Macroscopic Quantum-Mechanical Contributions to Radiative Polarization in Electron Storage Rings

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We present a complete first-order quantum electrodynamics calculation of the spin polarization in a relativistic electron storage ring. The result differs from previous semiclassical calculations of other authors in several respects. Under ideal conditions a maximum polarization of 99.2% may be obtained.

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The emission of synchrotron radiation in electron storage rings can lead to radiative polarization of the electrons.[1] By systematically isolating the effect of the external (microscopic) electromagnetic fields from the effect of the emitted photon (radiative) fields, we calculate a new and completely general formula for the asymptotic value of polarization. This result exhibits explicitly how long-range coherence of the spin-wave function controls the polarization.

The complete Hamiltonian for an electron in a storage ring may be written[2] as $H = H^{\text{orbit}} + H^{\text{spin}}$, where ($\hbar = c = 1$ and we use the Coulomb gauge) $H^{\text{orbit}} = H_0^{\text{orbit}} - e\boldsymbol{\beta}\cdot\mathbf{A}^{\text{rad}}$ and $H_0^{\text{orbit}} = [(\mathbf{p} - e\mathbf{A}_{\text{ext}})^2 + m^2]^{1/2}$ (m refers to the electron rest mass, e to the electronic charge, and $\boldsymbol{\beta}$ to its velocity.) The vector potential $\mathbf{A}_{\text{ext}}$ refers to the external guide fields of the storage ring, while $\mathbf{A}^{\text{rad}}$ refers to the fields of the emitted synchrotron radiation photons. Only terms of the first order in $\mathbf{A}^{\text{rad}}$ are kept in deriving this expression. Similarly H^{spin} may be written as $H^{\text{spin}} = -\frac{1}{2}\boldsymbol{\sigma}\cdot\boldsymbol{\Omega}^{\text{orbit}} - \frac{1}{2}\boldsymbol{\sigma}\cdot\boldsymbol{\Omega}^{\text{rad}}$, where $\boldsymbol{\Omega}^{\text{orbit}}$ contains only the magnetic field on the closed orbit and $\boldsymbol{\Omega}^{\text{rad}}$ contains all effects on the spin which are driven by the radiation fields. This expression may include terms involving magnetic field gradients in the storage ring. In a recent paper[3] Bell and Leinaas have pointed out that, for times long after injection of electrons into a storage ring, all deviations of electron trajectories from the closed-orbit equilibrium are driven by the radiation fields of the emitted photons. All such radiation effects must be treated simultaneously to obtain the correct formula for the polarization.

Orbital Motion.—The orbital equations of motion for an electron in a storage ring are derived from the Hamiltonian H^{orbit} (with $\theta = \beta_0 ct/R$ as the independent variable):

$$\frac{1}{R^2}\left[\frac{d^2x}{d\theta^2} + 2\Gamma_x\frac{dx}{d\theta}\right] - K(\theta)x = \frac{e}{E_0}\left[F_x^{\text{rad}} + \frac{R}{\rho_x}\int_{-\infty}^{\theta}\boldsymbol{\beta}\cdot\mathbf{E}^{\text{rad}}d\theta'\right], \tag{1}$$

where $\mathbf{F}^{\text{rad}} \equiv \mathbf{E}^{\text{rad}} + \boldsymbol{\beta}\times\mathbf{B}^{\text{rad}}$ and $K(s) \equiv (e/p_0)\partial B_x^{\text{ext}}(s)/\partial z$. We have chosen a coordinate system in which $\hat{\mathbf{y}}$ is along $\boldsymbol{\beta}$, $\hat{\mathbf{x}}$ is along $-\dot{\boldsymbol{\beta}}$, and $\hat{\mathbf{z}} = \hat{\mathbf{x}}\times\hat{\mathbf{y}}$; ρ_x refers to the local radius of curvature in the x direction. E_0 and p_0 are the reference energy and momentum. In Eq. (1), we include the effect of radiation damping by introducing small terms proportional to $\Gamma_x dx/d\theta$ (Γ_x is an appropriate damping constant).

These equations can be solved in terms of the standard Twiss parameters α, β, γ of Courant and Snyder.[4] $\sqrt{\beta_x}\exp(\pm i\mu_x)$ is a solution of the homogeneous equation $d^2x/ds^2 - K(s)x = 0$. There is a similar equation for z. Setting $\delta^{\text{rad}}(\theta) \equiv (e/E_0)R\int_{-\infty}^{\theta}\boldsymbol{\beta}\cdot\mathbf{E}^{\text{rad}}d\theta'$ and noting that[5]

$$\eta_x = \frac{\sqrt{\beta_x}R}{2\sin\pi Q_x}\int_{\theta}^{\theta+2\pi}\frac{[\beta_x(\theta')]^{1/2}}{\rho_x(\theta')}\cos[Q_x\pi + \mu_x(\theta) - \mu_x(\theta')]d\theta',$$

we get

$$x(\theta) = \eta_x(\theta)\int_{-\infty}^{\theta}e^{-\Gamma_s(\theta-\theta')}\frac{d\delta^{\text{rad}}(\theta')}{d\theta'}\cos[Q_s(\theta-\theta')]d\theta'$$
$$-[\beta_x(\theta)]^{1/2}\int_{-\infty}^{\theta}\sqrt{\beta_x}\frac{d\delta^{\text{rad}}}{d\theta'}\left[\frac{\eta_x}{\beta_x}C_x + \left[\eta_x' + \frac{\alpha_x\eta_x}{\beta_x}\right]S_x\right]d\theta' + [\beta_x(\theta)]^{1/2}R\int_{-\infty}^{\theta}\sqrt{\beta_x}F_x^{\text{rad}}S_x\,d\theta', \tag{2}$$

$$S_x \equiv e^{-\Gamma_x(\theta-\theta')}\sin[\mu_x(\theta) - \mu_x(\theta')], \quad C_x \equiv e^{-\Gamma_x(\theta-\theta')}\cos[\mu_x(\theta) - \mu_x(\theta')].$$

We have replaced a continuous energy loss term $\eta\delta$ by the corresponding synchrotron oscillation term with tune Q_s and damping constant Γ_s. The factor e/E_0 has been absorbed into $F_x^{\rm rad}$ in Eq. (2).

From this expression for x in terms of integrals over radiation field operators, we calculated $\langle x^2\rangle$, using standard techniques[6] to evaluate vacuum expectation values of field-operator products as contour integrals. We obtained a result which agrees with the usual calculation,[7] plus additional small terms which are proportional to γ^{-1} and γ^{-2}. These small corrections arise from the $F_x^{\rm rad}$ term in Eq. (1). It will be shown below that a similar type of effect occurs for the spin. The polarization at an energy near a spin-orbit resonance can be significantly affected in some cases by $F_x^{\rm rad}$ and $F_z^{\rm rad}$.

Spin Hamiltonian.—The part of the spin Hamiltonian defined above depending only on the external guide fields is $-\frac{1}{2}\boldsymbol{\sigma}\cdot\boldsymbol{\Omega}^{\rm orbit}$, where

$$\boldsymbol{\Omega}^{\rm orbit}=-\frac{e}{m}\left[\left(a+\frac{1}{\gamma}\right)\mathbf{B}_{\rm ext}-\left(a+\frac{1}{\gamma+1}\right)\boldsymbol{\beta}\times\mathbf{E}_{\rm ext}-\frac{a\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\mathbf{B}_{\rm ext})\right].$$

$\mathbf{B}_{\rm ext}$ and $\mathbf{E}_{\rm ext}$ are the external electromagnetic fields along the closed orbit, $a=(g-2)/2$, and g is the electron magnetic moment. This term in the Hamiltonian leads to the usual Thomas-Bargmann-Michel-Telegdi equation[8] for the spin precession on the closed orbit. The fact that only closed-orbit fields appear in $\boldsymbol{\Omega}^{\rm orbit}$ is a direct consequence of the fact that all deviations away from the closed orbit are due to radiation emission and thus must be considered as part of $\boldsymbol{\Omega}^{\rm rad}$. We write

$$H_{\rm rad}^{\rm spin}=-\tfrac{1}{2}\boldsymbol{\sigma}\cdot(\boldsymbol{\Omega}_{\rm vac}+\boldsymbol{\Omega}_{\rm traj}+\boldsymbol{\Omega}_{\delta\beta});$$

$\boldsymbol{\Omega}_{\rm vac}$ is the Bargmann-Michel-Telegdi expression with $\mathbf{B}_{\rm ext}$, $\mathbf{E}_{\rm ext}$ replaced by $\mathbf{B}_{\rm rad}$, $\mathbf{E}_{\rm rad}$; $\boldsymbol{\Omega}_{\rm traj}$ is due to the deviation in fields experienced because of trajectory deviations from the closed orbit in the presence of magnetic field gradients, while $\boldsymbol{\Omega}_{\delta\beta}=\sum_i(\partial\boldsymbol{\Omega}_{\rm vac}/\partial\beta_i)\delta\beta_i$ represents the change in $\boldsymbol{\Omega}_{\rm vac}$ due to changes in $\boldsymbol{\beta}$ because of photon emission. To first order in deviations from the closed orbit, in a strong-focusing machine $\boldsymbol{\Omega}_{\rm traj}$ arises because of quadrupole fields and hence can be set equal to $-(e/m)\times(a+\gamma^{-1})G_Q(\theta)(z\hat{\mathbf{i}}-x\hat{\mathbf{k}})$.

Radiative Polarization.—We use $H^{\rm spin}$ to calculate the electron polarization after many revolutions of the electron around the storage ring. A nonzero polarization results, because, as first calculated by Sokolov and Ternov,[1] the rates of spin flip up and down are not equal. A number of corrections to the Sokolov-Ternov formula in the case of inhomogeneous storage-ring fields have been derived by Derbenev and Kondratenko,[2] Chao,[9] Chao and Yokoya,[10] and Bell and Leinaas.[3] By noting that the entire $\boldsymbol{\sigma}\cdot\boldsymbol{\Omega}^{\rm rad}$ drives spin-flip transitions, a general expression may be obtained which reduces to the earlier calculations for special cases. It will be clear from the result that this can only be derived within the framework of a quantum-mechanical calculation.

The transition rates, in which we sum over photon polarization (a) and momentum (k), have the form

$$\Gamma_\pm\equiv\frac{1}{T_0}\sum_{a,k}\left|\int_{-T_0/2}^{T_0/2}d\tau\langle\chi_\pm(\tau)\,|\,\tfrac{1}{2}\boldsymbol{\sigma}\cdot\boldsymbol{\Omega}^{\rm rad}(\tau)\,|\,\chi_\mp(\tau)\rangle\right|^2. \tag{3}$$

T_0 is a time which can be taken as arbitrarily long. The χ's satisfy the Schrödinger equation $-id\chi_\pm/ds=H_0^{\rm spin}\chi_\pm$, with $H_0^{\rm spin}$ as defined above. We also require that the χ's, which are two-component spinors, be eigenvectors of the operator $\boldsymbol{\sigma}\cdot\mathbf{n}$: $\boldsymbol{\sigma}\cdot\mathbf{n}\chi_\pm=\pm\chi_\pm$. This condition can be satisfied if $d\mathbf{n}/d\theta=\boldsymbol{\Omega}^{\rm orbit}\times\mathbf{n}$. Since $\boldsymbol{\Omega}^{\rm orbit}$ is periodic around the ring, we can choose $\mathbf{n}(\theta)$ to be a periodic reference axis for the spin.[11] We note that $\mathbf{n}(\theta)$ is not necessarily the direction of polarization. A convenient way to parametrize the spinors is $\chi_\pm(\theta)=\exp\{-i\xi(\theta)[\boldsymbol{\sigma}\cdot\mathbf{n}(\theta)/2]\}\hat{\chi}_\pm(\theta)$;

$$\hat{\chi}_+(\theta)=\begin{pmatrix}\exp(-\frac{1}{2}i\phi_s)\cos\frac{1}{2}\Theta_s\\ \exp(\frac{1}{2}i\phi_s)\sin\frac{1}{2}\Theta_s\end{pmatrix},\quad \hat{\chi}_-(\theta)=\begin{pmatrix}-\exp(-\frac{1}{2}i\phi_s)\sin\frac{1}{2}\Theta_s\\ \exp(\frac{1}{2}i\phi_s)\cos\frac{1}{2}\Theta_s\end{pmatrix}. \tag{4}$$

$\chi_\pm(\theta)$ are periodic functions of θ, since Θ_s and ϕ_s are taken to be the polar angles of $\mathbf{n}(\theta)$. In order for $\chi_\pm$ to satisfy the Schrödinger equation, the phase factor ξ must satisfy the differential equation $d\xi/d\theta=\boldsymbol{\Omega}^{\rm orbit}\cdot(\mathbf{n}+\mathbf{n}_0)/(1+\mathbf{n}\cdot\mathbf{n}_0)$, where $\mathbf{n}_0=\mathbf{n}(0)$. We set $\xi(\theta+2\pi)-\xi(\theta)=2\pi(\nu_{\rm sp}+1)$ and identify $\nu_{\rm sp}$ with the spin tune. With the fact that the integrand in Eq. (3) is periodic except for a factor $e^{\mp i\xi(\theta)}$, the expression for the spin-flip transition rates is

$$\Gamma_\pm=\frac{1}{2\pi}\int_0^{2\pi}\tilde{\Gamma}_\pm(\theta')d\theta'. \tag{5}$$

To calculate the integral we approximate the spinor by a constant during photon emission: $\xi(\theta'+\frac{1}{2}\theta)\simeq\xi(\theta'-\frac{1}{2}\theta)$. The integral becomes

$$\tilde{\Gamma}_\pm(\theta')=\frac{1}{4}\sum_{j,k}N_{jk}^\pm\int_{-\infty}^{\infty}d\theta\langle 0\,|\,\Omega_j^{\dagger\,\rm rad}(\tfrac{1}{2}\theta)\Omega_k^{\rm rad}(-\tfrac{1}{2}\theta)\,|\,0\rangle,\quad N_{jk}^\pm=\delta_{jk}-n_jn_k\mp i\epsilon_{jkl}n_l. \tag{6}$$

$\mathbf{\Omega}^{\rm rad}$ is proportional to the photon fields. We evaluate the vacuum expectation values by standard methods.[12]

To do the final integral over θ, the integrand is expanded in powers of γ^{-1}. We resort to the computer program SMP[13] to keep track of the algebra, which is quite complicated. Remarkable cancellations take place, and the coefficients of 1, γ^{-1}, γ^{-2}, and γ^{-3} are all zero. In addition, all terms resulting from $\mathbf{\Omega}_{\delta\beta}$ give no contribution to the leading order, which is γ^{-4}. The final result is

$$\tilde{\Gamma}_{\pm}(\theta')=\Gamma_0(P_{\rm even}\pm P_{\rm odd}),$$
$$\Gamma_0=\frac{5\sqrt{3}}{8}\omega_0^3\gamma^5\frac{e^2}{m^2}=\frac{5\sqrt{3}}{8}\left[\frac{r_e\bar{\lambda}_c}{2\pi}\right]\gamma^5\frac{c}{|\rho|^3}. \quad (7)$$

r_e is the classical electron radius, $\omega_0=c/|\rho|$. The polarization can then be calculated, by use of detailed balance, to be

$$P=\frac{\int_0^{2\pi}(P_{\rm odd}/\rho^3)\,d\theta'}{\int_0^{2\pi}(P_{\rm even}/|\rho|^3)\,d\theta'}. \quad (8)$$

For horizontal bends

$$P_{\rm odd}=-(8/5\sqrt{3})\{w-{\rm Im}[(g_x^*+g_z^*)(\langle\sigma_z^\updownarrow\rangle+\tfrac{2}{3}f_x)-\tfrac{1}{6}f_z^*\langle\sigma_y^\updownarrow\rangle]\}, \quad (9)$$

$$P_{\rm even}=1-\tfrac{2}{9}v^2+\tfrac{11}{18}|g_x+g_z|^2-\tfrac{4}{9}{\rm Re}(f_x^*\langle\sigma_z^\updownarrow\rangle)-\tfrac{1}{18}{\rm Re}(f_z^*\langle\sigma_x^\updownarrow\rangle)+\tfrac{29}{180}|f_x|^2+\tfrac{13}{360}|f_z|^2, \quad (10)$$

$$\langle\sigma_x^\updownarrow\rangle\equiv\hat{\chi}_+^\dagger\sigma_x\hat{\chi}_-=\frac{wu+iv}{(1-w^2)^{1/2}},\quad \langle\sigma_y^\updownarrow\rangle\equiv\hat{\chi}_+^\dagger\sigma_y\hat{\chi}_-=\frac{wv-iu}{(1-w^2)^{1/2}},\quad \langle\sigma_z^\updownarrow\rangle\equiv\hat{\chi}_+^\dagger\sigma_z\hat{\chi}_-=-(1-w^2)^{1/2}. \quad (11)$$

u, v, w are the direction cosines of $\mathbf{n}$ with the z axis vertical and the y axis along the beam direction. The functions $f_{x,z}$ and $g_{x,z}$ are periodic in θ with period 2π, and

$$f_z(\theta)=2ai\{b_z^+D^+(z)-b_z^-D^-(z)\}, \quad (12)$$

$$g_z(\theta)=(\nu_{\rm sp}+1)\{a_zb_z^+D^+(z)+a_z^*b_z^-D^-(z)+c_z^+D^+(s)+c_z^-D^-(s)\}, \quad (13)$$

in which

$$D^{\pm}(\alpha)=\{1-\exp(-2\pi\Gamma_\alpha)\exp[2\pi i(\nu_{\rm sp}\pm Q_\alpha)]\}^{-1},\quad \alpha=x,z,s;$$

$$a_z=\eta_z/\beta_z-i(\eta_z'+\alpha_z\eta_z/\beta_z),$$

$$b_z^{\pm}=-\tfrac{1}{2}e^{-i[\xi(\theta)\pm\mu_z(\theta)]}R\sqrt{\beta_z}\int_\theta^{\theta+2\pi}\sqrt{\beta_z}K\langle\sigma_x^\updownarrow\rangle e^{i(\xi\pm\mu_z)}d\theta', \quad (14)$$

$$c_z^{\pm}=-\tfrac{1}{2}e^{-i[\xi(\theta)\pm Q_s\theta]}R\int_\theta^{\theta+2\pi}\eta_zK\langle\sigma_x^\updownarrow\rangle e^{i(\xi\pm Q_s\theta')}d\theta'. \quad (15)$$

The formulas for g_x and f_x for horizontal bends can be obtained by the substitutions $g_z\to g_x$ and $f_z\to f_x$ if $K\to -K$ and x is interchanged everywhere with z in Eqs. (12) through (15). The expressions for vertical bends can be obtained by the substitutions $f_{x,z}\to -f_{z,x}$, $g_{x,z}\to -g_{z,x}$ as well as $\langle\sigma_x^\updownarrow\rangle\to -\langle\sigma_z^\updownarrow\rangle$ and $\langle\sigma_z^\updownarrow\rangle\to\langle\sigma_x^\updownarrow\rangle$ in Eqs. (8) through (10) but not in (14) and (15). In $P_{\rm odd}$, w should be replaced by u.

The f terms arise from the Lorentz-force terms $\mathbf{F}^{\rm rad}$ and are the contributions to polarization arising from the electron recoiling in the photon field, while the g terms arise from the energy-loss terms $\delta^{\rm rad}$ [see Eq. (1)].

Normally the relative strength of the g's and the f's is of the order of $f_z/g_z\sim\beta_z/\gamma\eta_z$. In some realistic cases this ratio can be $\simeq 1$ in a machine where η_z is due to alignment errors. In general, terms involving f_z cannot be neglected *a priori*.[14] Because of our use of $\langle\sigma_{x,z}^\updownarrow\rangle$, these expressions are exact in spin to this order of the perturbation expansion of the Hamiltonian. No approximations are made for the spin motion as in the semiclassical treatments of Chao[9] and Chao and Yokoya.[10]

Discussion.— In the case of a perfectly planar storage ring the result is equivalent to that of Chao and Yokoya as modified by Bell and Leinaas. In this case $\langle\sigma_x^\updownarrow\rangle=1$, $\langle\sigma_z^\updownarrow\rangle=0$, and $\eta_z=\eta_z'=0$, everywhere in the machine. Only the f_z's are nonzero, and we obtain the Bell-Leinaas calculation generalized to a strong-focusing machine. If the f's were zero, the formula would reduce formally to the expression calculated by Derbenev and Kondratenko. Generally the integrals involving $b^{\pm}$ and $c^{\pm}$ play the same role as the spin-diffusion integrals of Chao and Yokoya, up to phase factors which may be important in this case. If the storage ring is designed so that all of the $b^{\pm}$'s and $c^{\pm}$'s are zero, then all f's and g's are zero also, and (without reverse bends or spin rotators) the polarization equals the Sokolov-Ternov limit of $8/5\sqrt{3}=92.4\%$. Our calculation thus can be used to define "spin-transparency" conditions which are identical to those defined by Chao.[10,15]

The presence of interference terms involving f, g, and fg in P_{odd} leads to the possibility of using the resonance behavior of f and g to increase the polarization beyond 92.4%. It can be shown that the maximum possible polarization in the case where $g_{x,z}=0$ is 99.2%. This is implicit in the work by Bell and Leinaas,[3] but we have extended their result to a nonplanar strong-focusing machine without x-z coupling. Our calculation also shows that with a very simple generalization of the formulas presented above, time-dependent electromagnetic fields could be incorporated in the theory and, for example, radio-frequency quadrupoles tuned to $\nu_{\text{sp}} \pm Q_{x,z}$ times the revolution frequency could be used to alter the f's and g's without significant effects on the orbital motion. Only further investigation will reveal whether such a technique will be of practical use in actual storage rings.

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[13]SMP (Symbolic Manipulation Program) is a program copyrighted by the Inference Corporation.

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[15]This was elegantly proved to us by Professor J. Buon (private communication).