CONTACT INTERACTIONS AND THE CALLAN–GROSS RELATION

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Quark–lepton contact interactions, which are invariant under the chiral $SU(2)_w \times U(1)_Y$ standard model gauge group, may flip helicities. We study the constraints from rare processes on these interactions which, in general, lead to a deviation from the Callan–Gross relation.

Deep-inelastic electron–proton scattering experiments have played an important role in unravelling the structure of the proton. By now the quark–gluon content of nucleons is rather well known, and the current interest in electron–proton scattering at high momentum transfer concerns mainly the electron–quark and electron–gluon interactions. Possible deviations from the predictions of the standard model could be due to new currents, i.e., the exchange of new heavy particles or to a further substructure of electrons and quarks.

At energies small compared to the new mass scale the effects of possible new interactions can be systematically investigated by means of an operator analysis. For electron–proton scattering operators of current–current type [1]

$$\mathscr{L}^{cc}_{eff} = \pm (g^2/\Lambda^2)\, \bar{e}\gamma^\mu e\, \bar{q}\gamma_\mu q \qquad (1)$$

have been extensively studied. Here e is a left- or right-handed electron, and q is a left- or right-handed u- or d-quark. The interaction (1) conserves fermion chirality at lepton and quark vertex which is a natural Ansatz for lepton and quark masses small compared to the new interaction scale Λ.

If one accepts the standard model as the correct low energy effective lagrangian new interactions should be invariant under local $SU(3)_c \times SU(2)_w \times U(1)_Y$ transformations and the only relevant chiral symmetry is the electroweak $SU(2)_w \times U(1)_Y$ gauge group. This imposes one constraint on current–current operators. Since the right-handed electron is an $SU(2)_w$ singlet it can only couple to the isosinglet combination of left-handed quarks

$$O_{RL} = \bar{e}_R \gamma^\mu e_R (\bar{u}_L \gamma_\mu u_L + \cos^2\theta_c \bar{d}_L \gamma_\mu d_L), \qquad (2)$$

i.e., the number of independent operators is reduced from eight to seven. The Cabibbo angle enters in the transition from weak- to mass-eigenstates.

The chiral $SU(2)_w \times U(1)_Y$ symmetry also allows interactions which flip the helicity at the lepton and quark vertex. There are three independent operators [2] involving quarks and leptons of the first family. The corresponding effective lagrangian may be written as

$$\mathscr{L}^{HF}_{eff} = (g^2/\Lambda^2)\,[c_1(\bar{\ell}_L e_R)(\bar{d}_R q_L) + c_2(\bar{\ell}_L e_R)(\bar{q}_L u_R) + c_3(\bar{\ell}_L u_R)(\bar{q}_L e_R) + \text{c.c.}], \qquad (3)$$

where

$$\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad q_L = \begin{pmatrix} u_L \\ d'_L \end{pmatrix}, \quad d'_L = \cos\theta_c\, d_L + \sin\theta_c s_L\,.$$

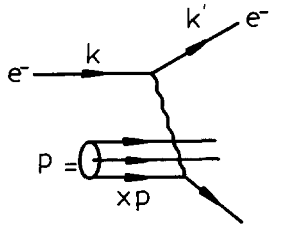

Fig. 1. The process ep→eX.

Tensor operators can be expressed in terms of the scalar operators (3) by means of a Fierz transformation. The last two terms in (3) are induced for instance through the exchange of SU(5)-type leptoquarks [3].

The differential cross section for the reaction ep→eX has the general form (cf. fig. 1)

$$d\sigma^{\mp}/dx\,dy=(4\pi\alpha^2/x^2y^2s)\,[y^2xF_1(x,Q^2)+(1-y)\,F_2(x,Q^2)\pm(y-\tfrac{1}{2}y^2)\,xF_3(x,Q^2)], \tag{4a}$$

where

$$s=(k+p)^2,\quad t=(k'-k)^2=-Q^2=-xys,\quad u=(k'-xp)^2=-x(1-y)s, \tag{4b, c, d}$$

if electron and quark masses are neglected. For spin-1/2 quarks γ- and Z-boson exchange gives structure functions which, up to QCD corrections [4], satisfy the Callan–Gross relation [5]

$$2xF_1=F_2, \tag{5}$$

i.e., the ratio R of longitudinal over transverse absorption cross section vanishes:

$$R=(2xF_1-F_2)/2xF_1=0. \tag{6}$$

Scalar partons inside the proton would contribute only to F_2 and not to xF_1. Hence they would give a negative correction to (6), $\Delta R_s<0$. One easily verifies that an arbitrary current–current interaction (1) does not contribute to R. This, however, is not the case for the helicity-flip operators (3) which yield a positive correction $\Delta R_{HF}>0$.

Before we compute this correction let us first examine the constraints from rare processes on the coefficients c_1, c_2, c_3. Using the Fierz identity

$$(\bar\chi_{1L}\chi_{2R})(\bar\chi_{3L}\chi_{4R})=-\tfrac{1}{2}\,[(\bar\chi_{1L}\chi_{4R})(\bar\chi_{3L}\chi_{2R})+\tfrac{1}{4}\,(\bar\chi_{1L}\sigma^{\mu\nu}\chi_{4R})(\bar\chi_{3L}\sigma_{\mu\nu}\chi_{2R})], \tag{7}$$

one obtains from (3)

$$\begin{aligned}\mathcal{L}^{HF}_{eff}&=(g^2/\Lambda^2)\{(\bar\nu_L e_R)\{c_1(\bar d_R u_L)+(c_2-\tfrac{1}{2}c_3)[\cos\theta_c(\bar d_L u_R)+\sin\theta_c(\bar s_L u_R)]\}\\&-\tfrac{1}{8}c_3(\bar\nu_L\sigma^{\mu\nu}e_R)[\cos\theta_c(\bar d_L\sigma_{\mu\nu}u_R)+\sin\theta_c(\bar s_L\sigma_{\mu\nu}u_R)]+\tfrac{1}{8}c_3(\bar e_L\sigma^{\mu\nu}e_R)(\bar u_L\sigma_{\mu\nu}u_R)\\&+(\bar e_L e_R)[c_1\cos\theta_c(\bar d_R d_L)+c_1\sin\theta_c(\bar d_R s_L)-(c_2-\tfrac{1}{2}c_3)(\bar u_L u_R)]+\text{h.c.}\}.\end{aligned} \tag{8}$$

The interactions (8) lead to helicity unsuppressed decays of pseudoscalar mesons to lepton pairs: $\pi^+\to e^+\nu$, $K^+\to e^+\nu$, $\pi^0\to e^+e^-$, and $K^0_L\to e^+e^-$. Neglecting the electron mass the decay width of a meson M containing the quarks q_1 and q_2 is given by (cf., e.g., ref. [3])

$$\Gamma(M)=(f^2_M m^3_M/64\pi)\,[m_M/(m_{q_1}+m_{q_2})]^2\,|A|^2, \tag{9}$$

where f_M and m_M are meson decay constant and mass, and A is the coefficient of the contributing operator in (8). The agreement between experimental data and standard model predictions yields upper bounds for the contribution of the new interactions (8) to meson decays. In table 1 these upper bounds are listed together with the corresponding lower bounds on Λ (with the convention $g^2=4\pi$). For $c_1\neq0$ and $c_3\neq2c_2$ these bounds

Table 1
Bounds on the coefficients c_1, c_2 and c_3 from rare decays.

Process	Branching ratios [a]	Lower bounds on Λ [TeV]
$\pi^0 \to e^+e^-$	$<1.8\times10^{-7}$	$>1.2[c_1 \cos\theta_c + (c_2 - c_3/2)]^{1/2}$
$K_L^0 \to e^+e^-$	$<2\ \times10^{-7}$	$>130\, c_1^{1/2}$
$\pi^+ \to e^+\nu$	$<1.2\times10^{-4}$	$>31[c_1 - (c_2 - c_3/2)\cos\theta_c]^{1/2}$
$K^+ \to e^+\nu$	$<1.5\times10^{-5}$	$>30(c_2 - c_3/2)^{1/2}$

[a] Ref. [6].

are so restrictive that an observable effect of these operators on the structure functions in electron–proton scattering can be excluded at HERA energies, i.e., momentum transfers $Q^2 < (300\ \mathrm{GeV})^2$. Only for $c_1 = 0$, $c_3 = 2c_2$ low values of Λ are possible. This case corresponds to the exchange of a spin-2 particle whose quark couplings always involve a right-handed u-quark. Due to the $SU(2)_w \times U(1)_Y$ symmetry there is no corresponding operator with a right-handed d-quark. It is possible to partially circumvent the bounds on Λ listed in table 1 by adding to (8) further operators involving quarks of the second family. This, however, requires a fine tuning of parameters dependent on the Cabbibo angle, which we consider unnatural.

Let us now consider the effect of the operators (3) on the structure functions in the case $c_1 = 0$, $c_2 = c_3 = 1$. As the interactions (3) flip the lepton and quark helicities they do not interfere with photon and Z-boson exchange. A straightforward calculation gives the result

$$d\sigma(e^-_{L,R}p)/dx\,dy = (4\pi\alpha^2/x^2y^2s)[y^2 x\Delta F_1(x,Q^2) + (1-y)\,\Delta F_2(x,Q^2) + (y - \tfrac{1}{2}y^2)\,x\Delta F_3(x,Q^2)], \tag{10a}$$

$$2x\Delta F_1(x,Q^2) = (5/2\alpha^2)(Q^4/\Lambda^4)\,x[u(x,Q^2) + \bar{u}(x,Q^2)], \tag{10b}$$

$$\Delta F_2(x,Q^2) = (1/\alpha^2)(Q^4/\Lambda^4)\,x[u(x,Q^2) + \bar{u}(x,Q^2)], \tag{10c}$$

$$x\Delta F_3(x,Q^2) = -(2/\alpha^2)(Q^4/\Lambda^4)\,x[u(x,Q^2) - \bar{u}(x,Q^2)]. \tag{10d}$$

As one might expect the helicity-changing contact interactions give a nonvanishing contribution to R (cf. eq. (6)):

$$R(x,Q^2) = \frac{(3/2\alpha^2)(Q^4/\Lambda^4)\,x[u(x,Q^2) + \bar{u}(x,Q^2)]}{F_2^{SM}(x,Q^2) + (5/2\alpha^2)(Q^4/\Lambda^4)\,x[u(x,Q^2) + \bar{u}(x,Q^2)]} > 0. \tag{11}$$

F_2^{SM} is the standard model structure function,

$$F_2^{SM}(x,Q^2) = 2xF_1^{SM}(x,Q^2) = \sum_q A_q(Q^2)\,x[q(x,Q^2) + \bar{q}(x,Q^2)], \tag{12a}$$

$$A_q(Q^2) = e_q^2 - 2e_q v_e v_q\, Q^2/(Q^2 + m_Z^2) + (v_e^2 + a_e^2)(v_q^2 + a_q^2)\,[Q^2/(Q^2 + m_Z^2)]^2, \tag{12b}$$

where e_q is the electric charge of the quark q, and v_e, a_e, v_q and a_q are the Z-boson vector and axial vector couplings to electron and quark, respectively. We note that for arbitrary coefficients c_1, c_2, c_3 one obtains deviations from the Callan–Gross relation proportional to c_1^2, c_2^2 and c_2c_3.

Approximating F_2^{SM} by the electromagnetic part with $u(x) \approx 2d(x)$ one obtains from (11) the x-independent expression

$$R(Q^2) \approx [(3/\alpha^2)\,Q^4/\Lambda^4]/[1 + (5/\alpha^2)\,Q^4/\Lambda^4]. \tag{13}$$

Hence an experimental upper bound $R < 0.1$ at $x = 0.5$ and maximal Q^2 would give for the new interaction scale the lower bound $\Lambda > 5$ TeV. Fig. 2 shows $R(x, Q^2)$ at $x = 0.5$ as a function of Q^2 for the complete expression (11). For a given value of Q^2 one can read off the lower bound on Λ which follows from an upper bound on

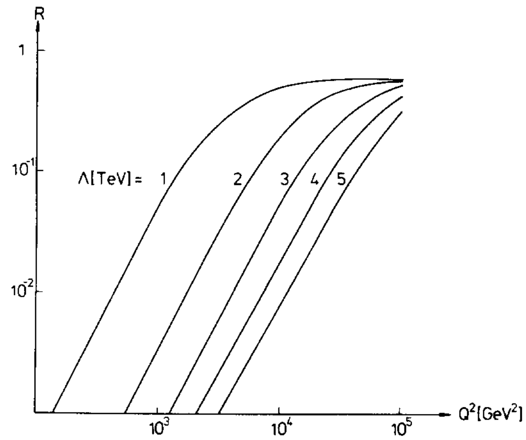

Fig. 2. The Q^2-dependence of R as given in eq. (11) for $x=0.5$.

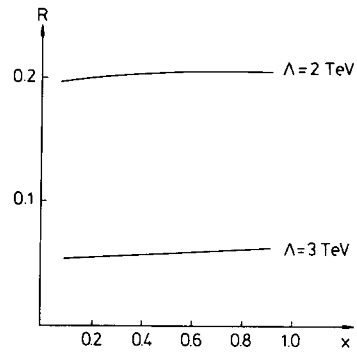

Fig. 3. The x-dependence of R as given in eq. (11) for $Q^2=10^4$ GeV^2.

R. Fig. 3 shows $R(x, Q^2)$ as a function of x for $Q^2=10^4$ GeV^2 and $\Lambda=2,3$ TeV. R is essentially x-independent as indicated already by (13). This is in contrast to the QCD correction to R [4] which is significantly different from zero only for $x<0.5$. We note that the contribution to R from supersymmetric radiative corrections [7,8] is expected to be smaller than 0.1.

Let us summarize our results. Contact interactions which are invariant under the chiral $SU(2)_w \times U(1)_Y$ symmetry of the standard model may change lepton and quark helicities. They give rise to a deviation from the Callan–Gross relation, $\Delta R>0$, and electron–proton scattering experiments at HERA energies will be sensitive to new, helicity-changing, interaction scales up to $\Lambda \approx 2$ TeV [9]. For such low values of Λ rare processes restrict the three possible helicity changing interactions to one linear combination which corresponds to the exchange of heavy spin-2 particles and the absence of scalar particles with the same internal quantum numbers. Since we are not aware of any theory with such a low energy effective lagrangian we do not expect that deviations from the Callan–Gross relation will be observed at HERA. This, however, may just be a theoretical prejudice.

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