MONOPOLE CONDENSATION AND COLOR CONFINEMENT

A.S KRONFELD [a], M.L. LAURSEN [b], G SCHIERHOLZ [a c], and U -J WIESE [d]

[a] Deutsches Elektronen-Synchrotron DESY, D-2000 Hamburg 52, Fed Rep Germany
[b] NORDITA, DK-2100 Copenhagen Ø, Denmark
[c] Institut für Theoretische Physik der Universitat Kiel, D-2300 Kiel, Fed Rep Germany
[d] II Institut für Theoretische Physik der Universitat Hamburg, D-2000 Hamburg 50, Fed Rep Germany

Received 11 September 1987

Monopole condensation is responsible for confinement in U(1) lattice gauge theory Using numerical simulations and the abelian projection, we demonstrate that this mechanism persists in SU(2) nonabelian gauge theories Our results support the picture of the QCD vacuum as a dual superconductor

The lattice formulation of quantum chromodynamics (QCD) provides a tool for exploring the dynamics of the QCD vacuum. In particular, it enables us to test current ideas on color confinement 't Hooft [1] and Mandelstam [1] have conjectured that this phenomenon can be understood in terms of a color magnetic superconductor, in which color magnetic monopoles condense and color electric charges are confined This picture is dual to the ordinary superconductor [2], in which electric charges condense and magnetic monopoles would be confined through the Meissner effect

These ideas have been successful in understanding the mechanism of confinement and the deconfinement phase transition in four-dimensional compact U(1) gauge theory, which contains monopoles [3,4]. To extend this to nonabelian gauge theories, it is crucial to formulate the theory in terms of its relevant abelian degrees of freedom, which are color magnetic monopoles, color electric charges and "photons" This can be achieved by fixing to a gauge such that the gauge freedom of the maximal abelian (Cartan) subgroup remains This gauge fixing is called the abelian projection [5-7] Also, one should choose a gauge which is renormalizable [5], and in which the abelian degrees of freedom describe the long-distance properties of the vacuum

In a recent paper [7] we provided the framework for quantitative analysis by constructing the abelian projection on the lattice. We also presented results of a Monte Carlo calculation of monopole densities at various couplings β (and temperatures) for gauge groups SU(2) and SU(3) However, these calculations were restricted to nonrenormalizable gauges which are contaminated by unphysical short-distance artefacts [#1]

In this letter we test the above picture of confinement quantitatively in four-dimensional SU(2) gauge theory. To relate the SU(2) theory to the better understood U(1) theory, we study the Georgi–Glashow model, which interpolates between the two.

The action on an $(L_s^3 \times L_t)$ lattice is

$$S=\beta_{\mathrm{G}}\sum_{p}[1-\tfrac{1}{2}\mathrm{Tr}\,U(\mathrm{p})] +\beta_{\mathrm{H}}\sum_{s\,\mu}[1-\tfrac{1}{2}\mathrm{Tr}(\phi(s)U(s,\hat{\mu})\phi(s+\hat{\mu})U^{+}(s,\hat{\mu}))]\,, \tag{1}$$

where $U(\mathrm{p})$ is the product of parallel transporters $U(s,\hat{\mu})$ around a plaquette p and $\phi(s)=\phi^a(s)\sigma_a$ is the fixed length $(\phi^a(s)\phi^a(s)=1)$ adjoint Higgs field For $\beta_{\mathrm{H}}=0$ eq (1) reduces to the pure SU(2) theory For $\beta_{\mathrm{H}}=\infty$ eq (1) reduces to the U(1) theory, which

[#1] The nonrenormalizable gauges are easier to implement numerically can be seen most easily in the unitary gauge $\phi(s)=\sigma_3$, a finite action then requires $U(s,\hat{\mu})$ to be diagonal and hence abelian.

A renormalizable, maximally abelian gauge is obtained by performing a local gauge transformation $\tilde{U}(s,\hat{\mu})=V(s)U(s,\hat{\mu})V(s+\hat{\mu})^{-1}$ such that

$$R=\sum_{s\,\mu}\mathrm{Tr}(\sigma_3\tilde{U}(s,\hat{\mu})\sigma_3\tilde{U}^+(s,\hat{\mu})) \tag{2}$$

is maximized $V(s)$ is only determined up to left multiplication by $d=\mathrm{diag}(\exp[i\alpha(s)], \exp[-i\times\alpha(s)])$, which represents the residual U(1) gauge invariance. Following ref [7] we perform the abelian projection in this gauge, i e we decompose the parallel transporters

$$\tilde{U}(s,\hat{\mu}) = \begin{pmatrix} (1-|c(s,\hat{\mu})|^2)^{1/2} & -c^*(s,\hat{\mu}) \\ c(s,\hat{\mu}) & (1-|c(s,\hat{\mu})|^2)^{1/2} \end{pmatrix} \times \begin{pmatrix} u(s,\hat{\mu}) & 0 \\ 0 & u^*(s,\hat{\mu}) \end{pmatrix}, \tag{3}$$

where $u(s,\hat{\mu})=\exp[i\arg\tilde{U}_{11}(s,\hat{\mu})]$ are abelian parallel transporters, and the coset fields $c(s,\hat{\mu})\in\mathrm{SU}(2)/\mathrm{U}(1)$ represent color electric charges Under a general SU(2) gauge transformation of the original gauge field, $u(s,\hat{\mu})$ and $c(s,\hat{\mu})$ transform in the desired fashion.

$$u'(s,\hat{\mu})=\exp[i\alpha(s)]\,u(s,\hat{\mu})\exp[-i\alpha(s+\hat{\mu})]\,,$$
$$c'(s,\hat{\mu})=c(s,\hat{\mu})\exp[-2i\alpha(s)]\,. \tag{4}$$

The color magnetic monopoles of the theory manifest themselves as half-integer valued magnetic currents on the dual lattice.

$$m(*s,\hat{\mu})=\frac{1}{4\pi}\sum_{p\in\partial f(s+\hat{\mu},\mu)}\arg u(p)=0,\ \pm\tfrac{1}{2},\ \ , \tag{5}$$

where $u(p)$ is the product of abelian parallel transporters $u(s,\hat{\mu})$ around a plaquette p, and $f(s+\hat{\mu},\mu)$ is the three-cube with origin $s+\hat{\mu}$ perpendicular to the μ-direction, dual to the link from $*s$ to $*s+\hat{\mu}$ on the dual lattice The monopole current is topologically conserved on the dual sites $*s$ $\sum_\mu[m(*s,\hat{\mu})-m(*s-\hat{\mu},\hat{\mu})]=0$ Consequently, the monopole currents form closed loops on the dual lattice

To understand confinement in terms of the ideas cited at the outset of this letter, it is helpful to investigate the different phases of the theory and the nature of the accompanying transitions The phase diagram at finite temperature ($T=(L_t a)^{-1}$) is shown in fig 1 The theory has a deconfinement phase tran-

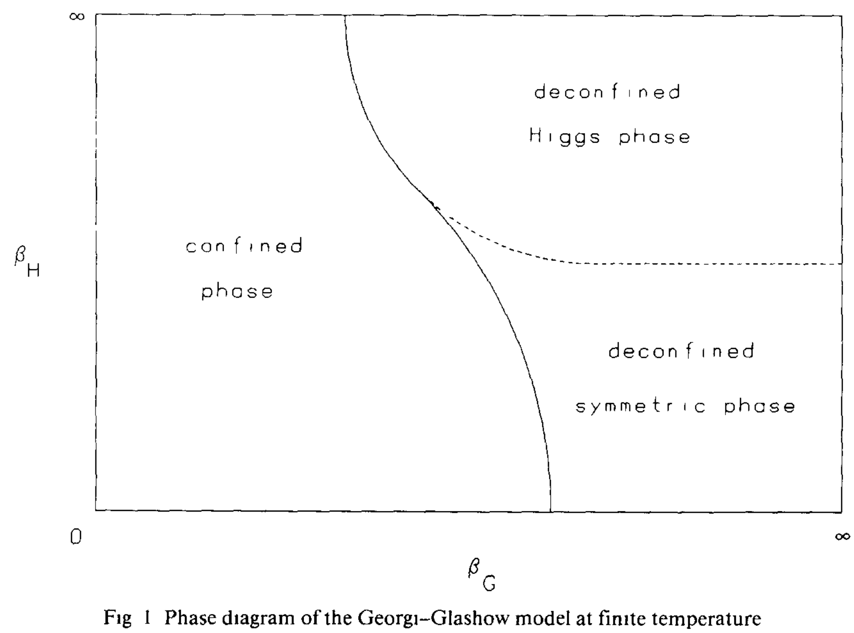

Fig 1 Phase diagram of the Georgi–Glashow model at finite temperature

sition extending from U(1) ($\beta_H=\infty$) to SU(2) ($\beta_H=0$), and the Polyakov loop

$$P=\tfrac{1}{2}\mathrm{Tr}\prod_{t=0}^{L_t-1} U(s+t\hat{4},\hat{4}) \qquad (6)$$

is the order parameter of the transition At finite β_H and large β_G, there is also a transition to a deconfined Higgs phase We use numerical simulations on a $10^3\times5$ lattice at various values of β_G, β_H to analyze the properties of the monopoles in the three phases (Simulations on 5^4 lattices yield similar results.) We generate the configurations according to standard methods, and then maximize R in eq (2) iteratively [#2] for the configurations in the Monte Carlo ensemble In all cases the statistical errors are smaller than the symbols plotted

The U(1) theory indicates that the confined phase is a coherent monopole plasma, characterized by a high density of long, entangled monopole loops In the deconfined phase monopoles are dilute and their loops are small In fig 2 we show a two-dimensional projection of the monopole currents for typical gauge field configurations at $\beta_H=8$, which corresponds essentially to U(1), and at $\beta_H=0$, which is SU(2) Consider first the U(1) case depicted in figs 2a, 2b In the confined phase ($\beta_G=1\ 1$) the monopole loops are so entangled that it is difficult to distinguish individual loops However, we have verified that the dominant portion of the magnetic currents is in long, intersecting loops In the deconfined phase ($\beta_G=1\ 3$) the monopole loops are small and have almost disappeared Now consider the SU(2) case depicted in figs 2c, 2d Remarkably, the behavior of the monopoles in the two phases is just as before

To quantify this picture we consider the perimeter density of monopole loops

[#2] In principle this procedure is critically slowed down, but this can be alleviated by Fourier acceleration See ref [8]

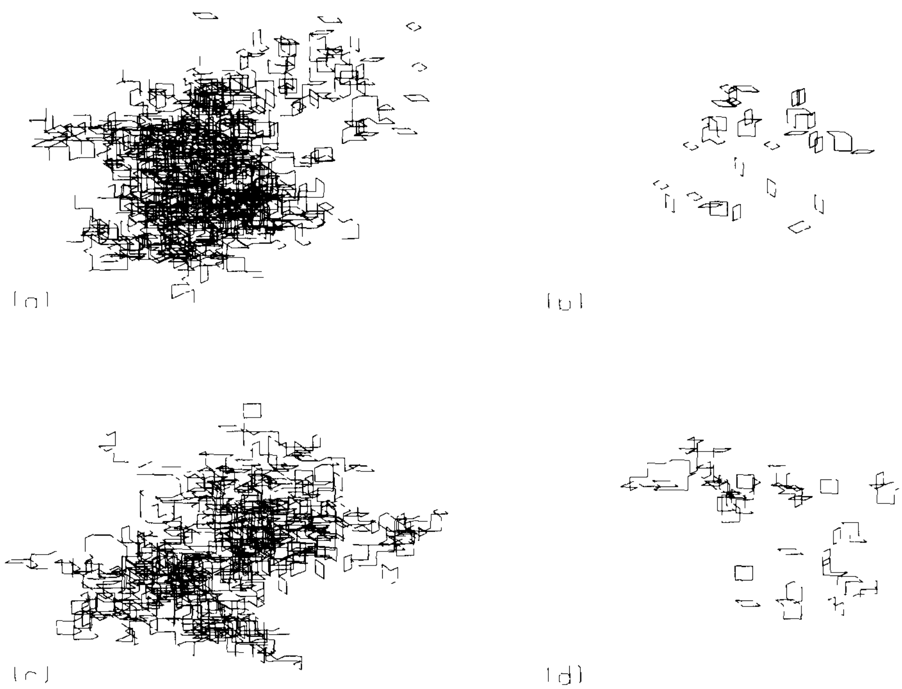

Fig 2 Two-dimensional perspective projection of the color magnetic monopole currents Apparently open loops are in fact closed due to the periodic boundary conditions The empty regions are illusory because we try to show long loops in their entirety and thereby occasionally leave the lattice (a) Confined phase close to the U(1) limit $\beta_G=1\ 1$, $\beta_H=8$, (b) deconfined phase close to the U(1) limit $\beta_G=1\ 3$, $\beta_H=8$ (c) Confined phase of the pure SU(2) theory $\beta_G=2\ 2$, $\beta_H=0$, (d) deconfined phase of the pure SU(2) theory $\beta_G=2\ 6$, $\beta_H=0$

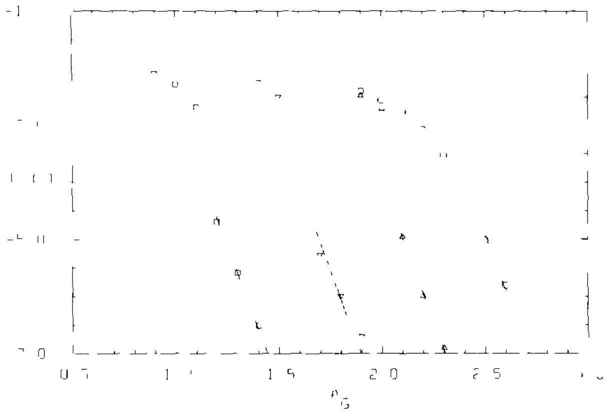

Fig 3 Perimeter density $\ln(l)$ of color magnetic monopoles as a function of β_G, for ($\square$) $\beta_H=8$, (∇) $\beta_H=2$, ($\triangle$) $\beta_H=1$ and ($\bigcirc$) $\beta_H=0$ The solid lines indicate exponential fall-off with slope $-\pi^2$

$$l=\frac{1}{4V}\sum_{*s,\mu}|m(*s,\hat{\mu})|\,, \qquad (7)$$

and the number density $\rho_{m\bar{m}}$ of monopole–antimonopole pairs in adjacent spatial cubes Having seen fig 2 one expects the physics of the monopoles in SU(2) to be similar to the U(1) case In particular, the deconfined phase is characterized by $l\propto\exp(-\pi^2\beta_G)$ and $\rho_{m\bar{m}}\propto\exp(-\pi^2\beta_G)$ in the Villain form of the U(1) theory [4]

In fig 3 we present Monte Carlo data for $\ln(l)$ as a function of β_G for $\beta_H=0$, 1, 2 and 8 They clearly indicate the deconfinement phase transition This occurs at the same critical β_G where the Polyakov loop gets a nonvanishing vacuum expectation value Our data suggest that the transition is first order at $\beta_H=8$, 2, 1 and second order at $\beta_H=0$ In the deconfined phase the exponential fall-off of l shows that the monopoles form a dilute gas, as in the U(1) theory The slope is compatible with $-\pi^2$ *independent* of β_H, as indicated by the solid lines in fig 3 [#3] The same is true for $\rho_{m\bar{m}}$ also Thus the dilute gas approximation of the U(1) theory correctly describes the physics of the monopoles in the deconfined phase of the SU(2) theory as well The abelian Polyakov loop, composed as in eq (6) from abelian parallel transporters, is also an order parameter of the deconfinement phase transition [#4], and it rises more dramatically at the transition than the nonabelian Polyakov loop This demonstrates again the relevance of the abelian degrees of freedom

For the Higgs phase transition it is also interesting to investigate the role of the 't Hooft–Polyakov monopoles They are defined in the unitary gauge, $\bar{\phi}(s)=W(s)\phi(s)W(s)^{-1}=\sigma_3$, $\bar{U}(s,\hat{\mu})=W(s)U(s,\hat{\mu})\times W(s+\hat{\mu})^{-1}$ Replacing $\tilde{U}(s,\hat{\mu})$ in eq. (3) by $\bar{U}(s,\hat{\mu})$ we repeat the construction of magnetic currents for the 't Hooft–Polyakov monopoles Fig 4 shows the perimeter density of 't Hooft–Polyakov monopole loops both as a function of β_G at $\beta_H=0$ 5,

#3 This is also in agreement with results of a recent study of the SU(2) vacuum at finite temperature which yielded a monopole action $S=\pi^2\beta_G$. See ref [9]

#4 The abelian Polyakov loop is nonzero in the deconfined phase, because the center Z_2 is a subgroup of U(1) See ref [10] for a review

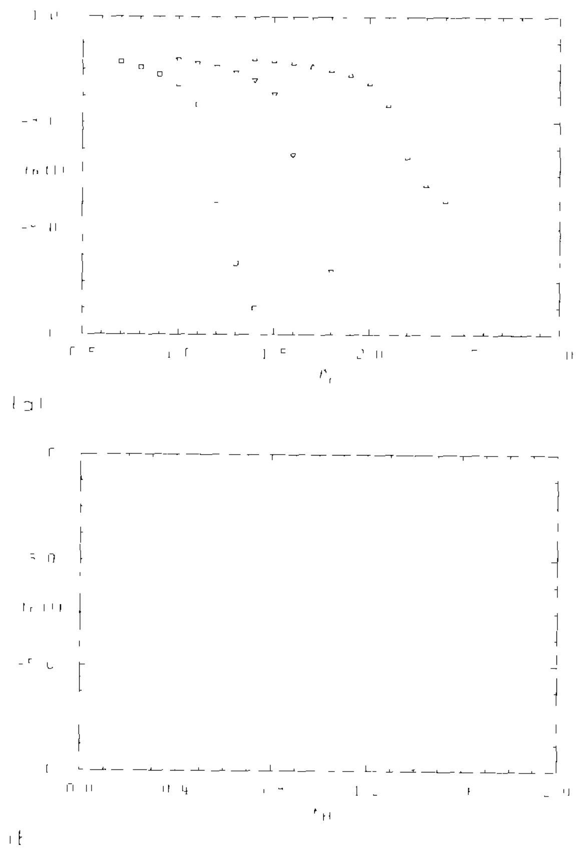

Fig 4 Perimeter density $\ln(l)$ of 't Hooft–Polyakov monopoles (a) as a function of β_G for ($\square$) $\beta_H=8$, (∇) $\beta_H=2$, ($\triangle$) $\beta_H=1$ and ($\diamond$) $\beta_H=0$ 5, and (b) as a function of β_H for $\beta_G=2$ 4

1, 2, 8 and as a function of β_H at $\beta_G=2.4$ For $\beta_H=1$, 2, 8 we find a dense state of long entangled 't Hooft–Polyakov monopole loops in the confined phase, whereas they become dilute and small in the Higgs phase. However, the slope of the exponential fall-off of l is $-\pi^2$ *only* at $\beta_H=8$, where the theory is essentially U(1) At this coupling the Higgs phase transition occurs at $\beta_G=1.1$, which is consistent with a $1/\beta_H$ expansion around the U(1) theory [11] At $\beta_H=0.5$ we cross from the confined to the deconfined symmetric phase, where the 't Hooft–Polyakov monopoles are not dilute We therefore do not observe an exponential fall-off of l in this case This is confirmed in fig 4b, where we cross the phase transition from the deconfined symmetric to the deconfined Higgs phase at fixed $\beta_G=2.4$.

The results presented in this letter suggest the following picture of the phases of the Georgi–Glashow model The Higgs phase transition is well described in terms of 't Hooft–Polyakov monopoles. in the Higgs phase they are heavy and therefore dilute, whereas they condense in the SU(2) symmetric phases On the other hand, the deconfinement phase transition can be understood in terms of color magnetic monopoles defined in the maximally abelian gauge In the deconfined phases the color magnetic monopoles are well described by the dilute gas approximation of the U(1) theory In the confined phase color magnetic monopoles condense causing color confinement by the dual Meissner effect

Finally, the importance of the abelian degrees of freedom may be relevant in numerical simulations It is possible to accelerate the update procedure in abelian gauge theories, but nonabelian gauge theories are more problematic [12]. Perhaps the abelian projection can be used to formulate nonabelian theories such that accelerated abelian algorithms apply.

We would like to thank M Luscher, M Marcu and F Wagner for their continued interest

References

[1] G 't Hooft, in High energy physics, ed A Zichichi (Editrice Compositori, Bologna, 1976),
S Mandelstam, Phys Rep 23 (1976) 245
[2] J G Bednorz and K A Muller, Z Phys B 64 (1986) 189
[3] T Banks, R Myerson and J Kogut, Nucl Phys B 129 (1977) 493,
J L Cardy, Nucl Phys B 170 (1980) 369
[4] T De Grand and D Toussaint, Phys Rev D 22 (1980) 2478
[5] G 't Hooft, Nucl Phys B 190 (1981) 455
[6] G Mack, Fortschr Phys 29 (1981) 135,
J Polonyi, MIT report CTP 1452 (1987)
[7] A S Kronfeld, G Schierholz and U -J Wiese, Nucl Phys B 293 (1987) 461
[8] C T H Davies et al , Cornell report (1987)
[9] M L Laursen and G Schierholz DESY report 87-061 (1987)
[10] B Svetitsky, Phys Rep 132 (1986) 1
[11] R C Brower et al , Phys Rev D 25 (1982) 3319
[12] C T M Davies et al , Cornell report, in preparation