PHOTON–PION CHARGE ASYMMETRY IN e^+e^- REACTIONS: A LABORATORY FOR PERTURBATIVE QCD PHASES

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Received 10 August 1987

The charge asymmetry in the production of a photon and a meson in e^+e^- annihilation is studied in perturbative QCD. This quantity measures the interference of amplitudes governed by different momentum scales. It is thus a powerful tool to probe strong interaction phases at high energy and in the context of Sudakov exponentiation and the chromo Coulomb phase. We find a null result at the lowest non-trivial order off α_s in the entire kinematic region described by perturbative QCD.

1. The applications of QCD to strong interaction physics have gone through a unique evolution. In the first generation of Born term phenomenology, it was sufficient to establish perturbative QCD as having predictive power. However, the strong interactions present many observables which are not straightforward to predict within perturbation theory. Among these, interference effects coming from the coupled real and imaginary terms in the amplitudes provide a basic source of information on the dynamics of quarks and gluons. Although the study of phase shifts, e.g., was a central problem of strong interaction physics at one time, the analytic structure of QCD amplitudes is often put aside as a complication. It seems appropriate to take advantage of physical situations where the relative phases of amplitudes can be measured. Here we will study the charge asymmetry in $e^+e^-\to\gamma\pi^\pm X$ and show that in general the process hinges directly on interference between amplitudes. Moreover, it is possible to arrange the kinematics so that perturbative QCD should apply in regions where all momentum invariants are large. Provided there are more than one distinct subprocesses, there is no a priori reason for strong interaction phases to cancel. Within perturbation theory, destructive interference and coherence are commonplace and one expects a non-zero phase effect in the charge asymmetry. Our surprising conclusion, presented below, provides evidence for a null effect at the lowest non-trivial order of α_s.

2. Color coherence is usually exhibited when the systematic cancellations in high orders of perturbation theory are studied. The famous exponentiation of infrared singular terms, the so-called Sudakov corrections, is completed with certain imaginary parts implied by analyticity. The chromo Coulomb phase (CCP) of the generic form $\exp[\mathrm{i}k\ln\ln(Q^2/\Lambda^2_{\mathrm{QCD}})]$ provides vivid evidence for coherence within perturbative QCD [1]. The CCP occurs because quarks at high energies are never free particles but distort each other's propagation in a nearly eikonal manner. In the case of proton–proton elastic scattering at fixed angle, the CCP interferes to produce an additive term oscillating with energy. This behavior was observed experimentally [2]. In the $\gamma\pi$ charge asymmetry studied here, one can ask whether similar oscillating effects could occur. The amplitude for the production of a real direct photon in e^+e^- is described by two distinct subprocesses shown in fig. 1: lepton emission (a) and quark (b) emission [3]. Neglecting weak interaction effects, the charge asymmetric quantity [4]

$$\Delta\sigma=\mathrm{d}\sigma(e^+e^-\to\gamma\pi^+X)-\mathrm{d}\sigma(e^+e^-\to\gamma\pi^-X) \quad (1)$$

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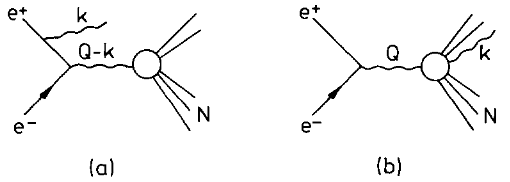

Fig. 1. Photon emission at the (a) hadronic level, (b) leptonic level.

receives contributions only from the interference between the two subprocesses. Taking the difference (1) of course eliminates background due to π^0 decays, etc. More importantly, the asymmetry is by its very nature well suited for exploring the high-energy phase dependence of the quark creation processes. The essential ingredient we need – a kinematical situation with two large scales – is present in the so-called back-to-back region where the photon and the pion have a relative transverse momentum which is small with respect to the total energy $\sqrt{s}$ but large with respect to the scale Λ of QCD. This is a particular case of the back to back jets program

$$e^+e^- \to h_A h_B X$$

which has been analysed in great detail [5].

3. Phase information is lost in many experiments which interfere two short distance amplitudes occurring at the same spacetime point, cancelling phases in a trivial way. The process (1) measures interference between two amplitudes that are, physically speaking

(a) quark creation at a given point (x),

(b) quark creation an adjustable spacetime interval ($x'-x$) away.

This feature is illustrated by contrasting the matrix elements different experiments measure. In $e^+e^- \to h_A + h_B + X$, (h_A, h_B *not* photons), one measures [5]

$$W^{\mu\nu} = \sum_N d^4x \exp(iQx) \langle 0|J^\mu(0)|N; h_A, h_B\rangle \langle N; h_A, h_B|J^\nu(x)|0\rangle$$

$$= \sum_N (2\pi)^4 \langle 0|J^\mu(0)|N; h_A, h_B\rangle \langle N; h_A, h_B|J^\nu(0)|0\rangle (2\pi)^4 \delta^4(p_A + p_B + p_N - Q) , \tag{2}$$

where J^μ is the electromagnetic current of the virtual photon of momentum Q^μ. Usually the simple contractions (e.g., W^μ_μ) are considered, in which the phase dependence in $\langle N; h_A, h_B|J^\mu(0)|0\rangle$ cancels against its complex conjugate.

The quantity (1), in contrast, singles out the interference of matrix elements

$$V^{\mu\nu\lambda} = \sum_N \int d^4x\, d^4x' \exp(iQx - iQ'x') \langle 0|J^\mu(x')|N, h^+\rangle \langle N, h^+|T(J^\lambda(0)J^\nu(x))|0\rangle + \text{c.c.}$$

$$-(h^+ \to h^-) \tag{3}$$

in which the separation of points x and x' is crucial. Phase information is preserved because of an intrinsic mismatch of different scales Q and $Q' \equiv Q-k$. Note that one measures the *difference* of strong interaction phases associated with emission of the real photon from the electron (fig. 1a) or quark beams (fig. 1b). Our analysis indicates that the phase difference, or more precisely the large Q^2 rate of evolution of the phase difference, is a well-defined perturbative problem. In simple language, taking the difference is sufficient to kill logarithmic infrared divergence in perturbation theory.

Let us now outline our strategy for computing the phase effect in the charge asymmetric quantity (1). Being a difference of cross sections, $\Delta\sigma$ will be proportional to the cosine of the difference of phases which are at best of $O(\alpha_s)$. Thus, the phase effect on $\Delta\sigma$ will only appear in $O(\alpha_s^2)$ calculations – i.e., two-loop diagrams.

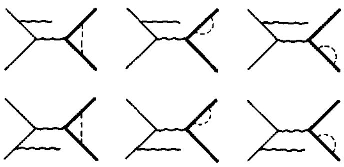

Fig. 2. One-loop diagrams for photon emission off the lepton line.

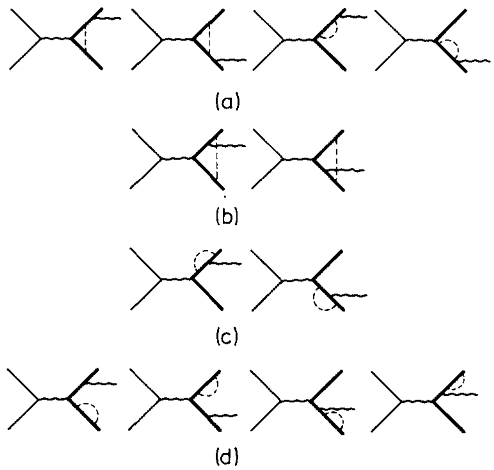

Fig. 3. One-loop diagrams for photon emission off the quark line.

However, an educated guess on the structure of the amplitudes significantly simplifies our problem. We are looking for a chromo Coulomb phase which is an effect of the infrared sensitivity of processes depending on two large scales. We expect the leading-log effects to factorize and exponentiate as

$$\mathcal{M} = \exp(-S) \exp\{i\alpha_s[\ln f(x_\gamma, x_T) + \sigma]\} \mathcal{M}^{(0)} + \text{non leading}, \tag{4}$$

where $\mathcal{M}^{(0)}$ is the Born amplitude, S is an infrared divergent real factor which leads to the usual Sudakov expression at $O(\alpha_s)$, δ is an infrared divergent phase which will be cancelled when a physical quantity is calculated, and x_γ, x_T are ratios of the large momenta as defined in fig. 1, with

$$x_\gamma = 2k_0/Q, \quad x_T = 2k_T/Q.$$

If there were an oscillating CCP effect, it should be sufficient to deduce the coefficients of the exponentiating terms in (4) from the one-loop imaginary parts

$$\text{Im}\,\mathcal{M}_i = \alpha_s \ln f_i(x_\gamma, x_T) \mathcal{M}_i^{(0)}. \tag{5}$$

The leading-log functions f_i have to be calculated separately for the different subprocesses (a), (b). Up to the kinematic factors one would then have

$$\Delta\sigma \propto \text{Re}(\mathcal{M}_e \mathcal{M}_q^*) \propto \mathcal{M}_e^{(0)} \mathcal{M}_q^{(0)} \cos \alpha_s\, [\ln f_a(x_\gamma, x_T) - \ln f_b(x_\gamma, x_T)]. \tag{6}$$

In the difference of the logarithms the infrared (imaginary) divergences have the opportunity to cancel out. Unfortunately, we will find that $f_a = f_b$ at $O(\alpha_s)$, an amazing coincidence between unrelated subprocesses.

If this strategy is *not* sufficient, there is no alternative within perturbative QCD to see the coherence effect. That is because the phase differences, by definition, come in an exponent: they generally require an infinite series of contributions to be summed.

4. Now we present the results of calculating the $O(\alpha_s)$ one-loop corrections to the basic brehmsstrahlung amplitudes $\mathcal{M}_e^{(0)}$ (photon off electron line) and $\mathcal{M}_q^{(0)}$ (photon off quark line). The imaginary parts of these amplitudes allow us to estimate the relative phase between the two emission processes:

(i) Photon off electron line. The result of calculating the one-loop diagrams shown in fig. 2 can be read off from the corresponding results for the one-loop corrections to $e^+e^- \to q\bar{q}$ given e.g. in ref. [6]. It is well-known that the result is ultraviolet (UV) finite, but infrared (IR) and mass (M) divergent if one works with massless quarks as we do. We choose to regularize the IR/M divergencies by dimensional regularization with dimension $n=4-2\epsilon$. One obtains

$$\mathcal{M}_e^{(1)} = C_F \frac{\alpha_s}{2\pi} \frac{1}{\Gamma(1-\epsilon)} \left(\frac{4\pi\mu^2}{-q(1-x_\gamma)}\right)^\epsilon \left(-\frac{1}{\epsilon^2} - \frac{3}{2}\frac{1}{\epsilon} - 4\right) \mathcal{M}_e^{(0)} , \tag{7}$$

where $C_F = \frac{4}{3}$ and q^2 is positive in the time-like region which we study. The imaginary part of (7) can be obtained by the expansion [we take $\ln(-q^2) = \ln q^2 + i\pi$]

$$\left(\frac{4\pi\mu^2}{-q^2(1-x_\gamma)}\right)^\epsilon = \left(\frac{4\pi\mu^2}{q^2}\right)^\epsilon \{1 - \epsilon \ln(1-x_\gamma) + \tfrac{1}{2}\epsilon^2 [\ln(1-x_\gamma) - \pi^2] - i\pi[\epsilon - \epsilon^2 \ln(1-x_\gamma)] + \theta(\epsilon^3)\} \tag{8}$$

leading to

$$\mathrm{Im}\, \mathcal{M}_e^{(1)} = C_F \frac{\alpha_s}{2\pi} \frac{\pi}{\Gamma(1-\epsilon)} \left(\frac{4\pi\mu^2}{q^2}\right)^\epsilon \left(\frac{1}{\epsilon} + \frac{3}{2} - \ln(1-x_\gamma)\right) \mathcal{M}_e^{(0)} . \tag{9}$$

The ϵ^{-2} and ϵ^{-1} singularities in the real part of (7) are cancelled by the corresponding (real) singularities in the squared tree graph contributions according to the Lee–Nauenberg theorem, whereas the ϵ^{-1} singularity in the imaginary part (9) must not show up in observable quantities if the perturbative treatment is self-consistent [#1].

(ii) Photon off the quark line. The $O(\alpha_s)$ one-loop contributions to the amplitude $\mathcal{M}_q$ are drawn in fig. 3. The result is UV finite which can be traced to the various groups of diagrams in fig. 3. Fig. 3a is UV finite due to the Ward identity for the virtual photon. Fig. 3b is UV finite from power counting. The UV infinities in 3c (vertex type) and 3d (quark wave function type) are proportional to the vertex renormalization constants Z_2 and the wave function renormalization constant Z_1 of QED, respectively, since the color factors of all the diagrams in fig. 3 are identical. Since $Z_1 = Z_2$ in QED, the UV singularities in 3c and 3d cancel. Using the results of ref. [7] one obtains

$$\mathcal{M}_q^{(1)} = C_F \frac{\alpha_s}{2\pi} \frac{1}{\Gamma(1-\epsilon)} \left(\frac{4\pi\mu^2}{-q^2}\right)^\epsilon \left(-\frac{1}{\epsilon^2} + \frac{1}{\epsilon}[\ln(1-x_\gamma) - \tfrac{3}{2}]\right) \mathcal{M}_q^{(0)} + \text{finite terms.} \tag{10}$$

The finite-term contributions indicated in (10) are not proportional to the Born term amplitude $\mathcal{M}_q^{(0)}$. Explicit expressions for the finite terms are given in ref. [7] but are of no relevance here. One finds from (10)

$$\mathrm{Im}\, \mathcal{M}_q^{(1)} = C_F \frac{\alpha_s}{2\pi} \frac{\pi}{\Gamma(1-\epsilon)} \left(\frac{4\pi\mu^2}{q^2}\right)^\epsilon \left(\frac{1}{\epsilon} + \frac{3}{2} - \ln(1-x_\gamma)\right) \mathcal{M}_q^{(0)} . \tag{11}$$

Comparing (9) and (11) one finds

$$\mathcal{M}_e^{(0)}\, \mathrm{Im}\, \mathcal{M}_q^{(1)} = \mathcal{M}_q^{(0)}\, \mathrm{Im}\, \mathcal{M}_e^{(1)} , \tag{12}$$

or, equivalently

$$\mathrm{Im}(\mathcal{M}_e \mathcal{M}_q^*) = 0 \quad \text{at } O(\alpha_s) . \tag{13}$$

Thus te interference of amplitude with protons off the electron and quark lines is purely real when calculated to $O(\alpha_s)$. This means that the phase difference in eq. (6) vanishes at $O(\alpha_s)$.

[#1] The ϵ^{-1} singularity in the absorptive part can be easily traced to the t-channel one-gluon pole in the cut diagram where one cuts through the quark and antiquark line.

5. It is not sufficient to invoke traditional logarithmic counting and factorization rules to explain why $\mathrm{Im}(\mathcal{M}_e \mathcal{M}_q^*) = 0$ at $O(\alpha_s)$. Such arguments could be used to relate the $1/\epsilon$ singularities, but not the details of cancellation of the two-scale ratios. That is, even though every scale is time-like not every logarithm is the same: only when all diagrams are summed does it turn out that the quark-emission process has just one interesting log.

Certainly the simple conclusion must break down when new analytic structure is added: finite quark masses would do this. Moreover there is a multitude of possibilities in higher orders, where it is not inconceivable that a separate pattern of next-to-leading exponentiation could occur. Experimental information could teach us whether the lowest-order null effect we calculate really applies when all of the competing physical processes are finally summed.

JPR acknowledges support from DOE Grant No. 85-ER40214.A002.

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