. Also the Goldstone-fields (1.4) in the 't Hooft-Feynman gauge are part of $\hat{\Sigma}^j$. $\tilde{\Sigma}^j$ contains all those contributions which are beyond the minimal model. Since in the general case of mixing in the neutral scalar sector none of H_0 and H_1 can be seperated as a "standard" Higgs in a natural way, the single Higgs part included in $\hat{\Sigma}^j$ has to be subtracted in $\tilde{\Sigma}^j$ in order to avoid double-counting. In the limiting situation discussed in [6] H_0 becomes the standard Higgs and H_1, H_2, $\phi^\pm$ decouple from H_0 in the loop diagrams.

With the definition of the mixing angle

$$s_W^2 \equiv \sin^2\theta_W = 1 - M_W^2/M_Z^2, \quad c_W^2 = 1 - s_W^2, \tag{2.5}$$

we list these additional contributions $\tilde{\Sigma}^j$:

γ self energy:

$$\tilde{\Sigma}^\gamma(k^2) = \frac{\alpha}{12\pi}\left[\frac{2k^2}{3} + H(k^2, M_{\phi^+}, M_{\phi^+})\right] \tag{2.6}$$

γ − Z mixing:

$$\tilde{\Sigma}^{\gamma Z}(k^2) = \tilde{\Sigma}^{\gamma Z}_{\text{fin}}(k^2) - k^2 \delta Z_2^{\gamma Z} \tag{2.7}$$

with

$$\tilde{\Sigma}^{\gamma Z}_{\text{fin}}(k^2) = \frac{\alpha}{4\pi}\frac{s_W^2 - c_W^2}{6 s_W c_W}\left[k^2 \log\frac{M_W^2}{M_{\phi^+}^2} + \frac{2k^2}{3} + H(k^2, M_{\phi^+}, M_{\phi^+})\right] \tag{2.8}$$

and the finite part of the renormalization constant

$$\delta Z_2^{\gamma Z} = \frac{c_W}{s_W}\left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right) \equiv \frac{c_W}{s_W}\,\text{Re}\left(\frac{\Sigma^Z_{\text{fin}}(M_Z^2)}{M_Z^2} - \frac{\Sigma^W_{\text{fin}}(M_W^2)}{M_W^2}\right), \tag{2.9}$$

$\Sigma^{Z,W}_{\text{fin}}$ from (2.12) and (2.13).

Z and W self energies:

$$\tilde{\Sigma}^Z(k^2) = \Sigma^Z_{\text{fin}}(k^2) - \Sigma^Z_{\text{fin}}(M_Z^2) + \delta Z_2^Z(k^2 - M_Z^2) \tag{2.10}$$

$$\tilde{\Sigma}^W(k^2) = \Sigma^W_{\text{fin}}(k^2) - \Sigma^W_{\text{fin}}(M_W^2) + \delta Z_2^W(k^2 - M_W^2) \tag{2.11}$$

with

$$\begin{aligned}\tilde{\Sigma}^Z_{\text{fin}}(k^2) = \frac{\alpha}{4\pi}\Bigg\{&\frac{(c_W^2 - s_W^2)^2}{12 c_W^2 s_W^2}\left[k^2\log\frac{M_W^2}{M_{\phi^+}^2} + \frac{2k^2}{3} + H(k^2, M_{\phi^+}, M_{\phi^+})\right] \\ &+ \frac{1}{12 s_W^2 c_W^2}k^2\left(\log\frac{M_W^2}{M_1 M_2} + \frac{2}{3}\right) \\ &+ \frac{\sin^2\zeta}{12 s_W^2 c_W^2}[H(k^2, M_1, M_Z) + H(k^2, M_0, M_2) \\ &- H(k^2, M_0, M_Z)] + \frac{\cos^2\zeta}{12 s_W^2 c_W^2}H(k^2, M_1, M_2) \\ &+ \frac{\sin^2\zeta}{s_W^2 c_W^2}M_Z^2\left[\log\frac{M_0}{M_1} + \bar{B}_0(k^2, M_1, M_Z) \right. \\ &\left. - \bar{B}_0(k^2, M_0, M_Z)\right]\Bigg\}\end{aligned} \tag{2.12}$$

$$\begin{aligned}\Sigma^W_{\text{fin}}(k^2) = \frac{\alpha}{4\pi}\frac{1}{12 s_W^2}\Bigg\{&k^2\left(\log\frac{M_W^2}{M_1 M_{\phi^+}} + \log\frac{M_W^2}{M_2 M_{\phi^+}} + \frac{4}{3}\right) \\ &+ \sin^2\zeta[H(k^2, M_1, M_W) + H(k^2, M_0, M_{\phi^+}) \\ &- H(k^2, M_0, M_W)] \\ &+ \cos^2\zeta\cdot H(k^2, M_1, M_{\phi^+}) + H(k^2, M_2, M_{\phi^+}) \\ &+ 12 M_W^2 \sin^2\zeta\left[\log\frac{M_0}{M_1} + \bar{B}_0(k^2, M_1, M_W)\right. \\ &\left. - \bar{B}_0(k^2, M_0, M_W)\right]\Bigg\}.\end{aligned} \tag{2.13}$$

The two mixing angles α, β in (1.3) appear only in the combination $\zeta = |\alpha - \beta|$. The field renormalization constants in (2.10–11) are:

$$\delta Z_2^Z = -\frac{\alpha}{12\pi}\log\frac{M_W}{M_{\phi^+}} + \frac{c_W^2 - s_W^2}{s_W^2}\left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right) \tag{2.14}$$

$$\delta Z_2^W = -\frac{\alpha}{12\pi}\log\frac{M_W}{M_{\phi^+}} + \frac{c_W^2}{s_W^2}\left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right). \tag{2.15}$$

For the functions H and $\bar{B}_0$ see the appendix.

3 The $M_W - M_Z$ interdependence

The masses in the parameter set (2.3) are not all independent. Specifying M_Z and the Higgs mass(*es*), M_W can be calculated from the well-known μ lifetime resp. in terms of the μ decay constant G_μ:

$$G_\mu = \frac{\pi\alpha}{\sqrt{2}M_W^2(1 - M_W^2/M_Z^2)(1 - \Delta r)}. \tag{3.1}$$

The quantity

$$\Delta r = \Delta\hat{r}(\alpha, M_W, M_Z, M_0) + \Delta\tilde{r}(\alpha, M_W, M_Z, M_0, M_1, M_2, M_{\phi^+}) \tag{3.2}$$

contains the standard correction [12]

$$\Delta\hat{r} = \frac{\hat{\Sigma}^W(0)}{M_W^2} + \frac{\alpha}{4\pi s_W^2}\left(6 + \frac{7 - 4s_W^2}{2s_W^2}\log c_W^2\right) \tag{3.3}$$

and the additional non-standard term

$$\Delta\tilde{r} = \frac{\tilde{\Sigma}^W(0)}{M_W^2} \tag{3.4}$$

with $\tilde{\Sigma}^W$ from (2.11).

The reward of (3.1) is twofold:
Firstly, it is interesting by itself since it allows a comparison of the $M_W - M_Z$ correlation with the

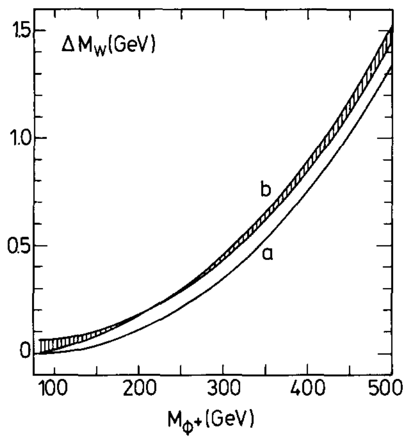

Fig. 1. Deviation of the W mass from the standard model value for additional neutral scalar/pseudoscalar masses: a) $M_1 = M_2 = M_Z$; b) $M_1 = 10\,\text{GeV}$, $M_2 = M_Z$. $M_Z = 93\,\text{GeV}$, $M_0 = M_Z$

directly measured masses, and secondly, it determines the value for M_W which can be used as input for calculating other observables of interest at the 1-loop level. Since M_Z will be known with ~ 20 MeV precision [18] this procedure makes use of the best known input parameters. The first aspect is discussed in this section (for earlier work see [9, 10]), whereas the second is utilized in the following applications to e^+e^- and ep processes.

(3.1) is solved numerically to give M_W in the standard model (with $\Delta\hat{r}$) and the 2-doublet model (with $\Delta\hat{r} + \Delta\tilde{r}$). The differences are shown in Fig. 1 (fixing $M_Z = 93\,\text{GeV}$) as functions of $M_{\phi^+} \geqq M_W$ for various choices of the neutral boson masses. Since we do not want to single out a specific mixing scheme, the plot contains the results where ζ varies from 0 to $\pi/2$ (shaded area). This includes possible large enhancement ($\zeta \to \pi/2$), but, as shown in [6], enhancement $\to O(10^2)$ does not affect the results in a visible way. Taking into account the restrictions from the B system [8], ζ is restricted to $\lesssim \pi/3$. The corresponding boundaries, however, lie very close to each other; thus it is of no practical importance what the actual limits of ζ are. For $M_0 = M_1$ the results become independent of ζ.

From the present M_W measurements with $\Delta M_W \simeq 2\,\text{GeV}$ no restrictive bounds on the mass splitting between neutral and charged sector can be derived. $\Delta M_W \simeq 100\,\text{MeV}$, as expected from LEP II [13], could restrict $M_{\phi^+} \lesssim 200\,\text{GeV}$ if the neutral Higgs masses are $\lesssim M_Z$.

Whereas in a, b the masses and mixing angles are treated as independent quantities, a supersymmetric Higgs sector would correlate them by (1.7). Specifying M_{ϕ^+} and fixing the lighter of the neutral scalars at 10 GeV, ζ and the other neutral masses are determined. With increasing M_{ϕ^+} also M_1 and M_2 increase; therefore the deviation from the minimal model (with $M_H = 10\,\text{GeV}$) remain smaller than the experimental uncertainty in M_W.

4 e^+e^- annihilation on the Z

In [6] we have shown that effects of the second Higgs doublet are small in the leptonic forward-backward asymmetries at $\sqrt{s} \sim 40\,\text{GeV}$. It is therefore more interesting to discuss those asymmetries on resonance. Large enhancement $\sim 10^2$ does not play a rôle in the on-resonance polarization and charge asymmetries, since the form factor contributions largely cancel. Therefore the source for loop effects from the Higgs sector are the gauge boson self energies Σ^Z and $\Sigma^{\gamma Z}$.

For $e^+e^- \to \mu^+\mu^-$ the general form of the differential cross section, including initial state longitudinal polarization, has been given in [6].

We discuss the following asymmetries ($s = M_Z^2$):

Forward-backward asymmetry:

$$A_{FB} = \frac{1}{\sigma}\left(\int_{\theta < \pi/2} d\sigma - \int_{\theta > \pi/2} d\sigma \right), \quad \theta = \sphericalangle(e^-, \mu^-) \quad (4.1)$$

Polarization (left-right) asymmetry:

$$A_{LR} = \frac{\sigma(e_L^-) - \sigma(e_R^-)}{\sigma(e_L^-) + \sigma(e_R^-)} \quad (4.2)$$

with left- and right-handed electrons $e_{L,R}^-$.

Under the assumption that v_1/v_2 is not too far from 1 (which means that Higgs contributions can also be neglected in the quark vertices) the *cross section for hadron production* at $s = M_Z^2$

$$R_h = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \quad (4.3)$$

can easily be expressed in the following way*:

$$R_h = 3 \sum_{f=\mu,\cdot\cdot b} \frac{1 + 4[I_3^f - 2s_W^2 Q_f + 2s_W c_W Q_f \Pi^{\gamma Z}]^2}{1 + [4s_W^2 - 1 - 4s_W c_W \Pi^{\gamma Z}]^2} + \delta R_h. \quad (4.4)$$

$\Pi^{\gamma Z} = \tilde{\Sigma}^{\gamma Z}(M_Z^2)/M_Z^2$ contains the additional Higgs contributions, (2.7), and δR_h summarizes all the other standard model contributions, including vertex, box and QED corrections.

The results shown in Figs. 2–4 show the deviations from the standard model predictions. They are obtained in the following way:

(i) For given M_Z, ζ, and Higgs masses a value for M_W is derived from (3.1).

(ii) With this value of M_W

$$\sin^2\theta_W = 1 - M_W^2/M_Z^2$$

is calculated and inserted into the 1-loop expressions for the quantities (4.1–3). Then the standard model result (with $M_0 = M_H$) is subtracted yielding the non-standard additional contributions.

These deviations thus have 2 sources: the shift in

* Here we will assume that $m_t > M_Z/2$

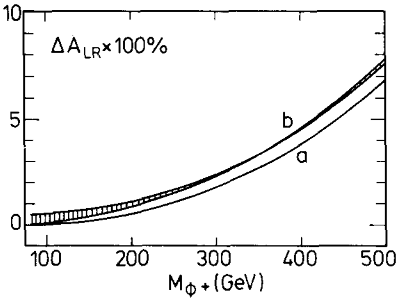

Fig. 2. Additional Higgs contribution to the e^+e^- polarization asymmetry at $\sqrt{s}=M_Z$. Some parameters as in Fig. 1

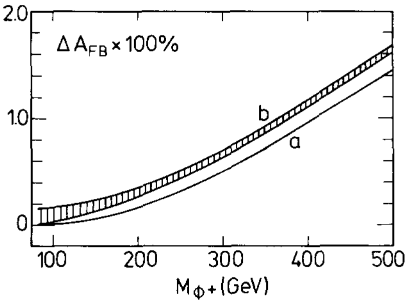

Fig. 3. Additional Higgs contributions to the $e^+e^-\to\mu^+\mu^-$ forward-backward asymmetry at $\sqrt{s}=M_Z$

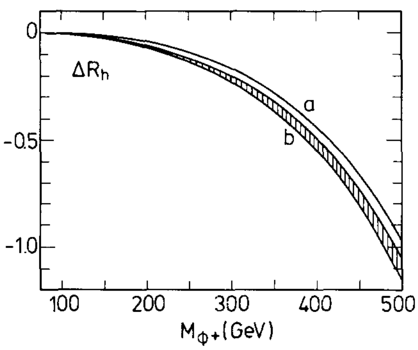

Fig. 4. Additional Higgs contributions to $R_h=\sigma(e^+e^-\to\text{hadrons})/\sigma(e^+e^-\to\mu^+\mu^-)$ at $\sqrt{s}=M_z$. Same parameters as in Fig. 1

the coupling constants, and the additional diagrams in $e^+e^- - f\bar{f}$.

In order to be on the safe side for R_h we restrict the ζ region to $\zeta\leq\pi/3$ avoiding large enhancement (shaded areas display the variation with ζ). This, however, leads only to marginal displacements of the boundary compared to $\zeta<\pi/2$.

The polarization asymmetry shows the best sensitivity to large mass splittings. The expected accuracy of $\Delta A_{LR}\approx 0.005$ at the SLC [14] can restrict $M_{\phi^+}\lesssim 200$ GeV if the neutral masses are around M_Z. This is comparable with the W mass measurement at LEP II. An equivalent bound can be derived from A_{FB} with $\Delta A_{FB}=0.0015$, and from R_h if R_h can be measured to ± 0.004, which corresponds to a relative accuracy of 0.2%.

If we again apply the SUSY constraints (1.7) and fix $M_1=10$ GeV the deviations from the minimal model results (with $M_H=10$ GeV) remain below the experimental sensitivity in all the observables:

$$\Delta A_{LR}\lesssim 0.003$$
$$\Delta A_{FB}\lesssim 0.001$$
$$\Delta R_h\lesssim 0.001.$$

If such a low lying Higgs state, which is predicted by minimal supergravity models [15], would be discovered it would therefore be practically indistinguishable from a single standard Higgs by means of studying radiative corrections.

5 Deep inelastic *ep* scattering

The *neutral current* differential cross section for $e^{\mp}p\to e^{\mp}X$ with longitudinally polarized $e^{\mp}$ beams can be written in the following way:

$$\frac{d^2\sigma^{\mp}}{dx\,dQ^2}=\frac{\pi\alpha^2}{s^2}\sum_f\frac{q_f(x,Q^2)}{x^2}\cdot[T_U^{\mp}(x,Q^2)\mp P_L T_L^{\mp}(x,Q^2)] \tag{5.1}$$

where $P_L=$ degree of longitudinal $e^{\mp}$ polarization, $Q^2=$ positive momentum transfer from the incoming to the outgoing $e^{\mp}$.

The contributions coming from standard radiative corrections can be summarized in terms of diagonal and non-diagonal propagators, vertex and box diagrams, and QED corrections. A complete list and numerical results have recently been given by Böhm and Spiesberger [16].

Those terms coming from the second Higgs doublet contribute via the bosonic 2-point functions if no large enhancement is present. In a compact notation, the quantities $T_U^{\mp}$ and $T_L^{\mp}$, including the extra Higgs terms, read (with $\hat{s}=xs$, $u=Q^2-\hat{s}$):

$$\begin{aligned}T_U^{\mp}&=2Q_f^2(\hat{s}^2+u^2)D_\gamma^2\\&\quad-4Q_f[VV_f(\hat{s}^2+u^2)\pm aa_f(\hat{s}^2-u^2)]D_\gamma D_Z\\&\quad+2[(V^2+a^2)(V_f^2+a_f^2)(\hat{s}^2+u^2)\\&\quad\pm VV_f aa_f(\hat{s}^2-u^2)]D_Z^2,\end{aligned} \tag{5.2}$$

$$\begin{aligned}T_L^{\mp}&=-4Q_f[aV_f(\hat{s}^2+u^2)\pm Va_f(\hat{s}^2-u^2)]D_\gamma D_Z\\&\quad+4[Va(V_f^2+a_f^2)(\hat{s}^2+u^2)\\&\quad\pm V_f a_f(V^2+a^2)(\hat{s}^2-u^2)]D_Z^2.\end{aligned} \tag{5.3}$$

If q_f refers to an anti-quark, the sign of the $\hat{s}^2-u^2$ terms have to be reversed. With the functions $\tilde{\Sigma}^j$ from (2.6–15) we have:

$$D_\gamma=[Q^2-\tilde{\Sigma}^\gamma(-Q^2)]^{-1}$$
$$D_Z=[Q^2+M_Z^2-\tilde{\Sigma}^Z(-Q^2)]^{-1}$$
$$V=v+\tilde{\Sigma}^{\gamma Z}(-Q^2)/Q^2$$
$$V_f=v_f-Q_f\tilde{\Sigma}^{\gamma Z}(-Q^2)/Q^2$$

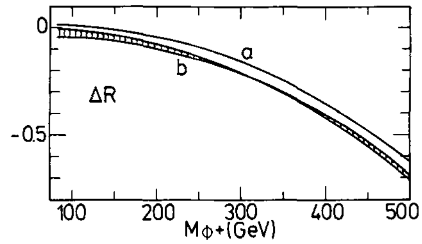

Fig. 5. Additional Higgs contributions to $R = \mathrm{NC/CC}$ in deep inelastic ep scattering at $s = 10^5\,\mathrm{GeV}$. Same parameters as in Fig. 1

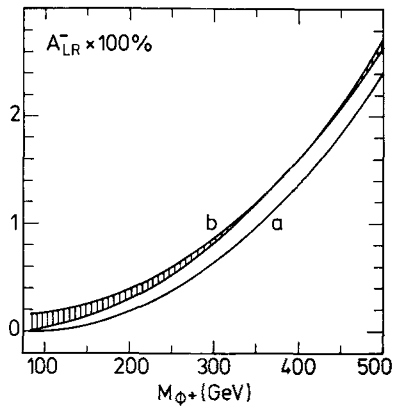

Fig. 6. Additional Higgs contributions to the polarization asymmetry in deep inelastic ep scattering at $s = 10^5\,\mathrm{GeV}$. Same parameters as in Fig. 1

$$a_f = I_3^f/(2s_W c_W)$$
$$v_f = (I_3^f - 2Q_f s_W^2)/(2s_W c_W)$$
$$v \equiv v_e, \quad a \equiv a_e. \tag{5.4}$$

For the *charged current process* $ep \to \bar{\nu}_e X$ the additional Higgs contributions enter the W propagator:

$$[Q^2 + M_W^2]^{-1} \to [Q^2 + M_W^2 - \tilde{\Sigma}^W(-Q^2)]$$

with $\tilde{\Sigma}^W$ from (2.11). In order to have a look on the effects of the additional Higgs bosons we calculate the integrated cross sections

$$\sigma = \int_{Q_0^2/s}^{1} dx \int_{Q_0^2}^{xs} dQ^2 \frac{d^2\sigma}{dx\,dQ^2} \tag{5.5}$$

and discuss the following measurable quantities:

NN/CC ratio:

$$R = \frac{\sigma^{\mathrm{NC}}}{\sigma^{\mathrm{CC}}} \tag{5.6}$$

NC polarization asymmetry:

$$A_{LR}^- = \frac{\sigma(e_L^-) - \sigma(e_R^-)}{\sigma(e_L^-) + \sigma(e_R)} \tag{5.7}$$

where e_L^- corresponds to $P_L = -1$ in (5.1).

Mixed charge-polarization asymmetry:

$$A_{LR}^{+-} = \frac{\sigma(e_L^+) - \sigma(e_R^-)}{\sigma(e_L^+) + \sigma(e_R^-)}. \tag{5.8}$$

The mixed asymmetry A_{LR}^{+-} was shown in [16] to be very insensitive to changes of the standard model parameters. On the other hand, it may indicate the presence of a second Z' boson in specific $SU(2) \times U(1) \times \tilde{U}(1)$ models [17]. Here A_{LR}^{+-} turns out to be practically insensitive to the enlarged Higgs sector. Deviations from the standard situation are $\sim 0(10^{-3})$, even for large mass splittings. This confirms A_{LR}^{+-} to be a probe for extra gauge bosons since it is also not affected by changing the particle content in the minimal gauge group.

The deviations from the standard model, ΔR and ΔA_{LR}^-, are displayed in Figs. 4 and 5 for a value $Q_0^2 = 5 \cdot 10^3\,\mathrm{GeV}^2$. The situations correspond to those that have been considered in Sect. 4. In particular, the standard model radiative corrections to A_{LR}^- are $\sim +4\%$ [16]; large mass splittings in 2-Higgs doublets enlarge this shift by a few percent more.

On the other hand, if we impose the SUSY constraints (1.7) and fix $M_1 = 10\,\mathrm{GeV}$, the deviations from the minimal model (with $M_H = 10\,\mathrm{GeV}$) become negligibly small.

Appendix

The function $\bar{B}_0$ is related to the scalar 2-point integral in the following way ($\Delta = 2/(4-D) - \gamma + \log 4\pi$):

$$\mu^{4-D} \int \frac{d^D q}{(2\pi)^D} \frac{1}{(q^2 - m_1^2)[(q+k)^2 - m_2^2]}$$
$$= \frac{i}{16\pi^2}\left[\Delta - \log\frac{m_1 m_2}{\mu^2} + \bar{B}_0(k^2, m_1, m_2)\right], \tag{A.1}$$

$$\bar{B}_0(k^2, m_1, m_2)$$
$$= -\int_0^1 dx \log\frac{x^2k^2 - x(k^2 + m_1^2 - m_2^2) + m_1^2 - i\varepsilon}{m_1 m_2}$$
$$= 1 - \frac{m_1^2 + m_2^2}{m_1^2 - m_2^2}\log\frac{m_1}{m_2} + F(k^2, m_1, m_2). \tag{A.2}$$

For the analytic result for F see [11]. F has the property: $F(0, m_1, m_2) = 0$.

The function H in the self energies reads:

$$H(k^2, m_1, m_2) = 2(m_1^2 - m_2^2)\log\frac{m_2}{m_1}$$
$$+ \frac{(m_1^2 - m_2^2)^2}{k^2} F(k^2, m_1, m_2)$$
$$+ (k^2 - 2m_1^2 - 2m_2^2)\bar{B}_0(k^2, m_1, m_2). \tag{A.3}$$

For $m_1 = m_2$ this can be simplified to

$$H(k^2, m, m) = (k^2 - 4m^2)F(k^2, m, m). \tag{A.4}$$

We also need the value at $k^2 = 0$:

$$H(0, m_1, m_2) = 2(m_1^2 - m_2^2)\log\frac{m_2}{m_1} + \frac{m_1^2 + m_2^2}{2} - 2\frac{m_1^2 m_2^2}{m_1^2 - m_2^2}\log\frac{m_1}{m_2} - 2(m_1^2 + m_2^2) \cdot \left[1 - \frac{m_1^2 + m_2^2}{m_1^2 - m_2^2}\log\frac{m_1}{m_2}\right] \quad \text{(A.5)}$$

if $m_1 \neq m_2$,

and

$$H(0, m_1, m_2) = 0 \quad \text{if} \quad m_1 = m_2.$$

References

1. G. Altarelli: Proceedings of the Berkeley Conference, 1986
2. H.E. Haber, G.L. Kane: Phys. Rep. 117 (1985) 75; J.F. Gunion, H.E. Haber: Nucl. Phys. B272 (1986) 1
3. A.J. Buras: Proceedings of EPS meeting, L. Nitti, G. Preparata (eds.) Bari 1985 and references therein
4. R.D. Peccei, H.R. Quinn: Phys. Rev. Lett. 38 (1977) 1440; Phys. Rev. D16 (1977) 1719
5. H.E. Haber, G.L. Kane, T. Sterling: Nucl. Phys. B161 (1979) 493
6. W. Hollik: Z. Phys. C—Particles and Fields 32 (1986) 291
7. L.F. Abbott, P. Sikivie, M.B. Wise, Phys. Rev. D21 (1980) 1393; M.D. Sher, D. Silverman: Phys. Rev. D31 (1985) 95
8. S. Komamiya, Proceedings of the Int. Symp. on Lepton and Photon Interactions, Kyoto 1985; CELLO Collaboration, DESY 87-30 (1987)
9. J.M. Frére, J. Vermaseren: Z. Phys. C—Particles and Fields 19 (1983) 63
10. S. Bertolini: Nucl. Phys. B272 (1986) 77
11. M. Böhm, W. Hollik, H. Spiesberger: Fortschr. Phys. 34 (1987) 687
12. M. Böhm, W. Hollik, H. Spiesberger: Z. Phys. C—Particles and Fields 27 (1985) 523
13. LEP 200 Workshop: Aachen 1986; A. Böhm, W. Hoogland, (eds.) CERN 87-08, ECFA 87/108 (1987)
14. D. Blockus et al.: Proposal for Polarization at the SLC, SLAC Proposal, 1986 existiert mur uls manuskript ohne weitere Augaben
15. M. Drees, M. Glück, K. Grassie: Phys. Lett. 159B (1985) 118; E. Reya: Preprint Dortmund University DO-TH 85/16 (1985)
16. M. Böhm, H. Spiesberger: Preprint Würzburg University, 1986
17. F. Cornet, R. Rückl: Phys. Lett. 184B (1987) 263
18. Physics at LEP, ed. J. Ellis, R. Peccei: CERN 86-02 (1986)