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An effective chiral Lagrangian model for the τ-decay into three charged pions*

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Abstract. An effective chiral Lagrangian for the π-ρ-A_1-system is used to study the decay $\tau^\pm \to \nu_\tau \pi^+ \pi^- \pi^\pm$. Demanding that the hadronic current $\langle 0|\mathbf{J}_\mu|3\pi\rangle$ agrees at the low energy limit with the one obtained from the corresponding pseudoscalar meson Lagrangian $\mathscr{L}(\pi)$, an ambiguity linked to free parameters in $\mathscr{L}(\pi, \rho, A_1)$ is eliminated. The decay spectrum and branching ratio of $\tau^\pm \to \nu_\tau \pi^+ \pi^- \pi^\pm$ are calculated as functions of the A_1-mass and compared with the experimental results. So conclusions can be drawn concerning the A_1-parameters, which are found to be

$$m_{A_1} = (1180 \pm 50)\,\mathrm{MeV}; \quad \Gamma_{A_1} = (450 \pm 100)\,\mathrm{MeV}.$$

The mass-value is smaller than recently published ones. This is attributed to the effect of suppressed decay channels not seen in the experiments up to now. A contribution from direct, non-($\rho^0\pi^\pm$) resonance A_1-decay is obtained, being about 15%.

1 Introduction

The process $\tau \to 3\pi\nu_\tau$, dominated by the A_1-meson, $J^{PC} = 1^{++}$, should be particular suited to determine the A_1-parameters (the new notation is a_1).

Until recently the hadronic reaction $\pi^- p \to A_1 X \to 3\pi X$ was taken for this purpose; however, a non-resonance (Deck-effect) background necessitates a delicate analysis to extract m_{A_1} and Γ_{A_1}.

The results are for $\pi^- p \to \pi^- \pi^+ \pi^- p$ [1]

$$m_{A_1} = (1280 \pm 30)\,\mathrm{MeV}; \quad \Gamma_{A_1} = (300 \pm 50)\,\mathrm{MeV} \qquad (1)$$

and for $\pi^- p \to \pi^- \pi^0 \pi^+ n$ [2]

$$m_{A_1} = (1240 \pm 80)\,\mathrm{MeV}; \quad \Gamma_{A_1} = (380 \pm 100)\,\mathrm{MeV}. \qquad (2)$$

During the last years, several collaborations [3, 4, 5] studied the decay of the τ-lepton into three charged pions. Their A_1-parameters, obtained by fitting the decay spectrum, are

	m_{A_1}/MeV	$\Gamma(A_1\rho\pi)$/MeV
ARGUS [3]	1046 ± 11	521 ± 27
DELCO [4]	$1056 \pm 20 \pm 15$	$476\,^{+132}_{-120} \pm 54$
MARK II [5]	$1194 \pm 14 \pm 10$	$462 \pm 56 \pm 30.$

As was mentioned in [4, 5], these parameters depend very much on the ansatz used to fit the spectrum. Bowler [7] reanalysed the data of [3, 5], considering the mass-dependance of Γ_{A_1} as well as a possible mass-dependance of the coupling between the hadronic and the leptonic current.

His result is consistent with (1, 2):

$$m_{A_1} = (1235 \pm 40)\,\mathrm{MeV}; \quad \Gamma_{A_1} = (400 \pm 100)\,\mathrm{MeV}. \qquad (3)$$

Törnquist [8] included the effects of the $K^*\bar{K} + \bar{K}^*K$ thresholds and a momentum-dependant correction to the A_1-mass, with the result

$$m_{A_1} = (1250 \pm 40)\,\mathrm{MeV}; \quad \Gamma_{A_1} = (600 \pm 100)\,\mathrm{MeV}. \qquad (4)$$

Remarkably, in these analyses it is assumed that the only decay channel is $\tau \to A_1\nu_\tau \to \rho\pi\nu_\tau \to 3\pi\nu_\tau$. However, the effective Lagrangian model used in this paper predicts small contributions from the decay via an off-shell pion $\tau \to \pi\nu_\tau \to \rho\pi\nu_\tau \to 3\pi\nu_\tau$ (approximately 10%) and $\tau \to A_1\nu_\tau \to 3\pi\nu_\tau$ (approximately 15%), which are neither seen nor excluded by the experiments up to now. These decay channels and the interferences belonging to them have an effect on the form of the decay spectrum which tends to lower the m_{A_1}-value derived from a fit.

Moreover, the model allows the use of the measured branching ratio as an additional means to determine m_{A_1}, which again comes out smaller when these decay channels are taken into account.

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Starting with a spontaneously broken $SU(2)_L \times SU(2)_R$-symmetry, we know that a non-linear effective Lagrangian in connection with a non-linear group realization describes the low energy dynamics of pions [9, 10]. The relation to QCD, the underlying dynamical theory, is still unresolved. Therefore the construction and interpretation of an effective Lagrangian describing meson-dynamics at energies of order 1 GeV is not straightforward; however, concerning the non-anomalous sector, a generally accepted procedure is known [9, 10].

Given the generating functional of connected Green's functions, vector meson fields are introduced by suitable Legendre transformations. They correspond to resonances generated by the underlying dynamics. It is assumed that only one particle states are excited by the sources; i.e. vector meson dominance in our case.

The $SU(2)_L \times SU(2)_R$-invariant terms of the effective action with lowest order of space-time derivatives together with symmetry-breaking mass terms are considered as a reasonable tool to describe low energy meson dynamics.

To eliminate an ambiguity linked to free parameters not fixed by invariance considerations it is natural to make sure that for vanishing hadronic momentum the physics described by the effective Lagrangian $\mathscr{L}(\pi, \rho, A_1)$ with (axial) vector mesons agrees with the one described by $\mathscr{L}(\pi)$. This aim is reached by choosing minimal momentum dependance for $\mathscr{L}(\pi, \rho, A_1)$.

In Sect. 2 a model of Golterman and Hari-Dass [11] is recapitulated to make this paper self-contained. The effective chiral Lagrangian is brought into a form suitable for further calculations.

Then, in Sect. 3, the relations for fixing the free parameters are given, whose values depend on the A_1-mass. The hadronic current $\langle 0|J_\mu^+|\pi^+\pi^+\pi^-\rangle$ is calculated using $\mathscr{L}(\pi, \rho, A_1)$ and these relations, the result is found to agree with the one derived from $\mathscr{L}(\pi)$.

In Sect. 4 the momentum dependance of the (axial) vector meson propagators occurring in the hadronic current is taken into account. This necessitates the knowledge of Γ_ρ and Γ_{A_1}, which are calculated using $\mathscr{L}(\pi, \rho, A_1)$. Γ_ρ comes out considerably smaller than the experimental value $\Gamma_\rho^{\exp} = (153 \pm 2)\,\mathrm{MeV}$ [6], so that $\Gamma_\rho^{\exp}$ is used for the current. Γ_{A_1} and the branching ratio $B(\tau^\pm \to \nu_\tau \pi^+\pi^-\pi^\pm)$ are given as functions of m_{A_1}. A comparision with the measured branching ratio and decay spectrum makes it possible to give an estimate for the A_1-parameters in Sect. 5.

2 The effective Lagrangian of Golterman and Hari-Dass

Without refering to the theoretical background mentioned in the Introduction, the derivation of the effective chiral Lagrangian published in [11] is recapitulated.

The procedure leaves little ambiguity, so that other effective Lagrangians (e.g. [13, 14]) should be equivalent. Assuming that chiral symmetry is realized in the Goldstone mode, the effective Lagrangian for pions is given in the σ-gauge by

$$\mathscr{L}(\pi) = -\tfrac{1}{2}(\partial_\mu \boldsymbol{\pi})^2 - \tfrac{1}{2}(\partial_\mu \sigma)^2 \tag{5}$$

with the constraint $\sigma^2 + \boldsymbol{\pi}^2 = f_\pi^2$, where $f_\pi(\simeq 93\,\mathrm{MeV})$ is the pion decay constant.

Demanding invariance of (5) under local chiral and isospin transformations

$$\begin{aligned} \delta\sigma &= -\boldsymbol{\omega}\cdot\boldsymbol{\pi}, \\ \delta\boldsymbol{\pi} &= -\boldsymbol{\eta}\times\boldsymbol{\pi} + \boldsymbol{\omega}\sigma, \end{aligned} \tag{6}$$

the (axial) vector meson fields $\boldsymbol{\alpha}_\mu$ and $\boldsymbol{\rho}_\mu$ are introduced, transforming

$$\begin{aligned} \delta\boldsymbol{\rho}_\mu &= \boldsymbol{\eta}\times\boldsymbol{\rho}_\mu + \boldsymbol{\omega}\times\boldsymbol{\alpha}_\mu + \frac{1}{g}\partial_\mu\boldsymbol{\eta}, \\ \delta\boldsymbol{\alpha}_\mu &= \boldsymbol{\eta}\times\boldsymbol{\alpha}_\mu + \boldsymbol{\omega}\times\boldsymbol{\rho}_\mu + \frac{1}{g}\partial_\mu\boldsymbol{\omega}, \end{aligned} \tag{7}$$

where $\boldsymbol{\omega} = \boldsymbol{\omega}(x)$ and $\boldsymbol{\eta} = \boldsymbol{\eta}(x)$ are the parameters of infinitesimal chiral and isospin transformations. As the minimal Lagrangian invariant under (6, 7) we have

$$\mathscr{L}_1 = -\tfrac{1}{2}[(D_\mu\boldsymbol{\pi})^2 + (D_\mu\sigma)^2] - \tfrac{1}{4}[\boldsymbol{\rho}_{\mu\nu}^2 + \boldsymbol{\alpha}_{\mu\nu}^2], \tag{8}$$

where the following definitions have been used:

$$\begin{aligned} D_\mu\boldsymbol{\pi} &\equiv \partial_\mu\boldsymbol{\pi} - g\boldsymbol{\rho}_\mu\times\boldsymbol{\pi} - g\sigma\boldsymbol{\alpha}_\mu, \\ D_\mu\sigma &\equiv \partial_\mu\sigma + g\boldsymbol{\pi}\cdot\boldsymbol{\alpha}_\mu, \\ \boldsymbol{\rho}_{\mu\nu} &\equiv \partial_\mu\boldsymbol{\rho}_\nu - \partial_\nu\boldsymbol{\rho}_\mu - g\boldsymbol{\rho}_\mu\times\boldsymbol{\rho}_\nu - g\boldsymbol{\alpha}_\mu\times\boldsymbol{\alpha}_\nu, \\ \boldsymbol{\alpha}_{\mu\nu} &\equiv \partial_\mu\boldsymbol{\alpha}_\nu - \partial_\nu\boldsymbol{\alpha}_\mu - g\boldsymbol{\rho}_\mu\times\boldsymbol{\alpha}_\nu - g\boldsymbol{\rho}_\nu\times\boldsymbol{\alpha}_\mu. \end{aligned} \tag{9}$$

However, to reach reasonable results, one has to add further terms [16], which are also invariant:

$$\begin{aligned} \mathscr{L}_2 = -\tfrac{1}{4}\kappa[(\boldsymbol{\rho}_{\mu\nu}^2 - \boldsymbol{\alpha}_{\mu\nu}^2)(1 - 2\boldsymbol{\pi}^2) + 4\sigma\boldsymbol{\pi}\cdot(\boldsymbol{\rho}_{\mu\nu}\times\boldsymbol{\alpha}_{\mu\nu}) \\ + 2(\boldsymbol{\pi}\cdot\boldsymbol{\rho}_{\mu\nu})^2 - 2(\boldsymbol{\pi}\times\boldsymbol{\alpha}_{\mu\nu})^2] \end{aligned} \tag{10}$$

and

$$\mathscr{L}_3 = -\tfrac{1}{2}\lambda\boldsymbol{\rho}_{\mu\nu}\cdot(D_\mu\boldsymbol{\pi}\times D_\nu\boldsymbol{\pi}) + \lambda\boldsymbol{\alpha}_{\mu\nu}\cdot D_\mu\boldsymbol{\pi}D_\nu\sigma. \tag{11}$$

Of course, there exists an unlimited number of invariant terms with higher orders of derivatives; it is assumed that they can be neglected for processes with energies of about 1 GeV.

Next, a symmetry breaking mass term for $\boldsymbol{\rho}_\mu$ and $\boldsymbol{\alpha}_\mu$ is introduced:

$$\mathscr{L}_4 = -\tfrac{1}{2}m^2(\boldsymbol{\rho}_\mu^2 + \boldsymbol{\alpha}_\mu^2). \tag{12}$$

The following redefinitions provide the link between $\boldsymbol{\pi}, \boldsymbol{\rho}_\mu$ and $\boldsymbol{\alpha}_\mu$ and the physical meson fields $\boldsymbol{\pi}^{\mathrm{ph}}$, $\boldsymbol{\rho}_\mu^{\mathrm{ph}}$ and $\mathbf{A}_{1\mu}^{\mathrm{ph}}$. Using the renormalization constant

$$z = \left(1 - \frac{g^2 f_\pi^2}{m^2}\right)^{-1}$$

we define:

$$\boldsymbol{\pi}^{\mathrm{ph}} \equiv z^{-1/2}\boldsymbol{\pi},$$

$$\boldsymbol{\rho}_\mu^{\mathrm{ph}} \equiv \sqrt{1+\kappa}\,\boldsymbol{\rho}_\mu,$$
$$\mathbf{A}_{1\mu}^{\mathrm{ph}} \equiv \sqrt{1-\kappa}\left(\boldsymbol{\alpha}_\mu - \frac{zg}{m^2+zg^2}\,\partial_\mu \boldsymbol{\pi}^{\mathrm{ph}}\right). \tag{13}$$

This leads to

$$m_\rho^2 = \frac{m^2}{1+\kappa},$$
$$m_A^2 = \frac{zm^2}{1-\kappa}. \tag{14}$$

Notice that the choice $\kappa=0$ and $z=2$ gives the KSFR-relation and Weinberg's second sum rule [17, 18]:

$$\frac{g^2 f_\pi^2}{m^2} = \frac{1}{2} \quad \text{and} \quad m_{A_1}^2 = 2m_\rho^2.$$

Here we introduce the following notation:

$$g_\rho^2 \equiv \frac{g^2}{1+\kappa},$$
$$g_A^2 \equiv \frac{g^2}{1-\kappa},$$
$$\alpha \equiv \frac{g_\rho^2 f_\pi^2}{m_\rho^2}, \tag{15}$$
$$x \equiv \sqrt{\frac{1+\kappa}{1-\kappa}},$$
$$\bar{\lambda} \equiv \lambda \frac{g_\rho f_\pi^2}{z\sqrt{1+\kappa}}.$$

The part of

$$\mathcal{L}(\boldsymbol{\pi}, \boldsymbol{\rho}_\mu, \boldsymbol{\alpha}_\mu) = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4$$

relevant for the process $\tau^\pm \to \nu_\tau \pi^+\pi^-\pi^\pm$ is given below; the derivation was accomplished by using the approximation $m_\pi^2 = 0$.

From now on all occuring fields are understood as physical, the "ph" will be suppressed.

$$\begin{aligned}
\mathcal{L} = &+ g_\rho \boldsymbol{\rho}_\mu \cdot \boldsymbol{\pi} \times \partial_\mu \boldsymbol{\pi} \\
&+ \frac{g_\rho}{m_\rho^2}\left(\tfrac{1}{2}\alpha + \tfrac{1}{2}\bar{\lambda}\left(1-\frac{1}{\alpha}\right)\right)(\partial_\mu \boldsymbol{\rho}_\nu - \partial_\nu \boldsymbol{\rho}_\mu)\cdot \partial_\mu \boldsymbol{\pi} \times \partial_\nu \boldsymbol{\pi} \\
&+ (\tfrac{1}{2}\alpha - \tfrac{1}{6})\frac{1}{f_\pi^2}(\boldsymbol{\pi} \times \partial_\mu \boldsymbol{\pi})^2 \\
&+ g_A\left(\frac{2}{3} - \frac{1}{3}\frac{\alpha}{1-\alpha}\right)\frac{1}{f_\pi}\mathbf{A}_{1\mu}\cdot \boldsymbol{\pi} \times (\boldsymbol{\pi} \times \partial_\mu \boldsymbol{\pi}) \\
&+ g_\rho^2 x\left(1 + \frac{\bar{\lambda}}{\alpha}\right) f_\pi \boldsymbol{\pi}\cdot(\boldsymbol{\rho}_\mu \times \mathbf{A}_{1\mu}) \\
&+ \tfrac{1}{2}x\left(\bar{\lambda} + \frac{m_\rho^2}{m_A^2}(1-x^2)\right) f_\pi \boldsymbol{\pi}\cdot(\partial_\mu \boldsymbol{\rho}_\nu - \partial_\nu \boldsymbol{\rho}_\mu) \\
&\times (\partial_\mu \mathbf{A}_{1\nu} - \partial_\nu \mathbf{A}_{1\mu}) + \cdots
\end{aligned} \tag{16}$$

3 Determination of α, κ, and $\bar{\lambda}$

Golterman and Hari-Dass extended their model to include the electroweak sector, so that the processes $\rho^0 \to e^+e^-$ and $\pi^+ \to \nu e^+ \gamma$ could be used to fix α, κ, and $\bar{\lambda}$. Their values for $m_{A_1} = 1275\,\mathrm{MeV}$ are

$$\alpha = 0.35 \pm 0.02,$$
$$\kappa = 0.28 \pm 0.02, \tag{17}$$
$$\bar{\lambda} = 0.32 \pm 0.05.$$

However, using $\rho^0 \to \mu^+\mu^-$ or $\rho \to \pi\gamma$ instead of the first process would lead to different values for α and κ, and the deduction of $\bar{\lambda}$ from the second one is also troubled by uncertainty [12].

In this paper, the obvious demand for a low energy behaviour of the theory in accordance with the corresponding theory based on $\mathcal{L}(\pi)$, (5), is chosen as a constraint. This means that (axial) vector meson effects should disappear for energies considerably lower than m_ρ.

Minimal momentum dependance of (16) is reached by setting the second and sixth line to zero. Together with (14), three equations are obtained for the determination of α, κ, and $\bar{\lambda}$.

1. $\bar{\lambda} = \dfrac{\alpha^2}{1-\alpha}$

2. $\bar{\lambda} = \dfrac{m_\rho^2}{m_{A_1}^2}\dfrac{2\kappa}{1-\kappa}$ (18)

3. $\dfrac{m_{A_1}^2}{m_\rho^2} = \dfrac{1+\kappa}{1-\kappa}\dfrac{1}{1-\alpha}$

For $m_{A_1} = 1275\,\mathrm{MeV}$ we have

$$\alpha = 0.389,$$
$$\kappa = 0.254, \tag{19}$$
$$\bar{\lambda} = 0.247,$$

to be compared with (17).

Using the relations (18), the hadronic current occuring in $\tau^\pm \to \nu_\tau \pi^+\pi^-\pi^\pm$ will now be calculated for the low energy limit.

With $\mathcal{L}(\boldsymbol{\pi}, \boldsymbol{\rho}_\mu, \boldsymbol{\alpha}_\mu)$ and (6, 7) we get

$$J_\mu^{\mathrm{axial},i} = -\frac{\delta \mathcal{L}}{\delta(\partial_\mu \omega^i)} = f_\pi \partial_\mu \pi^i + g_{A_1}\frac{1}{\alpha} f_\pi^2 A_{1\mu}^i. \tag{20}$$

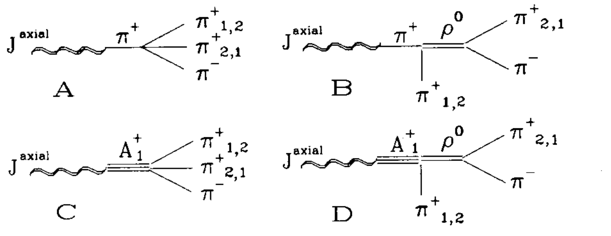

Fig. 1. Graphical representation of the contributions to the hadronic current $J_\mu(\pi^+\pi^+\pi^-)$

A graphical representation of $J_\mu(\pi^+\pi^+\pi^-)$ is shown in Fig. 1.

At the low energy limit, the vector meson propagators simply read $m^{-2}g^{\mu\nu}$. The propagator of the off-shell pion is $Q^{-2}g^{\mu\nu}$, with the total hadronic momentum $Q_\mu=(q_1+q_2+q_3)_\mu$. The expressions corresponding to diagrams A, B, C, and D are:

$$J_\mu^A=+(\tfrac{1}{2}\alpha-\tfrac{1}{6})\frac{1}{f_\pi}\frac{1}{Q^2}\partial_\mu\boldsymbol{\pi}\cdot(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})^2$$

$$J_\mu^B=-\tfrac{1}{2}\alpha\frac{1}{f_\pi}\frac{1}{Q^2}\partial_\mu\boldsymbol{\pi}\cdot(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})^2$$

$$J_\mu^C=+g_{A_1}^2\frac{1}{\alpha}f_\pi\frac{1}{m_{A_1}^2}\left(\frac{2}{3}-\frac{1}{3}\frac{\alpha}{1-\alpha}\right)\boldsymbol{\pi}\times(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})$$

$$J_\mu^D=+g_\rho^3 g_{A_1}f_\pi^3\frac{1}{\alpha}\frac{1}{m_\rho^2 m_{A_1}^2}x\left(1+\frac{\lambda}{\alpha}\right)\boldsymbol{\pi}\times(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})$$

Considering (14), (15) and (18) this can be simplified:

$$J_\mu^A+J_\mu^B=-\frac{1}{6}\frac{1}{f_\pi}\frac{1}{Q^2}\partial_\mu\boldsymbol{\pi}\cdot(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})^2,$$

$$J_\mu^C+J_\mu^D=+\frac{2}{3}\frac{1}{f_\pi}\boldsymbol{\pi}\times(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi}). \tag{21}$$

With the convention

$$\pi^\pm=\frac{1}{\sqrt{2}}(\pi_1\pm i\pi_2);\quad J_\mu^\pm=J_{\mu,1}\pm iJ_{\mu,2},$$

the current for the final state $|\pi^+\pi^+\pi^-\rangle$ is given by

$$J^\mu(\pi^+\pi^+\pi^-)=-i\tfrac{2}{3}\sqrt{2}\frac{1}{f_\pi}[q_1^++q_2^+-2q^-]_\nu\left[g^{\mu\nu}-\frac{Q^\mu Q^\nu}{Q^2}\right]. \tag{22}$$

This is in accordance with the current resulting from the Lagrangian $\mathscr{L}(\pi)$, (5), i.e. without any (axial) vector mesons [15].

4 Calculation of $B(\tau^+\to\nu_\tau\pi^+\pi^+\pi^-)$

First some formulas are given, taken from Fischer et al. [14, 15]. The T-matrix element for $\tau\to3\pi\nu_\tau$ reads

$$T=\frac{G}{\sqrt{2}}\cos\theta_C\bar{\tau}(p')\gamma_\mu(1+\gamma^5)\nu_\tau(p)J^\mu(q_1,q_2,q_3), \tag{23}$$

where $G\simeq1.023\times10^{-5}M_{\text{proton}}^{-2}$ is the coupling constant of the weak interaction. The corresponding decay rate is

$$\Gamma=\frac{1}{2m_\tau}\int dPS(p,p';q_1,q_2,q_3)\frac{1}{2}\sum_{\text{spin}}|T|^2. \tag{24}$$

Splitting the phase space into a leptonic and a hadronic part, using as variable

$$y\equiv\frac{Q^2}{m_\tau^2}$$

and introducing the pure leptonic decay width

$$\Gamma_L=\frac{G^2m_\tau^5}{192\pi^3},$$

we get

$$\Gamma=\Gamma_L\cos^2\theta_C\int_0^1 dy(1-y)^2(1+2y)a(y) \tag{25}$$

with

$$a=-\frac{2\pi}{Q^2}\int dPS(q_1,q_2,q_3)J_\mu^2 \tag{26}$$

and

$$dPS=\frac{1}{(2\pi)^5}\frac{1}{32Q^2}ds_{13}ds_{23}d\alpha\,d\cos\beta\,d\gamma,$$

$$s_{ij}\equiv(q_i+q_j)^2,\quad \alpha,\beta,\gamma:\text{Euler angles.}$$

Thus the problem is reduced to the calculation of $a(y)$. The hadronic current $J_\mu(\pi^+\pi^+\pi^-)$ obtained for the low energy limit (11) has to be modified according to the non-zero momentum of the (axial) vector mesons. We obtain the following currents, corresponding to the diagrams in Fig. 1:

$$J_\mu^A=+i2\sqrt{2}(+\tfrac{1}{3}-\alpha)\frac{1}{f_\pi}[Q-3q^-]^\nu\frac{Q_\mu Q_\nu}{Q^2}$$

$$J_\mu^B=+i2\sqrt{2}\alpha\frac{1}{f_\pi}\left[(q_1^+-q^-)^\nu\frac{m_\rho^2-im_\rho\Gamma_\rho}{s_{13}-m_\rho^2+im_\rho\Gamma_\rho}+(q_2^+-q^-)^\nu\frac{m_\rho^2-im_\rho\Gamma_\rho}{s_{23}-m_\rho^2+im_\rho\Gamma_\rho}\right]\frac{Q_\mu Q_\nu}{Q^2}$$

$$J_\mu^C=-i\sqrt{2}(\tfrac{2}{3}-\alpha)\frac{1}{f_\pi}\frac{m_{A_1}^2-im_{A_1}\Gamma_{A_1}}{Q^2-m_A^2+im_A\Gamma_A}(Q-3q^-)_\mu \tag{27}$$

$$J_\mu^D=-i\sqrt{2}\alpha\frac{1}{f_\pi}\frac{m_{A_1}^2-im_A\Gamma_{A_1}}{Q^2-m_{A_1}^2+im_{A_1}\Gamma_{A_1}}\cdot\left[(q_1^+-q^-)_\mu\frac{m_\rho^2-im_\rho\Gamma_\rho}{s_{12}-m_\rho^2+im_\rho\Gamma_\rho}+(q_2^+-q^-)_\mu\frac{m_\rho^2-im_\rho\Gamma_\rho}{s_{23}-m_\rho^2+im_\rho\Gamma_\rho}\right]$$

As the α-value is about 1/3, it is clear that J_μ^A is very small.

Γ_{A_1} is calculated using the matrix element

$$T=i\frac{\sqrt{2}}{f_\pi}\varepsilon_\mu(A_1)\left[g_{A_1}\left(\frac{2}{3}-\frac{1}{3}\frac{\alpha}{1-\alpha}\right)(Q-3q^-)^\mu+x\alpha g_\rho\left(1+\frac{\alpha}{1-\alpha}\right)\left((q_1^+-q^-)^\mu\frac{m_\rho^2-im_\rho\Gamma_\rho}{s_{13}-m_\rho^2+im_\rho\Gamma_\rho}+(q_2^+-q^-)^\mu\frac{m_\rho^2-im_\rho\Gamma_\rho}{s_{23}-m_\rho^2+im_\rho\Gamma_\rho}\right)\right] \tag{28}$$

resulting from the Lagrangian (16). In (27) and (28) the experimental value $\Gamma_\rho^{\exp}=(153\pm2)$ MeV [6] is

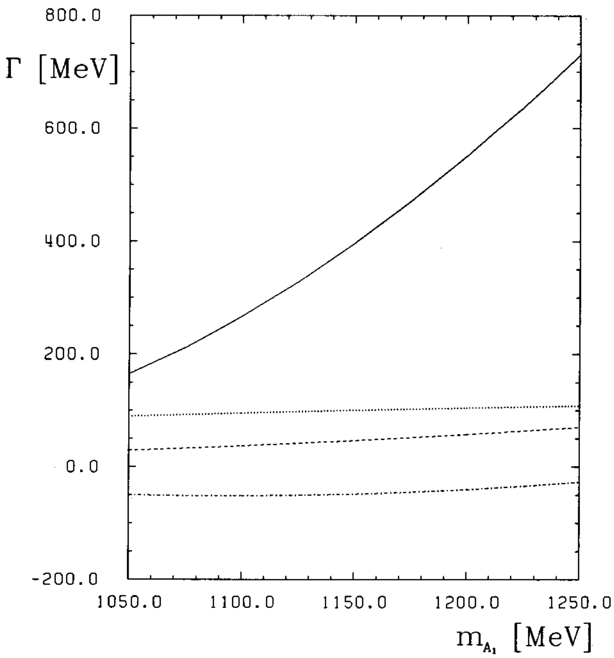

Fig. 2. Solid line: Γ_{A_1}. Dashed line: $\Gamma(A_1\pi\pi\pi)$. Dashed-dotted line: Interference between $\Gamma(A_1\rho\pi)$ and $\Gamma(A_1\pi\pi\pi)$. Dotted line: Γ_ρ. All decay widths as functions of m_{A_1}

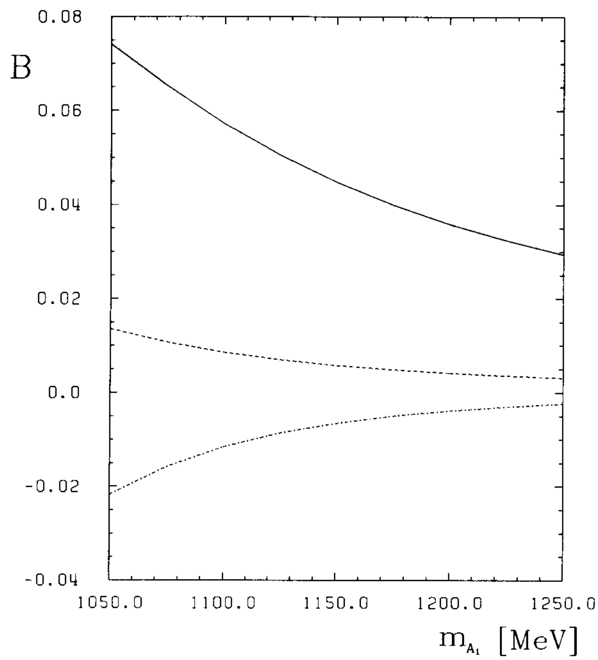

Fig. 3. Solid line: Branching ratio $B(\tau^\pm \to \nu_\tau\pi^+\pi^-\pi^\pm)$. Dashed line: Non-$(\rho^0\pi^\pm)$ resonance contribution. Dashed-dotted line: Contribution of interference between resonance- and non-resonance decay. All branching ratios as functions of m_{A_1}

taken instead of the calculated one, which is considerably smaller. This may be attributed to the approximation of minimal momentum dependence used to fix the parameters, where obviously the $\bar{\lambda}$-value comes out too low. For $\Gamma(A_1\rho\pi)$, this approximation tends to have the opposite effect ([11]; see also Ebert et al. [12] for $\Gamma(A_1\rho\pi) - m_{A_1}$ combinations). The calculations of decay widths and branching ratios presented here have been accomplished without the approximation $m_\pi^2 = 0$ used to derive (16).

In Fig. 2, the widths $\Gamma_\rho(m_{A_1})$ and $\Gamma_{A_1}(m_{A_1})$ are given. For $\Gamma_{A_1}(m_{A_1})$, the contribution of the direct decay $A_1 \to 3\pi$ and its interference with the decay $A_1 \to \rho\pi \to 3\pi$ is shown.

Figure 3 contains the branching ratio $B(\tau^\pm \to \nu_\tau\pi^+\pi^-\pi^\pm)$ as a function of m_{A_1}. The total τ-lifetime is supposed to be $\tau_\tau = 3.3 \times 10^{-13}$ s [6]. The momentum dependence of the decay widths Γ_{A_1} and Γ_ρ is taken into account; for Γ_{A_1}, this has an important effect on the form of the decay spectrum [7]. Again the non-$(\rho^0\pi^\pm)$ resonance contribution and its interference with the resonance decay is shown.

The experimental results for the branching ratio are

	$B(\tau^\pm \to \nu_\tau\pi^+\pi^-\pi^\pm)$
ARGUS [3]	$(5.6 \pm 0.7)\%$
DELCO [4]	$(5.0 \pm 1.0)\%$
MARK II [5]	$(7.8 \pm 0.5 \pm 0.8)\%$

Taking the value from [3], the A_1-mass is inferred from Fig. 3 to be

$$m_{A_1} = (1105 \pm 25)\,\text{MeV},$$

corresponding to an A_1-width of

$$\Gamma_{A_1} = (280 \pm 60)\,\text{MeV}.$$

The form of the calculated decay spectrum depends heavily on m_{A_1} and Γ_{A_1}, so that the comparison with the measured spectrum serves as a second means to deduce reasonable A_1-parameters.

In Fig. 4, the ARGUS-data [3] are shown together with three calculated spectra using

$$m_{A_1} = 1080\,\text{MeV};\quad \Gamma_{A_1} = 222\,\text{MeV},$$
$$m_{A_1} = 1105\,\text{MeV};\quad \Gamma_{A_1} = 279\,\text{MeV},$$
$$m_{A_1} = 1130\,\text{MeV};\quad \Gamma_{A_1} = 342\,\text{MeV}.$$

Their heights have been normalized to fit the experimental data.

Obviously, we have to take larger A_1-masses to obtain a better fit. Figure 5 shows the spectrum for

$$m_{A_1} = 1200\,\text{MeV};\quad \Gamma_{A_1} = 552\,\text{MeV}.$$

Additionally, the spectrum of all decay channels and their interferences without the contribution of pure $\tau \to A_1\nu_\tau \to \rho\pi\nu_\tau \to 3\pi\nu_\tau$ is given with the same normalization and, to illustrate their effect on the form of the spectrum, the contribution of this dominant decay channel for

$$m_{A_1} = 1200\,\text{MeV};\quad \Gamma_{A_1} = \Gamma(A_1\rho\pi) \equiv 552\,\text{MeV}$$

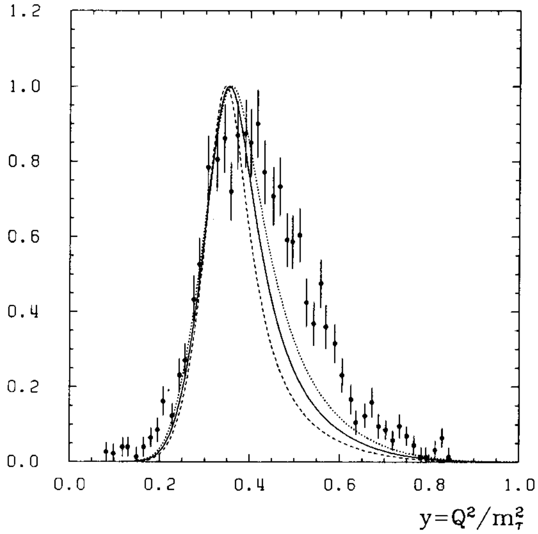

Fig. 4. Calculated decay spectra and experimental data from [3] (ARGUS). Solid line: $m_{A_1} = 1105$ MeV; $\Gamma_{A_1} = 279$ MeV. Dashed line: $m_{A_1} = 1080$ MeV; $\Gamma_{A_1} = 222$ MeV. Dotted line: $m_{A_1} = 1130$ MeV; $\Gamma_{A_1} = 342$ MeV

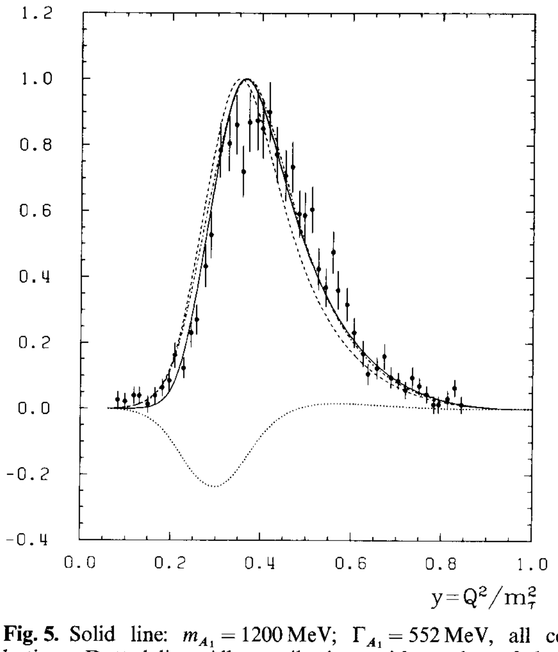

Fig. 5. Solid line: $m_{A_1} = 1200$ MeV; $\Gamma_{A_1} = 552$ MeV, all contributions. Dotted line: All contributions without that of the pure dominant decay channel, diagram D (Fig. 1), normalization as above. Dashed line: The dominant decay channel alone, with $m_{A_1} = 1200$ MeV; $\Gamma(A_1 \rho \pi) \equiv 552$ MeV. Dashed-dotted line: The dominant decay channel alone, with $m_{A_1} = 1230$ MeV; $\Gamma(A_1 \rho \pi) \equiv 552$ MeV. The last two curves have been normalized to make their maximum agree with that of the first curve

and

$$m_{A_1} = 1230\,\text{MeV}; \quad \Gamma_{A_1} = \Gamma(A_1 \rho \pi) \equiv 552\,\text{MeV}$$

with suitable normalizations. The calculated $\Gamma(A_1 \rho \pi)$-values for $m_{A_1} = 1200$ MeV and $m_{A_1} = 1230$ MeV are 534 MeV and 624 MeV.

5 Summary and conclusions

It has been shown that the effective chiral Lagrangian with ρ- and A_1-mesons as gauge bosons of $SU(2)_L \times SU(2)_R$, published by Golterman and Hari-Dass [11], is suited to describe the decay $\tau^\pm \to \nu_\tau \pi^+ \pi^- \pi^\pm$.

In contrast to previous papers [11, 13], the only condition used to determine the free parameters appearing in $\mathscr{L}(\pi, \rho, A_1)$ has been, that the current $\langle 0 | \mathbf{J}_\mu | 3\pi \rangle$ should agree at the low energy limit with the one resulting from $\mathscr{L}(\pi)$.

Considering the dependence of these parameters on the A_1-mass, the decay widths Γ_{A_1} and $\Gamma(\tau^\pm \to \nu_\tau \pi^+ \pi^- \pi^\pm)$ are calculated as functions of m_{A_1}. A comparison with the measured branching ratio of the τ-decay gives a first estimate for the A_1-parameters, which is checked by considering the corresponding decay spectra. Unfortunately, the fit is very poor. It is natural to put the blame on α, κ, and $\bar{\lambda}$. However, thanks to the combination of Γ_{A_1}- and $B(\tau^\pm \to \nu_\tau \pi^+ \pi^- \pi^\pm)$-calculations used in this paper the latter is stabilized against variations of the parameters. As was mentioned above, our way of determining their values probably results in A_1-widths which are too large. Therefore, it seems to be justified to use Γ_{A_1}-values in the B-calculation which are lower than suggested by Fig. 2, without changing the parameter-values; this corresponds to the way we handled Γ_ρ. The high uncertainty about the total decay width of A_1 (see also (3) and (4) taken from [7, 8]) on one hand and its strong effect on the B-value on the other makes Fig. 3 a questionable means for determining m_{A_1}. However, it should be noted that neglecting the decay channels $\tau \to \pi \nu_\tau \to \rho \pi \nu_\tau \to 3\pi \nu_\tau$ and $\tau \to A_1 \nu_\tau \to 3\pi \nu_\tau$ would lead to higher m_{A_1}-values for given B-values, because their effect together with all interferences is negative, as can be seen in Fig. 5.

Determining the A_1-mass by comparing the form of the calculated with the experimental spectrum is also made difficult by the uncertainty about Γ_{A_1}.

A reasonable result for the A_1-parameters is

$$m_{A_1} = (1180 \pm 50)\,\text{MeV}; \quad \Gamma_{A_1} = (450 \pm 100)\,\text{MeV},$$

with the A_1-mass being smaller than the values obtained by fitting the decay spectrum with carefully selected Breit-Wigner distributions under the assumption of constant coupling between the leptonic and the hadronic current [7, 8].

The main reason can be inferred from Fig. 5. Given a m_{A_1}-Γ_{A_1}-combination which provides a good fit to the experimental spectrum when all decay channels are considered, we have to use a higher m_{A_1}-value for a fit of similar quality when we take only the dominant decay channel $\tau \to A_1 \nu_\tau \to \rho \pi \nu_\tau \to 3\pi \nu_\tau$ into account. Just varying $\Gamma_{A_1} = \Gamma(A_1 \rho \pi)$ for fixed m_{A_1} does not suffice, because Γ_{A_1}-values which improve the fit for the ascending part of the spectrum make it worse for the descending part and vice versa.

Considering these effects it seems to be of little use to improve the precision of the fits as long as the issue about the contributions of suppressed decay channels is not settled. The relative and absolute values derived here depend on the special constraint used to fix α, κ, and $\bar{\lambda}$, corresponding to minimal momentum dependence. Obviously, this misses the right momentum dependence somewhat. Nevertheless, the best chance to learn something about suppressed decay channels may be the contribution from direct, non-resonance A_1-decay, i.e. $A_1 \rightarrow 3\pi$ in addition to $A_1 \rightarrow \rho\pi \rightarrow 3\pi$, predicted to be about 15% for $m_{A_1} = 1200$ MeV by our model. The experimentally acceptable upper limit stated in [3] (ARGUS) is about 10%, so that it may be possible to verify its existence.

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