GAUGINO-HIGGSINO MIXING AND SLEPTON-SQUARK PRODUCTION IN ep COLLISIONS

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We consider gaugino-higgsino mixing in the minimal supersymmetric extension of the standard model and determine the mass eigenstates and eigenvalues taking the physical masses of the lightest chargino and neutralino as input in the diagonalization procedure. Using the results we calculate cross sections and charge asymmetries for the associated production of sleptons and squarks in ep collisions and investigate the dependence on the chargino and neutralino properties and on the slepton and squark spectrum. The numerical analysis is performed within the minimal supergravity model as well as in a more general framework. Although predictions are made also for higher energies, the main focus is on the HERA energy range. We indicate the values of sparticle masses accessible to SUSY searches at HERA and illustrate the expected reach beyond the present bounds.

1. Introduction

The associated production of a scalar lepton (\(\tilde{e}\) or \(\tilde{\nu}\)) and a scalar quark (\(\tilde{q}\)) constitutes the most important process in the search for supersymmetry in ep collisions as it provides the clearest signatures and allows to explore the largest mass range of supersymmetric particles. The transitions of the initial electron and quark into sleptons and squarks proceed by \(t\)-channel exchange of gauginos and higgsinos, the spin-\(\frac{1}{2}\) partners of the electroweak gauge bosons and the Higgs bosons, respectively. In the minimal supersymmetric extension of the standard \(SU(3)_c \times SU(2)_L \times U(1)\) theory [1], which contains two Higgs doublets, the charged-current-type processes \(eq \rightarrow \tilde{e}q\) are mediated by wino \(\tilde{W}^\pm\) and charged higgsino \(\tilde{H}_1^-\) and \(\tilde{H}_2^-\) exchange, while the neutral-current-type processes \(eq \rightarrow \tilde{e}q\) involve the neutral \(SU(2)_L\) and \(U(1)\) gauginos \(\tilde{W}_3\) and \(\tilde{B}\) and the neutral higgsino fields \(\tilde{H}_1^0\) and \(\tilde{H}_2^0\).

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The couplings of the above SU(2) \(_L \times U(1)\) eigenstates are fixed by gauge invariance and supersymmetry and are related to the corresponding gauge and Higgs boson couplings. Since it is justified to take \(m_e, m_q \rightarrow 0\) in the case of ep \(\rightarrow \bar{q}qX\), we only have to deal with gaugino couplings later on. Masses can be generated as usual through spontaneous SU(2) \(_L \times U(1)\) breaking. However, in order to agree with observations supersymmetry must also be broken. In view of the missing experimental evidence for supersymmetry it is not surprising that the SUSY breaking mechanism is unclear. A possible dynamical framework is provided by supergravity models [1,2]. These models lead to effective low energy lagrangians with global supersymmetry broken by so-called soft breaking terms originating in gravitational interactions. Together with SU(2) \(_L \times U(1)\) breaking which can also be induced dynamically in such schemes, these terms generate the mass spectrum of the superpartners. As a characteristic feature, the mass eigenstates are mixtures of SU(2) \(_L \times U(1)\) eigenstates with mass eigenvalues and mixing angles depending on a more or less large number of model parameters. This fact renders predictions of SUSY processes somewhat uncertain.

The mixing of the scalar partners \(\tilde{f}_L, \tilde{f}_R\) of the left- and right-handed fermions \(f_L, f_R = \frac{1}{2}(1 - T - 3\gamma_5)f\) is expected [1] in supergravity models to be proportional to the fermion masses. Since only light leptons and quarks play an important role in ep \(\rightarrow \bar{q}qX\), \(\tilde{f}_L - \tilde{f}_R\) mixing will be neglected throughout this paper. Also flavor mixing has no essential influence on our analysis as will become clear later. On the other hand, the mixing of gauginos and higgsinos is typically substantial. The charged mass eigenstates \(\tilde{\chi}_i^\pm (i = 1, 2)\) being mixtures of \(\tilde{W}^- (\tilde{W}^+)\) and \(\tilde{H}_1^\pm (\tilde{H}_2^\pm)\) are called charginos, while the neutral mass eigenstates \(\tilde{\chi}^0_i (i = 1, 2, 3, 4)\) being mixtures of \(\tilde{B}, \tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0\) are called neutralinos. It is the main purpose of this paper to study chargino and neutralino spectra in the mass range accessible at HERA, to investigate the model-dependence of the cross sections for ep \(\rightarrow \bar{q}qX\), and to estimate the range of slepton and squark masses and other model parameters which can be explored at HERA energies.

Already some time ago, Jones and Llewellyn Smith [3] calculated cross sections for ep \(\rightarrow \bar{q}qX\) considering wino \((\tilde{W}^\pm)\), zino \((\tilde{Z})\) and photino \((\tilde{\gamma})\) exchange in the absence of gaugino-higgsino mixing. The above gauginos are the superpartners of the W and Z bosons and the photon, respectively, as it is evident from their definition

\[
\tilde{W}^\pm = \sqrt{\frac{1}{2}} (\tilde{W}^1 \pm i \tilde{W}^2),
\]

\[
\tilde{Z} = \cos \theta_w \tilde{W}^3 - \sin \theta_w \tilde{B},
\]

\[
\tilde{\gamma} = \sin \theta_w \tilde{W}^3 + \cos \theta_w \tilde{B}
\]

in terms of the SU(2) \(_L\) gaugino fields \(\tilde{W}^i (i = 1, 2, 3)\) and the U(1) gaugino \(\tilde{B}\) with \(\theta_w\) being the Weinberg angle. Although one can approximate this case by diagonal-
izing the gaugino/higgsino mass matrices under rather special assumptions, it does not occur as a typical solution and not for the mass eigenvalues $m_{\tilde{w}} \approx m_W$, $m_{\tilde{Z}} \approx m_Z$ and $m_{\tilde{\gamma}} = 0$ assumed in ref. [3], at least not in the minimal model as will be seen. Gaugino-higgsino mixing was later included by Harrison [4] who emphasized the considerable sensitivity of the production cross sections for $ep \rightarrow \tilde{q}X$ to mixing and demonstrated this point for a few special cases. Our aim is a more comprehensive study which improves, extends and corrects the earlier analyses in several respects:

(i) Instead of diagonalizing the gaugino/higgsino mass matrices for given values of the various mass parameters which appear in the effective lagrangian, we use the desired mass values of the lightest neutralino and chargino eigenstates as input in the diagonalization problem and determine the gaugino/higgsino composition of the eigenstates and the heavier masses from these directly observable parameters under assumptions on the remaining parameters which are suggested by renormalization group considerations within supergravity models and which will be specified later. The phenomenological advantages of such a procedure are quite obvious.

(ii) We investigate the total $ep \rightarrow \tilde{q}X$ cross sections for many interesting chargino and neutralino solutions and clarify the dependence of the cross sections on slepton and squark masses and on the chargino and neutralino spectra. This analysis is performed in a more general framework with basically free scalar masses as well as for sparticle spectra obeying the renormalization group mass relations [5] of the minimal supergravity model.

(iii) We indicate the region of the parameter space of the supergravity model which can be probed at HERA in comparison to the already existing experimental bounds.

(iv) Charge asymmetries derived from the cross sections for $e^+p \rightarrow \tilde{q}X$ are also investigated. These asymmetries provide particularly sensitive tests of the chargino and neutralino properties.

(v) We have recalculated the cross sections tabulated in ref. [4] using essentially the same values of parameters. In most cases we reproduce the results within a few percent and in some cases within 30%. However, in a few cases we disagree by a factor 2 and more.

The paper is organized as follows. Sect. 2 deals with gaugino/higgsino mixing and the determination of chargino and neutralino eigenstates and masses. In sect. 3 we summarize analytical formulas which are useful for cross section calculations. Our numerical results are presented and discussed in sect. 4. Sect. 5, finally, contains some concluding remarks.

2. Gaugino-higgsino mixing

In the minimal supersymmetric $\text{SU}(2)_L \times \text{U}(1)$ model [1,2], two $\text{SU}(2)_L$ Higgs doublets $H_1 = (H_1^0, H_1^-)$ and $H_2 = (H_2^+, H_2^0)$ exist with opposite $\text{U}(1)$ hypercharge.
The neutral Higgs fields $H_{i,2}^0$ acquire non-vanishing vacuum expectation values $v_{1,2}$ which break $SU(2)_L \times U(1)$ to $U(1)_{em}$. The higgsino superpartners $\tilde{H}_1 = (\tilde{H}_1^+, \tilde{H}_1^-)$ and $\tilde{H}_2 = (\tilde{H}_2^+, \tilde{H}_2^-)$ of the Higgs-doublets mix with the $SU(2)_L \times U(1)$ gauginos $\tilde{W}^i$ ($i = 1, 2, 3$) and $\tilde{B}$ through the spontaneous breakdown of $SU(2)_L \times U(1)$. Going from the two-component spinor notation tacitly assumed above to a four-component notation, one may choose the Dirac spinors

$$\tilde{\psi}_{C1} = \begin{pmatrix} \tilde{W}^- \\ \tilde{W}^+ \end{pmatrix}, \quad \tilde{\psi}_{C2} = \begin{pmatrix} \tilde{H}^-_1 \\ \tilde{H}^+_2 \end{pmatrix}$$

and the Majorana spinors

$$\tilde{\psi}_{N1} = \begin{pmatrix} \tilde{\gamma} \\ \tilde{\gamma} \end{pmatrix}, \quad \tilde{\psi}_{N2} = \begin{pmatrix} \tilde{Z} \\ \tilde{Z} \end{pmatrix}, \quad \tilde{\psi}_{N3} = \begin{pmatrix} \tilde{H} \\ \tilde{H} \end{pmatrix}, \quad \tilde{\psi}_{N4} = \begin{pmatrix} \tilde{H}' \\ \tilde{H}' \end{pmatrix}$$

as the basis of the gaugino-higgsino system, where $\tilde{W}^\pm$, $\tilde{Z}$ and $\tilde{\gamma}$ are defined in eq. (1) and

$$\tilde{H} = \cos \theta_v \tilde{H}_1^0 - \sin \theta_v \tilde{H}_2^0,$$

$$\tilde{H}' = \sin \theta_v \tilde{H}_1^0 + \cos \theta_v \tilde{H}_2^0$$

with

$$\tan \theta_v = \frac{v_2}{v_1}. \quad (5)$$

The most general gaugino-higgsino mass lagrangian [1] of the effective theory can then be written as follows:

$$\mathcal{L}_m = -\tilde{\psi}_{C1}^C \left( M_{ij}^C P_L + M_{ij}^C P_R \right) \tilde{\psi}_{Cj} - \frac{1}{2} \tilde{\psi}_{Ni} \left( M_{ij}^N P_L + M_{ij}^N P_R \right) \tilde{\psi}_{Nj}. \quad (6)$$

Here, $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$ are the chiral projection operators and the non-diagonal mass matrices $M_{ij}^C$ and $M_{ij}^N$ read

$$M^C = \begin{pmatrix} M_2 & i\sqrt{2} m_w \cos \theta_v \\ i\sqrt{2} m_w \sin \theta_v & -\mu \end{pmatrix} \quad (7)$$

and

$$M^N = \begin{pmatrix} M_{11} & M_{12} & 0 & 0 \\ M_{12} & M_{22} & i m_Z & 0 \\ 0 & im_Z & -\mu \sin 2\theta_v & \mu \cos 2\theta_v \\ 0 & 0 & \mu \cos 2\theta_v & \mu \sin 2\theta_v \end{pmatrix}. \quad (8)$$
The generally complex mass parameters $M_1$, $M_2$ and $\mu$ in the above are soft SUSY breaking parameters associated with the U(1) and SU(2)$_L$ gauginos and with the higgsinos, respectively, while $m_w$ and $m_z$ denote the physical W and Z boson masses.

The mass matrices (7) and (8) can be diagonalized by unitary matrices $U_L$, $U_R$ and $U_N$,

$$
(U_R^T M^C U_L)_{ij} = m_{C_i} \delta_{ij},
$$

$$
(U_N^T M^N U_N)_{ij} = m_{N_i} \delta_{ij},
$$

with the (positive) eigenvalues $m_{C_i}$ and $m_{N_i}$ being the masses of the chargino ($\tilde{\chi}_{C_i}$, $i = 1, 2$) and neutralino ($\tilde{\chi}_{N_i}$, $i = 1, 2, 3, 4$) mass eigenstates. These states are obtained from the basis defined in eqs. (2) and (3) by the transformations

$$
P_L \tilde{\chi}_C = U_L^T \tilde{\psi}_C, \quad P_R \tilde{\chi}_C = U_R^T \tilde{\psi}_C,
$$

$$
P_L \tilde{\chi}_N = U_N^T \tilde{\psi}_N, \quad P_R \tilde{\chi}_N = U_N^T \tilde{\psi}_N.
$$

The mass lagrangian (6) then takes the simple form

$$
\mathcal{L}_m = -m_{C_i} \tilde{\chi}_{C_i} \tilde{\chi}_{C_i} - \frac{1}{2} m_{N_i} \tilde{\chi}_{N_i} \tilde{\chi}_{N_i}.
$$

In order to determine the physical chargino and neutralino states and their masses one must know $M_1$, $M_2$, $\mu$ and $\theta_w$, in other words, the model must be further specified. As reasonable assumptions, we shall adopt the following three constraints:

(a) $\cos 2\theta_w = 0$,

(b) $3M_1 \cos^2 \theta_w = 5M_2 \sin^2 \theta_w$,

(c) $M_1$, $M_2$ and $\mu$ real.

Assumption (a) is suggested by a renormalization group analysis of a class of supergravity models [2, 5] for a top quark mass $m_t \approx 50$ GeV. Small deviations of $\cos 2\theta_w$ from zero do not alter our result significantly except in a small region of the
parameter space where one has a mass degeneracy. Assumption (b) applies if $M_1$ and $M_2$ are evolved according to the renormalization group [5] from equal values $M_1 = M_2 = m_{1/2}$ at a grand unification scale $M_X$ down to energies of O(1 TeV). In that case,

$$\frac{1}{g_x^2} m_{1/2} = \frac{1}{g^2} M_2 = \frac{3}{5 g'^2} M_1,$$

(16)

where $g_X$ is the unified gauge coupling at $M_X$ and $g$ and $g'$ are the usual effective SU(2)$_L$ and U(1) gauge couplings, respectively, with $g'/g = \tan \theta_W$. Finally, assumption (c) is employed merely for reasons of simplicity. Adding the specifications (15) to the model, one has to deal with only two unknown real parameters, $M_2$ and $\mu$.

Instead of choosing certain values for $M_2$ and $\mu$ and deriving the corresponding chargino and neutralino spectrum, we want to proceed in the opposite way and use two physical chargino and neutralino masses as input in the diagonalization problem. Substituting first assumption (15a) in the neutral mass matrix $M_N$, eq. (8), one immediately sees that the higgsino $\tilde{\psi}_{N4}$ defined in eqs. (3) and (4) does not mix with the other neutral fields $\tilde{\psi}_{Ni}$; $i = 1, 2, 3$. Hence, according to eqs. (11) and (13), the fourth neutralino eigenstate $\tilde{\chi}_{N4}$ remains a pure higgsino, $\tilde{\chi}_{N4} = \tilde{\psi}_{N4}$, with mass $m_{N4} = |\mu|$, while the other eigenstates $\tilde{\chi}_{Ni}$; $i = 1, 2, 3$ and their masses $m_{Ni}$ are found by diagonalizing the appropriate $3 \times 3$ submatrix of eq. (8). Ordering $\tilde{\chi}_{Ni}$; $i = 1, 2, 3$ and $\tilde{\chi}_{C1}$, $i = 1, 2$ such that $m_{N1} \leq m_{N2} \leq m_{N3}$ and $m_{C1} \leq m_{C2}$, we choose suitable values for $m_{N1}$ and $m_{C1}$ and solve the eigenvalue equations.

In table 1 we list solutions for a range of values of $m_{N1}$ and $m_{C1}$ which is of particular interest from the point of view of future searches for electroweak SUSY signals, that is the mass range up to O(1 TeV). Also quoted in table 1 are the values of $M_2$ (with the convention $M_2 \geq 0$) and $\mu$ associated with a given solution. The following features are noteworthy:

(i) For a fixed value of $m_{N1}$ solutions exist for $m_{C1}$ approximately in the range $m_{N1} \leq m_{C1} \leq \sqrt{4m_{N1}^2 + m_w^2}$.

(ii) For $0 = m_{N1} < m_{C1}$ there are two solutions which differ in $M_2$ and $\mu$ as well as in the other neutralino and chargino masses and in the composition of the mass eigenstates. On the other hand, for $m_{N1} = m_{C1}$ only one solution yields finite mass values for all states, while for $m_{N1} = 0$, $m_{C1} \leq m_w$ one has unique solutions.

(iii) The lightest state ($\tilde{\chi}_{N1}$ by definition) of the neutralinos $\tilde{\chi}_{Ni}$; $i = 1, 2, 3$ tends to be dominantly a photino or it normally contains at least a large photino component.

(iv) The approximate photino $\tilde{\chi}_{N1}$ is in most cases also lighter than the pure higgsino $\tilde{\chi}_{N4}$, i.e. $m_{N1} < m_{N4}$, except in the degenerate case $m_{N1} = m_{C1}$ where $m_{N4} < m_{N1}$.

(v) The mixing of zino and higgsino components in $\tilde{\chi}_{N2}$ and $\tilde{\chi}_{N3}$ is similar to the mixing of wino and higgsino components in $\tilde{\chi}_{C1}$ and $\tilde{\chi}_{C2}$, respectively. This similarity is also reflected in the mass pattern $m_{N2} \approx m_{C1}$ and $m_{N3} \approx m_{C2}$.
Table 1
Masses and eigenstates of neutralinos and charginos and the corresponding values of the SUSY breaking parameters $M_2$ and $\mu$ (masses in GeV).

<table>
<thead>
<tr>
<th>Neutralinos</th>
<th>Charginos</th>
<th>SUSY breaking</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{N_1} (U_{N_{11}}, U_{N_{21}}, U_{N_{31}})$</td>
<td>$m_{N_2} (U_{N_{12}}, U_{N_{22}}, U_{N_{32}})$</td>
<td>$m_{N_3} (U_{N_{13}}, U_{N_{23}}, U_{N_{33}})$</td>
</tr>
<tr>
<td>0(1.000, 0.0, 0.0)</td>
<td>38(0.000, -0.927, 0.375)</td>
<td>230(0.000, 0.375, -0.927i)</td>
</tr>
<tr>
<td>0(1.000, 0.0, 0.0)</td>
<td>60(0.000, 0.839, -0.544i)</td>
<td>143(0.000, -0.544i, 0.839)</td>
</tr>
<tr>
<td>0(1.000, 0.0, 0.0)</td>
<td>91(0.000, 0.713, -0.701i)</td>
<td>95(0.000, -0.701i, 0.713)</td>
</tr>
<tr>
<td>20(0.103, -0.277, -0.955i)</td>
<td>221(0.895, -0.393, 0.210i)</td>
<td>441(0.434, 0.877, -0.207i)</td>
</tr>
<tr>
<td>20(0.996, 0.047, 0.077i)</td>
<td>44(0.084i, -0.790i, 0.607)</td>
<td>149(0.032, 0.11, -0.791i)</td>
</tr>
<tr>
<td>20(0.972, -0.231, 0.050i)</td>
<td>52(0.237, 0.952, -0.192i)</td>
<td>429(0.003i, -0.198i, 0.980)</td>
</tr>
<tr>
<td>20(0.997, -0.099, 0.074i)</td>
<td>64(0.058i, -0.711i, 0.701)</td>
<td>123(0.047, 0.703, -0.709i)</td>
</tr>
<tr>
<td>20(0.996, -0.061, 0.069i)</td>
<td>88(0.089, 0.847, -0.524i)</td>
<td>120(0.026i, -0.528i, 0.849)</td>
</tr>
<tr>
<td>20(0.997, -0.032, 0.072i)</td>
<td>92(0.038i, -0.608i, 0.793)</td>
<td>101(0.069, 0.793, -0.605i)</td>
</tr>
<tr>
<td>50(0.982, -0.089, 0.164i)</td>
<td>64(0.087i, -0.563i, 0.822)</td>
<td>140(0.166, 0.822, -0.545i)</td>
</tr>
<tr>
<td>50(0.374, -0.635, -0.676i)</td>
<td>123(0.898, 0.066, 0.436i)</td>
<td>258(0.232, 0.770, -0.595i)</td>
</tr>
<tr>
<td>50(0.978, -0.145, 0.150i)</td>
<td>92(0.061i, -0.484i, 0.870)</td>
<td>131(0.200, 0.860, -0.469i)</td>
</tr>
<tr>
<td>50(0.872, -0.490, -0.008i)</td>
<td>100(0.490, 0.872, 0.014i)</td>
<td>6103(0.000, 0.015, -1.000i)</td>
</tr>
<tr>
<td>50(0.974, -0.175, 0.142i)</td>
<td>111(0.050i, -0.447i, 0.893)</td>
<td>127(0.219, 0.877, -0.427i)</td>
</tr>
<tr>
<td>100(0.931, -0.323, 0.170i)</td>
<td>109(0.058i, -0.329i, 0.942)</td>
<td>210(0.361, 0.887, -0.288i)</td>
</tr>
<tr>
<td>100(0.582, -0.624, -0.521i)</td>
<td>175(0.775, 0.231, 0.589i)</td>
<td>324(0.247, 0.746, -0.618i)</td>
</tr>
<tr>
<td>100(0.924, -0.354, 0.143i)</td>
<td>157(0.041i, -0.281i, 0.959)</td>
<td>209(0.379, 0.892, -0.245i)</td>
</tr>
<tr>
<td>100(0.872, -0.489, -0.014i)</td>
<td>200(0.490, 0.872, 0.026i)</td>
<td>3317(0.000, 0.030, -1.000i)</td>
</tr>
<tr>
<td>100(0.919, -0.375, 0.123i)</td>
<td>206(0.030i, -0.244i, 0.969)</td>
<td>208(0.393, 0.894, -0.213i)</td>
</tr>
</tbody>
</table>
The mass values $m_{N_1}$ and $m_{C_1}$ and the constraints eq. (15) are used as input in the diagonalization of the mass matrices $M^C$ and $M^N$ given in eqs. (7) and (8). The neutralino eigenstates $\tilde{\chi}_{N_i}$, $i=1,\ldots,4$ are characterized by the vectors $(U_{N_1}, U_{N_2}, U_{N_3}, U_{N_4})$ where the elements $U_{N_{j,i}}$, $j=1,\ldots,4$ of the diagonalization matrix in eq. (11) describe the $\tilde{\nu}$, $\tilde{Z}$, $\tilde{H}$ and $\tilde{H}^*$ admixture, respectively. The vanishing components $U_{N_{4,i}}$, $i=1,2,3$ and the unmixed state $\tilde{\chi}_{N_4}$ which has the mass $m_{N_4} = |\mu|$ are not shown. Similarly, the chargino eigenstates $\tilde{\chi}_{C_i}$, $i=1,2$ are given by the vectors $(U_{L_1}, U_{L_2})$ where the elements $U_{L_{j,i}} = U_{R_{j,i}}$, $j=1,2$ of the diagonalization matrices in eq. (10) refer to the $\tilde{W}^\pm$ and $\tilde{H}^\pm$ components, respectively. The state $\tilde{\chi}_{C_2}$ not shown can be obtained from $\tilde{\chi}_{C_1}$ as follows: $U_{L_{1,2}} = -i e U_{L_{2,1}} |U_{L_{1,1}}|/U_{L_{1,1}}$ and $U_{L_{2,2}} = -i e |U_{L_{1,1}}|$ with $e = 1$ for $M_2 + \mu > 0$ and $e = i$ for $M_2 + \mu < 0$. 

<table>
<thead>
<tr>
<th>$m_{N_1}$ ($U_{N_{11}}, U_{N_{21}}, U_{N_{31}}$)</th>
<th>$m_{N_2}$ ($U_{N_{12}}, U_{N_{22}}, U_{N_{32}}$)</th>
<th>$m_{N_3}$ ($U_{N_{13}}, U_{N_{23}}, U_{N_{33}}$)</th>
<th>$m_{C_1}$ ($U_{L_{11}}, U_{L_{12}}$)</th>
<th>$m_{C_2}$</th>
<th>$M_2$</th>
<th>$\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>200(0.893, -0.437, 0.105)</td>
<td>205(0.032i, -0.171i, 0.985)</td>
<td>404(0.449, 0.883, -0.139i)</td>
<td>200(0.136i, -0.991)</td>
<td>404</td>
<td>392.3</td>
<td>-188.8</td>
</tr>
<tr>
<td>200(0.760, -0.552, -0.343)</td>
<td>314(0.561, 0.292, 0.775i)</td>
<td>486(0.327, 0.782, -0.531i)</td>
<td>300(0.518, 0.855i)</td>
<td>484</td>
<td>434.7</td>
<td>349.4</td>
</tr>
<tr>
<td>200(0.891, -0.446, 0.085)</td>
<td>304(0.022i, -0.144i, 0.989)</td>
<td>404(0.454, 0.883, -0.118i)</td>
<td>300(0.117i, -0.993)</td>
<td>404</td>
<td>394.1</td>
<td>-290.4</td>
</tr>
<tr>
<td>200(0.873, -0.488, -0.023i)</td>
<td>400(0.488, 0.871, 0.047i)</td>
<td>2139(0.002, 0.052, -0.999i)</td>
<td>400(0.999, 0.047i)</td>
<td>2138</td>
<td>403.8</td>
<td>2133.9</td>
</tr>
<tr>
<td>200(0.889, -0.452, 0.071i)</td>
<td>403(0.016i, -0.124i, 0.992)</td>
<td>404(0.457, 0.883, -0.103i)</td>
<td>400(0.102i, -0.995)</td>
<td>404</td>
<td>395.4</td>
<td>-391.6</td>
</tr>
<tr>
<td>400(0.882, -0.469, 0.055i)</td>
<td>403(0.016i, -0.086i, 0.996)</td>
<td>804(0.472, 0.879, -0.068i)</td>
<td>400(0.068i, -0.998)</td>
<td>804</td>
<td>798.6</td>
<td>-394.4</td>
</tr>
<tr>
<td>400(0.840, -0.504, -0.204i)</td>
<td>609(0.347, 0.209, 0.914i)</td>
<td>853(0.418, 0.838, -0.350i)</td>
<td>600(0.345, 0.939i)</td>
<td>852</td>
<td>822.1</td>
<td>630.0</td>
</tr>
<tr>
<td>400(0.881, -0.471, 0.044i)</td>
<td>620(0.011i, -0.072i, 0.997)</td>
<td>804(0.473, 0.879, -0.058i)</td>
<td>600(0.058i, -0.998)</td>
<td>804</td>
<td>799.5</td>
<td>-595.2</td>
</tr>
<tr>
<td>400(0.874, -0.485, -0.031i)</td>
<td>800(0.486, 0.871, 0.076i)</td>
<td>1877(0.010, 0.081, -0.997i)</td>
<td>800(0.997, 0.076i)</td>
<td>1875</td>
<td>806.2</td>
<td>1869.2</td>
</tr>
<tr>
<td>400(0.880, -0.473, 0.037i)</td>
<td>802(0.008i, -0.062i, 0.998)</td>
<td>804(0.474, 0.879, -0.051i)</td>
<td>800(0.051i, -0.999)</td>
<td>804</td>
<td>800.2</td>
<td>-795.8</td>
</tr>
<tr>
<td>800(0.879, -0.477, 0.028i)</td>
<td>801(0.008i, -0.043i, 0.999)</td>
<td>1607(0.478, 0.878, -0.034i)</td>
<td>800(0.034i, -0.999)</td>
<td>1607</td>
<td>1604.5</td>
<td>-797.2</td>
</tr>
<tr>
<td>800(0.867, -0.486, -0.109i)</td>
<td>1205(0.188, 0.117, 0.975i)</td>
<td>1633(0.461, 0.866, -0.193i)</td>
<td>1200(0.192, 0.981i)</td>
<td>1633</td>
<td>1616.8</td>
<td>1216.0</td>
</tr>
<tr>
<td>800(0.878, -0.477, 0.022i)</td>
<td>1201(0.006i, -0.036i, 0.999)</td>
<td>1607(0.478, 0.878, -0.029i)</td>
<td>1200(0.029i, -1.000)</td>
<td>1607</td>
<td>1605.0</td>
<td>-1197.6</td>
</tr>
<tr>
<td>800(0.876, -0.482, -0.030i)</td>
<td>1600(0.482, 0.868, 0.117i)</td>
<td>2301(0.030, 0.117, -0.993i)</td>
<td>1600(0.993, 0.117i)</td>
<td>2300</td>
<td>1609.7</td>
<td>2289.9</td>
</tr>
<tr>
<td>800(0.878, -0.478, 0.019i)</td>
<td>1601(0.004i, -0.031i, 1.000)</td>
<td>1607(0.478, 0.878, -0.025i)</td>
<td>1600(0.025i, -1.000)</td>
<td>1607</td>
<td>1605.3</td>
<td>-1597.9</td>
</tr>
</tbody>
</table>

\[414\]
One can easily understand these properties by considering the limit $\sin^2 \theta_W \to 0$. In this limit, the photino $\tilde{\chi}^0_{N1}$ does not get mixed with the other neutral fields, i.e. $\tilde{\chi}^0_{N1} \equiv \tilde{\psi}^0_{N1}$, and the mass matrix of $\tilde{\psi}^0_{N2}$ and $\tilde{\psi}^0_{N3}$ becomes identical to the chargino mass matrix as can be seen from eqs. (7) and (8). Hence, the neutralino masses assume the values

$$m_{N1} = M_1, \quad m_{N2} = m_{C1}, \quad m_{N3} = m_{C2}, \quad m_{N4} = |\mu|,$$  \hspace{1cm} (17)

while the chargino masses are given by

$$m_{C1,C2} = \frac{1}{2} \left[ M_2 + \mu \pm \sqrt{(M_2 - \mu)^2 + 4m_W^2} \right]$$  \hspace{1cm} (18)

with the convention $m_{C1} \ll m_{C2}$. Furthermore, we note the approximate numerical relation

$$M_1 = \frac{1}{2} \tan^2 \theta_W \cdot M_2 = \frac{1}{2} M_2.$$

At small values of $m_{N1}$, the mass spectrum (17) is considerably distorted due to effects of the non-vanishing Weinberg angle $\sin^2 \theta_W = 0.23$. However, already for $m_{N1} \gtrsim 100$ GeV eq. (17) becomes a very good approximation. Similarly, one expects the neutralino and chargino eigenstates to approach the limits

$$\tilde{\chi}^0_{N1} \to \tilde{\chi}^0_{N1} \equiv \cos \theta_W \tilde{\chi}^0 - \sin \theta_W \tilde{\chi}^0,$$

$$\tilde{\chi}^0_{N2} \to \tilde{\chi}^0_{N2} \text{ or } \tilde{\psi}^0 = \sin \theta_W \tilde{\psi} + \cos \theta_W \tilde{\gamma},$$

$$\tilde{\chi}^0_{N3} \to \tilde{\psi}^0 \text{ or } \tilde{\psi} = \sqrt{1/2} (\tilde{H}^0_1 - \tilde{H}^0_2),$$

$$\left(\tilde{\chi}^0_{N4} = \tilde{\chi}^0_{N4} \equiv \sqrt{1/2} (\tilde{H}^0_1 + \tilde{H}^0_2)\right),$$

and

$$\tilde{\chi}^0_{C1} \to \tilde{\chi}^0_{C1,2} \text{ or } \tilde{\psi}^0, $$

$$\tilde{\chi}^0_{C2} \to \tilde{\psi}^0 \text{ or } \tilde{H}^0_{1,2},$$

respectively. This is obviously the case for the numerical solutions given in table 1.

3. Calculation of cross sections

Our next task is to derive cross-section formulas for slepton-squark production in ep collisions which take into account gaugino-higgsino mixing. To recapitulate, the couplings of the SU(2)$_L \times$ U(1) gauginos to fermion-sfermion pairs are related to the standard gauge boson-fermion couplings, while the higgsino-fermion-sfermion interactions are of Yukawa type similar to the familiar couplings of Higgs bosons to fermions. Since the latter are proportional to lepton and quark masses, higgsino
couplings can safely be neglected in eq. \( \bar{\tilde{t}} q \), the elementary processes we are dealing with. Then, writing the gaugino-fermion-sfermion lagrangian [1] of the effective theory in terms of the chargino and neutralino mass eigenstates defined in eqs. (12) and (13), one gets

\[
\mathcal{L}_{\text{int}} = -i \left( \eta^C_{uL} \right)_i \bar{f}_L^* \tilde{X}_C^* u_L - i \left( \eta^C_{dL} \right)_i \bar{u}_L^* \tilde{X}_C^* d_L \\
- i \left( \eta^N_{\tilde{f}_L} \right)_i \bar{f}_L^* \tilde{X}_N^* f_L - i \left( \eta^N_{\tilde{f}_R} \right)_i \bar{f}_R^* \tilde{X}_N^* f_R + \text{h.c.},
\]

with

\[
\left( \eta^C_{uL} \right)_i = \frac{e}{\sin \theta_w} (U_L)_{1i},
\]

\[
\left( \eta^C_{dL} \right)_i = \frac{e}{\sin \theta_w} (U_R^*)_{1i},
\]

\[
\left( \eta^N_{\tilde{f}_L} \right)_i = \sqrt{2} e \left[ Q_l (U_N)_{1i} + \frac{T_{3f} - Q_l \sin^2 \theta_w}{\sin \theta_w \cos \theta_w} (U_N)_{2i} \right],
\]

\[
\left( \eta^N_{\tilde{f}_R} \right)_i = \sqrt{2} e Q_l \left[ (U_N^*)_{1i} - \tan \theta_w (U_N^*)_{2i} \right].
\]

In the above, \( u \) and \( d \) denote the up-type fermions (\( \nu, u, \nu, c \); etc.) and the down-type fermions (\( e, d, \mu, s \); etc.), respectively, while \( f \) refers to both \( u \) and \( d \). The scalar partners are correspondingly denoted by \( \tilde{u}, \tilde{d} \) and \( \tilde{f} \). Summation over lepton and quark flavors is implied in eq. (22) and is obvious. Furthermore, the subscripts \( L \) and \( R \) mark the left- and right-handed fermion components \( f_{L,R} = \frac{1}{2}(1 \mp \gamma_5)f \) and their superpartners \( \tilde{f}_{L,R} \), whereas the labels \( C \) and \( N \) distinguish chargino and neutralino quantities. The field \( \tilde{\chi}^c \) is the charge conjugate of \( \tilde{\chi} \). Finally, the effective couplings given in eqs. (23) and (24) involve the electromagnetic coupling constant \( e \), the electromagnetic charges \( Q_l \) (with the convention \( Q_e = -1 \)), the third components of the weak isospin \( T_{3f} \) and elements of the diagonalization matrices \( U_L, U_R \) and \( U_N \) defined in eqs. (10)-(13).

More specifically, \( (U_L)_{1i} \) and \( (U_R)_{1i} \) in eq. (23) characterize the wino admixture in the chargino eigenstates \( \tilde{X}_C^*; \ i = 1, 2 \), while \( (U_N)_{1i} \) and \( (U_N)_{2i} \) in eq. (24) describe the photino and zino components in the neutralino eigenstates \( \tilde{X}_N^*; \ i = 1, 2, 3, 4 \), respectively. Since higgsino Yukawa couplings are neglected, the elements \( (U_L)_{2i} \), \( (U_R)_{2i} \) and \( (U_N)_{3i} \), \( (U_N)_{4i} \) associated with the higgsino admixtures do not enter eqs. (23) and (24). Further simplifications arise from assumption (15a) which implies \( U_R = U_L^* \) and \( (U_N)_{4i} = 0 \) for \( i = 1, 2, 3 \) reflecting the fact that \( \tilde{X}_{N4} \) is a pure higgsino. For reasons pointed out in the introduction we disregard in eq. (22) the possibility of mixing among the scalar partners \( \tilde{f}_L \) and \( \tilde{f}_R \) of the left- and right-handed
helicity components of a fermion field \( f \). Also flavor mixing is suppressed in eq. (22) since it is irrelevant for suitably defined total cross sections because of unitarity if the scalar quarks are mass degenerate. Such a degeneracy is approximately expected in supergravity models \([2, 5]\) and shall be assumed throughout this paper. Scalar top may be an exceptional case which, however, would not influence our numerical analysis of eq \( \ell \bar{q}X \) significantly.

With the effective lagrangian (22) it is rather straightforward to compute the differential cross sections for eq \( \ell \bar{q} \) according to the diagrams sketched in fig. 1. For an incident electron and quark with the same helicity \( a = L \) or \( R \), one obtains \([3]\)

\[
\frac{d\sigma}{d\hat{t}}(e^-_aq_a \rightarrow \ell \bar{q}_a) = \frac{1}{16\pi\hat{s}} \left| \sum_i (\eta_{e_a})_i (\eta_{q_a})_i \frac{1}{\hat{t} - m_i^2} \right|^2
\]

\[
= \frac{d\sigma^{aa}(e^-q \rightarrow \ell\bar{q})}{d\hat{t}},
\]

whereas for an incident electron and quark with opposite helicities \( a = L \), \( b = R \) or vice versa, one finds \([3]\)

\[
\frac{d\sigma}{d\hat{t}}(e^-_aq_b \rightarrow \ell \bar{q}_b) = \frac{1}{16\pi\hat{s}^2} \left| \sum_i (\eta_{e_a})_i (\eta_{q_b})_i \right|^2
\]

\[
\times \left[ -\hat{s} - (m^2_{\ell} - \hat{t})(m^2_{\bar{q}} - \hat{t}) \right]
\]

\[
= \frac{d\sigma^{ab}(e^-q \rightarrow \ell\bar{q})}{d\hat{t}}.
\]

The scattering variables are defined as usual by

\[
\hat{s} = (p_e + p_q)^2, \quad \hat{t} = (p_e - p_\ell)^2, \quad \hat{u} = (p_e - p_q)^2,
\]

(27)
with \( \hat{s} + \hat{t} + \hat{u} = m_\tilde{\tau}^2 + m_\tilde{q}^2 \), \( m_\tilde{\tau} \) and \( m_\tilde{q} \) being the appropriate slepton and squark masses and \( p \) denoting the appropriate particle four-momenta. In the case of the chargino exchange process \( e^- u \rightarrow \tilde{\tau} \tilde{d} \) (fig. 1a), one substitutes the chargino masses \( m_{\tilde{c}_i} \) for \( m_i \) and uses the fact that \( (\eta_{c_i}^C)_{i} = 0 \). Similarly, in the case of the neutralino exchange processes \( e^- q \rightarrow \tilde{\chi} \tilde{q} \) (fig. 1b) the appropriate substitutions are \( m_i = m_{N_i} \) and \( (\eta_{L_i}^{N})_{i} = (\eta_{R_i}^{N})_{i} \), where eq. (24) is to be used. Furthermore, the polarized differential cross sections for the processes

\[
\begin{align*}
  e^{-} q_b \rightarrow \tilde{\epsilon} \tilde{q}_b', \\
e^{+} q_b \rightarrow \tilde{\epsilon} \tilde{q}_b, \\
e^{+} \bar{q}_b \rightarrow \tilde{\epsilon} \tilde{q}_b',
\end{align*}
\]

(28)
can be obtained from \( \frac{d\sigma^{ab}(e^- q \rightarrow \tilde{\epsilon} \tilde{q})}{d\tilde{t}} \) given in eqs. (25) and (26) by the following replacements:

\[
(\eta_{L_i})_{i} \rightarrow (\eta_{R_i})_{i}^{*} \quad \text{if } f_L \rightarrow \tilde{f}_L,
\]

\[
(\eta_{R_i})_{i} \rightarrow (\eta_{L_i})_{i}^{*} \quad \text{if } f_R \rightarrow \tilde{f}_R.
\]

(29)

For clarity, we note that in our notation \( \tilde{f}_R \) and \( \tilde{f}_L \) are the scalar partners of the left- and right-handed antifermions \( f_L \) and \( f_R \), respectively, so that the subscripts \( a \) and \( a' \) (and, similarly, \( b \) and \( b' \)) specifying the processes (28) are just opposite to each other.

The integrated cross section for the production of a particular slepton-squark pair in unpolarized ep collisions at the c.m. energy \( \sqrt{s} \) is obtained from the above polarized differential cross sections as follows:

\[
\sigma^{ab}(ep \rightarrow \tilde{\tau} \tilde{q} X) = \int_{x_{\text{min}}}^{1} dx \int_{\tilde{t}_{\text{min}}}^{\tilde{t}_{\text{max}}} d\tilde{t} \frac{1}{4} \frac{d\sigma^{ab}(eq \rightarrow \tilde{\epsilon} \tilde{q})}{d\tilde{t}} q(x, Q^2)
\]

(30)

with the integration boundaries

\[
x_{\text{min}} = \left( m_\tilde{\tau} + m_\tilde{q} \right)^2 / s,
\]

\[
\tilde{t}_{\text{min}} = \frac{1}{2} \left( sx - m_\tilde{\tau}^2 - m_\tilde{q}^2 \pm \sqrt{(sx - m_\tilde{\tau}^2 - m_\tilde{q}^2)^2 - 4m_\tilde{\tau}^2m_\tilde{q}^2} \right).
\]

(31)

The factor \( \frac{1}{4} \) in eq. (30) arises from averaging over the incident lepton and quark polarizations. Furthermore, the function \( q(x, Q^2) \) denotes the appropriate quark density (or antiquark density in the case of \( e\bar{q} \rightarrow \tilde{\chi} \tilde{q} \)) of the proton, \( x \) being the
fraction of the proton momentum carried by the (anti)quark and $Q^2$ being the QCD evolution scale for which we take

$$Q^2 = -\hat{t}. \quad (33)$$

Finally, the various production channels are indicated by $a, b \in \{L, R\}$ according to the notation used in eqs. (25), (26) and (28).

4. Numerical results

Having at hand suitable examples of chargino and neutralino spectra as well as the necessary analytical expressions of cross sections, we are now ready for numerical investigations. Thereby, we shall concentrate on the maximum ep center-of-mass energy provided by HERA [6], that is $\sqrt{s} = 314$ GeV, but for completeness we shall also make a brief excursion to higher energies. For the chargino and neutralino spectra we exclusively use solutions for $m_{N1} \leq 100$ GeV derived with the constraints (15) as explained in sect. 2 and summarized in table 1. As far as the scalar masses are concerned, we shall study two cases. In the first case, slepton and squark masses are considered as basically free parameters in the sense that no theoretical mass relations are used. In the second case, we employ the renormalization group relations [5] for sparticle masses provided by the minimal supergravity model. Furthermore, for the quark densities we take set I of ref. [7] with the evolution scale (33). We have checked that the results do not change significantly, if the scale $Q^2 = \frac{1}{2}(\hat{t}_{\text{min}} + \hat{t}_{\text{max}})$ with $\hat{t}_{\text{min, max}}$ from eq. (32) is used as in refs. [3] and [4]. Finally, for the electroweak parameters we substitute the numerical values $\alpha = e^2/4\pi = 0.137$, $\sin^2\theta_W = 0.23$ and $m_Z = m_W/\cos\theta_W = 93$ GeV.

4.1. PRODUCTION CROSS SECTIONS FOR UNCONSTRAINED SCALAR MASSES

Taking the attitude that slepton and squark masses are unknown parameters to be determined by experiment, we are free to make the choice

$$m_{\tilde{\ell}} = m_{\tilde{\ell} L} = m_{\tilde{\ell} R} = m_{\tilde{\ell}},$$

$$m_{\tilde{u}_L} = m_{\tilde{u}_R} = m_{\tilde{d}_L} = m_{\tilde{d}_R} = m_{\tilde{q}}, \quad (34)$$

where $\tilde{u}$ and $\tilde{d}$ stand for all up- and down-type squark flavors, respectively. This simplification suffices for illustrative purposes. Furthermore, we define unpolarized total cross sections

$$\sigma(ep \rightarrow \tilde{\ell}\tilde{q}X) = \sum_{a=L,R} \sum_{b=L,R} \sum_{q} \sigma_{ab}(ep \rightarrow \tilde{\ell}\tilde{q}X) \quad (35)$$

by summing the cross sections for $\tilde{\ell}\tilde{q}$ production given in eq. (30) with respect to the
quark flavors present in the proton and the L- and R-sfermion species. Explicit
calculation reveals that the cross sections depend, to a very good approximation,
only on the sum $m_\tilde{f} + m_\tilde{q}$ of slepton and squark masses. This is expected from eq.
(31) and the fact that the dominant contributions to the integral over $x$ in eq. (30)
come from the region $x = x_{\text{min}}$. Only if $m_\tilde{f} \ll m_\tilde{q}$ or $m_\tilde{q} \ll m_\tilde{f}$ deviations from this
simple behaviour are observed. Therefore, one may conveniently take

$$m_\tilde{f} = m_\tilde{q}$$

in numerical calculations without losing much of generality. More precisely, the
cross sections for $m_\tilde{f} \neq m_\tilde{q}$ are the same as the ones for equal masses as long as
$m_\tilde{f} + m_\tilde{q}$ takes the same value and $m_\tilde{f}$ and $m_\tilde{q}$ are not too different.

Fig. 2 shows predictions on the total cross sections (35) versus $m_\tilde{f} + m_\tilde{q}$ for
HERA. Plotted are the results for four chargino/neutralino spectra selected from
the solutions of table 1 with $m_{N_1} \leq 100$ GeV including those which lead to
maximum and minimum cross sections. Whereas for $e^+p$ collisions the relative
magnitude of the cross sections (a)-(d) follows the pattern one would naively expect
from the masses of the wino-dominated chargino and the photino-dominated
neutralino states of the spectra (a)-(d), the relative magnitude of the $e^-p$ cross
sections is less easy to explain due to a rather subtle interplay of valence and sea
quark contributions. We note that the current lower limits on sparticle masses [8], to
wit

$$
m_\tilde{e} \gtrsim \begin{cases} 
20 \text{ GeV for } m_\tilde{f} = 20 \text{ GeV} \\
60 \text{ GeV for } m_\tilde{f} = 0
\end{cases}
,$$

$$m_\tilde{q} \gtrsim 60 \text{ GeV},$$

$$m_\tilde{W} \gtrsim 20 \text{ GeV},$$

still allow cross sections at HERA as large as 10 pb. Here, the $\tilde{\gamma}$ and $\tilde{W}$ bounds
should be applied to $\tilde{X}_{N_1}$ and $\tilde{X}_{C_1}$, respectively, except in the case (b) where
$\tilde{X}_{N_1}$ is essentially a higgsino and $\tilde{\gamma} = \tilde{\chi}_{N_2}$.

Another important question concerns the minimum production rates which are
required for detection. The answer to this question depends first and foremost on
the dominant decay modes of sleptons and squarks. Rather clear signatures are
provided by the two-body decays $\tilde{e} \rightarrow e + \text{LSP}$ and $\tilde{q} \rightarrow q + \text{LSP}$ where LSP denotes
the lightest supersymmetric particle which in the usual models is assumed to be
stable. Since the LSP is invisible, the above decays give rise to large energy-momen-
tum imbalances and thus allow a very efficient separation of SUSY events from the
ordinary deep-inelastic scattering background due to $e p \rightarrow e qX$. This has been
convincingly demonstrated in ref. [9] for the case $e p \rightarrow \tilde{e} \tilde{q}X$; $\tilde{e} \rightarrow e \tilde{\gamma}$, $\tilde{q} \rightarrow q \tilde{\gamma}$ where
the LSP is identified with the (massless) photino. It was concluded that the standard NC background can be eliminated by suitable cuts without losing more than about 20% of the signal. In that case, a rate of ten such events per year should be sufficient for detection. This implies a minimum cross section of 0.1 pb for the luminosity $10^{31}$ cm$^{-2}$ s$^{-1}$ designed for HERA, provided $\tilde{e} \rightarrow e\bar{\gamma}$ and $\tilde{q} \rightarrow q\bar{\gamma}$ are the dominant decay modes. Although this situation is not unlikely, it is by no means guaranteed. Many other and more complicated decays may occur. For example, the LSP may be a higgsino or a sneutrino in which case one would expect $\tilde{e}$ and $\tilde{q}$ to decay through some cascades. Moreover, the gluino may be sufficiently light so that squarks decay dominantly through $\tilde{q} \rightarrow q\bar{g}$ followed by $\tilde{g} \rightarrow q\bar{q} + \text{LSP}$. Therefore, one should reckon with the possibility that a clear signal may only be obtained for cross sections somewhat larger than 0.1 pb. This particularly applies to the chargino exchange process $e^+p \rightarrow \tilde{\nu}qX$ with $\tilde{\nu} \rightarrow \nu + \text{LSP}$ since both $\tilde{\nu}$ decay products are
From the above remarks it is clear that one cannot just straightforwardly deduce detection limits from the theoretical cross sections presented in fig. 2. One would rather need detailed Monte Carlo studies of the dominant decay processes for a given sparticle spectrum in order to draw definite conclusions. Nevertheless, it may be useful to at least indicate the reach of HERA by quoting the range of slepton and squark masses for which the larger one of the two cross sections \( \sigma(e^+ p \rightarrow \tilde{e}^+ \tilde{q}X) \geq 0.1 \text{ pb} \). The so-defined detection limits are

\[
m_{\tilde{e}} + m_{\tilde{q}} \lesssim \begin{cases} 
180 \text{ GeV} & \text{(a), (c), (d)} \\
160 \text{ GeV} & \text{(b)}
\end{cases}
\tag{38}
\]

where (a)–(d) refer to the chargino and neutralino models considered in fig. 2. In fact, in the cases (a) and (c) the lightest neutralino \( \tilde{X}_{N1} \) being a candidate for the LSP is essentially a photino. Hence, if \( m_{\tilde{g}} > m_{\tilde{q}} \) one is in the favorable situation analyzed in ref. [9] which confirms the limit (38). On the other hand, in scenario (b) the lightest neutralino is approximately a higgsino while in scenario (d) the lightest neutralino \( \tilde{X}_{N1} = \tilde{\gamma} \) is so heavy that either \( \tilde{e} \rightarrow e\gamma \) or \( \tilde{q} \rightarrow q\gamma \) is forbidden for \( m_{\tilde{e}} + m_{\tilde{q}} \leq 200 \text{ GeV} \). Thus, in these two cases the limit (38) may be somewhat too optimistic.

4.2. PRODUCTION CROSS SECTIONS IN THE MINIMAL SUPERGRAVITY MODEL

In the framework of the minimal supergravity model, the \( SU(3)_C \times SU(2)_L \times U(1) \) gaugino mass parameters \( M_3, M_2, \text{ and } M_1 \) are related to a single mass parameter \( m_{1/2} \) by renormalization group equations \([2, 5]\) such as eq. (16). Assuming

\[
M_3 = M_2 = M_1 = m_{1/2}
\tag{39}
\]

at the grand unification scale \( M_X = 2.4 \times 10^{16} \text{ GeV} \) and using \( g^2_X/4\pi = \frac{1}{24} \), \( \alpha(m_W) = \frac{1}{128} \) and \( \alpha_s(m_W) = 0.12 \) one obtains

\[
M_3 = 2.9m_{1/2} \quad \text{and} \quad M_2 = 2M_1 \approx 0.82m_{1/2}
\tag{40}
\]

at energy scales of \( O(m_W) \). \( M_2 \) and \( M_1 \) enter the chargino and neutralino mass matrices given in eqs. (7) and (8) and \( M_3 = m_{\tilde{g}} \) is the effective gluino mass.

The model also provides renormalization group relations for scalar masses \([5]\). For equal Higgs vacuum expectation values \( v_1 = v_2 \) as assumed in (15a), these
relations read

\[ m_{\tilde{e}}^2 = m_{\tilde{e}_L}^2 = m_0^2 + 0.23M_1^2 + 0.73M_2^2, \]
\[ m_{\tilde{e}_R}^2 = m_0^2 + 0.91M_1^2. \]
\[ m_{\tilde{\mu}_L}^2 = m_{\tilde{\mu}_L}^2 = m_0^2 + 0.025M_1^2 + 0.73M_2^2 + 0.79M_3^2, \]
\[ m_{\tilde{\mu}_R}^2 = m_0^2 + 0.4M_1^2 + 0.79M_3^2, \]
\[ m_{\tilde{d}_R}^2 = m_0^2 + 0.1M_1^2 + 0.79M_3^2, \]

(41)

where the scalar mass parameter \( m_0 \) defined at \( M_X \) is the gravitino mass and \( M_1, M_2 \) and \( M_3 \) are as given in eq. (40). Contributions from Yukawa couplings to eq. (41) are neglected. These effects mainly shift the mass of the scalar top quark \( \tilde{t} \) away from \( m_{\tilde{t}} \) and induce small mixing between sfermions with the same charge. Combining eqs. (40) and (41) one arrives at the approximate slepton and squark mass relations [2]

\[ m_{\tilde{\tau}_L}^2 = m_0^2 + 0.5m_{1/2}^2, \quad m_{\tilde{\tau}_R}^2 = m_0^2 + 0.15m_{1/2}^2, \]
\[ m_{\tilde{\mu}_L}^2 = m_0^2 + 7m_{1/2}^2. \]

(42)

To proceed, we choose values for the lightest neutralino and chargino masses \( m_{N_1} \) and \( m_{\chi_1} \), diagonalize the mass matrices eqs. (7) and (8) under the assumptions (15) and determine the remaining neutralino and chargino masses, the eigenstates and the parameters \( M_2 \) and \( \mu \) as explained in sect. 2. From \( M_2 \) and eq. (40) we get \( m_{1/2} \) which is then substituted in eq. (42). Finally, using eq. (42) we compute the cross sections defined in eq. (35) as a function of \( m_0 \). Numerical results are depicted in fig. 3 for four choices (a)–(d) of \( m_{N_1} \) and \( m_{\chi_1} \) which are specified in the figure caption together with the corresponding values of \( m_{1/2} \) and \( \mu \). From current SUSY searches one has deduced various limits [10] on the parameters \( m_{1/2} \) and \( m_0 \) such as

\[ m_0 \gtrsim 55 \text{ GeV} \quad \text{for} \quad m_{1/2} \approx 20 \text{ GeV} \quad (\text{a and b}), \]
\[ m_0 \gtrsim 15 \text{ GeV} \quad \text{for} \quad m_{1/2} \approx 40 \text{ GeV} \quad (\text{c and d}). \]

(43)

These bounds are taken into account in fig. 3.

One sees that the maximum cross sections compatible with the existing constraints on the present model are of the order of 1 pb. On the other hand, as argued in subsect. 4.1 detection of slepton-squark production at HERA [9] should be possible for cross sections as small as about 0.1 pb. Thus, using the same criterion as
Fig. 3. Slepton-squark production cross sections at $\sqrt{s} = 314$ GeV versus the scalar mass parameter $m_0$ for the mixing scenarios (masses in GeV) (a) $m_{N1} = 10$, $m_{C1} = 30$ ($M_2 = 17.3$, $\mu = -493.2$); (b) $m_{N1} = 10$, $m_{C1} = 80$ ($M_2 = 16.4$, $\mu = -24.7$) (c) $m_{N1} = 20$, $m_{C1} = 30$ ($M_2 = 32.1$, $\mu = 77.3$); (d) $m_{N1} = 20$, $m_{C1} = 50$ ($M_2 = 35.5$, $\mu = -410.0$). The corresponding gaugino mass parameter $m_{1/2}$ is obtained from eq. (40) and takes the values (in GeV): (a) 21.0, (b) 20.0, (c) 39.1, (d) 43.3. The scalar masses $m_{\tilde{z}}$ and $m_{\tilde{q}}$ are related to $m_0$ and $m_{1/2}$ by the renormalization group equations (42). Cases incompatible with current experimental constraints are marked by dashed curves.

In eq. (38) in order to estimate detection limits from fig. 3 and eq. (42) one finds

$$m_{\tilde{z}} \leq 80 \text{ GeV}, \quad m_{\tilde{q}} \leq 100 \text{ GeV} \quad \text{for (a) and (b)},$$

$$m_{\tilde{z}} \leq 60 \text{ GeV}, \quad m_{\tilde{q}} \leq 120 \text{ GeV} \quad \text{for (c) and (d)}.$$  \hspace{1cm} (44)

These values are consistent with the result (38) obtained for the previous model which is slightly more general. In addition, the gluino masses associated with the scalar masses (44) are fixed in the present model by eq. (40) yielding

$$m_{\tilde{g}} = 60 \text{ GeV} \quad \text{for (a) and (b)},$$

$$m_{\tilde{g}} = m_{\tilde{q}} \quad \text{for (c) and (d)}.$$ \hspace{1cm} (45)
Fig. 4. Limits of SUSY searches at HERA ($\sqrt{s} = 314$ GeV) in the framework of a minimal supergravity model. The full curves indicate the accessible range of the SUSY breaking parameters $m_{1/2}$, $m_0$ and $\mu$ if $\sigma(e^+p \rightarrow \tilde{t}QX) \approx 0.1$ pb is the smallest cross section for which a signal can be detected. The region in $(m_{1/2}, m_0)$ below the dashed curves is already excluded by present-day limits on sparticle masses.

Hence, in the cases (a) and (b) of fig. 3 squarks with masses $m_{\tilde{q}} \approx 100$ GeV decay via $\tilde{q} \rightarrow q\tilde{g}$ and $\tilde{g} \rightarrow q\bar{q} + \text{LSP}$ producing a missing momentum signal which is less striking than the one from the direct decay $\tilde{q} \rightarrow q + \text{LSP}$. As a consequence, we expect the detection limit for (a) and (b) to be somewhat lower than the one quoted in eq. (44).

The prospects of testing the minimal supergravity model at HERA are summarized in fig. 4. This figure shows the contours in the $(m_{1/2}, m_0)$-plane for which the cross sections defined by eq. (35) take the value $\sigma(e^+p \rightarrow \tilde{t}QX) = 0.1$ pb. Here, the higgsino mass parameter $\mu$ is restricted to the range $|\mu| \lesssim 100$ GeV. However, as expected and quantified in fig. 4, the value of $\mu$ does not have a decisive influence on the size of the cross section. For comparison, we also indicate the current limits [10] on $m_{1/2}$ and $m_0$ derived from the non-observation of the processes $e^+e^- \rightarrow \tilde{e}^+\tilde{e}^-$, $e^+e^- \rightarrow \gamma\gamma\gamma$ and of $\tilde{g}$ production at the CERN collider. It becomes quite evident from our analysis that the minimal supergravity model can be tested at HERA only in a relatively small region of the parameter space beyond the present bounds.

4.3. UNPOLARIZED ASYMMETRIES

If a sufficiently strong signal of slepton-squark production is found, there are several ways to extract information on the exchanged chargino and neutralino
states. As obvious from figs. 2 and 3 some insight could be gained directly from the size of the production cross sections. However, since the latter are steep functions of the slepton and squark masses, such an attempt would require a sufficiently precise mass determination in addition to the knowledge of the branching ratios of the observed decay modes.

Another possibility, which does not need beam polarization, is investigated in fig. 5. These plots show the asymmetries

\[
A_N = \frac{\sigma(e^- p \rightarrow \tilde{e}^- \tilde{q}X) - \sigma(e^+ p \rightarrow \tilde{e}^+ \tilde{q}X)}{\sigma(e^- p \rightarrow \tilde{e}^- \tilde{q}X) + \sigma(e^+ p \rightarrow \tilde{e}^+ \tilde{q}X)},
\]

\[
A_C = \frac{\sigma(e^- p \rightarrow \tilde{e}^- \tilde{q}X) - \sigma(e^+ p \rightarrow \bar{\tilde{q}}X)}{\sigma(e^- p \rightarrow \tilde{e}^- \tilde{q}X) + \sigma(e^+ p \rightarrow \bar{\tilde{q}}X)}.
\]

in the total slepton-squark production in $e^- p$ versus $e^+ p$ collisions as specified in
eq. (35). From this demonstration in which all neutralino and chargino spectra of
table I with \( m_{N1} \leq 50 \text{ GeV} \) are considered we learn that the above charge asymme-
tries are very sensitive to neutralino and chargino properties, while they depend only
relatively weakly on the slepton and squark masses in the interesting range \( m_{\tilde{\tau}^+} + m_{\tilde{q}} \geq 100 \text{ GeV} \). Both facts make these observables particularly useful for testing the
neutralino and chargino sector at HERA in case a \( \tilde{\tau} \tilde{q} \) signal is observed. Moreover,
tests based on ratios such as eqs. (46) and (47) profit by cancellations of uncertain-
ties in the experimental cross section determinations arising from systematic errors
and a priori unknown \( \tilde{\tau} \) and \( \tilde{q} \) branching fractions. It is clear from fig. 5 that even a
rough measurement of \( A_N \) and \( A_C \) would sort out a particular class of neutralino
and chargino solutions and thus provide valuable information on gaugino-higgsino
mixing, that is on SUSY and electroweak symmetry breaking.

Finally, we want to mention the existence of various asymmetries in \( \tilde{\tau} \tilde{q} \) produc-
tion with longitudinally polarized \( e^\pm \) beams [4,11]. Measurements of polarization
asymmetries, which may indeed become possible at HERA [6], would shed light on
further details of the neutralino/chargino sector and on the mass difference
between \( \tilde{\epsilon}_L \) and \( \tilde{\epsilon}_R \).

4.4. SLEPTON-SQUARK PRODUCTION AT HIGHER ENERGIES

We conclude our numerical studies with a brief outlook for slepton and squark
production at energies beyond the HERA c.m. energy range [12]. Fig. 6 exemplifies
the rise of the production cross sections with the ep collision energy. The parameters
of these examples are chosen in accordance with the specifications of subsect. 4.1,
but in such a way that they are also roughly consistent with the mass relations (40)
and (42) of the minimal supergravity model considered in subsect. 4.2. The ap-
propriate values of \( m_{1/2} \) and \( m_0 \) to be substituted in these relations are

(a) \( m_{1/2} = 40 \text{ GeV}, \quad m_0 = 40 \text{ GeV} \),
(b) \( m_{1/2} = 100 \text{ GeV}, \quad m_0 = 70 \text{ GeV} \),
(c) \( m_{1/2} = 230 \text{ GeV}, \quad m_0 = 100 \text{ GeV} \). \hspace{1cm} (48)

For definiteness, we shall concentrate on the c.m. energy \( \sqrt{s} = 1.3 \text{ TeV} \) which
would be provided by collisions of 50 GeV electrons from LEP with 8.5 TeV
protons from LHC, the pp collider project in the LEP tunnel. For this ep option we
assume the luminosity \( L = 10^{32} \text{ cm}^{-2} \text{ s}^{-1} \) as suggested by detailed machine studies
[13]. It is then reasonable to take \( 10^{-2} \text{ pb} \) as the smallest cross section for which a
signal can be detected. In that case, one would be able to reach sparticle masses up
to

\[ m_{\tilde{\tau}} + m_{\tilde{q}} \approx 700 \text{ GeV} \] \hspace{1cm} (49)
as indicted by fig. 6.
Fig. 6. Slepton-squark production cross sections versus the ep c.m. energy $\sqrt{s}$ for the following mixing scenarios (see table 1) and scalar masses (in GeV): (a) $m_{N1} = 20$, $m_{C1} = 50$ ($M_2 = 33$, $\mu = 31$), $m_\tau = 50$, $m_\chi = 100$; (b) $m_{N1} = 50$, $m_{C1} = 100$ ($M_2 = 87$, $\mu = -64$), $m_\tau = 100$, $m_\chi = 250$; (c) $m_{N1} = 100$, $m_{C1} = 200$ ($M_2 = 190$, $\mu = -183$), $m_\tau = 150$, $m_\chi = 600$.

5. Concluding remarks

In ep collisions, the occurrence of processes such as $e\gamma \rightarrow \tilde{\chi}_1$, $e\gamma \rightarrow \tilde{\tau}X$, $\gamma q \rightarrow \tilde{q}q$ and $\gamma g \rightarrow \tilde{q}\bar{q}$ would be a clear manifestation of supersymmetry. Additional, but less direct evidence would be provided by effects of squarks and gluinos on the running of the strong coupling constant $\alpha_s(Q^2)$, and by changes to the deep-inelastic structure functions due to the evolution of a q and g sea inside the proton. However, in view of the current limits on sparticle masses some of these possibilities appear to be beyond or already quite close to the limit of observability [14]. It is the associated production of sleptons and squarks which will probably play the most important role in SUSY searches at future ep colliders.

In agreement with other studies we find that at the c.m. energy $\sqrt{s} = 314$ GeV and with the luminosity $L = 10^{31}$ cm$^{-2}$ s$^{-1}$ provided by HERA one may be able to reach slepton and squark masses up to $m_\tau + m_\chi = 180$ GeV. Our estimates further indicate that ep collisions at $\sqrt{s} = 1.3$ TeV should give access to scalar masses in the range $m_\tau + m_\chi = 700$ GeV provided the luminosity is increased to $L = 10^{32}$ cm$^{-2}$ s$^{-1}$. In order to give more precise discovery limits one must pay attention to the considerable model-dependence of the cross sections for ep $\rightarrow \tilde{e}qX$ and carefully investigate the decay signatures of $\tilde{e}$ and $\tilde{q}$. On the theory side, the main uncertainties arise from the unknown masses and mixing angles of the neutralino and chargino states and from the gluino mass. We have clarified the problem concerning
the production cross sections in the minimal supersymmetric extension of the standard model with and without supergravity mass relations. The numerical examples shown give a fairly detailed account of slepton-squark production for the experimentally relevant range of parameters.

Finally, it is interesting to compare the discovery potential of HERA with the prospects for detecting selectrons at LEP and squarks at the Tevatron, although such a comparison should be made with caution. Similarly as present-day $e^+e^-$ machines, LEP will allow to test the existence of selectrons with masses almost as large as the beam energy that is $m_\tilde{e} = 40$ GeV at LEP I [15] and $m_\tilde{e} = 90$ GeV at LEP II [16]. Stronger limits are possible, but more model-dependent. Squarks, on the other hand, are expected [17] to be detectable at the Tevatron $\bar{p}p$ collider up to masses $m_\tilde{q} = 120-200$ GeV. There will thus be a considerable overlap of SUSY searches at these machines and at HERA which is useful to establish a clear signal or to put new stringent bounds on sparticle masses. Not to forget, these searches are also complementary in the sense that they test different fundamental couplings among ordinary and supersymmetric particles.

Note added

An initial brief account of our work was given in ref. [14]. While this paper was written up, we received a preprint by Bartl et al. [18], in which the production and decay of selectrons and squarks in $e^p$ collisions is studied and numerical results are given for three cases of gaugino-higgsino mixing with $v_1 = 0.9v_2$.

References

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