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Bag model approach to the intermediate states in hadronic transitions in heavy $Q\bar{Q}$ systems*

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Abstract. Instead of the quark confining string model (QCS) used in [4], a bag model approach to the intermediate states in hadronic transitions in heavy $Q\bar{Q}$ systems is studied. The calculation is based on a spherical bag model with Born-Oppenheimer approximation. It is shown that the spectra of intermediate states in this approach are quite different from those obtained in the QCS model, while normalized by the experimental data of $\Gamma(\psi' \to \psi\pi\pi)$ the two approaches give very similar results for most of the hadronic transition rates in the $b\bar{b}$ system. This shows that the results in [4] are not sensitive to the models of the intermediate states. The present approach can also be applied to the study of intermediate states including light quarks.

Hadronic transitions play important roles in heavy quarkonium physics. They are dominant decay modes of ψ' and Υ', and most promising processes for reaching the interesting $^1P_1$ states of $c\bar{c}$ and $b\bar{b}$ are related to hadronic transitions [1]. In most hadronic transition processes, the emitted light hadrons are soft, so that perturbative QCD does not work. Due to the smallness of the sizes of heavy quarkonium states, multipole expansion in QCD has proved to be useful [2]. A general formalism of multipole expansion in QCD describing hadronic transitions has been given by Yan [3]. For example, the $E1-E1$ transition amplitude for $\Phi_i \to \Phi_f + \pi + \pi$ is [3]

$$M_{E1E1} = i(g^2/6)\langle \Phi_f \pi\pi| \cdot \mathbf{x} \cdot \mathscr{E}(E_i - H_8 - iD_0)^{-1} \mathbf{x} \cdot \mathscr{E} |\Phi_i\rangle \quad (1)$$

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where Φ_i and Φ_f are, respectively, the initial and final quarkonium states, $\mathbf{x}$ is the separation between the heavy quark Q and antiquark $\bar{Q}$, $\mathscr{E}$ is the colour electric field, E_i is the energy of Φ_i, H_8 is the Hamiltonian of the colour octet $Q\bar{Q}$ system, and the covariant derivative D_0 is

$$D_0 = \partial_0 - g A_0. \quad (2)$$

The diagram for this process is dipicted in Fig. 1.

There are two fundamental difficulties in the evaluation of (1), namely:

i) Lack of the information about the intermediate states. The intermediate state contains a soft gluon and a colour-octet $Q\bar{Q}$ pair. It is difficult to treat this $Q\bar{Q}g$ system from the first principle of QCD, nor can the knowledge of it be obtained from the known static potential for colour-singlet $Q\bar{Q}$ pair.

ii) The mechanism for the conversion of gluons into light hadrons (hadronization) is not known.

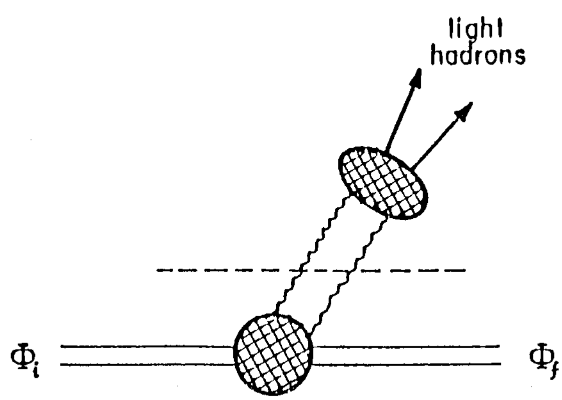

Fig. 1. Amplitude for hadronic transition between heavy quarkonium states Φ_i and Φ_f

In [4], quark confining string (QCS) model [5] is adopted to describe the intermediate states, and therein the gluon degree of freedom resides in the vibration of the string. Spectrum and wave functions of the intermediate states are obtained by solving the Schrödinger equation for $Q\bar{Q}$ system with an effective static potential given by the first vibrational mode of the string. In this approach (1) is factorized into a heavy quark factor and a hadronization factor

$$M_{E1E1}=i(g^2/6)\langle\Phi_f|\,x_k(E_i-H_v)^{-1}\,x_l\,|\Phi_i\rangle \cdot\langle\pi\pi|\,\mathscr{E}_k P\mathscr{E}_l\,|0\rangle \quad (3)$$

where H_v and P are the effective $Q\bar{Q}$ Hamiltonian and the projection operator for the sector of the first vibrational mode of the string, respectively. The heavy quark factor in (3) is completely calculable and the hadronization factor can be treated with the soft pion technique. Using the experimental data of $\Gamma(\psi'\to\psi\pi\pi)$ as input to determine the overall unknown constant in (3) one can predict $E1-E1$ transition rates between various $b\bar{b}$ states. Similar treatment can be applied to calculate η-transitions like $\Upsilon'\to\Upsilon+\eta$. In [4] many predictions are made in this way and they are in good agreement with experiments [1, 4].

At present, no string vibrational states have been observed. One may ask if the existence of such vibrational states is crucial for the success in [4]. Furthermore, when coupled channel effect is considered, light quark pair $q\bar{q}$ can be created in the intermediate states and therefore states like $Q\bar{q}g$ are relevant to hadronic transitions. It is difficult to deal with such $Q\bar{q}g$ states in the framework of QCS model. In view of all these, alternative approaches to the intermediate states are worth investigating. In this paper, we use the MIT bag model to calculate the $Q\bar{Q}g$ states as an alternative approach to the intermediate states. We shall see that the spectra of intermediate states for $c\bar{c}$ and $b\bar{b}$ systems in bag model approach are quite different from those obtained from the QCS model, while normalized by the data of $\Gamma(\psi'\to\psi\pi\pi)$ the two approaches give very similar results for most of the hadronic transition rates in the $b\bar{b}$ system. (The only exception is $\Gamma(\Upsilon''\to\Upsilon\pi\pi)$ which depends strongly on the shape of the bag and should be treated more carefully.) This means that the success of [4] does not depend strongly on the specific spectrum of the intermediate states. Since in MIT bag model $Q\bar{q}g$ states are also calculable, this approach gives the possibility of studying the details of coupled channel effect in hadronic transitions.

In bag model, the first emitted gloun (colour-octet) is put in a bag together with the colour-octet $Q\bar{Q}$ pair to form a colour-singlet system. This kind of "hybrid" has been studied by several authors [6–8]. The calculation is based on the Born-Oppenheimer approximation which contains two steps. In the first step, the heavy Q and $\bar{Q}$ are treated as fixed sources with separation $r=|\mathbf{x}|$; the gluon field inside the bag is obtained by solving the classical field equations with the bag boundary condition; and the shape of the bag is determined by minimizing the total static energy of the system with respect to the variation of the shape. This determines the shape as a function of r. The second step is to solve the non-relativistic Schrödinger equation for $Q\bar{Q}$ with the total static energy (as a function of r) as the effective static potential, and this will give the spectrum and wave functions of the intermediate states. With this treatment, the transition amplitude (1) is also factorized

$$M_{E1E1}=i(g^2/6)\langle\Phi_f|\,x_k(E_i-H_B)^{-1}\,x_l\,|\Phi_i\rangle \cdot\langle\pi\pi|\,\mathscr{E}_k\mathscr{E}_l\,|0\rangle. \quad (4)$$

where H_B is the above mentioned effective $Q\bar{Q}$ Hamiltonian in bag model approach. The hadronization factor can again be treated with the soft pion technique and so the only thing is to calculate the heavy quark factor. Let $|KL\rangle_B$ be the intermediate state calculated from bag model with principle quantum number K and angular momentum L. Equation (4) may be written as

$$M_{E1E1}=i(g^2/6)\sum_{K,L}\frac{\langle\Phi_f|\,x_k\,|KL\rangle_{BB}\langle KL|\,x_l\,|\Phi_i\rangle}{E_i-E^B_{KL}} \cdot\langle\pi\pi|\,\mathscr{E}_k\mathscr{E}_l\,|0\rangle, \quad (5)$$

with E^B_{KL} the eigenvalue of H_B. M_{E1E1} can be evaluated once E^B_{KL} and $|KL\rangle_B$ are known.

For simplicity, we consider here only the simplest spherical bag with radios R. This is good for small $Q\bar{Q}$ separations. In most of the interesting transition processes, the first few low lying $|KL\rangle_B$ states dominate in (5). So that the spherical bag model is a good approach. The only problemable process is $\Upsilon''\to\Upsilon+\pi+\pi$ in which there is a delicate cancellation between various terms in (5) and higher $|KL\rangle_B$ states give significant contributions. Thus bag shape deformation may be important in this process and the simple spherical bag result cannot be taken seriously.

It has been shown that from the phenomenological point of view the surface energy of the bag can be ignored [6]. Thus in Coulomb gauge the Hamiltonian of the system with Q and $\bar{Q}$ at rest is [9]

$$H=2m_Q+(4\pi/3)R^3B +\tfrac{1}{2}\int d^3x\,(\mathscr{E}^a_L\cdot\mathscr{E}^a_L+\mathscr{E}^a_T\cdot\mathscr{E}^a_T+\mathscr{B}^a\cdot\mathscr{B}^a) \quad (6)$$

where B is the volume energy density required to create the bag phase and the transverse and longitudinal components of $\mathscr{E}^a$ are defined by

$$\nabla \cdot \mathscr{E}^a_T = 0, \quad \nabla \times \mathscr{E}^a_L = 0. \tag{7}$$

To determine B, let us consider the colour-singlet $Q\bar{Q}$ system. In general, when r becomes large compared with $B^{-1/4}$, the spherical bag should deform into cylinderlike shape like a string. The string tension λ is related to B through [6]

$$\lambda = [(32/3)\,\pi\,\alpha_s B]^{1/2}, \tag{8}$$

where $\alpha_s = g_s^2/4\pi$ is the fine structure constant of QCD. In this paper we take the Cornell Coulomb plus linear potential model for the colour-singlet $Q\bar{Q}$ system

$$\begin{aligned} &V(r) = -\kappa/r + \lambda r \\ &\lambda = 1/a^2, \quad a = 2.34\ \mathrm{GeV}^{-1} \\ &\kappa = (4/3)\,\alpha_s = \begin{cases} 0.52, & c\bar{c} \\ 0.48, & b\bar{b} \end{cases} \\ &m_c = 1.84\ \mathrm{GeV}, \quad m_b = 5.17\ \mathrm{GeV}. \end{aligned} \tag{9}$$

This leads to

$$B = \begin{cases} 2.55 \times 10^{-3}\ \mathrm{GeV}^4, & c\bar{c} \\ 2.76 \times 10^{-3}\ \mathrm{GeV}^4, & b\bar{b}. \end{cases} \tag{10}$$

Next we consider the gluon field energy $\frac{1}{2}\int d^3x(\mathscr{E}^a_T \cdot \mathscr{E}^a_T + \mathscr{B}^a \cdot \mathscr{B}^a)$ for the $Q\bar{Q}g$ system. In bag model, perturbative QCD is assumed to be applicable inside the bag. To zeroth order, quark-gluon and gluon-gluon interactions are neglected and this field energy can be obtained by solving the Maxwell equations in the spherical bag with bag boundary conditions. It has been shown that [6]

$$\tfrac{1}{2}\int d^3x(\mathscr{E}^a_T \cdot \mathscr{E}^a_T + \mathscr{B}^a \cdot \mathscr{B}^a) = \chi/R. \tag{11}$$

The value of χ depends on the eigen modes of the fields. For the lowest TE and TM modes,

$$\chi = \begin{cases} 2.744, & \text{lowest } TE \text{ mode } (J^P = 1^+) \\ 4.493, & \text{lowest } TM \text{ mode } (J^P = 1^-). \end{cases} \tag{12}$$

In $E1-E1$ transitions, the gluon in the intermediate states should have the quantum number $J^P = 1^-$. Therefore we should take the lowest TM mode with $\chi = 4.493$.

Finally, the Green's function satisfying the spherical bag boundary condition is [6, 9]

$$G(\mathbf{x}, \mathbf{x}') = \frac{1}{4\pi|\mathbf{x}-\mathbf{x}'|} + \frac{1}{4\pi R}\sum_{l=1}^{\infty}\frac{l+1}{l}\left(\frac{|\mathbf{x}|\,|\mathbf{x}'|}{R^2}\right)^l \cdot P_l\left(\frac{\mathbf{x}\cdot\mathbf{x}'}{|\mathbf{x}|\,|\mathbf{x}'|}\right). \tag{13}$$

With the help of this Green's function, the longitudinal colour electric field energy $\frac{1}{2}\int d^3x\,\mathscr{E}^a_L \cdot \mathscr{E}^a_L$ can be written as

$$\begin{aligned} &\tfrac{1}{2}\int d^3x\,\mathscr{E}^a_L \cdot \mathscr{E}^a_L \\ &= (g_s^2/2)\int d^3x\,d^3x'[\rho^a(\mathbf{x}) + f_{abc}\mathscr{E}^b_T(\mathbf{x})\cdot\mathbf{A}^c(\mathbf{x})] \\ &\quad \cdot G(\mathbf{x}, \mathbf{x}')[\rho^a(\mathbf{x}') + f_{abc}\mathscr{E}^b_T(\mathbf{x}')\cdot\mathbf{A}^c(\mathbf{x}')] \end{aligned} \tag{14}$$

where f_{abc} is the colour $SU(3)$ structure constant, $\rho^a(x)$ is the colour charge density of the heavy quarks. For simplicity, we can approximately take the gluon colour charge density $f_{abc}\mathscr{E}^b_T(\mathbf{x})\cdot\mathbf{A}^c(\mathbf{x})$ to be uniform inside the bag. It has be been shown that this approximation is adequate within a few MeV [7]. With elementary calculations (14) can be written as [6, 7]

$$\begin{aligned} &\tfrac{1}{2}\int d^3x\,\mathscr{E}^a_L \cdot \mathscr{E}^a_L = (1/6)(\alpha_s/r) \\ &\quad + (\alpha_s/R)\left[-6 + (3/8)(r/R)^2 + \frac{\left(\frac{7}{24}\left(\frac{r}{R}\right)^2 + \frac{3}{2}\right)}{1 - \frac{r^4}{16R^4}} + \frac{7}{12}\ln\frac{4R^2 + r^2}{4R^2 - r^2} - (3/4)\ln\left(1 - \frac{r^4}{16R^4}\right)\right]. \end{aligned} \tag{15}$$

The Hamiltonian (6) for the $Q\bar{Q}g$ system is then

$$\begin{aligned} &H(r, R) = 2m_Q + (4\pi/3)R^3 B + \chi/R + (1/6)(\alpha_s/r) \\ &\quad + (\alpha_s/R)\left[-6 + (3/8)(r/R)^2 + \frac{\left(\frac{7}{24}\left(\frac{r}{R}\right)^2 + \frac{3}{2}\right)}{1 - \frac{r^4}{16R^4}} + \frac{7}{12}\ln\frac{4R^2 + r^2}{4R^2 - r^2} - (3/4)\ln\left(1 - \frac{r^4}{16R^4}\right)\right]. \end{aligned} \tag{16}$$

In Born-Oppenheimer approximation, the bag radius R is determined by the stationary condition

$$\frac{\partial H(r, R)}{\partial R} = 0. \tag{17}$$

Equation (17) can be solved numerically to get $R = R(r)$ and the function $R(r)$ is shown in Fig. 2. The effective static potential between Q and $\bar{Q}$ in the intermediate states is then

$$V_B(r) = H(r, R(r)). \tag{18}$$

The shape of $V_B(r)$ is shown in Fig. 3. For large r, $V_B(r)$ increases more rapidly than a linear potential does as r increases.

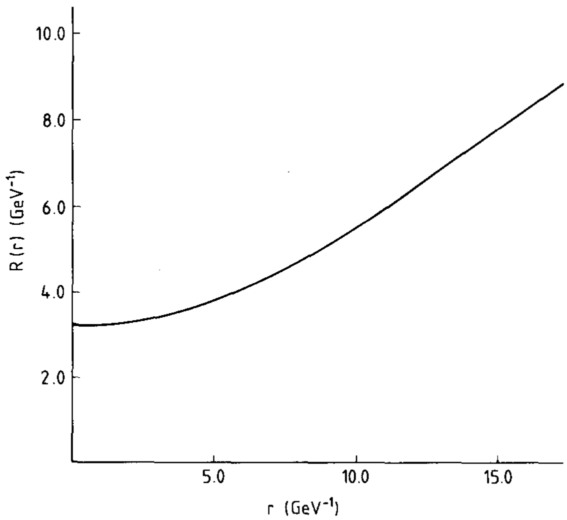

Fig. 2. The function $R=R(r)$ obtained from (18)

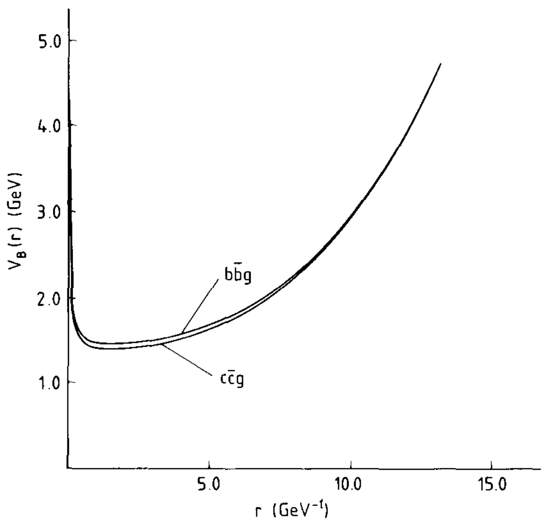

Fig. 3. The effective static potential $V_B(r)$ for intermediate states in the $c\bar{c}$ and $b\bar{b}$ systems. The constant term $2m_Q$ is not included

Table 1. Comparison of the spectra of intermediate states obtained from the bag model and the QCS model for the $c\bar{c}$ and $b\bar{b}$ systems

		E_{KL}^B (GeV)			
		$1S$	$1P$	$2P$	$3P$
$c\bar{c}$	BAG	4.83	5.48	5.83	6.21
	QCS	4.03	4.30	4.64	4.95
$b\bar{b}$	BAG	11.92	12.00	12.19	12.40
	QCS	10.46	10.75	10.98	11.19

The next step is to solve the Schrödinger equation numerically with the static potential $V_B(r)$ and get all E_{KL}^B and $|KL\rangle_B$ needed in (5). The values of various E_{KL}^B's are listed in Table 1. We see that they are all much higher than the corresponding values in the QCS model [4].

Let $\psi_{n_i l_i}(r)$, $\psi_{n_f l_f}(r)$ and $\psi_{KL}^B(r)$ be the radial wave functions of the initial quarkonium state, final quarkonium state and the intermediate state, respectively. Define

$$f_{n_i l_i, n_f l_f}^L \equiv \sum_K f_{n_i l_i, n_f l_f}^{KL} \equiv \sum_K \frac{\langle \psi_{n_f l_f} | r | \psi_{KL}^B \rangle \langle \psi_{KL}^B | r | \psi_{n_i l_i} \rangle}{M_i - E_{KL}^B}. \quad (19)$$

With the soft pion treatment of the hadronization factor in (5), the transitions rates for various initial and final quarkonium states are [4]:

$$\Gamma(n_i^3 S_1 \to n_f^3 S_1 + \pi + \pi) = |C_1|^2 \, G \, |f_{n_i 0, n_f 0}^1|^2 \quad (20)$$

$$\Gamma(1^3 D_1 \to 1^3 S_1 + \pi + \pi) = (4/15) \, |C_2|^2 \, H \, |f_{12,10}^1|^2 \quad (21)$$

$$\Gamma(n_i^3 P_0 \to n_f^3 P_0 + \pi + \pi) = (1/9) \, |c_1|^2 \, G \, |f_{n_i 1, n_f 1}^0 + 2 f_{n_i 1, n_f 1}^2|^2 \quad (22)$$

$$\Gamma(n_i^3 P_0 \to n_f^3 P_2 + \pi + \pi) = (10/27) \, |C_2|^2 \, H \, |f_{n_i 1, n_f 1}^0 + (1/5) f_{n_i 1, n_f 1}^2|^2 \quad (23)$$

$$\Gamma(n_i^3 P_0 \to n_f^3 P_1 + \pi + \pi) = \Gamma(n_i^3 P_1 \to n_f^3 P_0 + \pi + \pi) = 0 \quad (24)$$

$$\Gamma(n_i^3 P_1 \to n_f^3 P_1 + \pi + \pi) = \Gamma(n_i^3 P_0 \to n_f^3 P_0 + \pi + \pi) + (1/4) \, \Gamma(n_i^3 P_0 \to n_f^3 P_2 + \pi + \pi) \quad (25)$$

$$\Gamma(n_i^3 P \to n_f^3 P_2 + \pi + \pi) = (3/4) \, \Gamma(n_i^3 P_0 \to n_f^3 P_2 + \pi + \pi) \quad (26)$$

$$\Gamma(n_i^3 P_2 \to n_f^3 P_0 + \pi + \pi) = (1/5) \, \Gamma(n_i^3 P_0 \to n_f^3 P_2 + \pi + \pi) \quad (27)$$

$$\Gamma(n_i^3 P_2 \to n_f^3 P_1 + \pi + \pi) = (9/20) \, \Gamma(n_i^3 P_0 \to n_f^3 P_2 + \pi + \pi) \quad (28)$$

$$\Gamma(n_i^3 P_2 \to n_f^3 P_2 + \pi + \pi) = \Gamma(n_i^3 P_0 \to n_f^3 P_0 + \pi + \pi) + (7/20) \, \Gamma(n_i^3 P_0 \to n_f^3 P_2 + \pi + \pi) \quad (29)$$

$$\Gamma(n_i^3 S_1 \to n_f^3 S_1 + \eta) = (8/27)(M_f/M_i)(\pi^2/m_Q^2) \, |C_3|^2 \, |f_{n_i 0, n_f 0}^1|^2 \, q_\eta^3 \quad (30)$$

where G and H are phase-space integrals

$$G = (3/4)(M_f/M_i) \, \pi^3 \int dM_{\pi\pi}^2 \cdot K \left(1 - \frac{4m_\pi^2}{M_{\pi\pi}^2}\right)^{1/2} (M_{\pi\pi}^2 - 2m_\pi^2)^2 \quad (31)$$

$$H=(1/20)(M_f/M_i)\pi^3\int dM_{\pi\pi}^2 \cdot K\left(1-\frac{4m_\pi^2}{M_{\pi\pi}^2}\right)^{1/2}\left[(M_{\pi\pi}^2-4m_\pi^2)^2\left(1+\frac{2}{3}\frac{K^2}{M_{\pi\pi}^2}\right)+(8/15)\left(\frac{K^4}{M_{\pi\pi}^4}\right)(M_{\pi\pi}^4+2m_\pi^2 M_{\pi\pi}^2+6m_\pi^4)\right] \quad (32)$$

with

$$K\equiv(1/2M_i)[(M_i+M_f)^2-M_{\pi\pi}^2]^{1/2} \cdot [(M_i-M_f)^2\text{-}M_{\pi\pi}^2]^{1/2}, \quad (33)$$

q_η is the momentum of η, C_1, $|C_2|=3|C_1|$ and C_3 are unknown constants. Equations (24)–(29) are derived from the general property of multipole expansion.

The unknown constants $|C_1|$ and $|C_3|$ can be determined by using the known results

$$\begin{aligned}&\Gamma(\psi'\to\psi\,\pi\pi)=\mathrm{BR}(\psi'\to\psi\,\pi\pi)\cdot\Gamma_{\mathrm{tot}}(\psi')\\&\Gamma(\psi'\to\psi\,\eta)=\mathrm{BR}(\psi'\to\psi\,\eta)\cdot\Gamma_{\mathrm{tot}}(\psi')\\&\Gamma_{\mathrm{tot}}(\psi')=215\pm40\ \mathrm{keV}\\&\mathrm{BR}(\psi'\to\psi\,\pi\pi)=(50\pm4)\%\\&\mathrm{BR}(\psi'\to\psi\,\eta)=(2.7\pm0.4)\%\end{aligned} \quad (34)$$

as inputs. The so calculated transition rates (20)–(30) and the corresponding branching ratios are listed in Table 2. The total widths of various quarkonium states needed for calculating the branching ratios are obtained by adding the hadronic transition rates to the partial widths given in Table 3 of [4]. We also list in Table 2 the corresponding results obtained from the QCS model [4] and some known experimental data for comparison. We see that except for the special process $\Upsilon''\to\Upsilon\pi\pi$, the two different approaches give very similar results. This means that the results obtained are not sensitive to the change of the models for the intermediate states (spectra of intermediate states). Thus the success of [4] does not depend on the existence of the specific string vibrational states. It is interesting to note that all E_{KL}^B's for the $b\bar{b}$ system lie beyond the scanned region in present experiments.

Table 2. Hadronic transition rates and branching ratios in the $b\bar{b}$ systems. The total widths needed for computing branching ratios are obtained by adding hadronic transition rates in the table to the partial widths given in Table 3 of [4]

	BAG		QCS		Experiment
	Γ (keV)	BR (%)	Γ (keV)	BR (%)	BR (%)
$\Upsilon'\to\Upsilon\pi\pi$	8	30	7	27	27.3 ± 2.1
$\Upsilon''\to\Upsilon'\pi^+\pi^-$	0.2	1	0.4	2	$3\ \pm2$
$\Upsilon''\to\Upsilon\pi^+\pi^-$	4	16	0.2	1	4.5 ± 0.8
$\Upsilon''\to\Upsilon\eta$	0.006	0.03	0.009	0.04	0.2
$2\,^3P_0\to1\,^3P_0\pi\pi$	0.4	0.06	0.4	0.06	
$2\,^3P_0\to1\,^3P_2\pi\pi$	0.04	0.006	0.04	0.006	
$2\,^3P_1\to1\,^3P_1\pi\pi$	0.4	0.3	0.4	0.3	
$2\,^3P_1\to1\,^3P_2\pi\pi$	0.03	0.02	0.03	0.03	
$2\,^3P_2\to1\,^3P_0\pi\pi$	0.01	0.004	0.01	0.004	
$2\,^3P_2\to1\,^3P_1\pi\pi$	0.02	0.01	0.02	0.01	
$2\,^3P_2\to1\,^3P_2\pi\pi$	0.4	0.2	0.4	0.2	
$1\,^3D_1\to\Upsilon\pi\pi$	22		24		

Table 3. Values of $f_{n_il_i,n_fl_f}^{KL}$ for $\Upsilon'\to\Upsilon\pi\pi$, $\Upsilon''\to\Upsilon'\pi\pi$ and $\Upsilon''\to\Upsilon\pi\pi$

	Intermediate states					
	$1P$	$2P$	$3P$	$4P$	$5P$	$6P$
$f_{20,10}^{K1}$	0.730	0.076	−0.051	−0.045		
$f_{30,20}^{K1}$	3.01	−0.46	0.08	−0.02		
$f_{30,10}^{K1}$	−0.783	0.852	0.122	−0.021	−0.032	−0.020

The calculated $\mathrm{BR}(\Upsilon''\to\Upsilon\pi^+\pi^-)$ is larger than the experimental value by roughly a factor of four. This process is relatively complicated. Because Υ'' and Υ are not neighbouring states, there are delicate cancellations between various $f_{30,10}^{K1}$'s in (19) and the summation converges rather slowly, so that contributions of large K states are relatively important. This can be seen from the first six $f_{30,10}^{K1}$'s listed in Table 3. In states with large K, the separation between Q and $\bar{Q}$ are large and so the spherical bag approach is not good for such states. Therefore bag shape deformation should be considered in this process and the problem is rather subtle. In general, when r becomes large, the spherical bag is supposed to deform into a cylinder-like one and the effective static potential will deform into a linear-like potential which increases more slowly than the present $V_B(r)$ does as r increases. Therefore E_{KL}^B's of higher K states will be smaller than those obtained from the simple spherical bag. Thus we expect a more serious cancellation in (19) when bag shape deformation is taken into account and this will make $\Gamma(\Upsilon''\to\Upsilon\pi\pi)$ smaller. Quantitative discussion on the bag shape deformation effect will be presented elsewhere.

The present approach can be applied to intermediate states including light quark q, say $Q\bar{q}g$, which will appear when coupled channel effect is taken into account. The coupled channel effect in hadronic transition processes is more complicated than it is in studying the static properties of the colour-singlet $Q\bar{Q}$ systems. Detailed investigations of coupled channel effect in hadronic transitions will be presented in a separate paper.

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