Mirror families and radiative $SU_2 \times U_1$ breaking

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We examine the possibility that a heavy fourth family with mirror quantum numbers is the driving force for radiative $SU_2 \times U_1$ breaking in the framework of supersymmetric grand unified theories coupled to $N=1$ supergravity. We compare the results to the case of a sequential fourth family.

I. INTRODUCTION

We consider a supersymmetric grand unified theory coupled to $N=1$ supergravity with four generations of quarks and leptons. It is conceivable that the fourth generation is a mirror family, which is the case, for instance, in supersymmetric σ models based on the exceptional group E_8.[1] Mirror fermions[2] are fermions with $SU_3 \times SU_2 \times U_1$ quantum numbers identical to those of the known quarks and leptons but with opposite handedness.

We assume that supergravity is broken and that the matter fields of the grand unified theory (GUT) feel the supersymmetric breaking at the scale M_W (=the weak boson mass).[3–7] Formally this can be arranged as follows. The effects of the breaking appear in the effective Lagrangian as explicit soft breaking terms with coefficient of $O(m_{3/2})$, where $m_{3/2}$ is the gravitino mass.[6] Therefore, one simply has to assume $m_{3/2}$ to be of the order of M_W.

By looking at the one-loop renormalization-group equations evolving from the Planck scale (M_P) to M_W several groups have shown that with this assumption the $SU_2 \times U_1$ invariance of the Weinberg-Salam theory can be broken radiatively to electromagnetic U_1 (Refs. 3 and 7). In fact they have shown that M_W can be fitted to its experimental value by considering $m_{3/2}$ as a free parameter and choosing it adequately. Those groups work with three families. In general the $SU_2 \times U_1$ breaking is driven by the biggest Yukawa coupling. Therefore, lower limits on the possible value of the t-quark mass could be "derived."

With a fourth family that is not possible anymore. A reasonable assumption might seem that now the top of the fourth generation should drive $SU_2 \times U_1$ breaking.[4] If the fourth family is a mirror family, even this is questionable. Therefore, we have scanned through all initial values for the Yukawa couplings of the fourth generation. We also considered the possibility that these are not too far from the corresponding value for the top quark; i.e., we did not neglect the influence of the third family on the breaking.

II. RENORMALIZATION-GROUP EQUATIONS

Besides the four families we work with a minimal set of other fields. This means just two additional Higgs doublets H and H' with opposite U_1 charge. For the case that all four families are ordinary families the renormalization-group equations have been given in Appendix B of Ref. 5. We call this "case N." Let u_3 be the Yukawa coupling of the t quark and u_4, d_4 be the quark Yukawa couplings of the fourth family. We assume that all other Yukawa couplings have no influence on the $SU_2 \times U_1$ breaking. Then the superpotential is[5]

$$f_N \simeq u_3 U_3 Q_3 H' + \mu H H' + u_4 U_4 Q_4 H' + d_4 D_4 Q_4 H \ . \quad (1)$$

Here Q_n are the left-handed quark doublets of the nth family and U_n, D_n the corresponding right-handed singlets. Now we assume the fourth family to be a mirror family ("case M"). One has

$$f_M \simeq u_3 U_3 Q_3 H' + \mu H H' + u_4 U_4 Q_4 H + d_4 D_4 Q_4 H' \ , \quad (1')$$

since the only difference between the two cases are the opposite U_1 charges. In (1′) we have excluded a term $\sim Q_3 Q_4$, which would give rise to a pathological phenomenology of the mirror fermions.[2] The renormalization-group equations depend only on the squares of the charges and ask only for what types of particle couple to what. Therefore, the renormalization-group equations for case M can be almost read off from those for case N and will not be written down here in full. We give only one characteristic example: Consider the mass $m_{H'}$ of the Higgs boson H' in the soft breaking terms. For case N we have

$$\begin{aligned} 8\pi^2 \frac{dm_{H'}^2}{dt} &= 3(m_{H'}^2 + m_{u_3}^2 + m_{Q_3}^2 + \eta_{u_3}^2) u_3^2 \\ &\quad + 3(m_{H'}^2 + m_{u_4}^2 + m_{Q_4}^2 + \eta_{u_4}^2) u_4^2 \\ &\quad - g_1^2 \mu_1^2 - 3 g_2^2 \mu_2^2 \ . \end{aligned} \quad (2)$$

Here $g_{1,2,3}$ are the coupling constants of the $SU_3 \times SU_2 \times U_1$ gauge group and $\mu_{1,2,3}$ the corresponding gaugino masses. t is related to the renormalization mass scale ξ via $t = \ln(\xi / M_P)$. m_i are the soft masses of the fields i and η_i are the coefficients of the corresponding trilinear soft terms.

For case M we have

$$8\pi^2\frac{dm_{H'}^2}{dt}=3(m_{H'}^2+m_{u_3}^2+m_{Q_3}^2+\eta_{u_3}^2)u_3^2+3(m_{H'}^2+m_{D_4}^2+m_{Q_4}^2+\eta_{D_4}^2)d_4^2-g_1^2\mu_1^2-3g_2^2\mu_2^2\ . \quad (3)$$

The reason is simply that one should sum over all contributions from particles which couple to H' in the superpotential. Some remarks are in order.

(i) For $u_3=0$ it is appropriate to say that the two cases differ from each other only by the interchange of H and H'. The reason is that in this case all the renormalization-group equations are identical up to $m_H \leftrightarrow m_{H'}$. *For $u_3=0$ there is no real difference between the two cases* from the standpoint of the renormalization-group equations. One must include the top to see a difference. The influence of the top quark will be bigger the smaller the mass difference between the third and fourth family is. We shall discuss this in detail in Sec. IV.

The equivalence of the two cases for $u_3\equiv 0$ extends to the masses of U_4 and D_4. This is obvious, since in case N the mass of U_4 is proportional to $\langle H'\rangle$, while in case M it is proportional to $\langle H\rangle$ [cf. Eq. (1)]. (Exchanging m_H and $m_{H'}$ means exchanging the values of $\langle H\rangle$ and $\langle H'\rangle$.) One may convert these considerations into an argument that in case M, $m(U_4)<m(D_4)$, whereas in case N, $m(U_4)>m(D_4)$ is to be expected. For this one needs the *assumption* that the mass relation $m(U_3)\gg m(D_3)$ for the third family is a hint for a hierarchy $|\langle H'\rangle|>|\langle H\rangle|$ and is only partly due to a hierarchy of the top and bottom Yukawa couplings at the Planck scale.

For $u_3\neq 0$ a difference between case M and case N arises. First, u_3 is driven by d_4 in case M and by u_4 in case N and u_4 and d_4 behave differently because of their different charges. Second, u_3 contributes to H' in both cases, so that one cannot exchange the role of H and H' anymore. Therefore, the quark masses are not correlated anymore in the two cases.

(ii) We have excluded a direct mass term between the two heavy families in case M. Therefore, there is no mixing allowed between the third and the fourth family. Since we are interested in case N only for reasons of comparison, we neglect mixing also in case N.

(iii) Let us collect the free parameters that can be varied. At the Planck scale we put all soft masses equal to $m_{3/2}$. The trilinear soft breaking terms are all assumed to have coefficients $\eta_i(M_P)=a_0 m_{3/2}$, where a_0 is a free parameter in the range $0.5<a_0<3$. Similarly for the bilinear soft term one assumes a coefficient $\beta=b_0 m_{3/2}$ with $0.5<b_0<2$ at M_P. μ and the μ_i are also assumed to be of the order of the gravitino mass:

$$\mu(M_P)=c_0 m_{3/2},\quad \mu_i(M_P)=d_0 m_{3/2}$$

with $0.4<c_0$, $d_0<1.5$, and all μ_i being equal at M_P. These assumptions can be justified in the simplest of supergravity models.[6] In addition, there are the three Yukawa couplings, which, in principle, may take any value between 0 and, say, 5.

The procedure is now as follows. One picks up any of the possible values of a_0, b_0, c_0, and d_0 and keeps them fixed. Now one looks for values of $u_3(M_P)$, $u_4(M_P)$, and $d_4(M_P)$, which realize the breaking of $SU_2\times U_1$ (cf. Sec. III). This is done numerically. At this stage $m_{3/2}$ is chosen to be 100 GeV. This is no restriction, because the symmetry breaking does not depend on $m_{3/2}$. Having found appropriate values of $u_3(M_P)$, $u_4(M_P)$, and $d_4(M_P)$ one can change $m_{3/2}$ in such a way that the vacuum expectation values come out as they should $[\langle H\rangle^2+\langle H'\rangle^2=v^2=(174\ \text{GeV})^2]$.

In general the range of values of $u_3(M_P)$, $u_4(M_P)$, and $d_4(M_P)$, that give the desired symmetry breaking is quite restricted. From that one can deduce restrictions on the possible masses of U_3, U_4, and D_4 quark. The restrictions are weakened, however, as soon as one also varies a_0, b_0, c_0, and d_0. The results will be discussed in detail in Sec. IV.

III. SPONTANEOUS SYMMETRY BREAKING

We have the Higgs potential as usually assumed in the literature.[7] Spontaneous symmetry breaking sets in at that value of t where

$$S\equiv(m_H^2+\mu^2)(m_{H'}^2+\mu^2)-\beta^2\mu^2 \quad (4)$$

becomes negative. For the potential to be bounded from below,

$$C\equiv m_H^2+m_{H'}^2+2\mu^2-2|\beta\mu| \quad (5)$$

must remain positive over the whole range $M_W<\xi<M_P$. This usually implies that the point where S becomes negative is not far above M_W. It also implies that

$$\sin 2\theta=|2\beta\mu|/(m_H^2+m_{H'}^2+2\mu^2) \quad (6)$$

defines an angle θ. θ should be chosen in such a way that $\cos 2\theta<0$, if $m_H^2<m_{H'}^2$. v can be given by means of this angle:

$$v^2=\frac{2}{(g_1^2+g_2^2)|\cos 2\theta|}[\,|m_H^2-m_{H'}^2|-(m_H^2+m_{H'}^2+2\mu^2)|\cos 2\theta|\,]\ . \quad (7)$$

In fact, θ parametrizes the relative strength of the vacuum expectation values of H and H':

$$\langle H\rangle=v\sin\theta\ , \quad (8a)$$

$$\langle H'\rangle=v\cos\theta\ . \quad (8b)$$

Because of (1) and (8) the quark masses are

$$m_{U_3}=u_3 v|\cos\theta|\ , \quad (9a)$$

$$m_{U_4}=u_4 v|\cos\theta|\ , \quad (9b)$$

$$m_{D_4}=d_4 v|\sin\theta| \quad (9c)$$

TABLE I. SSB windows; the error is always 1 in the last digit.

Yukawa couplings at M_P	$a_0=1,\ b_0=0.8,\ c_0=0.5,\ d_0=0.8$ Case M	Case N	$a_0=3,\ b_0=2,\ c_0=1,\ d_0=1$ Case M	Case N
(a) $u_4 \gg d_4, u_3$ ($\Longleftrightarrow u_3 \gg d_4, u_4$)	0.174–0.362	0.174–0.362	0.226–0.343	0.226–0.343
(b) $d_4 \gg u_3, u_4$	0.172–0.384	0.172–0.384	0.243–0.357	0.243–0.357
(c) $u_4 = d_4 \gg u_3$	0.168–0.172	0.168–0.172	0.178	0.178
(d) $u_3 = u_4 = d_4$	0.111–0.120	0.111–0.120	0.135–0.136	0.135–0.136
(e) $u_3 = u_4 \gg d_4$	0.160	0.110–0.196	No window within the numerical error	0.152–0.189
(f) $u_3 = d_4 \gg u_4$	0.110–0.196	0.160	0.152–0.189	No window within the numerical error

in case N. To get case M one has to interchange cosine and sine in (9b) and (9c).

From Eq. (3) and the initial conditions (iii) one can deduce that $m_{H'}$ changes linearly with $m_{3/2}$. The same is true for m_H and μ. Therefore, according to Eq. (7) it is also true for v, as was anticipated at the end of Sec. II.

IV. RESULTS

First we discuss the features that are independent of whether one considers case N or M. (a) The values of the Yukawa couplings at M_W usually are rather independent from the choice of their initial values at M_P (cf. Fig. 1 of Ref. 4). (b) The point at which the symmetry breaking sets in, is mainly determined by the maximum of these initial values. For initial values above 0.5, S becomes negative already at high energies, so that at M_W the consistency condition $C>0$ is violated. (In such a case one has to calculate one-loop corrections to decide which is the true vacuum.[4,8] We will not pursue this possibility here, but stick to smaller values of the Yukawa couplings, where the symmetry is broken at the tree level.) (c) For initial values below 0.05, S never becomes negative.

The effect b produces an upper limit for the masses of the heavy quarks. If one scans through the $a_0-b_0-c_0-d_0$ parameter space one does not find bigger masses than 200 GeV. The combined effects a and b restrict the values of masses for a fixed set of parameters a_0, b_0, c_0, and d_0. If these are also varied, however, no quantitative predictions are possible anymore. Therefore, we will discuss only the qualitative features of our result. As a characteristic example we may choose $a_0=1$, $b_0=0.8$, $c_0=0.5$, $d_0=0.8$, $m_{3/2}=100$ GeV. We know that cases M and N are equivalent for $u_3\equiv 0$ up to a trivial interchange $\sin\theta \leftrightarrow \cos\theta$. Varying the ratio $U_4(M_P)/d_4(M_P)$ one can accommodate any mass ratio $m(U_4)/m(D_4)$ in both cases. There is no strict proportionality between the two ratios. In fact, the value of θ is always such that it makes

$$\max\left[\frac{m(U_4)}{m(D_4)},\frac{m(D_4)}{m(U_4)}\right]$$

larger than

$$\max\left[\frac{u_4(M_P)}{d_4(M_P)},\frac{d_4(M_P)}{u_4(M_P)}\right];$$

i.e., θ always supports the higher-mass quark. If one switches on a small $u_3(M_P)$, this produces not only a top mass, but also increases $m(U_4)$ and $m(D_4)$ by an amount of the order of the top mass. To examine the differences between the two cases M and N we consider certain specific cases. In case N we think it is an interesting phenomenological possibility that $m(U_4)\geq m(U_3) > m(D_4)$. Therefore we examined the extreme case $u_4(M_P)=u_3(M_P)\equiv c_N$, $d_4\equiv 0$. We found that there is a broad window $(0.12\leq c_N \leq 0.18)$ which allows for spontaneous symmetry breaking. $m(U_3)$ and $m(U_4)$ come out the same and vary in the range $100\leq m(U_{3,4})\leq 125$ GeV (Ref. 9). It is interesting that under the above assumptions only a very tiny spontaneous-symmetry-breaking (SSB) window exists in case M. In fact, for case M the interesting phenomenological possibility is $m(D_4)\geq m(U_3) > m(U_4)$. The associated extreme case is $d_4(M_P)=u_3(M_P)\equiv c_M, u_4\equiv 0$. This time there is a window $(0.12\leq c_M \leq 0.18)$ only in case M, but not in case N. The masses vary in the range $99\leq m(U_3)\leq 125$ GeV, 96 GeV $\leq m(D_4)\leq 121$ GeV (Ref. 9). [The D_4 mass always comes out slightly smaller than the top mass. This is not a significant effect. It can be reversed easily by choosing $d_4(M_P)$ slightly bigger than $u_3(M_P)$.]

In Table I we have listed the width of the SSB window for some interesting cases, among them also cases where M and N do not behave differently. [We know already from Sec. II that the rows (a), (b), and (c) of Table I must be symmetric under exchange of M and N. Now we see that the same is true for the row (d).]

V. CONCLUSIONS

We have examined the effect of a heavy mirror family on supergravity-induced breaking of $SU_2\times U_1$. As compared to a sequential family the role of fourth up- and down-type quarks in the alignment of the vacuum is reversed. Therefore, naively one expects the mass of the fourth up-type quark to be lower than that of the fourth down-type quark. We have discussed some renormalization-group arguments in favor of this expectation.

We have also examined the influence the top quark has on it. In particular, we have elaborated on the possibility that the mass of the top is higher than that of the fourth up-type quark. Our results should not be taken as quantitative predictions, because there are too many unknowns in the game. For instance, we did not discuss the electron of the fourth family, which may or may not have a bigger influence on the symmetry breaking than the top. Also we did not discuss in detail the effect of other values a_0, b_0, c_0, and d_0. We only note in passing that for all of them a similar picture arises. Varying them is only of use if one wants to fit the biggest quark masses to some future experimental value.

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[9]This range remains true even for other values of a_0, b_0, c_0, and d_0, such as, for instance, the supergravity-inspired (Ref. 5) combination $a_0=3$, $b_0=2$, $c_0=d_0=1$. Note that for these values the width of the windows is in general smaller than for those discussed in the main text.