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Electromagnetic corrections to deep inelastic scattering at HERA

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Abstract. The dominant electromagnetic radiative corrections to neutral current deep inelastic scattering at HERA energies have been calculated analytically in collinear approximation for unpolarized beams. The results may be used for the construction of an event generator and are applied to discuss the influence of cuts on the magnitude of the correction. For final state collinear photon emission the measurement of the resulting electromagnetic jet (electron plus collinear photon) reduces x- and y-dependence of the correction factor. Electromagnetic corrections to charged current interactions are discussed in leading logarithmic approximation.

1 Introduction

For a conclusive comparison of QCD predictions with results of deep inelastic scattering at HERA energies electroweak radiative corrections have to be taken into account. Large 1-loop corrections would require the inclusion of higher order terms; strong variations over the accessible kinematical range may cause systematic errors because of the finite resolution of detectors. In certain kinematical regions the corrections may be reduced by applying cuts on the momenta of the current-quark jet and/or the emitted photon. For detailed studies of event rates to be expected for a given detector (geometry, resolution) Monte–Carlo event generators are needed for sampling kinematically complete events.

For event simulation by Monte–Carlo methods the matrix element squared has to be numerically calculable with a reasonable accuracy in the whole phase space. There are, however, kinematical regions where different very large contributions to the cross section cancel each other to a large extent. A treatment of those cancellations on the numerical level may give numerically unstable results. To avoid that problem we isolate the dominant contributions and treat them analytically applying approximations appropriate to the corresponding kinematical situations. This procedure exhibits the characteristic behaviour of the non-integrated cross section allowing for an appropriate choice of variables for sampling events in generators. It also enables us to study the influence of some typical experimental cuts like the one on the energy of soft photons or on energy and angle of hard collinear photons.

We differ from older analysis of electromagnetic [1] and electro-weak [2] radiative corrections and two independent recent calculations [3, 4] by providing the required analytic results for the dominating electromagnetic contribution. Numerically all groups now essentially agree on magnitude and shape of these corrections. However, other groups only give results which are fully integrated over the whole phase space of the bremsstrahlung photon.

The outline of the paper is the following: In the next section we discuss the collinear approximation applied and give the analytical results for neutral-current reactions. Numerical results are presented in Sect. 3. There we also discuss the influence of cuts on magnitude and shape of the electromagnetic correction, and consider in leading logarithmic approximation the effect of real hard photon radiation for charged current reactions. Finally a brief summary of the results is given in Sect. 4.

2 Collinear approximation to deep-inelastic neutral-current *ep* scattering

We apply the collinear approximation in the following way: Soft photons are integrated over up to an energy

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cutoff ε; the integration over hard collinear photons is restricted to a small opening angle between the photon momentum and the momenta of the incident and scattered electron, resp. These kinematical regions give the infrared and mass singularities dominating the behaviour of electromagnetic corrections from the electron line. Power corrections in the cutoff parameters are neglected. The collinear and infrared singularities from the quark line should be absorbed into the definition of the structure functions, only slightly modifying their Q^2-dependence [5]. Otherwize, unphysical parameters (quark masses) appear in the final answer. It is most convenient to exploit the remaining freedom in the definition of structure functions in such a way that all the corrections from the quark line are included. Thus no quark line corrections would have to be applied to the extraction of structure functions from neutral current reactions. This problem will be discussed in more detail in [5]. Furthermore, in neutral current reactions box diagrams and interference between photon emission from electron and quark lines do not contain collinear singularities and, therefore, can be handled directly by a Monte Carlo procedure. So we can restrict our discussion to the gauge invariant subset of electromagnetic contributions from the electron line alone.

The following three sources of bremsstrahlung photons connected to the electron line dominate the electromagnetic corrections:

i) soft and virtual photons,

ii) hard photons collinear with the incoming electron,

iii) hard photons collinear with the scattered electron.

The 1-loop correction $\delta^{(1)}$ to the inclusive distribution is defined as

$$2E' \frac{d^3\sigma}{d^3p'} = \sigma(p_e, P; p') = \sigma_{\mathrm{Born}}(p_e, P; p') \times (1 + \delta^{(1)}) \quad (1)$$

with the Born cross section given by

$$\sigma_{\mathrm{Born}}(p_e, P; p') = \frac{8\pi\alpha^2}{yQ^4} [y^2 x F_1(x, Q^2) + (1-y) F_2(x, Q^2) + (y - y^2/2) x F_3(x, Q^2)] \quad (2)$$

with standard notation: p_e, P are the incoming electron and proton four-momenta, respectively, p' refers to the observed electromagnetic shower, and

$$q = p' - p_e, \quad s = (p_e + P)^2, \quad x = \frac{q^2}{2(Pq)},$$

$$Q^2 = -q^2 = xys.$$

In leading order QCD the structure functions F_i are related to the quark (antiquark) densities $q_f(\bar{q}_f)$ as

$$F_2(x, Q^2) = 2xF_1(x, Q^2) = \sum_f A_f(Q^2) x [q_f(x, Q^2) + \bar{q}_f(x, Q^2)] \quad (3)$$

$$xF_3(x, Q^2) = \sum_f B_f(Q^2) x [q_f(x, Q^2) - \bar{q}_f(x, Q^2)] \quad (4)$$

with

$$A_f(Q^2) = Q_f^2 - 2Q_f v_e v_f \frac{Q^2}{Q^2 + M_Z^2} + (v_e^2 + a_e^2)(v_f^2 + a_f^2)\left(\frac{Q^2}{Q^2 + M_Z^2}\right)^2 \quad (5)$$

$$B_f(Q^2) = -2Q_f a_e a_f \frac{Q^2}{Q^2 + M_Z^2} + 4 v_e v_f a_e a_f \left(\frac{Q^2}{Q^2 + M_Z^2}\right)^2. \quad (6)$$

Here Q_f, v_f and a_f denote the electromagnetic charges and the neutral-current vector and axialvector couplings, resp., with the convention $Q_e = -1$ and $a_e = -1/(2\sin 2\theta_W)$ (see, e.g., [6]).

We use dimensional regularization to treat the infrared singularities, and on-shell renormalization. For the soft plus virtual photon contribution we find the correction factor

$$\delta_i^{(1)} = \frac{\alpha}{\pi}\left[\frac{3}{2}\log\frac{Q^2}{m_e^2} - 2 - \frac{1}{2}\log\frac{E^2}{m_e^2}\log\frac{E^2}{\varepsilon^2} - \frac{1}{2}\log\frac{E'^2}{m_e^2}\log\frac{E'^2}{\varepsilon^2} - \frac{1}{2}\log\frac{E^2}{\varepsilon^2}\log\frac{Q^2}{E^2} - \frac{1}{2}\log\frac{E'^2}{\varepsilon^2}\log\frac{Q^2}{E'^2} + \frac{1}{2}\log\frac{E^2}{\varepsilon^2} + \frac{1}{2}\log\frac{E'^2}{\varepsilon^2} + R(p_e, p') + O(\varepsilon)\right] \quad (7)$$

with ε the soft-photon cutoff governed by the resolution of the detector for low-energy photons. R is small (of the order $O(1\%) \times \delta_i^{(1)}$ for typical values of x, Q^2) and independent of ε. It is calculated numerically:

$$R(p_e, p') = \frac{q^2}{2}[I_1 + I_2(p_e, p') + I_2(p', p_e)] \quad (8)$$

with

$$I_1 = \int_0^1 \frac{dx}{|\mathbf{p}_x|(E_x + |\mathbf{p}_x|)} \log\frac{(E_x + |\mathbf{p}_x|)^2}{m_e^2 - x(1-x)q^2}$$

$$I_2(p_e, p') = \int_0^{1/2} \frac{dx}{m_e^2 - x(1-x)q^2}$$

and p_x defined by

$$p_x = xp' + (1-x)p_e.$$

This soft plus virtual photon correction has a relatively large negative value (up to about $40\% \times \sigma_{\mathrm{Born}}$) smoothly depending on x and Q^2.

In the case of hard photon emission collinear with the incident electron we apply the approximation $k \simeq (1-\beta)p_e$, keeping the photon angle only in the small denominator $(p_e k)$. With this approximation the corresponding 1-loop correction is found to be

$$\delta_{ii}^{(1)} \times \sigma_{\text{Born}}(p_e, P; p') = \frac{\alpha}{2\pi} \int_{\beta_{\min}}^{\beta_{\max}} d\beta \left[\frac{1+\beta^2}{1-\beta} \log \frac{\theta_0^2 E_e^2}{m_e^2} - 2\frac{\beta}{1-\beta} \right] \times \sigma_{\text{Born}}(\beta p_e, P; p') + O(\theta_0) \tag{9}$$

with θ_0 defining the opening half-angle of the forward cone where the photon angle can be neglected for the final state kinematics (in the case of event generation) or cannot be detected (to calculate the correction for inclusive electron measurements). For the purpose of writing an event generator, ε and θ_0 should be chosen smaller than the experimental resolution.

Note that the Born cross section on the r.h.s. of (9) is to be computed with rescaled x, q^2 and s. The limits in β are

$$\beta_{\min} = \frac{1}{x}\frac{q^2 + xs}{q^2 + s} = \frac{1-y}{1-xy}, \quad \beta_{\max} = 1 - \frac{\varepsilon}{E_e}. \tag{10}$$

For $\beta_{\min} \geqq \beta_{\max}$ the contribution vanishes. For typical values of x and Q^2 this hard photon contribution is positive and relatively small as compared to the soft photon correction.

It is convenient to introduce an electron structure function $G_{e/e}$ in the usual way:

$$\sigma(p_e, P; p') \simeq \int d\beta\, G_{e/e}(\beta, p_e; p') \sigma_{\text{Born}}(\beta p_e, P; p') \tag{11}$$

$G_{e/e}$ contains essentially the mass singularities from (9) together with those terms of (7) cancelling the infrared singularities (the remaining singular terms of (7) are cancelled by contributions from final-state collinear photons):

$$G_{e/e}(\beta, p_e; p') = \delta(1-\beta)\left[1 + \frac{\alpha}{2\pi}\frac{3}{2}\log\frac{Q^2}{m_e^2}\right] + \frac{\alpha}{2\pi}\frac{1}{(1-\beta)_+}\left[(1+\beta^2)\log\frac{\theta_0^2 E^2}{m_e^2} - 2\beta\right] \tag{12}$$

with the definition

$$\int_{\beta_{\min}}^{1} \frac{d\beta}{(1-\beta)_+} f(\beta) \equiv \int_{\beta_{\min}}^{1} d\beta \frac{f(\beta) - f(1)}{1-\beta} + \log(1-\beta_{\min}) \times f(1).$$

Note that the leading behaviour of this structure function is known to be process independent. This property will be used to discuss the characteristics of charged-current ep reactions in the next section.

Final state radiation from the electron line is the dominant process generating hard photons collinear with the scattered electron. There are two different ways to analyze those final states. If one defines the kinematics by the scattered electron (momentum p_e') it has to be kept in mind that the underlying basic electron-quark scattering process

$$e + q \to (e' + \gamma) + q' \tag{13}$$

takes place at rescaled variables $\bar{x}$ and $\bar{Q}^2$. Consequently the 1-loop correction given below in (15) is obtained as convolution of the Born cross section for the effective $2 \to 2$ process (13) with an electron fragmentation function. From an experimental point of view problems may arise in the separation of signals from highly collinear energetic electrons and photons.

Therefore, in this kinematic situation the total 4-momentum p' of the final state electron and the collinear hard photon seems to be more appropriate to define the kinematics even for experimentally separable collinear $e-\gamma$ final states. This definition gives the correct x and Q^2 values for the basic subprocess (13), e.g. for the extraction of structure functions. As a consequence the corresponding contribution to the 1-loop cross section (comp. (1)) factorizes. The correction factor is given by

$$\delta_{iii}^{(1)} = \frac{\alpha}{2\pi}\left[\log\frac{E'^2}{\varepsilon^2}\left(\log\frac{\theta_0'^2 E'^2}{m_e^2} - 1\right) - \frac{3}{2}\log\frac{\theta_0'^2 E'^2}{m_e^2} - \frac{\pi^2}{3} + \frac{9}{2}\right] + O(\varepsilon) + O(\theta_0'). \tag{14}$$

Here the opening angle of the final state electromagnetic jet (electron and photon collinear) is restricted to be smaller than θ_0' characterizing the resolution of the detector.

If the kinematics is defined by the scattered electron this factorization does not hold, as discussed previously. In this case the correction is given by

$$\tilde{\delta}_{iii}^{(1)} \times \sigma_{\text{Born}}(p_e, P; p_e') = \frac{\alpha}{2\pi}\int_{\tilde{\beta}_{\min}}^{\tilde{\beta}_{\max}} \frac{d\tilde{\beta}}{\tilde{\beta}^2}\left[\frac{1+\tilde{\beta}^2}{1-\tilde{\beta}}\log\frac{\theta_0'^2 E_e'^2}{m_e^2} - 2\frac{\tilde{\beta}}{1-\tilde{\beta}}\right] \times \sigma_{\text{Born}}(p_e, P; p_e'/\tilde{\beta}) + O(\varepsilon) + O(\theta_0'). \tag{15}$$

The limits of integration are

$$\tilde{\beta}_{\min} = 1 - \frac{1-x}{x}\frac{Q^2}{s}, \quad \tilde{\beta}_{\max} = \frac{E_1'}{E_1' + \varepsilon} \tag{16}$$

where the correction again vanishes for $\tilde{\beta}_{\min} \geqq \tilde{\beta}_{\max}$.

3 Numerical results for neutral and charged current reactions

The characteristic behaviour of the 1-loop electromagnetic correction for neutral current reactions as obtained in our approximation is represented in Fig. 1 for several x-values: They can become large and strongly depend on the kinematical region. Our results agree in shape with those of the other groups [3, 4] with the exception of a small kinematic region at large x and small y, where the results of the Dubna group show a qualitatively different behaviour.

Comparing the absolute normalization our curves are systematically below the results of the exact calculations. This is a consequence of the restricted integration over the photon phase space in our approximation, as discussed in the preceding Section.

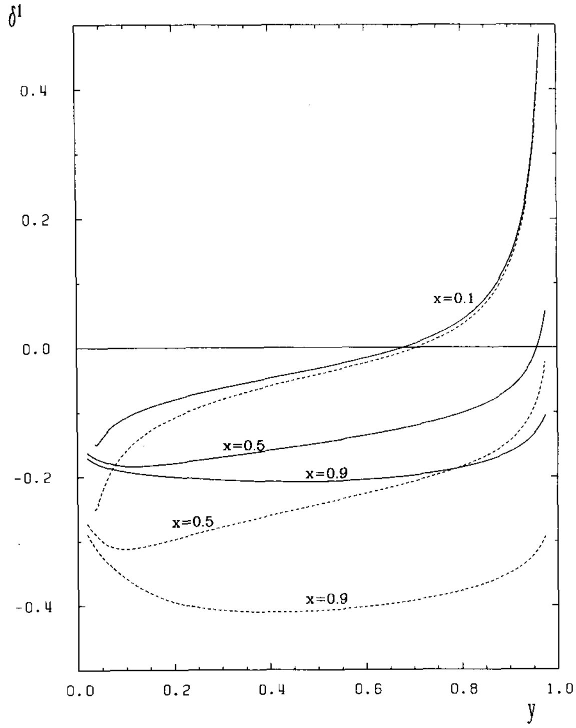

Fig. 1. x- and y-dependence of the electromagnetic 1-loop correction $\delta^{(1)}$ for neutral-current ep reactions in collinear approximation. Full lines: Kinematics for final-state electromagnetic jet. Dashed lines: Kinematics for inclusive electron measurement

The excluded integration region does not give mass or infrared singularities, but its contribution has to cancel the dependence on the cutoff parameters ε, θ_0 and θ_0' in our formulae (comp. (7, 9, 14, 15)). Therefore, the leading terms from the excluded photon phase space are doubly logarithmic in ε and θ_0 (θ_0') and not negligible.

We have checked our formulae by removing the cuts on the photon phase space and working in the leading logarithmic approximation. In this case we agree with the exact calculations within a few percent.

Furthermore, Fig. 1 also demonstrates that x- and y-dependence of the correction are reduced if the more appropriate jet kinematics is chosen.

A distinctive feature of the electromagnetic radiative correction is the pronounced peak at small x and large y. For $x \leqq 0.05$ and $y \geqq 0.95$ the magnitude of the correction may exceed the Born cross section. This $x-y$ range corresponds to final states with a very hard photon in forward direction, the electron being scattered to extremely large angles. In (9) for the 1-loop correction in the case of forward photon emission this means small β and a small effective momentum transfer $\tilde{Q}^2 \simeq \beta Q^2$ in the basic electron-quark scattering process. Consequently the photon exchange contribution to the Born cross section on the rhs of (9) gives rise to the strong peak at large y.

The formulae given in Sect. 2 allow for a direct inclusion of hard-photon cuts into the analysis. To

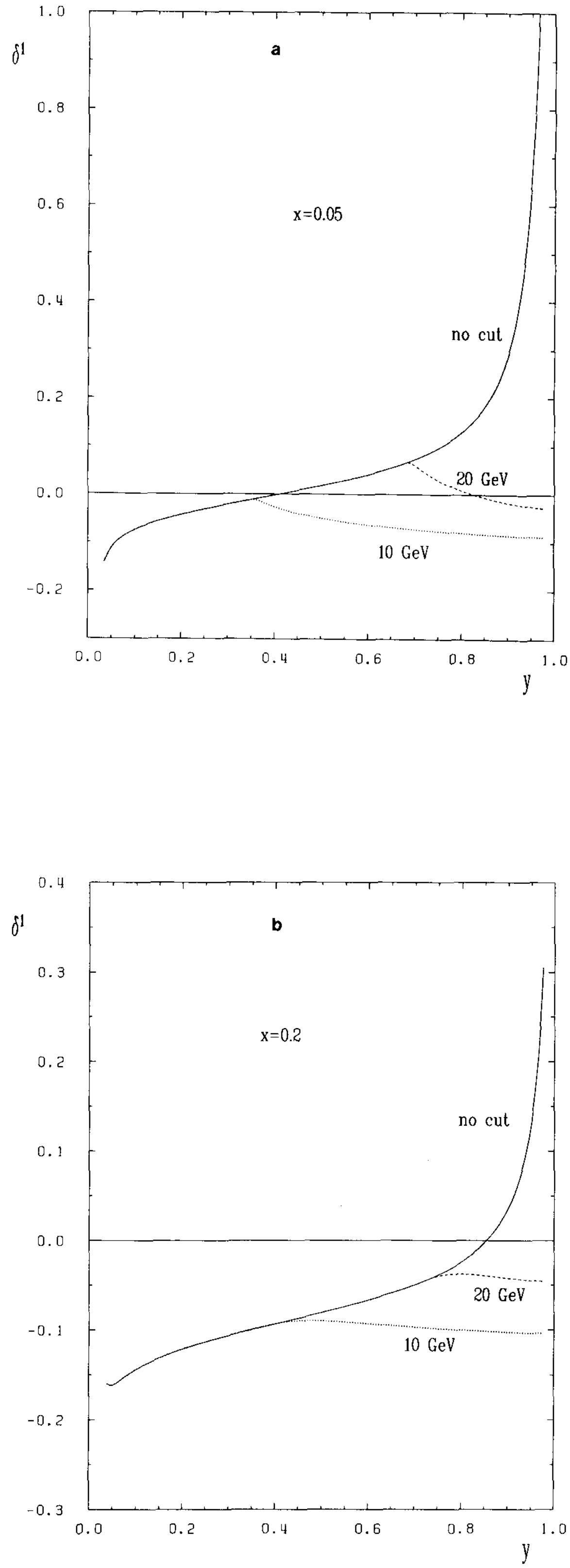

Fig. 2a, b. Influence of the energy cut E_γ^{cut} for hard photons in forward direction on the 1-loop electromagnetic correction $\delta^{(1)}$ for neutral-current reactions (kinematics for final-state electromagnetic jet) **a** $x = 0.05$, **b** $x = 0.2$

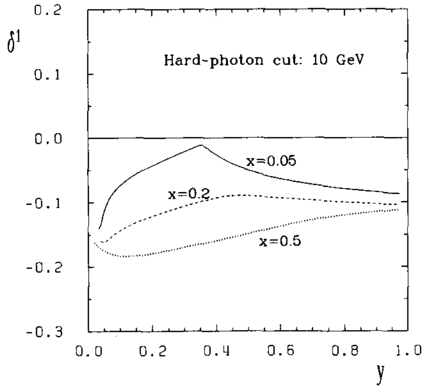

Fig. 3. x- and y-dependence of the electromagnetic 1-loop correction for neutral-current reactions with a hard-photon cut $E_\gamma^{\rm cut} = 10\,{\rm GeV}$ imposed

exclude photons with energies larger than $E_\gamma^{\rm cut}$ within a forward cone characterized by θ_0 we simply have to replace the lower limit of the β-integral in (9) by

$$\beta_{\min} = {\rm Max}\left(\frac{1}{x}\frac{q^2+xs}{q^2+s}; 1-\frac{E_\gamma^{\rm cut}}{E_e}\right). \quad (17)$$

In Fig. 2a, b we demonstrate the influence of different hard-photon cuts on the behaviour of the electromagnetic correction at $x = 0.05$, 0.2: For $E_\gamma^{\rm cut} = 20\,{\rm GeV}$ the peak at high y completely disappears; for $E_\gamma^{\rm cut} = 10\,{\rm GeV}$ the remaining correction is of order 10% and only weakly depends on y. At the latter value of $E_\gamma^{\rm cut}$ there is no strong x-dependence, too, as shown in Fig. 3.

Experimentally those promising cuts could be realized by tagging the hard forward photon and/or measuring the current-quark jet. Indeed the HERA experiments have planned small-angle photon detectors for luminosity monitoring. However, due to the huge background from synchrotron radiation and quasi-real photon scattering processes the detection of those rare very hard photons from higher-order deep inelastic scattering with the required efficiency may be difficult. Similarly, acceptance and energy resolution may limit the accuracy of the measured current-quark momentum in the interesting kinematical region. The discrimination of multi-jet events could improve the latter situation. A final conclusion on the feasibility of hard-photon cuts for the reduction of radiative corrections clearly requires a detailed Monte Carlo analysis taking the detector properties into account.

For charged currents it may be even more difficult to tag hard photons in the forward direction because the current-quark jet serves to define the kinematic variables now and thus cannot be used to discriminate events with hard photons. However, for W exchange the Born cross section does not show a low-Q^2 peak anyway, and therefore no large-y peak is expected to arise from hard photon emission.

In our discussion of the neutral current reaction it has been mentioned that a leading-log treatment of the electron mass singularities gives a rather precise estimate of the expected electromagnetic correction. This should be the case for charged current reactions as well. In Feynman gauge, leading-log contributions now also come from a photon insertion connecting the lepton and quark line, as well as from diagrams involving a triple γWW vertex. The various contributions add up to an expression which can again be summarized in terms of a universal electron structure function

$$\sigma(p_e, P; p'_q) = \int d\beta\, G_{e/e}(\beta, p_e)\, \sigma^{\rm c.c.}_{\rm Born}(\beta p_e, P; p'_q). \quad (18)$$

Here $G_{e/e}$ stands for the leading-log terms from the electron structure function as defined for neutral current reactions in (12). (Note that in leading logarithmic approximation there is no dependence on the cutoff angle θ_0.) Forward emission is now the only hard-photon contribution to electron mass singularities.

The invariant Born cross section for charged current ep scattering is applied in a somewhat simplified form,

$$\sigma^{\rm c.c.}_{\rm Born}(p_e, P; p'_q) = \frac{\pi\alpha^2}{2\sin^4\theta_W (Q^2 + M_W^2)^2}\frac{x}{y} \cdot [u(x, Q^2) + (1-y^2)\bar{d}(x, Q^2)] \quad (19)$$

where only u and d quarks have been taken into account (comp., e.g., [5] for the exact expression). Equation (18) could be obtained most directly by using a physical (axial) gauge.

Figure 4 shows results for the 1-loop electromagnetic correction to charged current reactions in leading logarithmic approximation, (18). They exhibit the qualitatively expected behaviour, i.e. a smooth x- and y-dependence as compared to the neutral-current case without pronounced structures.

Our numerical results were obtained with the parametrization of structure functions given by Duke and Owens [7]. From a comparison with corresponding

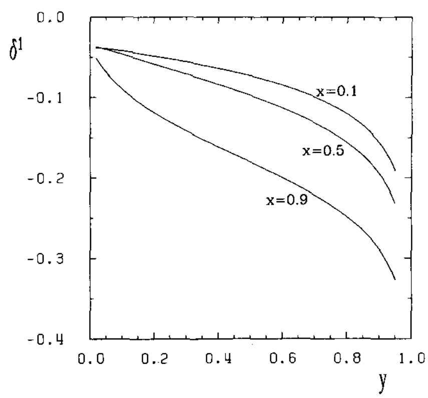

Fig. 4. x- and y-dependence of the electromagnetic 1-loop correction in leading logarithmic approximation for charged-current ep reactions

calculations using another set of structure functions from [8] we estimate a general uncertainty of O(1%) due to different parametrizations of parton densities.

4 Summary

The dominant contributions to the 1-loop electromagnetic radiative corrections to neutral-current deep-inelastic *ep* scattering have been isolated and treated analytically with the aim of writing an event generator. Both the magnitude of the correction and their dependence on the kinematical variables can be reduced by

i) using the appropriate final state variable (the electromagnetic jet energy including collinear final-state photon emission), and

ii) imposing an energy-cut on hard photons collinear with the incident electron.

If the hard-photon cuts are experimentally feasible the remaining radiative correction only smoothly depends on x and y and will not pose problems for the study of QCD structure functions in neutral current reactions. The desired more detailed Monte Carlo studies are in preparation.

For charged currents it is less important (and more difficult) to impose a hard-photon cut since for W boson exchange there is no pronounced structure in the shape of the 1-loop radiative corrections.

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