Z. Phys. C – Particles and Fields 37, 321–327 (1988)

Zeitschrift für Physik C Particles and Fields

Perturbation theory for the anomaly-free chiral Schwinger model

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Received 7 July 1987

Abstract. We apply perturbation theory to the gauge invariant version of the chiral Schwinger model. The cancellation of anomalies is shown explicitly in terms of Feynman diagrams. We calculate the exact propagators for the gauge field, for the Wess–Zumino field and for the mixing between these fields. Using these propagators, we demonstrate the existence of a massive state.

I Introduction

Since it has been shown that the chiral Schwinger model [1] leads to a consistent quantum theory [2], this model attracted a lot of attention [3–17]. Another reason for this renewed interest is the general development in the field of anomalous gauge theories, namely the discovery that a gauge invariant formulation is possible in spite of an apparent anomaly [12, 18, 19]. This development might give rise to the hope that even anomalous gauge theories can be quantized consistently, which has also been expressed in [20–22]. The chiral Schwinger model served as a playground to demonstrate how an apparently anomalous theory can be formulated gauge invariantly. This gauge invariant formulation is extremely useful in order to establish that there are no genuine anomalies which could spoil gauge invariance. The reason for this freedom of anomalies is the fact that the anomalous contributions of the fermionic sector are cancelled by those of the Wess–Zumino scalars, which are automatically present in the gauge invariant formulation of the quantum theory [12, 18, 19]. The mechanism of anomaly cancellation has been investigated using nonperturbative methods like bosonization or solving the equations of motion [2–5, 10–15]. Most of these works use the $\theta=0$ gauge (θ is the Wess–Zumino scalar), which is identical to the earlier "anomalous" formulation without gauge invariance. It has been claimed that perturbation theory can not be applied since the exact photon propagator contains inverse powers of the gauge coupling constant [2, 3]. This feature, however, is a gauge artefact, which can be avoided in other gauges.

In the present work we want to develop the perturbative approach to the chiral Schwinger model in two different gauges: i) the Lorentz gauge and ii) the so-called Jackiw–Rajaraman (JR) gauge. For these gauges there exist operator solutions [13] such that it is possible to compare the exact photon propagators. Besides these we are going to calculate the $\theta-\theta$ and the photon-θ mixing propagators to all orders in the coupling constant. Furthermore we show the anomaly cancellation in terms of Feynman diagrams and we present the effective action of the gauge field which results from integrating out both, the fermion and the Wess–Zumino scalar. Our motivation to consider the perturbative approach is twofold: Firstly, the Feynman diagram calculation shows us explicitly how the cancellation of anomalies occurs and therefore might be more convincing than formal arguments. Secondly, it seems that the above mentioned nonperturbative methods are not available in a realistic four (or higher?) dimensional world so that perturbation theory is the only possible approach.

In Sect. 2 we calculate the vacuum polarization diagrams, using a modified dimensional regularization for the fermion loop. Here the cancellation of the anomalous contributions coming from the fermionic and the bosonic sectors is explicitly shown. In Sect. 3 we sum up the perturbation series in order to obtain the completely corrected propagators for the boson fields. To this aim we use two different methods, namely the explicit summation of Feynman diagrams and a simple inversion of the kinetic operator in the effective action after integrating out the fermion fields. In Sect. 4 we indicate how the spectrum can be read off from the full propagators, and we discuss the special case of the covariant anomaly.

* Supported by Bundesministerium für Forschung und Technologie, 05 4HH 92P/3, Bonn, FRG

II Vacuum polarization

Our starting point is the Lagrangian density

$$\mathscr{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\gamma^\mu[i\partial_\mu + e\sqrt{\pi}A_\mu(1+i\gamma_5)]\psi - \frac{a-1}{2}\theta'\Box\theta' + e\theta'\partial_\mu[(a-1)g^{\mu\nu} + \varepsilon^{\mu\nu}]A_\nu + \mathscr{L}_{GF}, \tag{1}$$

where the following notation is used:

$$g_{00} = -g_{11} = 1, \quad \varepsilon^{01} = -\varepsilon_{01} = 1, \quad \gamma_5 = i\gamma^0\gamma^1 \Rightarrow i\gamma_\mu\gamma_5 = \varepsilon_{\mu\nu}\gamma^\nu. \tag{2}$$

$\mathscr{L}_{GF}$ is the (unspecified) gauge fixing term, which is assumed to depend on the gauge field only. The action (1) is the so-called standard action [10, 12] of the chiral Schwinger model with gauge fixing term. Integration over the fermion fields yields the action of [5, 10, 12], provided that the regularization of the fermion determinant is taken into account appropriately by the parameter "a" [8, 9, 12]. It is one of our aims to see how the regularization dependent parameters of the fermionic and bosonic sectors are related to each other in perturbation theory. In order to have a conventional kinetic term for the Wess–Zumino scalar, we rescale $\theta = \sqrt{a-1}\theta'$. This is possible for $a > 1$, only; $a = 1$ is a special case to be treated separately (cf. Sect. 4), and for $a < 1$ θ becomes a complex field with kinetic term $+\tfrac{1}{2}\theta^+\Box\theta$, which has the wrong sign. Hence for the moment we restrict ourselves to $a > 1$, then the action reads:

$$S = \int d^2x \left\{ -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\gamma^\mu[i\partial_\mu + e\sqrt{\pi}A_\mu(1+i\gamma_5)]\psi - \tfrac{1}{2}\theta\Box\theta + e\sqrt{a-1}\,\theta\left[\partial_\mu - \frac{1}{a-1}\varepsilon_{\mu\nu}\partial^\nu\right]A^\mu + \mathscr{L}_{GF} \right\}. \tag{3}$$

From here the Feynman rules may be read off: the free fermion and scalar propagators are as usual:

$$D^0_\psi = \frac{i}{\gamma_\mu k^\mu}, \tag{4}$$

$$D^0_\theta = \frac{i}{k^2}, \tag{5}$$

and the $A-\theta$ vertex is given by

$$iV_\mu = -e\sqrt{a-1}\left(k_\mu - \frac{1}{a-1}\varepsilon_{\mu\alpha}k^\alpha\right). \tag{6}$$

For the time being we leave the photon propagator unspecified, since it depends on the choice of gauge. Finally, there is a speciality concerning the $A\bar{\psi}\psi$ vertex for the following reason. We want to use dimensional regularization and, additionally, we would like to exhibit the full regularization arbitrariness. Since dimensional regularization is gauge invariant, it necessarily leads to the covariant anomaly [23], which corresponds to $a = 1$ (see (18), below) and does not reflect the regularization arbitrariness. Hence one is led to modify the fermion photon vertex in d dimensions to [16]:

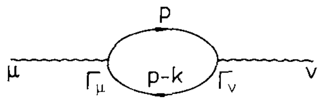

Fig. 1. Fermion loop diagram

$$\Gamma_\mu = ie\sqrt{\pi}(r\gamma_\mu + is\theta_{\mu\nu}\gamma^\nu\gamma_5)(1+i\gamma_5), \tag{7}$$

where

$$r + s = 1 + O(2-d), \tag{8}$$

$$\theta_{\mu\nu} = -\theta_{\nu\mu} = \varepsilon_{\mu\nu} + O(2-d), \tag{9}$$

such that in two dimensions the usual vertex is recovered

$$r\gamma_\mu + is\theta_{\mu\nu}\gamma^\nu\gamma_5 = \gamma_\mu + O(2-d). \tag{10}$$

Usual dimensional regularization corresponds to $r = 1$, $s = 0$, any deviation from these values means an unequal treatment of the light-cone components of the gauge field, i.e. a violation of gauge invariance, which is necessary to avoid the covariant anomaly.

The contribution $\Pi^\psi_{\mu\nu}$ of the fermion loop to the vacuum polarization tensor is shown in Fig. 1. It is given by the following expression in d dimensions:

$$\Pi^\psi_{\mu\nu}(k) = -\pi e^2 \int \frac{d^dp}{(2\pi)^d}\frac{1}{(k-p)^2p^2} \cdot \mathrm{Tr}\{(r\gamma_\mu + is\theta_{\mu\alpha}\gamma^\alpha\gamma_5)(1+i\gamma_5)[\gamma\cdot(p-k)] \cdot (r\gamma_\nu + is\theta_{\nu\beta}\gamma^\beta\gamma_5)(1+i\gamma_5)[\gamma\cdot p]\}. \tag{11}$$

Using standard integrals in d dimensions with integrands $p_\rho/p^2(p-k)^2$ and $p_\sigma p_\rho/p^2(p-k)^2$ reported in [24] we obtain for (11)

$$\Pi^\psi_{\mu\nu}(k) = \frac{ie^2\pi^{1-d/2}}{2^d}(-k^2)^{d/2-2}\frac{\Gamma^2\left(\frac{d}{2}\right)\Gamma\left(2-\frac{d}{2}\right)}{\Gamma(d)} \cdot \left\{\Gamma_{\mu\sigma\nu\rho}k^\sigma k^\rho - \Gamma_{\mu\sigma\nu\rho}g^{\sigma\rho}\frac{k^2}{2-d}\right\}, \tag{12}$$

where

$$\Gamma_{\mu\sigma\nu\rho} = \mathrm{Tr}\{(r\gamma_\mu + is\theta_{\mu\alpha}\gamma^\alpha\gamma_5)(1+i\gamma_5)\gamma_\sigma \cdot (r\gamma_\nu + is\theta_{\nu\beta}\gamma^\beta\gamma_5)(1+i\gamma_5)\gamma_\rho\}. \tag{13}$$

In (12) only the second term has a pole in $(2-d)$. Therefore we can evaluate the trace of the first term directly in two dimensions with the result [25]

$$k^\sigma k^\rho \Gamma_{\mu\sigma\nu\rho} = 8k_\mu k_\nu - 4k^2 g_{\mu\nu} + 4\varepsilon_{\mu\alpha}k^\alpha k_\nu + 4\varepsilon_{\nu\alpha}k^\alpha k_\mu. \quad (14)$$

In the second term we have to calculate the trace of the γ matrices in d dimensions. This is problematic in the terms which involve γ_5 linearly. In these terms one cannot assume the anticommutativity of γ_5 with the γ matrices. General schemes to do the evaluation correctly have been developed by several authors [26, 27]. These schemes are not needed here since we need not anticommute γ_5 with γ in the trace with one single γ_5. We use the cyclic commutativity of the trace instead. Then we obtain [25]

$$g^{\sigma\rho}\Gamma_{\mu\sigma\nu\rho} = 4(r^2 - s^2)(2-d)g_{\mu\nu} \quad (15)$$

where the $(2-d)$ factor comes from $\gamma_\sigma\gamma_\nu\gamma^\sigma = (2-d)\gamma_\nu$. Substituting (14) and (15) into (12) and performing the limit $d \to 2$ yields the fermion loop contribution

$$\Pi^\psi_{\mu\nu} = \frac{ie^2}{k^2}\{(r^2 - s^2 + 1)k^2 g_{\mu\nu} - 2k_\mu k_\nu - \varepsilon_{\mu\alpha}k^\alpha k_\nu - \varepsilon_{\nu\alpha}k^\alpha k_\mu\}. \quad (16)$$

This implies the effective action:

$$W^\psi[A] = \frac{e^2}{2}\int d^2x\left\{aA_\mu A^\mu - A^\mu(g_{\mu\alpha} + \varepsilon_{\mu\alpha})\frac{\partial^\alpha\partial^\beta}{\Box}(g_{\beta\nu} - \varepsilon_{\beta\nu})A^\nu\right\}. \quad (17)$$

By making the identification

$$r^2 - s^2 = 1 - 2s = 2r - 1 = a \quad (18)$$

we establish the desired relation between the regularization parameter of the fermionic sector (since $r + s = 1$ there is only one such parameter) and the regularization parameter of the Wess–Zumino sector. It is motivated by the definition of the 1-cocycle, which gives the action for the Wess–Zumino field θ as [10, 12]

$$S^\theta = W^\psi\left[A - \frac{1}{e\sqrt{a-1}}\partial\theta\right] - W^\psi[A]. \quad (19)$$

The coincidence of (17) with the result of the fermion integration in the path integral [2, 8, 9] exhibits the well known fact that in two dimensions the one-loop polarization diagram is the only fermionic contribution to the polarization tensor.

The contribution $\Pi^\theta_{\mu\nu}$ of the θ-exchange to $\Pi_{\mu\nu}$ is shown in Fig. 2, a simple calculation gives

$$\Pi^\theta_{\mu\nu} = \frac{-ie^2}{k^2}\left[\frac{-1}{a-1}k^2 g_{\mu\nu} + \left(a - 1 + \frac{1}{a-1}\right)k_\mu k_\nu - \varepsilon_{\mu\alpha}k^\alpha k_\nu - \varepsilon_{\nu\alpha}k^\alpha k_\mu\right]. \quad (20)$$

So the complete one loop polarization tensor reads

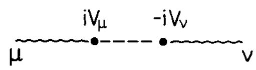

Fig. 2. θ-exchange diagram

$$\Pi_{\mu\nu} = \Pi^\psi_{\mu\nu} + \Pi^\theta_{\mu\nu} = \frac{im^2}{k^2}(k^2 g_{\mu\nu} - k_\mu k_\nu), \quad (21)$$

with

$$m^2 = \frac{e^2 a^2}{a-1}. \quad (22)$$

From here it is clear that the anomalous contributions from the fermion and from the scalar sector cancel each other:

$$k^\mu \Pi^\psi_{\mu\nu} = ie^2[(a-1)k_\nu - \varepsilon_{\nu\beta}k^\beta] \quad (23)$$

but

$$k^\mu \Pi_{\mu\nu} = 0. \quad (24)$$

In this way we recover the most important result of the nonperturbative treatment within perturbation theory, namely that there is no genuine anomaly. As mentioned above, ordinary dimensional regularization of the fermion loop corresponds to $a = 1$, i.e. to the covariant anomaly ($\sim \varepsilon_{\mu\nu}F^{\mu\nu} \sim \varepsilon_{\mu\nu}\partial^\nu A^\mu$). The case $a = 1$, however, is excluded in our procedure since in this case the θ' field does not propagate, hence the θ-exchange does not exist. This again elucidates the necessity to deviate from ordinary dimensional regularization in order to exhibit anomaly cancellation perturbatively. In Sect. 4 we also discuss $a = 1$.

Equation (21) may be used to integrate out both the fermion and Wess–Zumino field, this results in the complete effective action:

$$S_{\text{eff}} = \int d^2x\left\{-\tfrac{1}{4}F_{\mu\nu}\left(1 + \frac{m^2}{\Box}\right)F^{\mu\nu} + \mathcal{L}_{GF}\right\}. \quad (25)$$

Equation (25) coincides with the result of the path integral treatment of [12], it contains a complete summation of the perturbation series for the photon propagator. We shall make this summation more explicit in the next section.

III Boson propagators

In this section we are going to calculate the completely corrected propagators of the boson fields A_μ and θ. This can be done using two different methods: The first one consists of summing up all Feynman diagrams built out of Figs. 1 and 2. The second method relies on the fact that the effective action after fermion integration is purely quadratic in the boson fields. Then the corrected boson propagators may be obtained by simply inverting the matrix between the fields. We want to present both procedures for two gauge choices, namely the Lorentz gauge $\partial_\mu A^\mu = 0$ and the so-called JR gauge [13] $\partial_\mu(g^{\mu\nu} + (1/(a-1))\varepsilon^{\mu\nu})A_\nu = 0$.

The first one is the most popular gauge, the second one is motivated by the fact that, classically, θ and A_μ decouple in this gauge. At this point we want to stress that it is not possible to use the $\theta=0$ gauge within perturbation theory since this gauge does not allow to define a free photon propagator.

IIIa) Summation of Feynman diagrams

i) In the Lorentz gauge the gauge fixing term in the Lagrangian reads:

$$\mathscr{L}_{GF}=\frac{-1}{2\alpha}(\partial_\mu A^\mu)^2 \tag{26}$$

with α being the gauge parameter. This implies the free photon propagator

$$D^0_{\mu\nu}=-\frac{i}{k^2}\left(g_{\mu\nu}-(1-\alpha)\frac{k_\mu k_\nu}{k^2}\right). \tag{27}$$

The full A^μ propagator is given by (cf. Fig. 3)

$$D_{\mu\nu}=D^0_{\mu\alpha}\cdot\sum_{n=0}^{\infty}[(\Pi\cdot D^0)^n]^\alpha{}_\nu. \tag{28}$$

Using (21) for $\Pi_{\mu\nu}$, we find

$$(\Pi\cdot D^0)^\alpha{}_\beta=\frac{m^2}{k^2}\left(g^\alpha_\beta-\frac{k^\alpha k_\beta}{k^2}\right) \tag{29}$$

which may be inserted into (28) to yield

$$D_{\mu\nu}=-\frac{i\alpha}{k^4}k_\mu k_\nu-\frac{i}{k^2}\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)\sum_{n=0}^{\infty}\left(\frac{m^2}{k^2}\right)^n$$
$$=\frac{-i}{k^2-m^2}\left\{g_{\mu\nu}-\left[1-\alpha\left(1-\frac{m^2}{k^2}\right)\right]\frac{k_\mu k_\nu}{k^2}\right\}, \tag{30}$$

which agrees with the result of the operator solution [13]. The Wess–Zumino propagator is shown in Fig. 4, here no iteration is necessary since the full photon propagator is used:

$$D_\theta=D^0_\theta+D^0_\theta(-iV_\mu)D^{\mu\nu}(iV_\nu)D^0_\theta$$
$$=\frac{i}{k^2(k^2-m^2)}\left\{k^2-\frac{m^2}{a^2}\left[a^2-1+\alpha\cdot\left(1-\frac{m^2}{k^2}\right)\right](a-1)^2\right\}. \tag{31}$$

Finally, the full $A-\theta$ propagator is (cf. Fig. 5)

$$D_\mu=D_{\mu\nu}iV^\nu D^0_\theta$$
$$=\frac{-e\sqrt{a-1}}{k^2(k^2-m^2)}\left[\alpha\left(1-\frac{m^2}{k^2}\right)k_\mu-\frac{1}{a-1}\varepsilon_{\mu\alpha}k^\alpha\right]. \tag{32}$$

This completes our diagrammatic calculation of the exact boson propagators, the results may be collected in the matrix:

$$\mathscr{P}_L=\begin{pmatrix}D^{\mu\nu} & D^\mu\\ -D^\nu & D_\theta\end{pmatrix}=\frac{i}{k^2(k^2-m^2)}\begin{pmatrix}-k^2g^{\mu\nu}+\left[1-\alpha\left(1-\frac{m^2}{k^2}\right)\right]k^\mu k^\nu & ie\sqrt{a-1}\left[\alpha\left(1-\frac{m^2}{k^2}\right)k^\mu-\frac{1}{a-1}\varepsilon^{\mu\alpha}k_\alpha\right]\\ -ie\sqrt{a-1}\left[\alpha\left(1-\frac{m^2}{k^2}\right)k^\nu-\frac{1}{a-1}\varepsilon^{\nu\beta}k_\beta\right] & k^2-\frac{m^2}{k^2}\left[a^2-1+\alpha\left(1-\frac{m^2}{k^2}\right)(a-1)^2\right]\end{pmatrix} \tag{33}$$

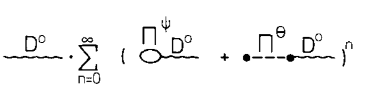

Fig. 3. full $A-A$—propagator

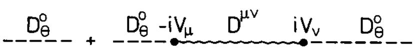

Fig. 4. full $\theta-\theta$—propagator

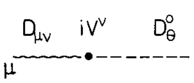

Fig. 5. full $A-\theta$—propagator

ii) In the JR gauge the gauge fixing term is given by:

$$\mathscr{L}_{GF}=\frac{-1}{2\alpha}\left[\partial_\mu\left(g^{\mu\nu}+\frac{1}{a-1}\varepsilon^{\mu\nu}\right)A_\nu\right]^2. \tag{34}$$

In order to calculate the free photon propagator, the quadratic term in A contained in the action has to be written in the form $\frac{1}{2}A^\mu M^0_{\mu\nu}A^\nu$, the free propagator is i times the inverse of $M^0_{\mu\nu}$. In the present case M^0 is of the form

$$M^0_{\mu\nu}=ag_{\mu\nu}+bk_\mu k_\nu+c(\varepsilon_{\mu\alpha}k^\alpha k_\nu+\varepsilon_{\nu\alpha}k^\alpha k_\mu), \tag{35}$$

then $(M^0)^{-1}$ can be parametrized as

$$((M^0)^{-1})^{\mu\nu}=xg^{\mu\nu}+yk^\mu k^\nu+z(\varepsilon^{\mu\alpha}k_\alpha k^\nu+\varepsilon^{\nu\alpha}k_\alpha k^\mu) \tag{36}$$

with

$$x=\frac{a+bk^2}{a(a+bk^2)+c^2k^4},\quad y=\frac{-b}{a(a+bk^2)+c^2k^4},$$

$$z=\frac{-c}{a(a+bk^2)+c^2k^4}. \tag{37}$$

This may be used to derive the free photon propagator:

$$D^0_{\mu\nu}=\frac{-i}{k^4}\left[k^2 g_{\mu\nu}+\left(\alpha-1-\frac{1}{(a-1)^2}\right)k_\mu k_\nu + \frac{1}{a-1}(\varepsilon_{\mu\alpha}k^\alpha k_\nu+\varepsilon_{\nu\alpha}k^\alpha k_\mu)\right]. \tag{38}$$

Precisely as in the case of the Lorentz gauge, the full propagator is given by the diagram of Fig. 3 (we employ an obvious index free notation):

$$D=D^0\cdot\sum_{n=0}^{\infty}(\Pi D^0)^n. \tag{39}$$

The sum can be performed, if one observes that

$$D^0\Pi D^0=\left(D^0+i\alpha\frac{kk}{k^4}\right)\Pi\left(D^0+i\alpha\frac{kk}{k^4}\right) =\frac{m^2}{k^2}\left(D^0+i\alpha\frac{kk}{k^4}\right). \tag{40}$$

This implies

$$D=-i\alpha\frac{kk}{k^4}+\left(D^0+i\alpha\frac{kk}{k^4}\right)\cdot\sum_{n=0}^{\infty}\left(\frac{m^2}{k^2}\right)^n. \tag{41}$$

Reinserting indices, we find for the full photon propagator:

$$D_{\mu\nu}=\frac{-i}{k^2-m^2}\left\{g_{\mu\nu}-\left[1+\frac{1}{(a-1)^2}-\alpha\left(1-\frac{m^2}{k^2}\right)\right]\frac{k_\mu k_\nu}{k^2} +\frac{1}{(a-1)k^2}(\varepsilon_{\mu\alpha}k^\alpha k_\nu+\varepsilon_{\nu\alpha}k^\alpha k_\mu)\right\}. \tag{42}$$

Again, this agrees with the result of the operator solution [13]. The full $\theta-\theta$ and $A-\theta$ propagators are given in Fig. 4 and 5, respectively, the calculation yields the complete propagator matrix

$$\mathscr{P}_{JR}=\frac{i}{k^2(k^2-m^2)}\begin{pmatrix} -k^2g^{\mu\nu}+\left[1+\frac{1}{(a-1)^2}-\alpha\left(1-\frac{m^2}{k^2}\right)\right]k^\mu k^\nu - \frac{1}{a-1}(\varepsilon^{\mu\alpha}k_\alpha k^\nu+\varepsilon^{\nu\alpha}k_\alpha k^\mu) & ie\sqrt{a-1}\,\alpha\left(1-\frac{m^2}{k^2}\right)k^\mu \\ -ie\sqrt{a-1}\,\alpha\left(1-\frac{m^2}{k^2}\right)k^\nu & (k^2-m^2)\left[1-\frac{\alpha}{a^2}(a-1)^2\frac{m^2}{k^2}\right] \end{pmatrix} \tag{43}$$

We note that the $A-\theta$ propagator is proportional to the gauge parameter, it can be eliminated by choosing $\alpha=0$. This is the quantum analogue of the classical statement that A_μ and θ decouple in this gauge.

As promised in the preceding section, we want to clarify the procedure of integrating out the fermion and the scalar field. The resulting effective action may be achieved by inverting the full photon propagator. Let $D^0_{\mu\nu}$ be the free photon propagator in any gauge, then the inverse of the full photon propagator is:

$$D^{-1}=\left[D^0\cdot\sum_{n=0}^{\infty}(\Pi\cdot D^0)^n\right]^{-1} =[D^0\cdot(\mathbb{1}-\Pi D^0)^{-1}]^{-1} =D^{0^{-1}}-\Pi=iM^0-\Pi \tag{44}$$

M^0 gives the free action for the gauge field while Π leads to $-\frac{1}{4}F_{\mu\nu}(m^2/\Box)F^{\mu\nu}$ such that (25) is valid for any gauge which depends only on the gauge field. The restriction to these gauges is necessary since otherwise the vacuum polarization tensor would become gauge dependent which is out of the scope of the present work. Equation (25) can also be verified in the Lorentz and JR gauge by explicitly inverting the full photon propagators of (30) and (42), respectively.

IIIb) Full propagators via matrix inversion

When the fermion field is integrated out, the fermionic part of the action is replaced by $W^\psi[A]$ (cf. (17)), then the Lagrangian is quadratic in (A_μ,θ) and may be written according to:

$$\mathscr{L}=\tfrac{1}{2}(A^\mu,\theta)\begin{pmatrix} M_{\mu\nu} & V_\mu \\ -V_\nu & M^\theta\end{pmatrix}\begin{pmatrix}A^\nu\\ \theta\end{pmatrix} =\tfrac{1}{2}(A^\mu,\theta)\mathscr{M}\begin{pmatrix}A^\nu\\ \theta\end{pmatrix} \tag{45}$$

where

$$M^\theta=k^2 \tag{46}$$

V_μ is given by (6), and $M_{\mu\nu}$ contains the contributions of the classical gauge field action, of $W^\psi[A]$ and of the gauge fixing term. Now we define $\mathscr{P}$ to be i times the inverse of $\mathscr{M}$, we use the parametrization:

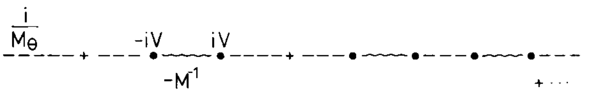

Fig. 6. diagrammatic representation of id (~~~~ contains fermion loops)

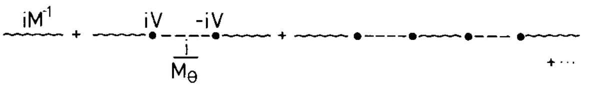

Fig. 7. diagrammatic representation if $ia^{\mu\nu}$ (~~~~ contains fermion loops)

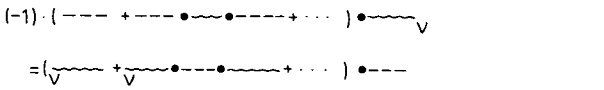

Fig. 8. diagrammatic representation of ib^{ν} (~~~~ contains fermion loops)

$$\mathscr{P}=i\mathscr{M}^{-1}=i\begin{pmatrix} a^{\mu\nu} & b^{\mu} \\ -b^{\nu} & d \end{pmatrix} \tag{47}$$

with

$$d=[M^\theta+V_\mu(M^{-1})^{\mu\nu}V_\nu]^{-1} \tag{48}$$

$$(a^{-1})_{\mu\nu}=M_{\mu\nu}+V_\mu\frac{1}{M^\theta}V_\nu \tag{49}$$

$$b^\nu=-dV_\mu(M^{-1})^{\mu\nu}=-a^{\nu\mu}V_\mu\frac{1}{M^\theta} \tag{50}$$

Equations (48) and (49) can be rewritten in terms of geometric series:

$$i\cdot d=\frac{i}{M^\theta}\sum_{n=0}^{\infty}\left[-iV_\mu(iM^{-1})^{\mu\nu}iV_\nu\frac{i}{M^\theta}\right]^n, \tag{51}$$

$$i\cdot a^{\mu\nu}=(iM^{-1})^{\mu\alpha}\cdot\sum_{n=0}^{\infty}\left\{\left[(iV)\frac{i}{M^\theta}(-iV)(iM^{-1})\right]^n\right\}_\alpha^{\ \nu} \tag{52}$$

which may be represented diagrammatically (Figs. 6 and 7, respectively), where the photon lines already include all fermionic corrections. Hence id is the full $\theta-\theta$ propagator D_θ and $i\cdot a^{\mu\nu}$ is the full $A-A$ propagator $D^{\mu\nu}$. Inserting some factors of i into (50), $i\cdot b^\nu$ is recognized as the full $A-\theta$ propagator D^ν:

$$i\cdot b^\nu=-(id)(-iV_\mu)(iM^{-1})^{\mu\nu}=ia^{\nu\mu}(iV_\mu)\frac{i}{M^\theta}. \tag{53}$$

This is shown diagrammatically in Fig. 8.

Hence we may conclude: the completely corrected propagators, containing all powers of the gauge coupling, can be achieved by simply inverting the matrix $\mathscr{M}$. We also performed this calculation of the exact propagators in the Lorentz and JR gauge, and the results are identical to $\mathscr{P}_L$ and $\mathscr{P}_{JR}$, respectively. There is only one point worth to be mentioned in this calculation, namely the inverse of $M_{\mu\nu}$. This is given by:

Lorentz gauge:

$$(M^{-1})^{\mu\nu}=\frac{1}{-e^2(a-1)(k^2-m^2)+\frac{1}{\alpha}k^2(k^2-e^2(a+1))}\cdot\left\{\left[-\frac{1}{\alpha}k^2+e^2(a-1)\right]g^{\mu\nu}-\left[\left(1-\frac{1}{\alpha}\right)k^2-2e^2\right]\frac{k^\mu k^\nu}{k^2}-\frac{1}{k^2}(\varepsilon^{\mu\alpha}k_\alpha k^\nu+\varepsilon^{\nu\alpha}k_\alpha k^\mu)\right\}, \tag{54}$$

JR gauge:

$$(M^{-1})^{\mu\nu}=\frac{1}{k^2(k^2-m^2)}\left\{-k^2g^{\mu\nu}-\frac{1}{a-1}\cdot(\varepsilon^{\mu\alpha}k_\alpha k^\nu+\varepsilon^{\nu\alpha}k_\alpha k^\mu)+\frac{k^2a^2\left(1+\frac{1}{(a-1)^2}\right)-\alpha[k^2a^2-2m^2(a-1)]}{k^2a^2-\alpha m^2(a-1)^2}k^\mu k^\nu\right\} \tag{55}$$

In both cases the limit $\alpha\to\infty$ (i.e. no gauge fixing) reproduces $-iG^{\mu\nu}$ of [2], which is the exact photon propagator in the $\theta=0$ gauge:

$$G^{\mu\nu}=\frac{i}{k^2-m^2}\left\{-g^{\mu\nu}+\frac{1}{(a-1)k^2}\left[\left(\frac{k^2}{e^2}-2\right)k^\mu k^\nu-\varepsilon^{\mu\alpha}k_\alpha k^\nu-\varepsilon^{\nu\alpha}k_\alpha k^\mu\right]\right\}. \tag{56}$$

$G^{\mu\nu}$ has a pole for $e\to 0$. This is related to the fact that in the $\theta=0$ gauge a free photon propagator does not exist and therefore a perturbative treatment is not possible. Finite values for α allow for a free photon propagator (cf. (27) and (28)) and hence remove the pole for $e\to 0$, thus allowing for a perturbative approach.

IV Discussion and conclusions

The propagator matrices can be used to establish the presence of a massive state perturbatively. To this aim we study the residue matrix of $\mathscr{P}$ at the pole $k^2=m^2$:

$$\mathrm{Res}\,[\mathscr{P}_L, k^2=m^2]=\frac{i}{m^2}\begin{pmatrix} -m^2g^{\mu\nu}+k^\mu k^\nu & \frac{-ie}{\sqrt{a-1}}\varepsilon^{\mu\alpha}k_\alpha \\ \frac{ie}{\sqrt{a-1}}\varepsilon^{\nu\alpha}k_\alpha & \frac{m^2}{a^2} \end{pmatrix} \tag{57}$$

$$\mathrm{Res}\,[\mathscr{P}_{JR}, k^2 = m^2] = \frac{i}{m^2}\begin{pmatrix} -m^2 g^{\mu\nu} + \left[1 + \frac{1}{(a-1)^2}\right] k^\mu k^\nu & 0 \\ -\frac{1}{a-1}(\varepsilon^{\mu\alpha}k_\alpha k^\nu + \varepsilon^{\nu\alpha}k_\alpha k^\mu) & \\ 0 & 0 \end{pmatrix} \quad (58)$$

In both gauges, the residue matrices of $-i\mathscr{P}$ have three eigenvalues, two of them vanish and the third one is positive for $m^2 > 0$, exhibiting one physical state of mass m. Unfortunately, it is not so easy to count the massless modes by looking at $-i\,\mathrm{Res}\,[\mathscr{P}, k^2 = 0]$, since one is accustomed to find more nonvanishing eigenvalues than physical states. We found two nonzero eigenvalues with different sign. If the usual procedure [28] can be transcribed to our case, too, this means that there is no massless state in the boson sector. Then the massless state, found in the bosonized version [2, 3, 15], has to be interpreted as the translation of the fermionic pair of left and right moving states. Making these statements more precise, however, would require a detailed investigation of the physical subspace of the Hilbert space like in the Gupta–Bleuler quantization [29, 30], this we did not intend to do.

Our final item is the case $a = 1$. As we already pointed out, this case is achieved by ordinary dimensional regularization which implies the covariant anomaly. For $a = 1$ the Lagrangian reads

$$\mathscr{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\gamma^\mu[i\partial_\mu + e\sqrt{\pi}A_\mu(1 + i\gamma_5)]\psi + e\theta'\partial_\mu\varepsilon^{\mu\nu}A_\nu + \mathscr{L}_{GF}. \quad (59)$$

Here anomaly cancellation can not be exhibited diagrammatically, since the θ-exchange (Fig. 2) does not exist because θ' does not propagate. Functional integration over θ' gives $\delta(\partial_\mu\varepsilon^{\mu\nu}A_\nu)$, this means that this case is anomaly-free, too, since the covariant anomaly is proportional to $\varepsilon_{\mu\nu}F^{\mu\nu}$ with $F^{\mu\nu} = 0$ due to the δ-function. The latter can be exponentiated as usual to give

$$\mathscr{L}_{\theta'} = \frac{1}{2\xi}(\partial_\mu\varepsilon^{\mu\nu}A_\nu)^2 = \frac{-1}{4\xi}F_{\mu\nu}F^{\mu\nu} \quad (60)$$

with $\xi \to 0$ in order to reproduce the δ-function. This implies the free photon propagator in the Lorentz gauge:

$$D^0_{\mu\nu} = \frac{-i}{(\xi - 1)k^2}\left[\xi g_{\mu\nu} - (\xi - \alpha\xi + \alpha)\frac{k_\mu k_\nu}{k^2}\right] \underset{\xi \to 0}{\longrightarrow} \frac{-i\alpha}{k^4}k_\mu k_\nu. \quad (61)$$

There are no loop corrections since:

$$D^0 \Pi^\psi D^0 = 0. \quad (62)$$

This means $D_{\mu\nu} = D^0_{\mu\nu}$, which vanishes in the Landau gauge ($\alpha = 0$). Hence the quantum system, defined by (59) together with the prescription to regularize gauge invariantly, does not contain any degree of freedom in the boson sector. Again the massless boson, which has been found in earlier works [2, 3, 10], has to be construed as the bosonized version of the fermionic degree of freedom.

In conclusion, we have studied the perturbative approach to the gauge invariant version of the chiral Schwinger model. Precisely as in the nonperturbative treatment it is the Wess–Zumino scalar field which makes the absence of anomalies transparent. Gauge invariance, which is not a feature of the action, can be read off from conservation of the vacuum polarization tensor $\Pi_{\mu\nu}$. Hence in this case we have just the contrary of the usual approach to anomalous gauge theories: there the action is gauge invariant and the corresponding quantum theory is not, here the action is not gauge invariant but the quantum theory is. This feature, which is astonishing at first sight, can be understood in the path integral approach: there the gauge variation of the classical action cancels that of the fermion measure [12, 18, 19].

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