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TOP MASS CORRECTIONS TO WEAK BOSON DECAYS

T. ALVAREZ, A. LEITES* and J. TERRÓN

Universidad Autónoma de Madrid, Departamento de Física Teórica, C-XI, 28049 Madrid, Spain

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We compute and study the tb and tt decays of the W and Z^0 to one QCD loop including finite mass corrections.

1. Introduction

A calculation of the second order QCD corrections to the decays $W \to t\bar{b}$ and $Z^0 \to t\bar{t}$ is presented. The calculation takes into account exactly the effects due to the finite mass of the top quark. This should be of interest in view of the foreseen accuracy of future experiments for Γ_{Z^0} [1], Γ_W [2], to which our corrections represent a sizeable contribution for the top ratio, 14% to 30% for top mass between 25 GeV and 40 GeV for the Z^0 and from 10% to 30% for top masses between 25 GeV and 70 GeV in the W case.

The paper is divided into two main parts. In the first part we will concentrate on the Z^0 case. We will compute its partial width coming from the channels $Z^0 \to t\bar{t}$ and $Z^0 \to t\bar{t}g$. Once we have this partial width, we will compare it with the previous approximated result given in [4]. In the second part we will follow the same steps as for the Z^0 case. For the W boson there exist the channels $W \to t\bar{b}(g)$, $W \to t\bar{s}(g)$, $W \to t\bar{d}(g)$ but we will only calculate the first one because the magnitude of the Kobayashi-Maskawa elements favor it, $V_{tb} = 0.999$, $V_{ts} = 0.045$ and $V_{td} = 0.010$, see [3]. Nevertheless, the contributions from the other channel, involving "down" and "strange" quarks could be obtained straightforwardly from the one we will calculate.

* Mailing address: DESY F1, Notkestraße 85, D 2000 Hamburg 52, FR Germany.

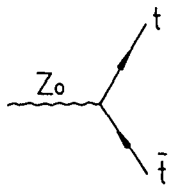

Fig. 1. Diagram of Z^0 decay to leading order in the standard model.

2. Z^0 decay

At the tree level the $Z^0 \to t\bar{t}$ width comes from the computation of the diagram showed in fig. 1 giving the well known result:

$$\Gamma_0(Z \to t\bar{t}) = \frac{\alpha_{em} M_{Z_0}}{4\sin^2 2\theta_w} \frac{\beta}{2} \left[(3-\beta^2) v^2 + 2\beta^2 a^2 \right], \tag{1}$$

where α_{em} is the electromagnetic coupling constant, $\beta = \sqrt{1 - 4m^2/M_{Z_0}^2}$ and m is the top quark mass supposed to be $m < \frac{1}{2}M_{Z_0}$. The constants v, a are defined so that the $Z^0\bar{t}t$ vertex is $ie\gamma^\mu(v + a\gamma_5)/2\sin 2\theta_w$, θ_w is the weak angle.

Next to leading terms in the perturbation expansion involve terms proportional to $\alpha_w\alpha_s$, $\alpha_w\alpha_{em}$ and α_w^2, where α_w, α_s and α_{em} are respectively the weak, strong and electromagnetic coupling constants. All these terms are represented in fig. 2 by means of their Feynman diagrams.

We have computed explicitly the QCD corrections, the diagrams of figs. 2a and 2b that contain gluon emission and gluon interchange, the corresponding QED ones can easily be obtained making trivial changes in the expression we will obtain; as will be indicated.

In the calculation of these corrections one has to deal with both ultraviolet and infrared divergences. For the first ones we will use dimensional regularization and the "on-shell" scheme for the renormalization procedure. For this specific calculation we only need to renormalize the quark propagator because the Z^0 propagator does not get corrections to this order.

Explicit expressions for the counterterms can be found in the appendix. To regularize infrared divergences we have given a fictitious mass (λ) to the gluon. With this prescription the only Feynman rule we have to modify is the one related with the gluon propagator. The new gluon propagator reads

$$a \overset{k}{\to} b, \qquad \Pi^{\mu\nu} = \frac{-ig^{\mu\nu}\delta_b^a}{k^2 - \lambda^2}. \tag{2}$$

Using a gluon mass λ one only regularizes soft gluon emission divergences, colinear ones are avoided with the top quark mass.

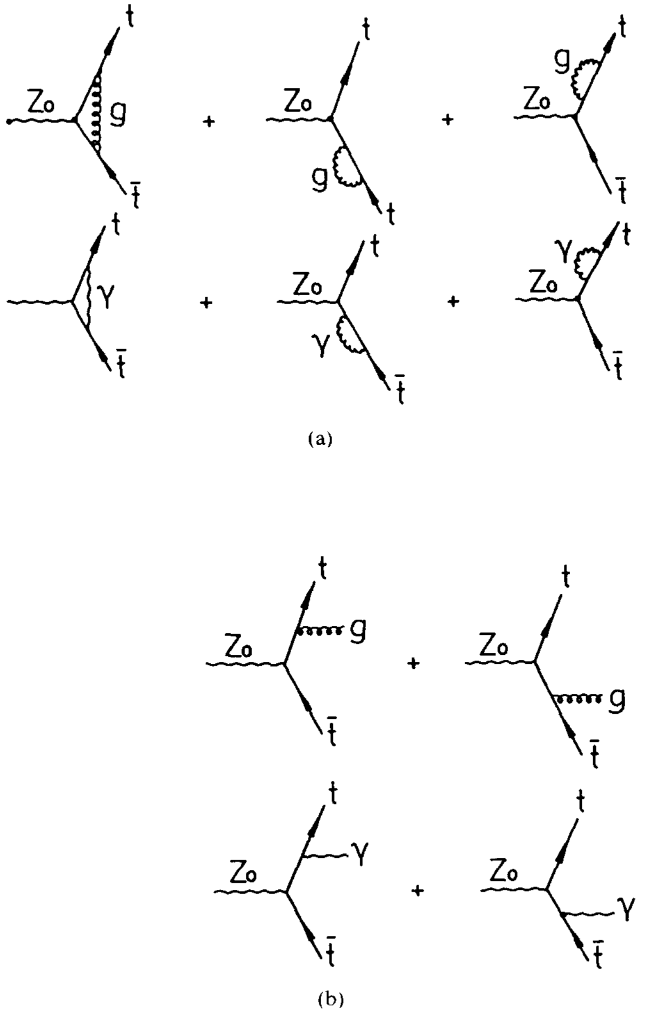

Fig. 2. Next to leading order in the Z^0 decay. (a) One QCD and QED loop diagrams, (b) Bremsstrahlung diagrams, (c) One-loop weak diagrams.

Thus as a consequence in intermediate computations, like phase-space or Feynman integrals, the limits $\lambda \to 0$ and $m \to 0$ neither commute nor are finite. The criterion that must be used solving all the integrals is to neglect always terms of the type λ^n with $n > 0$. Doing all the integrals and collecting all their results one obtains the final expression for the QCD correction, which is:

$$\Gamma_{\mathrm{QCD}}(\mathrm{Z} \to \mathrm{t\bar{t}} + \mathrm{Z} \to \mathrm{t\bar{t}g}) = \frac{\alpha_{\mathrm{em}} \alpha_{\mathrm{s}} C_{\mathrm{F}} M_{\mathrm{Z_0}}}{16\pi \sin^2 2\theta_{\mathrm{w}}} \left(v^2 A + a^2 B \right), \tag{3}$$

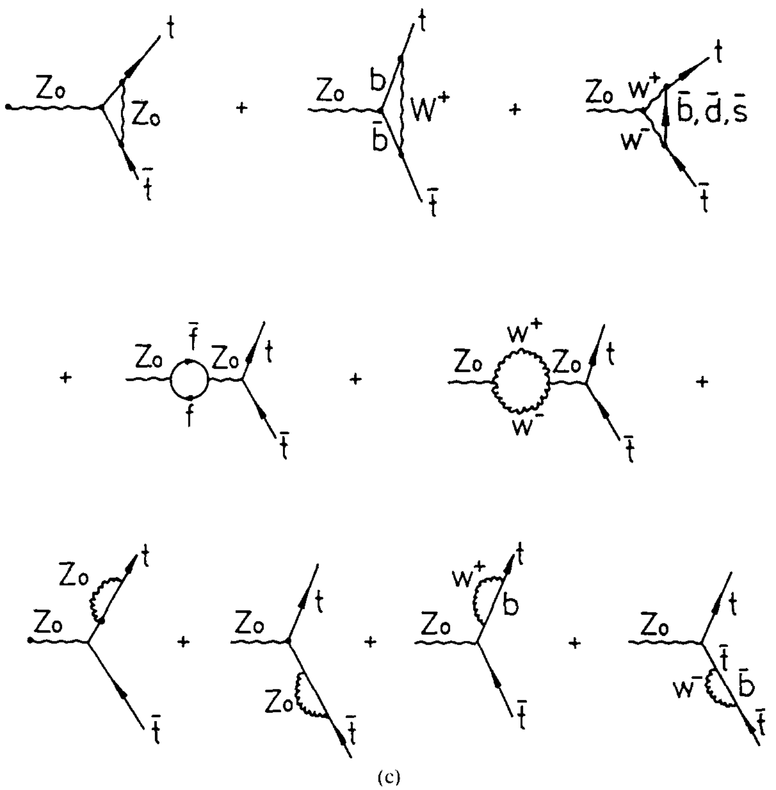

Fig. 2. (Continued)

where

$$A = 3\beta + \frac{1-\beta^2}{4}\left\{\frac{3}{2}\left[\left(\beta^2-2\right)^2+5\right]\log\frac{1+\beta}{1-\beta} + 3\beta\left(7-\beta^2\right)\right\} + 2\Xi\left(3-\beta^2\right),$$

$$B = 3\beta + \frac{1-\beta^2}{4}\left\{\frac{1}{2}\left[\left(\beta^2-9\right)^2-72\right]\log\frac{1+\beta}{1-\beta} - 2\beta\left(15+\beta^2\right)\right\} + 4\Xi\beta^2,$$

$$\begin{aligned}\Xi = & -4\beta\log\beta + 6\beta\log\frac{1+\beta}{2} + (1-\beta)(1-2\beta)\log\frac{1+\beta}{1-\beta}\\ & + \frac{1+\beta}{2}\left\{\frac{\pi^2}{3} + 2\,\mathrm{Li}_2(-\beta) - 2\,\mathrm{Li}_2(\beta) + \mathrm{Li}_2\left(\frac{1+\beta}{2}\right) - \mathrm{Li}_2\left(\frac{1-\beta}{2}\right)\right.\\ & \quad + 6\,\mathrm{Li}_2\left(\frac{1-\beta}{1+\beta}\right) + 4\,\mathrm{Li}_2\left(-\frac{1-\beta}{1+\beta}\right)\\ & \quad \left. + 5\log\frac{1+\beta}{2}\log\frac{1+\beta}{1-\beta} - 2\log\beta\log\frac{1+\beta}{1-\beta}\right\},\end{aligned} \tag{4}$$

where $C_F = \frac{4}{3}$ is the SU(3) color factor, $v = 1 - \frac{8}{3}\sin^2\theta_w$ and $a = 1$. For the QED correction, replace $\alpha_s C_F$ by $\frac{4}{9}\alpha_{em}$. The function $\mathrm{Li}_2(x)$ is the dilogarithm function and is defined in the appendix. Our exact result must be compared with the approximate one given in ref. [4]

$$(\Gamma_{QCD})_{(approx)} = \frac{\alpha_{em}\alpha_s C_F M_{Z_0}}{4\sin^2 2\theta_w}\frac{\beta}{2}\left\{(3-\beta^2)\left[\frac{\pi}{2\beta} - \frac{3+\beta}{4}\left(\frac{\pi}{2} - \frac{3}{4\pi}\right)\right]v^2 + 2\beta^2\left[\frac{\pi}{2\beta} - \left(\frac{19}{10} - \frac{22\beta}{5} + \frac{7\beta^2}{2}\right)\left(\frac{\pi}{2} - \frac{3}{4\pi}\right)\right]a^2\right\}. \quad (5)$$

They use the same approximation that Schwinger did in QED [5]. Both expressions give the same results when taking the limits $\beta \to 0$ and $\beta \to 1$.

$$\lim_{\beta\to 1}\Gamma_{QCD} = \lim_{\beta\to 1}(\Gamma_{QCD})_{(approx)} = \frac{\alpha_{em}M_{Z_0}}{4\sin^2 2\theta_w}\frac{\alpha_s}{\pi}(v^2 + \alpha^2),$$

$$\lim_{\beta\to 0}\Gamma_{QCD} = \lim_{\beta\to 0}(\Gamma_{QCD})_{(approx)} = \frac{\alpha_{em}M_{Z_0}}{4\sin^2 2\theta_w}\alpha_s\pi v^2. \quad (6)$$

In fig. 3 we have plotted both expressions (3) and (5), normalized to its value for $\beta = 1$ as functions of the top mass m, supposing that α_s is a constant. Both curves are very similar giving differences always less than 9%. It is important to notice that for top mass values between 30–40 GeV the differences with the massless correction are very large, about 50%. For the total width $Z^0 \to t\bar{t}g$ the difference between both results is always less than 1%, so from now on we will only use the exact expression we have calculated.

It is worth noting that these QCD corrections introduce a significant deviation of the width from the leading order if m approaches $\frac{1}{2}M_{Z_0}$, $(\beta \to 0)^\star$. It has been demonstrated [6] that the QCD corrected cross section coincides very well with the production cross section for $t\bar{t}$ resonances. One can thus be confident that general characteristics of the mass dependence of the partial width are adequately described by eq. (3) or eq. (5).

In fig. 4 we have plotted $\Gamma(Z \to t\bar{t}\,(g))$ (including the QCD and QED corrections) versus the top mass, showing the different contributions, e.g., the leading and next to leading terms. To illustrate the different contributions we will separate them in

* For higher order corrections, potentially divergent, one might hope that they sum up to modify the leading term only by a factor $(4\pi\alpha_s/3\beta)/(1 - e^{-4\pi\alpha_s/3\beta})$, similar to the result in QED.

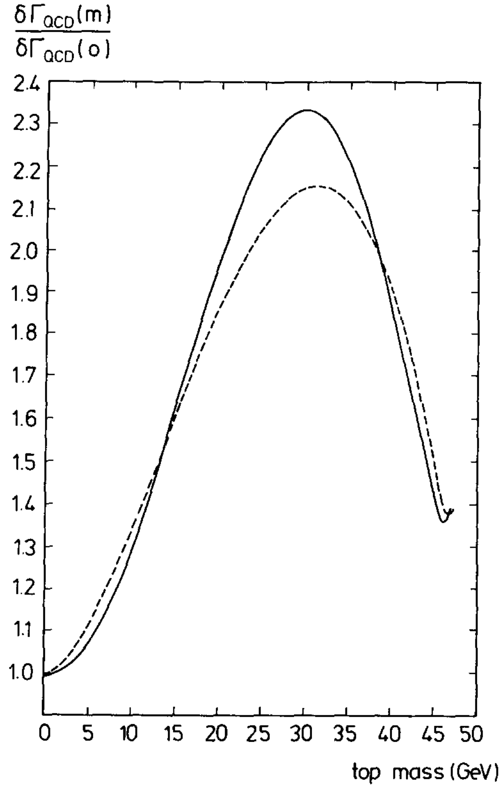

Fig. 3. Comparation between the exact and the approximate QCD correction to top ratio for Z^0 decay.

the following way

$$\Gamma = \Gamma_0\big(1 + \delta\Gamma_{\alpha_s}(0) + \delta\Gamma_{\alpha_s}(m_t) + \delta\Gamma_{\alpha_{em}}(0) + \delta\Gamma_{\alpha_{em}}(m_t)\big). \tag{7}$$

For $m_t = 35$ GeV, $\sin^2\theta_w = 0.22$, $\alpha_s = 0.14$ we get

$$\Gamma = 0.1413 \text{ GeV},$$

$$\Gamma_0 = 0.1089 \text{ GeV},$$

$$\Gamma_{QCD} = \Gamma_0\big(\delta\Gamma_{\alpha_s}(0) + \delta\Gamma_{\alpha_s}(m_t)\big) = 0.0318 \text{ GeV},$$

$$\Gamma_{QED} = \Gamma_0\big(\delta\Gamma_{\alpha_{em}}(0) + \delta\Gamma_{\alpha_{em}}(m_t)\big) = 0.0005 \text{ GeV},$$

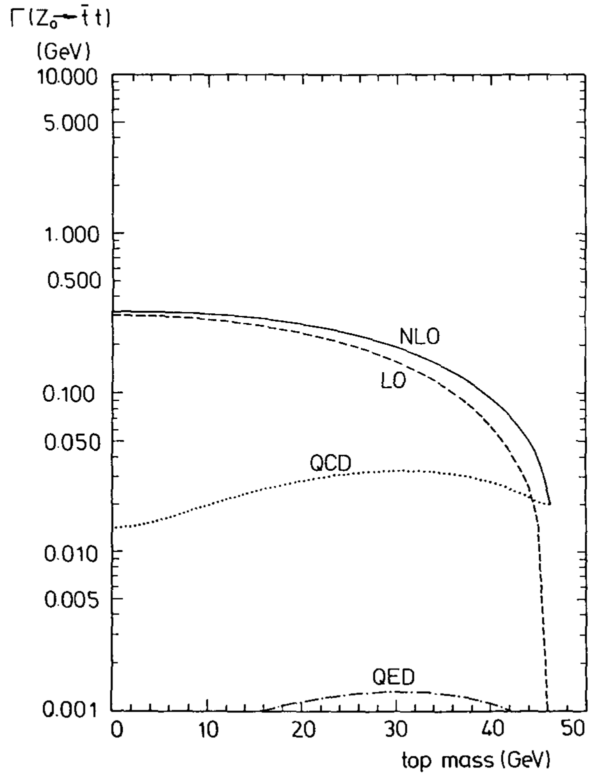

Fig. 4. Leading order (LO) and next to leading order (NLO) for top ratio in Z^0 decay.

with

$$\delta\Gamma_{\alpha_s}(0) = 0.1293,$$

$$\delta\Gamma_{\alpha_s}(m_t) = 0.1630,$$

$$\delta\Gamma_{\alpha_{em}}(0) = 0.0024,$$

$$\delta\Gamma_{\alpha_{em}}(m_t) = 0.0027. \qquad (8)$$

Thus the QCD corrections represent a sizeable contribution of the order of 22.5% while the QED ones are of the order of 0.4%.

In absolute numbers these QCD and QED corrections represent a difference of 33 MeV with respect to the leading term, a correction probably accessible for future experiments.

3. W^+ decay

To calculate the QCD corrections to the process $W^+ \to t\bar{b}(g)$, we have neglected the bottom mass, but we have remained it different from zero for the leading term ($m_b = 4.5$ GeV). The leading term for this partial width comes from the diagram of fig. 5 and it is found to be

$$\Gamma_0(W^+ \to t\bar{b}) = \frac{\alpha_{em} M_W}{8 \sin^2\theta_w} \sqrt{\frac{\left(M_W^2 - m_t^2 - m_b^2\right)^2 - 4m_b^2 m_t^2}{M_W^4}} \times \left(2 - \frac{m_t^2 + m_b^2}{M_W^2} - \frac{\left(m_t^2 - m_b^2\right)^2}{M_W^4}\right), \tag{9}$$

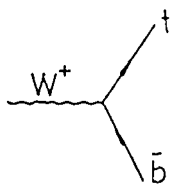

Fig. 5. Diagram of W^+ decay to leading order in the standard model.

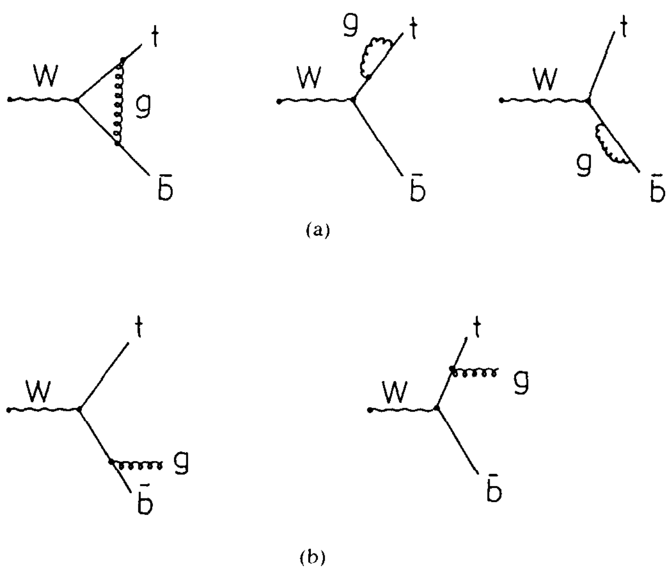

Fig. 6. Diagrams of the QCD correction to the top ratio of the W^+ decay. (a) One-loop QCD. (b) Bremsstrahlung.

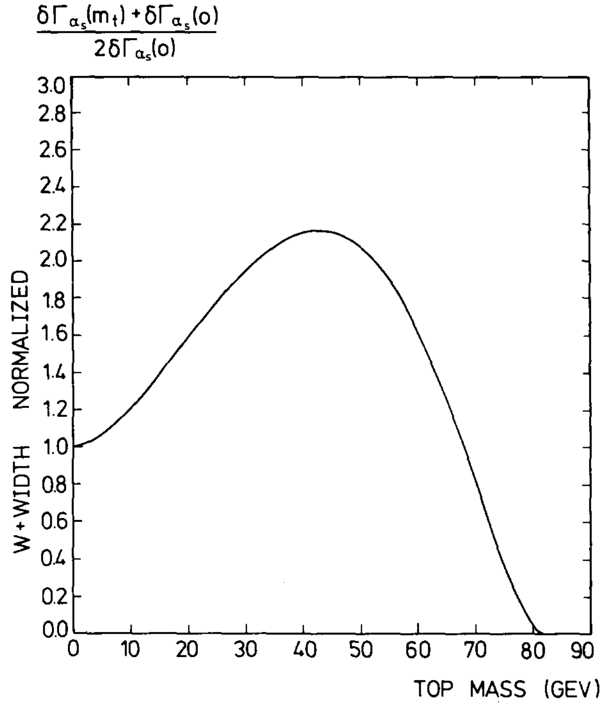

Fig. 7. QCD correction to $W^+ \to t\bar{b}$ normalized to its value for $m_t = 0$, $(\delta\Gamma_{\alpha_s}(m) + \delta\Gamma_{\alpha_s}(0))/2\delta\Gamma_{\alpha_s}(0)$.

where M_W, m_t and m_b are, respectively, the masses of the W boson, the top and the bottom quark. QCD corrections to the leading order involve the diagrams represented in fig. 6. Almost all the comments we have made for the Z^0 case are valid for the W^+ one except that now the fictitious gluon mass regularizes, besides the soft gluon emission divergences, the colinear divergences that appear due to the assumption of a massless bottom. We would like to make some emphasis in one of the terms that appear in the amplitude for the diagrams 8b, which is (following the notation of the appendix),

$$|T|^2_\lambda = -\frac{1}{2}\left(1-\frac{m^2}{s}\right)\frac{(2s+m^2)\lambda^2}{\left[\left(s+\lambda^2-s_1\right)-\left(s_3-m^2\right)\right]^2}. \tag{10}$$

Even though this term is proportional to λ^2, it must not be neglected because it gives the contribution

$$\Gamma_\lambda = -\frac{\alpha_{em}\alpha_s C_F M_W}{16\pi \sin^2\theta_w}\beta^4\frac{3-\beta^2}{2}, \tag{11}$$

with $\beta^2 = (M_W^2 - m^2)/M_w^2$, a contribution that is clearly different from zero. For the entire calculation we have followed the same procedure as in the Z^0 case.

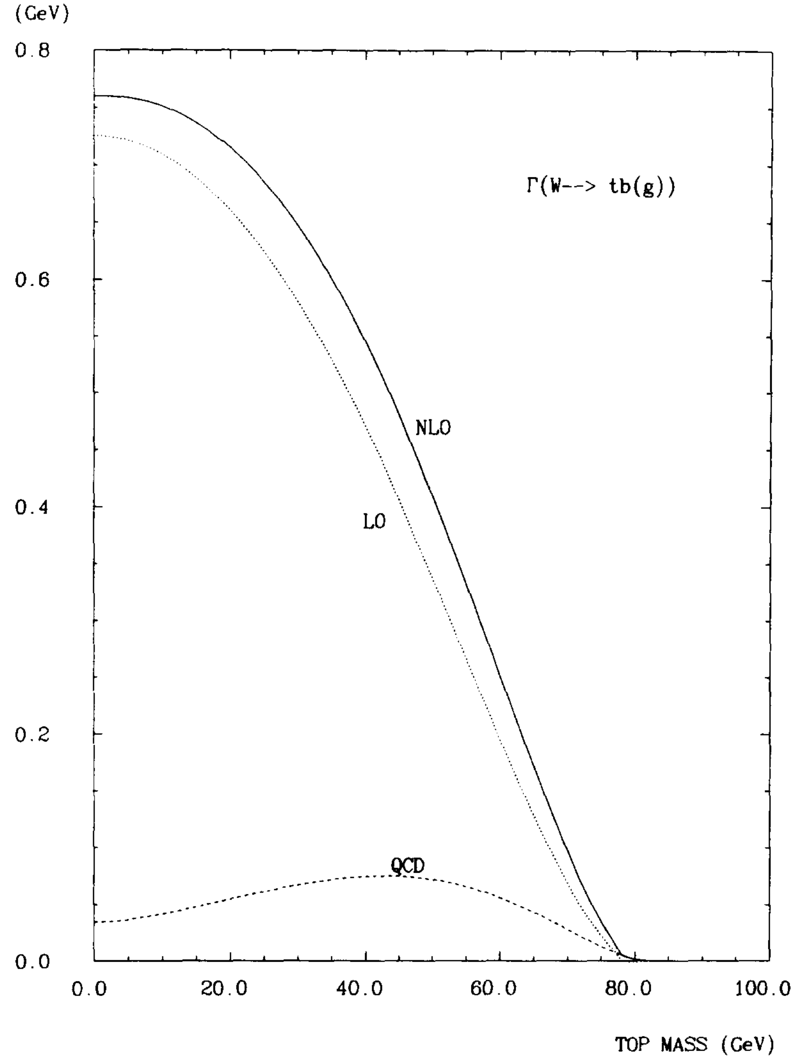

Fig. 8. Leading and next to leading order for top ratio in W^+ decay.

The final result for the QCD correction to this process is

$$\Gamma_{QCD}(W^+ \to t\bar{b} + W^+ \to t\bar{b}g)$$
$$= \frac{\alpha_{em}\alpha_s C_F M_W}{16\pi \sin^2\theta_w}\Big((\beta^2-4)(1-\beta^2)\beta^2 - (9-5\beta^2)\beta^4 \log\beta^2 - (1-\beta^2)$$
$$\times(4+6\beta^2-5\beta^4)\log(1-\beta^2) + (3-\beta^2)\beta^4$$
$$\times\big(\tfrac{3}{2} + 4\,\mathrm{Li}_2(1-\beta^2) + 2\log\beta^2 \log(1-\beta^2)\big)\Big). \quad (12)$$

It is very easy to check that the limit $m \to 0$ or $\beta \to 1$ gives the well known massless QCD correction.

$$\lim_{\beta \to 1} \Gamma_{QCD} = \frac{\alpha_{em} M_W \alpha_s}{4 \sin^2\theta_w \pi} . \tag{13}$$

In fig. 7 we have represented this QCD correction normalized to its value for $m = 0$ versus m. For values of m between 30 and 55 GeV the QCD corrections are very large, about 48% of difference with respect to the massless case.

In fig. 8 we have plotted the leading term with $m_b = 4.5$ GeV, the QCD correction and their sum versus the top mass for a constant value of $\alpha_s = 0.15$.

For top mass values around 45 GeV we find the largest corrections, being of the order of 16% which means a correction of 75 MeV for the width, that might be greater than the experimental errors [2]. We will illustrate this corrections expressing the width in the following way

$$\Gamma = \Gamma_0\left(1 + \delta\Gamma_{\alpha_s}(0) + \delta\Gamma_{\alpha_s}(m_t)\right). \tag{14}$$

As an example for $m_t = 45$ GeV, $\alpha_s = 0.15$, $\alpha_{em} = \frac{1}{128}$, $\sin^2\theta_w = 0.22$, one obtains

$$\Gamma = 0.481 \text{ GeV},$$

$$\Gamma_0 = 0.406 \text{ GeV},$$

$$\Gamma_{QCD} = \Gamma_0\left(\delta\Gamma_{\alpha_s}(0) + \delta\Gamma_{\alpha_s}(m)\right) = 0.075 \text{ GeV},$$

with

$$\delta\Gamma_{\alpha_s}(0) = 0.080,$$

$$\delta\Gamma_{\alpha_s}(m) = 0.104. \tag{15}$$

4. Conclusions

We have studied those decays of the weak bosons which involve the top quark, if kinematically possible, and have calculated the QCD corrections for these decays. For the Z^0 case we have also calculated the QED ones. We have shown that for both bosons and for top mass values around 35 GeV these corrections are rather large and might be accessible for future experiments.

In the Z^0 case previous approximated QCD corrections have been compared with our exact ones.

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Appendix

From the renormalized quark propagator in the "on-shell" scheme one obtains

$$Z_\psi = 1 + \frac{\alpha_s N_c C_F}{4\pi}\left(\bar{\gamma}_\varepsilon - \log\frac{m^2}{\nu^2} + 2\log\frac{\lambda^2}{m^2} + 4\right),$$

$$Z_{m_t} = 1 + \frac{\alpha_s N_c C_F}{4\pi}\left(3\bar{\gamma}_\varepsilon - 3\log\frac{m^2}{\nu^2} + 4\right), \tag{16}$$

for the top quark, and

$$Z_\psi = 1 + \frac{\alpha_s N_c C_F}{4\pi}\left(\bar{\gamma}_\varepsilon - \log\frac{\lambda^2}{\nu^2} - \frac{1}{2}\right),$$

$$Z_{m_b} = 1 \tag{17}$$

for the bottom quark. $N_c = 3$ is the color number, $C_F = (N_c^2 - 1)/2N_c = \frac{4}{3}$. The constant $\bar{\gamma}_\varepsilon$ is defined to be

$$\bar{\gamma}_\varepsilon = \frac{2}{(4-D)} - \log 4\pi + \gamma_\varepsilon . \tag{18}$$

$D = 4 - \varepsilon$ is the number of dimensions in the dimensional regularization and γ_ε is the Euler constant.

The dilogarithm function $\mathrm{Li}_2(x)$ is defined in the following way

$$\mathrm{Li}_2(x) = -\int_0^x \frac{\log(1-z)}{z}\,\mathrm{d}z . \tag{19}$$

Many properties and useful formulae related with this function could be found in ref. [7].

The explicit expression for the phase-space integral in the Z^0 case reads

$$\mathrm{d}\Gamma(Z \to t\bar{t}g) = \frac{\alpha_{em} C_F}{16\pi M_{Z_0} s \sin^2 2\theta_w}|T|^2\,\mathrm{d}s_2\,\mathrm{d}s_3 ,$$

$$|T|^2 = 4\left[(v^2+a^2) - 2m^2a^2\right]\left\{(s-2m^2)c_2c_3 - m^2(c_3^2+c_2^2)\right\} + \frac{8m^2}{s}a^2$$
$$-2s(v^2+a^2)\left\{(c_2+c_3)\left[2 - \frac{m^2}{s}\left(1+\frac{m^2}{s}\right)\right] - \frac{2}{s}\left(1+\frac{m^2}{s}\right)\right\}$$
$$+\frac{4m^2}{s}(v^2-a^2)\left\{6s(c_2+c_3) + \frac{c_2}{c_3} + \frac{c_3}{c_2}\right\} \tag{20}$$

in terms of the invariant variables $s_1 = (p_1 + p_2)^2 = (Q - k)^2$, $s_2 = (p_2 + k)^2 = (Q - p_1)^2$, $s_3 = (k + p_1)^2 = (Q - p_2)^2$ where the four vectors p_1, p_2, k and Q correspond to the top and the antitop quark, the gluon and the Z^0 weak boson respectively, and $c_i = (s_i - m^2)^{-1}$.

The limits of the integration variables are $s_3^- < s_3 < s_3^+$ and $(m + \lambda)^2 < s_2 < (\sqrt{s} - m)^2$, where

$$s_3^{\pm} = \lambda^2 - \frac{1}{2s_2}\left[\left(s_2 - s - \lambda^2\right)\left(s_2 + m^2\right) \mp K^{1/2}\left(s_2, s, \lambda^2\right)\left(s_2 - m^2\right)\right]. \quad (21)$$

$K(x, y, z)$ is the kinematical function defined as $K(x, y, z) = (x - y - z)^2 - 4yz$.

For the W case the bremsstrahlung contribution comes from the following integral

$$\mathrm{d}\Gamma\left(W^+ \to t\bar{b}g\right) = \frac{\alpha_{\mathrm{em}} C_F}{16\pi \sin^2\theta_w M_W s} |T|^2 \,\mathrm{d}s_1 \,\mathrm{d}s_3,$$

$$|T|^2 = -\left(1 - \frac{m^2}{s}\right)\left\{1 - \frac{(2s + m^2)}{\left(s + \lambda^2 - s_1\right) - \left(s_3 - m^2\right)} \times \left[\frac{s - s_3}{s_3 - m^2} - \frac{\lambda^2}{2\left(s + \lambda^2 - s_1\right) - \left(s_3 - m^2\right)}\right] + \frac{(2s + m^2)}{s_3 - m^2}\frac{s_3}{s_3 - m^2} - \frac{\left(s + \lambda^2 - s_1\right)^2}{2\left(\left(s + \lambda^2 - s_1\right) - \left(s_3 - m^2\right)\right)\left(s_3 - m^2\right)}\right\}$$
$$+ \frac{1}{2}\left(1 + \frac{s_3 - m^2 - \lambda^2}{\left(s + \lambda^2 - s_1\right) - \left(s_3 - m^2\right)} + \frac{s - 2m^2}{s}\frac{\left(s - s_1\right) - \left(s_3 - m^2\right)}{s_3 - m^2}\right), \quad (22)$$

where now the four vectors p_1, p_2, k and Q correspond to the top, the antibottom quark, the gluon and the W^+ weak boson respectively.

The limits of the integration variables are $s_3^- < s_3 < s_3^+$ and $m^2 < s_1 < (\sqrt{s} - \lambda)^2$, where

$$s_3^{\pm} = m^2 + \lambda^2 - \frac{1}{2s_1}\left[\left(s_1 - s + \lambda^2\right)\left(s_1 + m^2\right) \mp K^{1/2}\left(s_1, s, \lambda^2\right)\left(s_1 - m^2\right)\right]. \quad (23)$$

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