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HADRONIC STRUCTURE BY INTRODUCING CHEMICAL POTENTIAL

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Hadronic structure can be obtained dynamically by calculating baryon density correlation functions at finite chemical potential μ. Even though preliminary results show some structure, it is not yet clearly identifiable with nucleons due to the early onset with μ of thermodynamic quantities. Analyzing the free Fermi gas suggests to take a much finer lattice to improve these results.

1. INTRODUCTION

In this talk I will describe a method to find the fermionic structure for states generated by QCD on the lattice. Considering this theory at finite temperature and chemical potential, it will provide us with dynamical and extended states in a thermal superposition.

The chemical potential μ is coupled to the conserved baryon number B. The corresponding current density, $j_\nu(x)$, is derived from the improved action at finite chemical potential on the lattice, as introduced by Kogut et al. [1] and Hasenfratz and Karsch [2], in order to obtain the proper continuum limit for various thermodynamic quantities [1-3].

The baryon current can then be used to find Hadronic structure emerging from QCD, by evaluating equal-time correlation functions

$$c(r) = \langle j_0(r) \; j_0(0) \rangle_{\beta,\mu} \tag{1}$$

in a thermal state at finite temperature $T = \beta^{-1}$ and chemical potential μ. The thermal state is obtained as usual [4] by considering the functional integral over fields with (anti-)periodic boundary conditions for the Euclidean time β.

For any Operator like the correlation, the energy density or the current density:

$$\langle O \rangle_{\beta,\mu} = 1/Z \sum_n \langle n \,|\, e^{-\beta(H-\mu B)} \, O \,|\, n \rangle \tag{2}$$

where the expectation for each dynamical state $|n\rangle$ is weighted by the appropriate Boltzmann factor for a given energy and net baryon number. For low temperature and zero μ, we expect just the vacuum state $|\Omega\rangle$ with some small admixture of lowest meson states $|m\rangle$ to contribute to the expectation value in Eq.(2). As we turn on the chemical potential μ, states with net fermion number become increasingly probable and even though baryon states $|b\rangle$ have a higher mass they will contribute to the thermal admixture. These $|b\rangle$ states should contribute significantly once we reach a μ of the order of the nucleon mass. The idea is then to study the correlation as a function of μ and from there extract the distribution of fermions in baryons. At higher μ one could study the deconfined correlations.

Contrary to these expectations it has been found [5], for quite coarse and quenched lattices, that several thermodynamic quantities behave with finite μ as if controlled by fermions with half the pion mass. In the strong coupling limit it has been found [6] that this behavior can be improved by considering the full unquenched problem, even though one still does not find

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baryons but a phase full of fermions. Attempts are being made, as reported in this meeting by Barbour and collaborators [7] to try to include the complex determinant at intermediate couplings which is very hard due to phase fluctuations.

On the other extreme of very weak coupling, the theory approaches the free fermi gas limit in which the determinant cancels trivially for any expectation value. For this case we will see that the proper continuum limit is reached for several thermodynamic quantities, but the rate of convergence as we take finer lattices is slow. In fact, for an 8^4 lattice the results are still off by 100 % from the continuum due to big a^2 corrections and the spectrum of states can be quite distorted on these coarse lattices. In the interacting case this implies that as we move from the strong coupling regime to the weak scaling one (while keeping a low temperature and big enough volume) the states could shift so as not to allow the state that produces the unexpected onset that seems controlled by a Goldstone mode. In fact for a coupling of 6.0 the results for the number density seem already compatible with a mass $m_N/3$ but for this coupling still larger lattices are required.

In the next Section we describe the method to obtain the observables on the lattice, in Sec. 3 we discuss the lattice artifacts for the free fermi gas and in Sec.4 we show results for interacting case.

2. LATTICE METHODS

Let me describe the procedure that was used to calculate on the lattice the expectation value for several operators, like for the correlation in Eq.(1):

$$c(r) = 1/Z \int Du\, D\chi\, D\bar{\chi} \;\; j_0(r)\, j_0(0)\, e^{-S(\beta,\mu)} \quad (3)$$

where the action S for the period β is

$$S = 6/g^2 \sum_{\text{plaq.}} (1 - 1/3 \text{ Re Tr } uuuu) + S_F \quad (4)$$

with the Kogut-Susskind fermionic action:

$$S_F = \sum_x \{ m\, \bar{\chi}_x \chi_x + 1/2 \sum_\nu \Gamma_\nu(x) \times \quad (5)$$

$$(e^{\mu a \delta^{\nu 4}}\, \bar{\chi}_x u_{x,\nu} \chi_{x+\nu} - e^{-\mu a \delta^{\nu 4}}\, \bar{\chi}_{x+\nu} u^{+}_{x,\nu} \chi_x)\}.$$

For $\mu = 0$ this is the usual action. Naively one would have considered just the linear term in μ as in the continuum. This can be shown[1-3] to give erroneous results for the energy density ε and the number density $\langle j \rangle$ for the free fermi gas. The exponentiated form gives the right continuum limit in the free case, behaves properly in the hamiltonian formalism and can be interpreted as an imaginary gauge field A_0. Unfortunately, as we will see, it reaches the continuum very slowly and it distorts the energy states at finite μ for coarse lattices by modifying the kinetic energy by the factor $\cosh(\mu a)$.

The conserved current due to U(1) invariance on the lattice gives the baryon density

$$j_4(x) = 1/2\, \Gamma_4(x) (e^{\mu}\, \bar{\chi}_x u_{x,4} \chi_{x+4} - e^{-\mu}\, \bar{\chi}_{x+4} u^{+}_{x,4} \chi_x) \quad (6)$$

whose expectation value can also be obtained as $1/\beta V\; \partial/\partial\mu \ln Z$.

The fermionic action S_F is quadratic in the χ fields and so we can perform the integration over fermions with the known results

$$\int D\chi\, D\bar{\chi}\;\; \bar{\chi}_i\, \chi_j\, e^{\bar{\chi} P^{-1} \chi} = P_{ji} \;\; \det P^{-1}$$

$$\int D\chi\, D\bar{\chi}\;\; \bar{\chi}_i\, \chi_j\, \bar{\chi}_k\, \chi_l\, e^{\bar{\chi} P^{-1} \chi} = (P_{ji} P_{lk} - P_{jk} P_{li}) \det P^{-1} \quad (7)$$

Both terms in the last expression have to be considered for the current correlation. Even in the quenched case, in which we neglect fermion loops (not proceeding from the currents), we could produce meson and baryon states at finite temperature and chemical potential as indicated

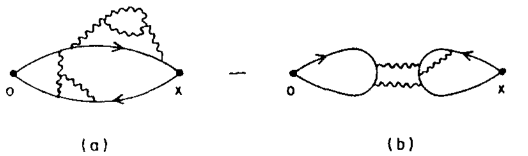

Fig.1. Vacuum diagrams for the correlation c(x).

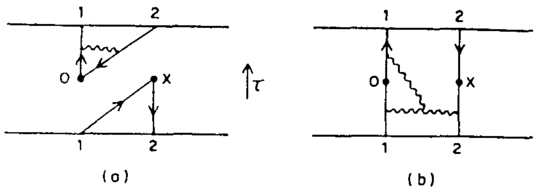

Fig.2. Meson contribution at finite temperature.

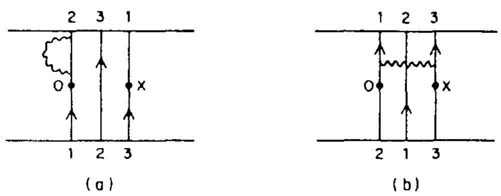

Fig.3. Baryons produced easier at finite μ and T.

diagrammatically in Figures 2 and 3. Although the determinant seems to be important at least in the strong coupling limit [6], I have worked in the quenched case partly due to the problems, for the moment[7], with its inclusion. At finite μ it is complex, so that standard Montecarlo methods cannot be applied and the seemingly crucial phase [8] fluctuates wildly. On the other hand, it is possible in the quenched case, that, as we move to weaker couplings to the scaling regime, and therefore to finer lattices, we could approach the right behavior for several quantities.

In the quenched case then, we generate equilibrium gauge configurations (U_i) with a heat bath method and calculate the expectation of operators as

$$\langle O \rangle = \{ O [P[U_i] , U_i] \}_{\text{averaged over conf.}} \tag{8}$$

Given the number of propagators P_{ij} needed to calculate the current correlation function c(r), for each U and μ, the most efficient method of inversion is the second order (P is nonhermitian) pseudo-fermion combined with heat bath, which can be explicitly solved and applied to $(P^+P)^{-1}$. By multiplying the inverted matrix by the original, one gets ~ 5% error after 4000 iterations.

3. FREE FERMI GAS

To be able to extract useful information from the data for the interacting case, it is necessary to compare it to the behavior of the free fermi gas on the lattice where the μ dependence can be easily studied. Furthermore, we will see that already in the free case there are large lattice artifacts that can distort strongly the expected states, as the current operator gets mixed with the kinetic energy term in the action. These a^2 effects are very strong for typical lattices in use and could even produce level crossing in the interacting theory, leaving unphysical configurations as the lowest states.

We have looked mainly at the thermodynamic quantities : fermion density j_0 and fermion energy density ε, and also at the chiral condensate $\langle\bar{\chi}\chi\rangle$ and the current correlation c(r). One finds in the free case that the finite volume effect is not so important (compare the highest pair of curves in Figs. 4-5), but even in the infinite volume limit there are big shifts due to the coarse lattice in the β direction. In fact, one can parametrize the current in terms of physical products comparable with the continuum, and finds in an a^2 expansion:

$$(\langle j \rangle \beta^3) = F(\mu\beta, m\beta) + 1/N_\beta^2\, G(\mu\beta, m\beta) + \dots \tag{9}$$

with $N_\beta = \beta/a$. Here the function F gives the continuum result and the corrections to it are sizable as can be seen in Figs. 4-5. By taking a twice finer lattice while keeping the physical parameters fixed, we converge substantially to the continuum result. The Eq.(9) is important in showing that we can directly compare the $\langle j \rangle a^3$ with the interacting case, by choosing the same N_β and μa (which does not renormalize) and fit for some effective mass ma.

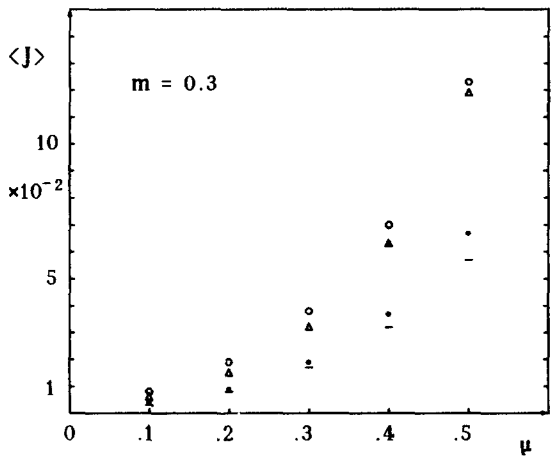

Fig.4. Fermion density versus μ for the free fermi gas, for several lattices as compared to the continuum (-). The sizes are: $8\times(8^2\times14)$ as (o), then $8\times(16^2\times26)$ as (▲) which is equal to ∞ volume, and $16\times(34^3)$ with half lattice spacing as (•). The 3×4 species factor is included in continuum. For larger μ the lattice curves reach saturation as if we had only one flavor . Similar shifts for other m.

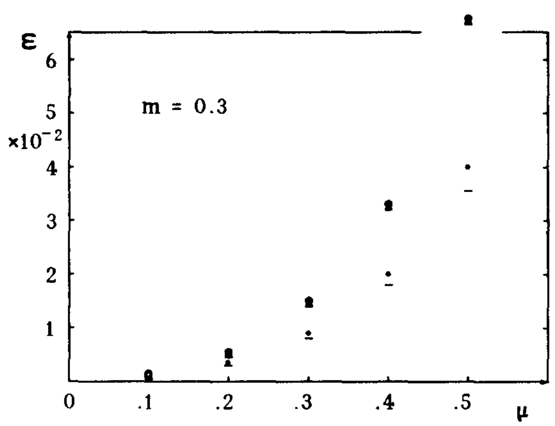

Fig.5. Energy density versus μ for the free fermi gas (with ▪ at zero μ substracted), for the same cases as in fig.4. The corrections to the continuum for the current, Δj, are given approx. by $\Delta\varepsilon/\mu$ as seen for several masses and temperatures.

4. RESULTS FOR INTERACTING CASE

For the moment, I have done simulations for an $8\times(8^2\times14)$ lattice, measuring $\langle j \rangle$, ε, $\langle\bar\chi\chi\rangle$ and the correlation c(r) in the spatial (14) direction. For coupling of 5.7, I have considered two quark masses: $m_q = .03$, for which [9] $m_\pi \sim .5$, $m_N \sim 1.5$ and for $m_q = .1$, where $m_\pi \sim .9$, $m_N \sim 2.1$. For a weaker coupling of 6.0, which is very close to deconfinement, I used $m_q = .04$ so that $m_\pi \sim .5$, $m_N \sim 1.1$. The results for the various densities are presented in Figs. 6-8 . I expect to calculate on a 16^4 finer lattice and at lower T in the near future.

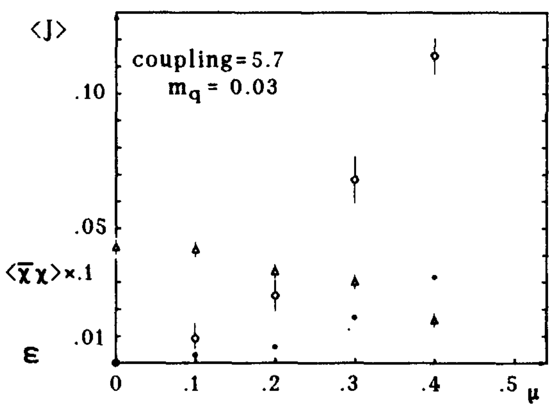

Fig.6. Current(o), energy(•) and chiral(▲) density versus μ for a coupling of 5.7. Compared to the free case, they behave as if controlled by $m_\pi/2$.

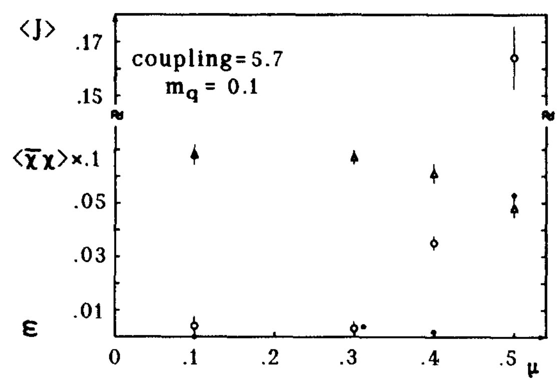

Fig.7. Same as in fig.6, but for higher quark mass. For $\mu < .4$, it is inconsistent with $m_\pi/2$ and more likely corresponds to a much higher mass.

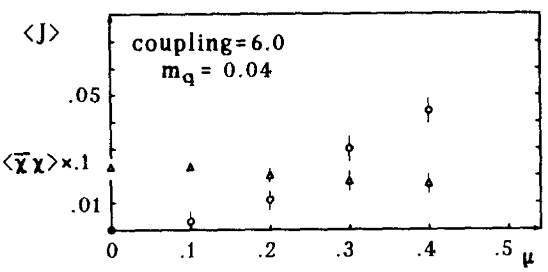

Fig.8. Current(○) and chiral(▲) density versus μ for a coupling of 6.0. For this finer lattice (but T close to deconfinement), the current behaves as if controlled by a mass of .4 $\sim m_n/3$ and not $m_\pi/2$.

It is important to check if this result at a finer lattice, in Fig.8 (that shows possibly the expected onset with m_N), will persist for larger N_β and V.

Let me present now the results for the baryon density correlation, c(r). For μ=0 we expect a curve corresponding to vacuum polarization plus a small admixture of pion states (finite T). As we turn on μ, we expect a contribution from states with net fermion number. If these states resemble baryons, there should be a positive Δc(r) over some distance where the other two quarks are present. In Figs. 9-10 we show $c_{\mu=0}(r)$ and $\Delta c(r) = c - c_0 - \langle j\rangle^2$.

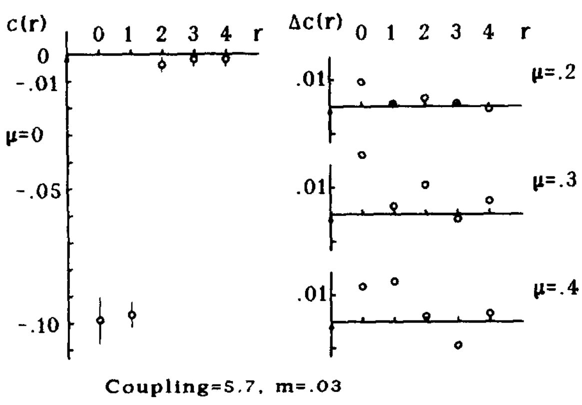

Fig. 9. Fermion density correlation versus r, at μ=0 and shifts from it as we increase μ. Even if the individual error bars are of the size of the signal for the shifts Δc(r), there is a consistent positive enhancement for short distances, possibly indicating baryonic state. With growing μ there sets in an oscillation which could be a lattice artifact.

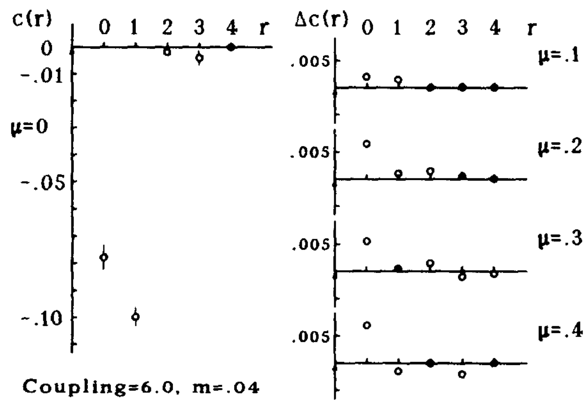

Fig.10. Same as in fig.9 but for weaker coupling. In this case there is less correlation, except at the origin, perhaps indicating free quarks (here we are almost at deconfinement). Oscillations as in fig.9.

5. CONCLUSIONS

I have presented a method to extract fermionic spatial distributions for the lowest baryon state in a thermal ensemble at finite μ. We have seen that the known problem of the early onset of the current density, could be due to the fact that one has been working with coarse lattices in the strong coupling regime. In fact, for one finer lattice (at 6.0) the expected onset seems compatible.

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