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How exotic is the a_0 (980)?

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Abstract. The mass and hadronic width of the scalar isovector meson $a_0(980)$ are estimated in QCD for two possible quark assignments: (*a*) $\bar{q}q$ and (*b*) $\bar{q}q\bar{q}q$. The two-photon width of the $a_0(980)$ is also discussed.

1 Introduction

The quark structure of the scalar isovector meson $a_0(980)$ [1] (previously called δ (980)) is still a matter of debate and speculation on account of the difficulties to explain all of its properties within a single assignment. The naive quark model suggests a $\bar{q}q$ interpretation but other classifications have been proposed, e.g. that the $a_0(980)$ could be a four-quark state [2] or even a $K\bar{K}$ "molecule" [3]. QCD sum rule estimates of the $a_0(980)$ mass and couplings [4], considerations based on $\eta' \to \eta\pi\pi$ [5], and $a_0(980)$ photoproduction [6], appear to support the naive quark model assignment. However, it has been claimed that some constituent $\bar{q}q$ potential models predict hadronic [7] and two-photon widths [3b] which are too large to comply with the experimental data. On the other hand, the problem of the $a_0(980)-f_0(975)$ near degeneracy together with the weak coupling of $f_0(1350)$ to strange quarks has prompted the suggestion of a $q^2\bar{q}^2$ classification which was studied in the framework of the MIT bag model [2].

In this paper we wish to reexamine both the $\bar{q}q$ and the $q^2\bar{q}^2$ interpretations of the $a_0(980)$ by estimating its hadronic and two-photon widths in the framework of QCD duality sum rules. This method is by now well established and thus we shall not go into details here (for a recent review see e.g. [8]). We only wish to stress that one of the advantages of this framework is that the QCD bound state spectrum can be directly related (through dispersive integrals) to the fundamental QCD Lagrangian parameters. These are the strong coupling constant α_s, the current quark masses and a set of quark and gluon vacuum condensates which appear in the Operator Product Expansion (OPE) of current correlators. These vacuum condensates characterize the distinctive features of confinement in QCD, among them e.g. spontaneous symmetry breaking, without any reference to the concept of potential for light quarks. One of the main differences with previous analyses done in the same framework [4a, 9] is that here we exploit the long distance realization of QCD in order to fix the normalization of the hadronic spectral functions pertinent to both the $\bar{q}q$ and the $q^2\bar{q}^2$ cases. When this fundamental constraint is imposed on a finite-width, Breit–Wigner resonance parametrization of the spectral functions we are left with only two physical parameters to determine: the mass M_{a_0} and the hadronic width Γ_{a_0}. Also, we use in our analysis both Laplace transform [10] and Finite Energy Sum Rules (FESR) [11]. This allows for a more systematic and stringent determination of M_{a_0} and Γ_{a_0}. In particular, in the $\bar{q}q$ case we use a new FESR [12] which uniquely relates the hadronic width to the sum of the light current quark masses.

The paper is organized as follows. In Sect. 2 we discuss the determination of M_a and Γ_a in the $\bar{q}q$ assignment, and in Sect. 3 we do it in the $q^2\bar{q}^2$ case. In Sect. 4 we estimate the two-photon width of the $a_0(980)$ ($\bar{q}q$) using Vector Meson Dominance and three-point function QCD sum rules; we argue that the results do not provide enough compelling evidence against a $\bar{q}q$ assignment. Finally, in Sect. 5 we summarize our results.

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2 Mass and hadronic width of the $a_0(980)$ in the $\bar{q}q$ assignment

We begin by considering the two-point function associated to the vector current divergence with $I^G = 1^-$, $J^P = 0^+$ quantum numbers

$$\psi(q^2) = i \int d^4x e^{iqx} \langle 0 | T(\partial^\mu V_\mu(x) \partial^\nu V_\nu^\dagger(0)) | 0 \rangle, \tag{1}$$

where

$$\partial^\mu V_\mu(x) = i(m_d - m_u) : \bar{d}(x) u(x) :, \tag{2}$$

and $m_{u,d}$ are the "current" quark masses. The function $\psi(q^2)$ satisfies the dispersion relation ($Q^2 \equiv -q^2 > 0$)

$$\psi(Q^2) = \frac{1}{\pi} \int_0^\infty ds \frac{\mathrm{Im}\,\psi(s)}{s + Q^2} + \text{subtractions}. \tag{3}$$

In the limit $Q^2 \to 0$, $\psi(Q^2)$ obeys the well known current algebra low energy theorem based on Ward identities

$$\psi(0)|_{\mathrm{QCD}} = -(m_u - m_d) \langle \bar{u}u - \bar{d}d \rangle + \psi(0)|_{\mathrm{A.F.}}, \tag{4}$$

where $\psi(0)_{\mathrm{A.F.}}$ is the purely perturbative contribution to $\psi(0)$. Taking $Q^2 = 0$ in (3) the asymptotic freedom piece of (4) cancels exactly with the corresponding piece in $\mathrm{Im}\,\psi(s)$ and one obtains the following FESR [12]

$$\psi(0)|_{\mathrm{N.P.}} \equiv -(m_u - m_d) \langle \bar{u}u - \bar{d}d \rangle = \int_0^{s_0} \frac{ds}{s} \frac{1}{\pi} \mathrm{Im}\,\psi(s) - \frac{3}{8\pi^2} [m_d(s_0) - m_u(s_0)]^2 \cdot s_0 [1 + R_1(s_0)], \tag{5}$$

where S_0 is the asymptotic freedom threshold, and the radiative corrections $R_n(s_0)$ are given by

$$R_n(s_0) = \frac{\alpha_s(s_0)}{\pi} \cdot \left[\frac{17}{3} + \frac{2}{n} - \frac{2}{\beta_1} \left(\gamma_2 - \gamma_1 \frac{\beta_2}{\beta_1} \right) + 2\gamma_1 \frac{\beta_2}{\beta_1^2} \ln \ln \frac{s_0}{\Lambda^2} \right] \tag{6}$$

with

$$\alpha_s(s_0) = \frac{2\pi}{-\beta_1 \ln(s_0/\Lambda^2)}, \tag{7}$$

and $\beta_1 = -11/2 + n_f/3$, $\beta_2 = -51/4 + 19\, n_f/12$, $\gamma_1 = 2$, $\gamma_2 = 101/12 - 5\, n_f/18$, ($n_f$ is the number of quark flavours). The non-perturbative value of $\psi(0)$ in the lhs of (5) is expected to be very small [8, 12, 13]. Notice that in addition to the explicit $SU(2)$ flavour symmetry breaking factor $(m_u - m_d)$, it contains the piece $\langle \bar{u}u - \bar{d}d \rangle$ which measures $SU(2)$ breaking in the QCD vacuum. Since we are interested here in estimating the hadronic width of the $a_0(980)$ we can safely neglect the lhs of (5) and obtain

$$\int_0^{s_0} \frac{ds}{s} \frac{1}{\pi} \mathrm{Im}\,\psi(s) \simeq \frac{3}{8\pi^2} [m_d(s_0) - m_u(s_0)]^2 \cdot s_0 [1 + R_1(s_0)]. \tag{8}$$

Turning to the spectral function appearing in (8), the lowest two-particle states contributing to it are $\eta\pi$ and $K\bar{K}$ which involve the matrix elements $|\langle 0 | \bar{u}d | \eta\pi \rangle|^2$ and $|\langle 0 | \bar{u}d | K\bar{K} \rangle|^2$, respectively. The threshold behaviour of the spectral function can be derived in several ways, e.g. using the long distance realization of QCD (chiral Lagrangian) [12, 13b]; through the connection to the $K^+ - K^0$ non-electromagnetic or "tadpole" mass difference [14]; or by means of soft-meson techniques and quark model commutators to reduce π, η, etc. from the external states. They all lead to the same answer which reads

$$\frac{1}{\pi} \mathrm{Im}\,\psi(s)|_{\mathrm{thr.}} \cong \frac{1}{16\pi} \mu_\pi^4 \left(\frac{m_d - m_u}{m_d + m_u} \right)^2 \cdot \{ \tfrac{2}{3} [(1 - s_+/s)(1 - s_-/s)]^{1/2} \theta(s - s_+) + (1 - 4\mu_K^2/s)^{1/2} \theta(s - 4\mu_K^2) \}, \tag{9}$$

where $S_\pm = (M_\eta \pm \mu_\pi)^2$. Imposing this threshold behaviour on a Breit–Wigner resonance form we obtain the following parametrization of the spectral function

$$\frac{1}{\pi} \mathrm{Im}\,\psi(s) = \frac{1}{24\pi^2} \mu_\pi^4 \left(\frac{m_d - m_u}{m_d + m_u} \right)^2 \cdot \{ [(1 - s_+/s)(1 - s_-/s)]^{1/2} \theta(s - s_+) + \tfrac{3}{2} (1 - 4\mu_K^2/s)^{1/2} \theta(s - 4\mu_K^2) \} \cdot \frac{(s_+ - M_{a_0}^2)^2 + M_{a_0}^2 \Gamma_{a_0}^2}{(s - M_{a_0}^2)^2 + M_{a_0}^2 \Gamma_{a_0}^2}, \tag{10}$$

where M_{a_0} and Γ_{a_0} are the mass and hadronic width of the $a_0(980)$, respectively, and we have normalized the Breit–Wigner form at the physical threshold $S_+ = (M_\eta + \mu_\pi)^2$ in order to account for the finite η and π masses consistently.

Inserting (10) into (8) the quark mass difference cancels out and we obtain

$$I(s_0) \equiv \int_0^{s_0} \frac{ds}{s} \frac{1}{[(s - M_{a_0}^2)^2 + M_{a_0}^2 \Gamma_{a_0}^2]} \cdot \{ [(1 - s_+/s)(1 - s_-/s)]^{1/2} \theta(s - s_+) + \tfrac{3}{2} (1 - 4\mu_K^2/s)^{1/2} \theta(s - 4\mu_K^2) \} \cong 9 \frac{[m_d(s_0) + m_u(s_0)]^2}{\mu_\pi^4} \frac{s_0 [1 + R_1(s_0)]}{(s_+ - M_{a_0}^2)^2}. \tag{11}$$

Using the experimental value of M_{a_0}, this equation determines the width Γ_{a_0} in terms of just the fundamental QCD parameters m_q and Λ, as well as the asymptotic freedom threshold S_0. The dependence on S_0 is a typical feature of FESR and predictions are meaningful only if they are stable against reasonable changes in S_0 within the so called duality region [12]. In the present case this means that the hadronic lhs of (11) should match the QCD rhs in a hopefully wide region of values for S_0. In Fig. 1 we show the behaviour of the rhs of (11) (vertical bars) versus S_0 for $\Lambda = 100$ MeV and $(m_u + m_d)(1\,\mathrm{GeV}) = (15.5 \pm 1.0)$ MeV [12]. The solid curves show the behaviour of the lhs of (11) for the two extreme values $\Gamma_{a_0} = 40$ MeV

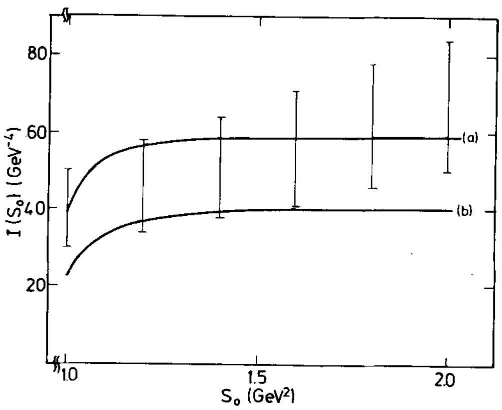

Fig. 1. The QCD rhs of (11) (vertical bars) and the hadronic lhs for $\Gamma_{a_0} = 40$ MeV (curve a) and $\Gamma_{a_0} = 60$ MeV (curve b)

(curve a), $\Gamma_{a_0} = 60$ MeV (curve b). Width values outside this range are not dual to QCD. As one may appreciate from Fig. 1 there is a wide duality region for $S_0 \simeq 1$–$2\,\text{GeV}^2$ which allows us to predict

$$\Gamma_{a_0} = (50 \pm 10)\,\text{MeV}, \tag{12}$$

in nice agreement with the experimental value $\Gamma_{a_0}|\text{exp} = (54 \pm 7)$ MeV.

The above determination of the width required the use of the experimental value of the $a_0(980)$ mass. In order to assess the reliability of this determination we proceed to estimate M_{a_0} in the same framework by using the additional FESR [12]

$$\int_0^{s_0} ds \frac{1}{\pi} \text{Im}\, \psi(s) = \frac{3}{16\pi^2} s_0^2 [m_d(s_0) - m_u(s_0)]^2 \cdot [1 + R_2(s_0) + 2C_4\langle O_4 \rangle / s_0^2], \tag{13}$$

where

$$C_4\langle O_4 \rangle = \frac{\pi}{3}\langle \alpha_s G^2 \rangle + 4\pi^2(m_u + m_d)\langle \bar{u}u \rangle. \tag{14}$$

Taking the ratio between (13) and (8) and making a narrow-width approximation in (10), well justified in view of (12), we find

$$M_{a_0}^2 \simeq \tfrac{1}{2} s_0 \left[\frac{1 + R_2(s_0) + 2C_4\langle O_4 \rangle / s_0^2}{1 + R_1(s_0)} \right] \tag{15}$$

Although the actual value of the gluon condensate appearing in (14) is somewhat controversial (for recent reviews see e.g. (8, 15]) it does not have a major impact here, and we take the conservative range $C_4\langle O_4 \rangle \simeq 0.05 - 0.15\,\text{GeV}^4$. Solving (15) we find M_{a_0} to be rather stable in the duality region $s_0 = 1$–$2\,\text{GeV}^2$, viz

$$M_{a_0} \simeq 0.8 - 1.0\,\text{GeV}, \tag{16}$$

which is in good agreement with experiment.

It is possible to estimate Γ_{a_0} in still another way by using the Laplace transform version of (5). Including vacuum condensates up to dimension $d = 6$ this sum rule reads

$$\int_0^{\infty} \frac{ds}{s} e^{-s\sigma} \frac{1}{\pi} \text{Im}\, \psi(s) = \psi(0) + \frac{3}{8\pi^2} \frac{\left[m_d\left(\frac{1}{\sigma}\right) - m_u\left(\frac{1}{\sigma}\right) \right]^2}{\sigma} \cdot \left\{ 1 + \frac{\alpha_s(1/\sigma)}{\pi} \left[\frac{17}{3} - \gamma_1 \psi(1) - 4\frac{\beta_2}{\beta_1^2} \ln\ln(\sigma\Lambda^2) - \frac{4}{\beta_1\gamma_1}(\gamma_2 - \gamma_1\beta_2/\beta_1) \right] + C_4\langle O_4 \rangle \sigma^2 + C_6\langle O_6 \rangle \sigma^3/2 + \cdots \right\} \tag{17}$$

where $\psi(1)$ is the digamma function, $C_4\langle O_4 \rangle$ was defined in (14), and the leading contribution to $C_6\langle O_6 \rangle$ comes from the four-quark condensate, which in the vacuum saturation approximation is given by [10]

$$C_6\langle O_6 \rangle|_{\text{v.s.}} = -\frac{2816}{81}\pi^3 \alpha_s \langle \bar{q}q \rangle^2. \tag{18}$$

Present uncertainties in $C_6\langle O_6 \rangle$ have no sizable influence here and we use $C_6\langle O_6 \rangle \simeq -0.04\,\text{GeV}^6$. The integral in the lhs of (17) is split as usual into two pieces, viz. in the interval $0 < s < s_0$ the spectral function is given by the hadronic parametrization (10), and for $s > s_0$ it takes the asymptotic freedom form which can be calculated using two-loop perturbative QCD [16]. Here the influence of s_0 is exponentially suppressed and thus the results are not so sensitive to this parameter, which we choose in the range $s_0 \simeq 1$–$2\,\text{GeV}^2$.

Neglecting $\psi(0)$ as before, and using the experimental value of M_{a_0} we have solved (17) inside the "sum rule window" $\sigma \simeq 0.5 - 1.3\,\text{GeV}^{-2}$ and obtained

$$\Gamma_{a_0} = (30 - 50)\,\text{MeV}, \tag{19}$$

which nicely confirms our previous estimate (12) based on the FESR.

In view of our results for M_{a_0} and Γ_{a_0} we conclude that the observed experimental mass and hadronic width of the $a_0(980)$ are not incompatible with the $\bar{q}q$ assignment.

3 Mass and hadronic width of a $q^2\bar{q}^2$ state

We consider the two-point function

$$\pi(q^2) = i\int d^4x\, e^{iqx} \langle 0 | T(J_E(x) J_E^\dagger(0)) | 0 \rangle, \tag{20}$$

where $J_E(x)$ is a local current built from four quark fields and which when acting on the vacuum creates the $E = \bar{q}q\bar{q}q$ colour singlet, spin zero state. In principle, several combinations in colour and spin of the quark fields could be used to construct the current operator $J_E(x)$. Following [9] we use the following combination

$$J_E = \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d)\bar{s}s + \frac{1}{\sqrt{2}}(\bar{u}i\gamma_5 u - \bar{d}i\gamma_5 d)\bar{s}i\gamma_5 s, \quad (21)$$

which, in addition to having the desired quantum numbers, has the important property of being free of troublesome $\ln(-q^2)/\varepsilon$ singularities ($d = 4 - \varepsilon$ is the number of space-time dimensions in the dimensional regularization scheme).

We begin by discussing the parametrization of the hadronic spectral function associated to the current operator (21). As in the $\bar{q}q$ case the lowest lying two-particle intermediate states are $\eta\pi$ and $\bar{K}K$ but now they involve the transition matrix elements $\langle 0|J_E|\eta\pi\rangle$ and $\langle 0|J_E|K\bar{K}\rangle$ which are not known a priori. In order to fix the scale of these matrix elements we have used soft-meson techniques and evaluated the resulting commutators of the axial-vector charges with the operator J_E by means of standard quark model commutation rules. In this way, and after appropriate Fierz rearrangements we have expressed the required matrix elements, in the $SU(3) \times SU(3)$ limit, in terms of vacuum expectation values of four quark operators. Finally, assuming vacuum saturation the latter can be related to the familiar quark condensate $\langle \bar{q}q\rangle$. After this lengthy but straightforward procedure the threshold behaviour of the hadronic spectral function is given by

$$\frac{1}{\pi}\operatorname{Im}\pi(s)|_{\text{thr.}} \cong \frac{3}{8\pi^2}\left(\frac{\langle \bar{q}q\rangle}{f_\pi}\right)^4 \cdot\{[(1 - s_+/s)(1 - s_-/s)]^{1/2}\theta(s - s_+) + \tfrac{1}{6}(1 - 4\mu_K^2/s)^{1/2}\theta(s - 4\mu_K^2)\}, \quad (22)$$

where we assumed also the flavour $SU(3)$ limit $\langle \bar{q}q\rangle = \langle \bar{u}u\rangle = \langle \bar{d}d\rangle = \langle \bar{s}s\rangle$, and $f_\pi = f_k = f_\eta = 93$ MeV. As usual, finite Goldstone boson masses are accounted for by using physical thresholds in (22). We point out that the procedure described above, always in the chiral limit, would lead to vanishing matrix elements in the case of colour octet-colour octet $\bar{q}q$ combinations. This case is an alternative to the singlet–singlet choice made in (21) to construct J_E. Imposing the threshold behaviour (22) on a Breit–Wigner resonance form and using the PCAC relation: $-\mu_\pi^2 f_\pi^2 = (m_u + m_d)\langle \bar{q}q\rangle$ we obtain

$$\frac{1}{\pi}\operatorname{Im}\pi(s) = \frac{3}{8\pi^2}\frac{\mu_\pi^4}{(m_u + m_d)^2}\langle \bar{q}q\rangle^2 \cdot\{[(1 - s_+/s)(1 - s_-/s)]^{1/2}\theta(s - s_+) + \tfrac{1}{6}(1 - 4\mu_K^2/s)^{1/2}\theta(s - 4\mu_K^2)\} \cdot\frac{(s_+ - M_E^2)^2 + M_E^2\Gamma_E^2}{(s - M_E^2)^2 + M_E^2\Gamma_E^2}, \quad (23)$$

where M_E, Γ_E are the mass and width of the lowest resonance state in (20), which we proceed to estimate and then compare with the mass and width of the $a_0(980)$.

The large Q^2 behaviour of (19) in QCD was obtained in [9] to lowest order in perturbation theory, with power corrections up to dimension $d = 10$, and to leading order in the strange quark mass m_s, with the result

$$\pi(Q^2) = -\ln(Q^2/\nu^2) \cdot[C_0 Q^8 + C_2 Q^6 + C_4\langle O_4\rangle Q^4 + C_6\langle O_6\rangle Q^2] + \frac{C_{10}\langle O_{10}\rangle}{Q^2}, \quad (24)$$

where

$$C_0 = 1/40960\pi^6 \cong 2.5 \times 10^{-8}, \quad (25.a)$$

$$C_2 = m_s^2/1024\pi^6 \cong (3.5 - 4.5) \times 10^{-8}\,\text{GeV}^2 \quad (25.b)$$

$$C_4\langle O_4\rangle = \frac{m_s\langle \bar{s}s\rangle}{128\pi^4} + \frac{\langle \alpha_s G^2\rangle}{64\pi^5} \cong (2 - 5) \times 10^{-6}\,\text{GeV}^4 \quad (25.c)$$

$$C_6\langle O_6\rangle = -\frac{m_s}{128\pi^4}\langle g_s\bar{s}\sigma\lambda G s\rangle - \frac{3}{8\pi^2}(\langle \bar{u}u\rangle^2 + \langle \bar{s}s\rangle^2) \cong -7.5 \times 10^{-6}\,\text{GeV}^6 \quad (25.d)$$

$$C_{10}\langle O_{10}\rangle = \tfrac{8}{3}m_s\langle \bar{s}s\rangle\langle \bar{u}u\rangle^2 \cong -(3 - 5) \times 10^{-7}\,\text{GeV}^{10}. \quad (25.e)$$

The numbers quoted in (25) were obtained using: m_s (1 GeV) $= 200 \pm 50$ MeV, $\langle \bar{u}u\rangle \simeq \langle \bar{s}s\rangle \simeq -0.01\,\text{GeV}^3$ (at a scale of 1 GeV), $\pi\langle \alpha_s G^2\rangle/3 \simeq 0.04 - 0.1\,\text{GeV}^4$, and $\langle g_s\bar{s}\sigma\lambda G s\rangle = 2M_0^2\langle \bar{s}s\rangle$ with $M_0^2 \simeq 0.5\,\text{GeV}^2$. Uncertainties in these QCD parameters induce lesser uncertainties in M_E and Γ_E and will not affect our conclusions.

Notice that in the present case, in contrast with the $\bar{q}q$ alternative, the spectral function contains in addition to the purely perturbative piece a number of non-perturbative vacuum condensates, viz

$$\frac{1}{\pi}\operatorname{Im}\pi(s)|_{\text{QCD}} = C_0 s^4 - C_2 s^3 + C_4\langle O_4\rangle s^2 - C_6\langle O_6\rangle s. \quad (26)$$

In any case, we have checked that with the choice of parameters as in (25) there is no contradiction with positivity. The Laplace transform QCD sum rules read

$$\int_0^{s_0} ds\, e^{-s\sigma}\frac{1}{\pi}\operatorname{Im}\pi(s) = 24\frac{C_0}{\sigma^5}(1 - f_4(s_0)) - 6\frac{C_2}{\sigma^4}(1 - f_3(s_0)) + 2\frac{C_4\langle O_4\rangle}{\sigma^3}(1 - f_2(s_0))$$

$$-\frac{C_6\langle O_6\rangle}{\sigma^2}(1-f_1(s_0))+C_{10}\langle O_{10}\rangle, \tag{27}$$

$$\int_0^{s_0} dss e^{-s\sigma}\frac{1}{\pi}\mathrm{Im}\,\pi(s)=120\frac{C_0}{\sigma^6}(1-f_5(s_0))$$
$$-24\frac{C_2}{\sigma^5}(1-f_4(s_0))+6\frac{C_4\langle O_4\rangle}{\sigma^4}(1-f_3(s_0))$$
$$-2\frac{C_6\langle O_6\rangle}{\sigma^3}(1-f_2(s_0)), \tag{28}$$

where

$$f_n(s_0)\equiv e^{-s_0\sigma}\sum_{k=0}^{n}\frac{(s_0\sigma)^k}{k!}$$
$$f_n(0)=1, \tag{29}$$

arising from the integration of the asymptotic freedom piece, i.e.

$$\int_u^{\infty} x^n e^{-\sigma x}dx=\frac{e^{-u\sigma}}{\sigma^{n+1}}n!\sum_{k=0}^{n}\frac{(u\sigma)^k}{k!} \tag{30}$$

We have written two sets of QCD sum rules in order to estimate both M_E and Γ_E. To check the consistency of the results it is useful to consider also FESR. Since the OPE has been truncated at dimension $d=10$ we can only write the single FESR

$$\int_0^{s_0} ds\frac{1}{\pi}\mathrm{Im}\,\pi(s)=C_{10}\langle O_{10}\rangle+C_0 s_0^5/5-C_2 s_0^4/4$$
$$+C_4\langle O_4\rangle s_0^3/3-C_6\langle O_6\rangle s_0^2/2 \tag{31}$$

Solving the Laplace transform QCD sum rules (27)–(28) inside the window $\sigma\cong(0.3-1.1)\,\mathrm{GeV}^{-2}$ and for values of s_0 in the range $s_0\cong(1-1.5)\,\mathrm{GeV}^2$, we find the remarkable stable value

$$M_E=(0.9-1.0)\,\mathrm{GeV}, \tag{32}$$

which is practically insensitive to changes in the values of the QCD parameters. Concerning the width, for a given value of s_0 and a given set of QCD parameters it is absolutely stable against changes in σ. It exhibits, though, some dependence on s_0 and on the QCD vacuum condensates. For instance, for fixed values of the latter Γ_E varies about 20% in the region $s_0=(1-1.5)\,\mathrm{GeV}^2$, while for fixed s_0 a change in the QCD parameters at the level of 30–40% induces a change in Γ_E of roughly the same amount. Taking into account all these uncertainties our prediction for the width is

$$\Gamma_E\approx(200-330)\,\mathrm{MeV}. \tag{33}$$

Next, using M_E as input we have solved the FESR (31) for Γ_E. Changing s_0 within the same range as before induces changes in Γ_E at the level of 10%. However, the values of the condensates have now a slightly bigger impact on Γ_E. All things considered we obtain

$$\Gamma_E\approx(200-360)\,\mathrm{MeV}, \tag{34}$$

which is nicely consistent with the Laplace estimate (33).

It should be clear, in view of our results for Γ_E, that a zero-width parametrization of the hadronic spectral function is not a sensible approximation. Given the rather large width of this $q^2\bar{q}^2$ state one could also question the choice of a Breit–Wigner parametrization. However, we can safely conclude that the interpretation of the $a_0(980)$ as a single pure $q^2\bar{q}^2$ state is inconsistent with the observed width $\Gamma_{a_0}=(54\pm7)\,\mathrm{MeV}$.

4 Two photon width of the $a_0(980)$ in the $\bar{q}q$ assignment

The two-photon width of the $a_0(980)$ has been measured recently by the Cristal Ball collaboration [17] with the result

$$\Gamma(a_0\to\gamma\gamma)\times\mathrm{BR}(a_0\to\eta\pi)$$
$$=\left(0.19\pm0.07\begin{matrix}+0.10\\-0.07\end{matrix}\right)\mathrm{keV}. \tag{35}$$

The theoretical description of two-photon meson decays through constituent quark annihilation diagrams coupled to $\bar{q}q$ hadronic bound state wave-functions, though successful for $\pi^0\to\gamma\gamma$ leads to a gross overestimate [18], i.e.

$$\Gamma(a_0\to\gamma\gamma)\cong1.5-3.8\,\mathrm{keV}, \tag{36}$$

A priori, this result appears to provide a serious motivation to cast doubt on the $\bar{q}q$ assignment of the $a_0(980)$. However, to assess how compelling this argument really is, one should keep in mind that quark model estimates of this kind are definitely guaranteed to work for $\pi^0\to\gamma\gamma$ on account of PCAC and the triangle anomaly. This is not necessarily the case for $a_0\to\gamma\gamma$ as these powerful theoretical concepts do not directly apply to scalars. In fact, in some alternative approaches to the quark model $a_0\to\gamma\gamma$ is predicted to be suppressed relative to the result in (36) even within the $\bar{q}q$ assignment [19].

We wish to argue here that some suppression of $a_0\to\gamma\gamma$ can also be obtained in the framework of QCD sum rules for vertex functions. Our arguments will closely follow those of [20]. However, our interpretation of the results is somewhat different and lead us to a much weaker conclusion against the $\bar{q}q$ assignment of the $a_0(980)$.

We begin by using Vector Meson Dominance (VMD), as illustrated in Fig. 2, to express the ratio of the $a_0\to\gamma\gamma$ and the $\pi^0\to\gamma\gamma$ widths

$$\frac{\Gamma(a_0\to\gamma\gamma)}{\Gamma(\pi^0\to\gamma\gamma)}=\frac{M_{a_0}^3}{\mu_\pi^3}\left(\frac{g_{\omega\rho a_0}}{g_{\omega\rho\pi}}\right)^2 \tag{37}$$

where $g_{\omega\rho a_0}$ and $g_{\omega\rho\pi}$ are the corresponding strong coupling constants, and the vector meson-photon couplings have cancelled in the ratio. Also, non-negligible off-mass shell corrections are expected to cancel out in this ratio [21]. Since $\Gamma(\pi^0\to\gamma\gamma)$ is well

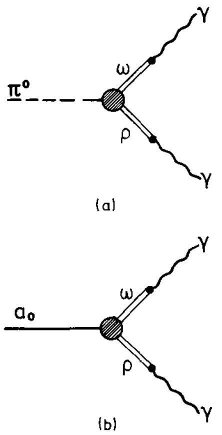

Fig. 2. Vector Meson Dominance diagrams for $\pi^0 \to \gamma\gamma$ and $a_0 \to \gamma\gamma$

known from experiment the problem then reduces to the estimate of the ratio of coupling constants in (37). This may be done e.g. by using three-point function QCD sum rules. This technique, essentially a generalization of the familiar procedure for two-point functions, allows one to relate trilinear hadronic couplings to the fundamental QCD parameters α_s, m_q and $\langle \bar{q}q \rangle$. However, due to increased complications over the two-point functions case, the intrinsic uncertainties are now expected to be bigger and to limit somewhat the accuracy of the estimates. With this due reservation in mind we proceed to adopt the version of vertex QCD sum rules proposed in [4b]. This yields, to leading order in the quark masses,

$$\left| \frac{g_{\omega\rho a_0}}{g_{\omega\rho\pi}} \right| \simeq \frac{(m_d - m_u)}{\frac{1}{2}(m_d + m_u)} \left(\frac{f_\pi \mu_\pi^2}{f_{a_0} M_{a_0}^2} \right). \tag{38}$$

The leptonic decay constant f_{a_0} appears in (38) on account of the fact that in this framework one has to treat resonances in the zero-width approximation from the outset. Referring back to (1)–(2) one has

$$\begin{aligned} \langle 0 | \partial^\mu V_\mu | a_0 \rangle &= i(m_d - m_u) \langle 0 | \bar{d} u | a_0 \rangle \\ &= \sqrt{2} f_{a_0} M_{a_0}^2, \end{aligned} \tag{39}$$

and

$$\frac{1}{\pi} \operatorname{Im} \psi(s) = 2 f_{a_0}^2 M_{a_0}^4 \delta(s - M_{a_0}^2). \tag{40}$$

Making a narrow-width approximation to the spectral function (10) and comparing with (40) one finds, to leading order in the quark masses as in (38),

$$f_{a_0} \simeq \frac{1}{4\sqrt{3\pi}} \mu_\pi^2 \left(\frac{m_d - m_u}{m_d + m_u} \right) \frac{1}{\sqrt{M_{a_0} \Gamma_{a_0}}}. \tag{41}$$

Using (41) in (38) leads to the prediction

$$\left| \frac{g_{\omega\rho a_0}}{g_{\omega\rho\pi}} \right| \simeq 8\sqrt{3\pi} \frac{f_\pi}{M_{a_0}} \sqrt{\frac{\Gamma_{a_0}}{M_{a_0}}} \simeq 0.5. \tag{42}$$

With $\Gamma(\pi^0 \to \gamma\gamma) = (7.8 \pm 0.4)$ eV from experiment, and BR $(a_0 \to \eta\pi) \approx 0.8$ as quoted in [1.a] and as found here in the $\bar{q}q$ assignment (cf. Sect. 2), we obtain from (37) and (42)

$$\Gamma(a_0 \to \gamma\gamma)\, \mathrm{BR}\,(a_0 \to \eta\pi) \approx 0.6\,\mathrm{keV}, \tag{43}$$

which is quite smaller than the non-relativistic quark model prediction (36). Given the intrinsic uncertainties of the QCD sum rule estimate (38), the error in (43) could be as large as 50%. With this in mind, we believe that a comparison with the experimental value (35) does not provide enough compelling evidence against the $\bar{q}q$ classification of the $a_0(980)$. On the other hand, using similar techniques the two-photon width of the $a_0(980)$ in the $q^2\bar{q}^2$ assignment has been predicted to be at the level of $(2-5) \times 10^{-4}$ keV [20], which is far too small to be reconciled with the data.

5 Summary

We have estimated here the mass, the hadronic and the two-photon width of the $a_0(980)$ scalar-isovector meson in the framework of QCD sum rules. A clear advantage of this approach is that particle masses and widths may be directly related to fundamental QCD parameters, i.e. α_s, m_q, and various vacuum matrix elements of quark and gluon fields. The latter characterize the fundamental features of quark confinement, such as e.g. spontaneous symmetry breaking, and are thus expected to be responsible for the rich and varied resonance structure observed at low energies. The only place where model-dependent parameters could eventually appear is in the parametrization of the hadronic spectral function. However, we have used here the realization of QCD at long distances (chiral Lagrangian) to get rid of this model dependency.

Assuming a $\bar{q}q$ assignment of the $a_0(980)$ we found $M_{a_0} \approx (0.8 - 1.0)$ GeV and $\Gamma_{a_0} = (50 \pm 10)$ MeV using FESR, as well as $\Gamma_{a_0} = (40 \pm 10)$ MeV using Laplace transform QCD sum rules. In the case of a $q^2\bar{q}^2$ interpretation we obtained, instead, $\Gamma_{a_0} \approx (200 - 360)$ MeV, which is clearly incompatible with the data.

Using QCD sum rules for three-point functions, together with VMD, we estimated the two-photon width of the $a_0(980)$ $(\bar{q}q)$. The result is somewhat bigger than the experimental value, but given the uncertainties of this method we do not find enough compelling evidence against the $\bar{q}q$ assignment. On the other hand, $\Gamma(E \to \gamma\gamma)$ in the $q^2\bar{q}^2$ case appears to be too small by several orders of magnitude [20].

We conclude that the observed mass and widths of the $a_0(980)$ can be understood in QCD if this particle is predominantly a $\bar{q}q$ state. We cannot rule out, however, admixtures of more exotic components, e.g. $q^2\bar{q}^2$. From this point of view the real nature of the $a_0(980)$, and in general the observed scalars, may still remain an open problem in meson spectroscopy.

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