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DUAL INTERPRETATION OF ORDER PARAMETERS FOR LATTICE GAUGE THEORIES WITH MATTER FIELDS*

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A dual interpretation of the vacuum overlap order parameter (VOOP) and of the the flux correlations order parameter (FCOP) is presented. For the FCOP we interchange one of the space dimensions with the euclidean time. In $3+1$ dimensions the resulting quantity is interpreted as the vacuum overlap of a *magnetic vortex*, i.e. of a gauge-invariant energy-regularized state containing a closed line of center-magnetic flux. In the confinement region the vortices are condensed in the vacuum. In the Higgs region only vortices of small size exist as excitations. The screening length for dynamical center-electric charge fluctuations can be reinterpreted as the characteristic length for vortex fragmentation. The vacuum correlations responsible for the existence of magnetic vortices as excitations are measured by a (purely spatial) quantity obtained from the VOOP by replacing the time direction with a spatial one. At finite temperatures this quantity can be used as an order parameter for the symmetry restauration transition. In $2+1$ dimensions the dual interpretation of our order parameters is simpler: we just interchange VOOP and FCOP, states with center-electric charge and states carrying center-magnetic charge (monopoles), the confinement and the Higgs region.

1. INTRODUCTION

In recent years we have introduced several order parameters for lattice gauge theories with matter fields [1,2,3,4,5]. We asked three fundamental questions:

1. Do states with center-electric charges exist or is the charge screened?
2. What properties of the vacuum are responsible for this?
3. What is the mechanism for charge screening?

The *vacuum overlap order parameter* (VOOP) is the scalar product with the vacuum (i.e. the vacuum overlap) of a gauge-invariant energy-regularized electric dipole state. The *flux correlations order parameter* (FCOP) measures the correlations of center-electric flux in the vacuum. We would like to gain some new insight into the meaning of our order parameters by asking the *appropriate dual questions*. By duality we mean a variety of related concepts like the electric–magnetic duality in electrodynamics, the duality transformation for lattice models (see e.g. [6]), or the duality between the Wilson loop and the 't Hooft loop [7,8].

We start by recalling the definition and the physical interpretation of the VOOP (section 2) and the FCOP (section 3). In order to simplify the exposition we assume 4 space-time dimensions throughout sections 2-5. We define both order parameters using the canonically quantized formulation of the theory, and then rewrite them in terms of expectation values in the euclidean path integral formulation. By interchanging the time axis with one of the space axes, these expectation values can be reinterpreted in the canonically quantized formulation based on the new choice of the time direction. Thus we arrive at a dual interpretation for the FCOP (section 4) and for the VOOP (section 5), in which the three fundamental questions mentioned above are modified by replacing *center-electric charge* with *closed line of center-magnetic flux* (also called a *magnetic vortex*). A magnetic vortex is a gauge-invariant state whose energy has been regularized such that it is proportional to the length of the closed line of magnetic flux (and not to the area spanned by this line). The FCOP is now the vacuum overlap of a magnetic vortex, while the VOOP now measures the correlations in the vacuum which are responsible for the existence or screening of magnetic vortices. Thus in the dual interpretation the roles of the VOOP and FCOP are interchanged.

The dual interpretation of the FCOP leads to a generalization to the theory with matter fields of the idea that in 4 dimensions the confinement vacuum is a condensate of magnetic vortices [7,8,9]. In the Higgs region, some of the properties of the magnetic vortices resem-

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ble those of confinement-region quark-antiquark pairs: small vortices exist as excitations; at some characteristic length their overlap with the vacuum becomes large, which is reminiscent of the quark fragmentation (i.e. hadronization) phenomenon. Furthermore, the vortex fragmentation length is identical to the characteristic length for the screening of dynamical charge fluctuations [5].

In its dual interpretation the VOOP is a purely spatial quantity and can be defined at finite temperatures too. It is useful in the study of the symmetry restauration transition [10] (from the Higgs region to the confinement-deconfinement region). At short distance it behaves as perturbation theory predicts for a two-point function of the scalars. In the Higgs region this is also true at long distances. In the confinement-deconfinement region however, the VOOP goes to a constant at long distances [4]. Thus on the one hand the conventional picture for this transition is not destroyed, while on the other hand the order parameter itself (which is the infinite distance limit of the VOOP) is nonzero in both regions. This resolves the apparent contradiction between the conventional perturbative picture and the fact that the two regions may be analytically connected.

In the deconfinement region, both charges and vortices are condensed in the thermal equilibrium state. The original FCOP and the dual VOOP define two distinct characteristic lengths.

For simplicity we shall consider a theory with a scalar matter field in the fundamental representation of the gauge group G. As in [5] we use the notation:

$$S = -\beta \sum_p \chi(U_p) - \kappa \sum_l 2\, Re\, (\phi^\dagger U \phi)(l) + \sum_x V(\phi(x)) \tag{1}$$

where p, l and x are the plaquettes, links and sites of a d–dimensional euclidean lattice ($d \geq 3$), $U(l)$ are the gauge fields, $\phi(x)$ the matter fields, and χ is some character containing the fundamental character as an irreducible component. Geometrical objects in $d-1$ (space) dimensions will be underlined and time-zero operators will be hatted, like $\hat{U}(\underline{l})$ or $\hat{\phi}(\underline{x})$.

2. BRIEF REVIEW OF THE VOOP

The ideas leading to the VOOP, its definition and some fundamental properties have been discussed in [1,2,3,4,5]. Let us summarize the most relevant aspects.

A. Assume $\underline{L}_{\underline{x}\underline{x}'}$ is a spatial path from $\underline{x}$ to $\underline{x}'$, chosen for simplicity to be a straight line. A naïve candidate for a dipole state is:

$$\hat{\phi}^\dagger(\underline{x})\, \hat{U}(\underline{L}_{\underline{x}\underline{x}'})\, \hat{\phi}(\underline{x}')\, |0\rangle \tag{2}$$

For large separations the energy of this state is proportional to $|\underline{x}-\underline{x}'|$. It can be regularized by translating $\hat{U}(\underline{L}_{\underline{x}\underline{x}'})$ by n steps into euclidean time ($\hat{T}$ is the transfer matrix; to avoid any confusion we write the group indices a and b explicitely here):

$$|\underline{x}, \underline{x}', n\rangle := \hat{\phi}^\dagger(\underline{x})_a\, \hat{\phi}(\underline{x}')_b\, \hat{T}^n\, \hat{U}(\underline{L}_{\underline{x}\underline{x}'})_{ab}\, |0\rangle \tag{3}$$

is a state with bounded energy provided that for some constant c, $n \geq c\,|\underline{x}-\underline{x}'|$ as $|\underline{x}-\underline{x}'| \to \infty$. This result is model-independent (for nonabelian G the missing link in the proof was the perimeter law for Wilson loops; this has now been proven [11]). Notice that the quantity translated into euclidean time is not gauge-invariant, but has a source at each endpoint. For $n \to \infty$, $\hat{T}^n$ projects out the state with lowest energy for a given configuration of sources. If $\hat{T}^n$ acts on (2), the state projected out is the vacuum, so there is no electric flux from $\underline{x}$ to $\underline{x}'$.

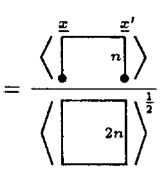
B. In the limit $\underline{x}' \to \infty$, $n \geq c\,|\underline{x}-\underline{x}'|$, the charge at $\underline{x}$ either becomes free or is screened. A quantity testing this is the vacuum overlap of the normalized dipole state (3):

$$\tilde{\rho}(|\underline{x}-\underline{x}'|, n) := \frac{\langle\, 0 \mid \underline{x}, \underline{x}', n\rangle}{\|\, |\underline{x}, \underline{x}', n\rangle\, \|} \tag{4}$$

A free charge is orthogonal to the vacuum Hilbert space. If the charge is screened and $\hat{\phi}(\underline{x})$ has no additional nontrivial quantum numbers, the vacuum overlap is nonzero in the limit. Thus the criterion for existence of charged states is:

$$\tilde{\rho}(\infty, \infty) = \begin{cases} 0 & \exists \text{ free charges} \\ \neq 0 & \text{charges are screened} \end{cases} \tag{5}$$

C. The vacuum overlap is easily expressed in terms of euclidean expectation values. It is convenient to redefine (4) by replacing in the denominator $|\underline{x}, \underline{x}', n\rangle$ with $\hat{T}^n \hat{U}(\underline{L}_{\underline{x}\underline{x}'})\, |0\rangle$. Denoting the resulting quantity by ρ instead of $\tilde{\rho}$,

$$\rho(|\underline{x}-\underline{x}'|, n) =$$  (6)

The criterion (5) also holds for ρ (with ρ a related interpretation is also possible: (5) tests whether the charge of a source is screened or not). The cancellation of perimeter contributions between the numerator and denominator of (6) is one of the main ingredients in proving (5). The order parameter is $\rho(\infty,\infty)$, but by an abuse of language we shall call (6) the VOOP.

D. $\rho(|\underline{x}-\underline{x}'|,\infty)$ is a gauge-invariant two-point function of the *dressed* charged field. For large β it behaves similarly to the matter field two-point function of the pure matter ($\beta=\infty$) theory. In this region it is relatively easily accessible by numerical methods. In the free charge phase it can be used to compute the mass of the charged particles. In the Higgs region it offers a method to compute the Higgs expectation value [12]. In the *confinement region* however, $\rho(|\underline{x}-\underline{x}'|,\infty)$ rapidly decreases with $|\underline{x}-\underline{x}'|$ at small distances (corresponding to the Coulomb plus linear region of the potential), where a charge-anticharge (quark-antiquark) pair can exist as an excitation, while at large distances it goes to a constant (after the potential becomes flat). Thus it should have a dip [4] around the *characteristic length for hadronization (fragmentation)*, which is roughly:

$$R_c \sim \frac{E_q}{\sigma} \tag{7}$$

(E_q is the energy of a source, σ is the string tension computed from the linear part of the potential).

E. The VOOP cannot be defined at finite temperatures since in this case it is impossible to let $n\to\infty$.

3. BRIEF REVIEW OF THE FCOP

The definition and basic properties of the FCOP have been discussed in [1,3,5]. As opposed to the VOOP, the FCOP tries to answer the question of charged states by investigating properties of the vacuum. Let us again summarize the most relevant aspects.

A. In a *massive* gauge theory with matter fields a charged state can be created by acting on the vacuum with an operator localized inside a *spatial cone* [13] (this charged state does not contain a *thin* electric flux tube; the flux is not isotropic as by letting $\underline{x}\to\infty$ in (3), but the state with the cone is in the same superselection sector; using the same methods as in [1] we explicitely constructed such a state for $G=Z_2$, the idea being to regularize the energy somewhat differently than in (3)). The charge can be determined by *Gauss' law*, i.e. by measuring the total flux through an arbitrarily large closed surface. For a charged state the asymptotic direction of the cone should not be observable, i.e. we should not be able to determine the charge by measuring the electric flux through an open surface around the cone. This implies that there are strong electric flux correlations in the vacuum that delocalize the flux. For an additive charge (e.g. $G=U(1)$) the electric flux through a surface is a sum of local operators, so in a massive phase, where all two-point functions decay exponentially, there are no strong flux correlations and therefore no charged states (Swieca's theorem [14]). In general however, Gauss' law only holds for the center C_G of G. If C_G is discrete, a multiplicative charge may exist in a massive phase provided the flux correlations in the vacuum are strong enough.

B. Let us denote by $\hat{\mathcal{E}}_C(\underline{l})$ the left multiplication operator by $C\in C_G$ for the oriented link $\underline{l}$. The electric flux $\hat{\mathcal{E}}_C(\underline{S})$ through a spatial surface $\underline{S}$ (actually $\underline{S}$ is a coconnected set of spatial links which forms a surface in the dual lattice) is defined as the product over oriented links $\underline{l}\in\underline{S}$ of $\hat{\mathcal{E}}_C(\underline{l})$. Consider a spatial volume $\underline{\Lambda}$ (a sphere or a parallelepiped) and denote the right and left halves of its surface by $\underline{S_r}$ and $\underline{S_l}$ (thus as sets of spatial links $\partial^*\underline{\Lambda}=\underline{S_r}\cup\underline{S_l}$), and by $\underline{S_m}$ the minimal surface with the same boundary as $\underline{S_r}$ and $\underline{S_l}$ ($\partial^*\underline{S_m}=\partial^*\underline{S_r}=\partial^*\underline{S_l}$ as sets of plaquettes). A quantity suitable for testing the electric flux correlations in the vacuum is:

$$F_C(\underline{\Lambda}) := \frac{\langle 0\,|\,\hat{\mathcal{E}}_C(\underline{S_l})\,|\,0\rangle\;\langle 0\,|\,\hat{\mathcal{E}}_C(\underline{S_r})\,|\,0\rangle}{\langle 0\,|\,\hat{\mathcal{E}}_C(\partial^*\underline{\Lambda})\,|\,0\rangle} \tag{8}$$

In the free charge phase there are many closed electric flux lines in the vacuum, as can be seen from the fact that the expectation value of Wilson loops is relatively large. $\hat{\mathcal{E}}_C(\underline{S_r})$ and $\hat{\mathcal{E}}_C(\underline{S_l})$ are affected by the closed flux lines that intersect $\underline{S_m}$ once, but $\hat{\mathcal{E}}_C(\partial^*\underline{\Lambda})$ is not. Therefore, by arguments similar to those used to prove exponentiation in convergent expansions, we expect an area law for $F_C(\underline{\Lambda})$. In the confinement region there are few closed flux lines in the vacuum. In the Higgs region the closed flux lines in the vacuum cannot play a distinguished role since open flux lines (between charge-anticharge pairs) are also condensed in the vacuum (and screen the electric flux). Thus in the confinement-Higgs phase we expect the numerator and denominator of (8) to be roughly the same, up to a perimeter contribution at $\partial^*\underline{S_m}$. Denoting by $r(\underline{\Lambda})$ the linear dimension of $\underline{\Lambda}$, the criterion for existence of charged states in a *massive* phase is that for $r(\underline{\Lambda}) \to \infty$

$$F_C(\underline{\Lambda}) \sim \begin{cases} \exp(-c_1 \left|\underline{S_m}\right|) & \exists \text{ free charges} \\ \exp(-c_2 \left|\partial^* \underline{S_m}\right|) & \text{charges are screened} \end{cases} \tag{9}$$

(c_1 and c_2 are constants).

C. Let us denote by $\underline{S} \times \{0,1\}$ the set of timelike plaquettes with spatial projection in $\underline{S}$. Then

$$\langle 0 \,|\, \hat{\mathcal{E}}_C(\underline{S}) \,|\, 0 \rangle = \left\langle \prod_{p \in \underline{S} \times \{0,1\}} \exp \beta \{\chi(CU_p) - \chi(U_p)\} \right\rangle \tag{10}$$

Thus we have expressed (8) in terms of euclidean expectation values. One of the main ingredients in proving (9) is the cancellation of surface contributions between the numerator and the denominator of (8). By an abuse of language we shall call $F_C(\underline{\Lambda})$ the FCOP.

D. For small κ, $F_C(\underline{\Lambda})$ behaves similarly to the 't Hooft loop [7,8] of the pure gauge ($\kappa = 0$) theory. At finite $r(\underline{\Lambda})$ the FCOP defines a characteristic length R_H in the *Higgs region*: the asymptotic perimeter law sets in only at $r(\underline{\Lambda}) > R_H$, while at $r < R_H$ we have the area law. We interpret R_H as the *screening length for the center-electric charge* (in the vacuum: for dynamical charge fluctuations). In the case of nonadditive charges the screening for the charge and that for the potential between two sources are different concepts, since the equation $\Delta V = \rho$ that relates them in usual electrodynamics no longer holds. For $G = Z_2$ for example, $R_H \to \infty$ as $\beta \to \infty$, while the screening length for the potential becomes zero in the same limit [5].

E. The FCOP is a purely spatial quantity and can be defined at finite temperatures too. In the deconfinement region we expect it to define the characteristic length R_H in a similar way to the Higgs region (for $G = Z_2$ see [5]), since charge-anticharge pairs connected by an open flux line are here condensed in the thermal equilibrium state too. Thus we can use the FCOP to investigate the confinement-deconfinement transition (or crossover).

In view of duality considerations, it is amusing to note that using the VOOP and the FCOP we can immediately see that at small κ the intermediate-β phase of Z_n models is massless. Using the known results for the pure gauge theory [8,15] and Griffith inequalities, one can easily show that the VOOP is zero, so there are free charges, and the FCOP has perimeter law, so either there are no free charges (which is ruled out by the VOOP) or the theory is massless.

4. DUAL INTERPRETATION OF THE FCOP

Consider a surface $\underline{S}$, chosen for simplicity to lie in a coordinate plane. In the pure gauge theory, the electric flux operator $\hat{\mathcal{E}}_C(\underline{S})$ creates from the vacuum a candidate for a *vortex*, i.e. a state containing a closed line of center-magnetic flux at the boundary $\partial^* \underline{S}$ of $\underline{S}$ (this is the *induction law* and it follows from the commutation relations with the Wilson loop operator $\hat{U}(\underline{L})$, $\partial \underline{L} = \emptyset$) [7,8]. The state has an energy proportional to the perimeter $|\partial^* \underline{S}|$, and its vacuum overlap $\langle 0 \,|\, \hat{\mathcal{E}}_C(\underline{S}) \,|\, 0 \rangle$ is usually called a *'t Hooft loop*. An area law for the 't Hooft loop means the state is an excitation (it is almost orthogonal to the vacuum), whereas a perimeter law means the vortices are condensed in the vacuum [7,8,9] (the scalar product with the vacuum is as large as we can expect in this case).

In order to generalize these ideas to the theory with matter fields we need vortex-type sources. It is possible to perform a duality transformation for the center degrees of freedom alone [8]. The gauge transformation operators of the dual model are localized on the links of the dual lattice, i.e. on the plaquettes $\underline{p}$ of the original lattice. In the original model their eigenvalues can be interpreted as external center-magnetic dipoles. We can enlarge the algebra of gauge-variant operators by adding the magnetic dipole creation operators $\hat{\mathcal{M}}_C(\underline{p})$ (they are left multiplication operators by $C \in \mathcal{C}_G$).

Let us now mimick the discussion in section 2 for the VOOP, this time however for vortices and not charges.

A. A naïve candidate for a vortex state is:

$$\hat{\mathcal{E}}_C(\underline{S})\,|0\rangle \tag{11}$$

For large separations the energy of this state is proportional to the area $|\,\underline{S}\,|$. We can regularize the energy by translating the gauge-variant object $\hat{\mathcal{M}}_C(\partial^* \underline{S})\,\hat{\mathcal{E}}_C(\underline{S})$ by n steps into euclidean time (as usual, $\hat{\mathcal{M}}_C(\partial^* \underline{S})$ is defined as a product over oriented plaquettes of $\hat{\mathcal{M}}_C(\underline{p})$):

$$|\underline{S}, n\rangle := \hat{\mathcal{M}}_C^\dagger(\partial^* \underline{S})\,\hat{T}^n \hat{\mathcal{M}}_C(\partial^* \underline{S})\,\hat{\mathcal{E}}_C(\underline{S})\,|0\rangle \tag{12}$$

is a gauge-invariant state with energy proportional to $|\partial^* \underline{S}|$ provided n grows rapidly enough with the linear size $r(\underline{S})$ of $\underline{S}$ (we proved this rigorously only for $G = Z_2$ up to now). Notice that this time the source translated into euclidean time is a *closed line of ex-*

ternal center-magnetic dipoles, or, in other words, a *closed solenoid*.

B. For large $r(\underline{S})$ the vortex is either free or screened. A quantity testing this is the vacuum overlap of the normalized vortex state, which, as discussed for the pure gauge theory, has an area law if vortices exist and a perimeter law if they are screened:

$$\frac{\langle\, 0 \,|\, \underline{S}, n \rangle}{\|\, |\underline{S}, n\rangle \,\|} \sim \begin{cases} \exp(-c_1\,|\underline{S}|) & \exists \text{ free vortices} \\ \exp(-c_2\,|\partial^*\underline{S}|) & \text{vortices are screened} \end{cases} \tag{13}$$

C. In terms of euclidean expectation values the l.h.s. of (13) turns out to be nothing else than the square root of $F_C(\underline{\Lambda})$, where $\underline{\Lambda}$ is a cylinder with basis $\underline{S}$ and height n (and $\underline{S_m} = \underline{S}$), the height being in euclidean time rather than in one of the space directions. Thus *the* FCOP *is the* VOOP *for center-magnetic vortices.*

D. For small κ the l.h.s. of (13) behaves similarly to the 't Hooft loop at $\kappa = 0$. In the free charge phase (if there is any) the vortices are free excitations. In the confinement region they condense into the vacuum (for electric charges this happens in the Higgs region). In the *Higgs region* the vortices exist for $r(\underline{S}) < R_H$, while for $r(\underline{S}) > R_H$ their vacuum overlap becomes large. In analogy to the situation for charges in the confinement region, we call this phenomenon *vortex fragmentation*. For the Z_2 model the vortex fragmentation length R_H obeys:

$$R_H \sim \frac{e_{solenoid}}{\sigma_s} \tag{14}$$

where $e_{solenoid}$ is the linear energy density of a closed line of external center-magnetic dipoles and σ_s is the surface tension computed from the denominator of (8) and (13), which at small $r(\underline{\Lambda})$ behaves (similarly to the pure matter theory) as $\exp(-\sigma_s|\underline{\Lambda}|)$.

E. Eq. (13) cannot be defined at finite temperatures since in this case it is impossible to let $n \to \infty$.

It is not yet clear to us what role the condition of nonzero mass gap plays for the dual interpretation of the FCOP. The crirerion (13) for deciding whether a vortex is orthogonal to the vacuum is probably not sensitive enough to the soft modes typical for massless phases.

5. DUAL INTERPRETATION OF THE VOOP

Let us ask a question similar to that leading to the FCOP, but now for the vortices: *what properties of the vacuum are responsible for the existence of vortex states?*

A. In analogy to section 2, assume that a vortex state can be created by acting on the vacuum with an operator localized inside a *spatial disc* that is thin at the perimeter but whose thickness at the center increases linearly with its diameter (this generalizes the cone of section 2; similarly to the charged state case, for $G = Z_2$ this construction can be carried out explicitely and it simply amounts to an energy regularization different from (12)). Although for vortices no structural results like those of [13] are available, we probably have to assume that the theory is in a massive phase (or at finite temperatures, where there are no infinite range correlations because of the thermal fluctuations). The vortex is localized at the perimeter of the disc, and its magnetic flux is measured by $\hat{U}(\underline{L})$, $\underline{L}$ being a closed line that winds around the perimeter. If the vortex state exists, the asymptotic orientation of the disc should not be observable (the disc could e.g. bend behind the moon), i.e. we should not be able to determine the magnetic flux by measuring a quantity localized around the intersection $\underline{L}'$ of $\underline{L}$ with the disc. One possible choice for this quantity is $\hat{\phi}^\dagger(\underline{x})\hat{U}(\underline{L}')\hat{\phi}(\underline{x}')$, where $\underline{x}$ and $\underline{x}'$ are now the endpoints of $\underline{L}'$. Let us denote by *vortex flux* the flux associated to the operator inside the disc, i.e. the flux created by acting with the electric flux operator on the vacuum (like in (11)). The conclusion is that there have to be strong correlations in the vacuum which delocalize the vortex flux.

B. Assume $\underline{L}$ is a rectangle in one of the coordinate planes and denote its left and right halves by $\underline{L_l}$ and $\underline{L_r}$ with $\underline{x}$ and $\underline{x}'$ as common endpoints, and by $\underline{L_m}$ the straight line from $\underline{x}$ to $\underline{x}'$. A quantity suitable for measuring the vacuum correlations described above is:

$$\frac{\langle\, 0 \,|\, \hat{\phi}^\dagger(\underline{x})\hat{U}(\underline{L_l})\hat{\phi}(\underline{x}') \,|\, 0 \,\rangle \;\langle\, 0 \,|\, \hat{\phi}^\dagger(\underline{x})\hat{U}(\underline{L_r})\hat{\phi}(\underline{x}') \,|\, 0 \,\rangle}{\langle\, 0 \,|\, \hat{U}(\underline{L}) \,|\, 0 \,\rangle} \tag{15}$$

In the free charge phase there are many closed vortex flux surfaces in the vacuum, since $\langle\, 0 \,|\, \hat{\mathcal{E}}_C(\partial^*\underline{\Lambda}) \,|\, 0 \,\rangle$ is relatively large. $\hat{\phi}^\dagger(\underline{x})\hat{U}(\underline{L_l})\hat{\phi}(\underline{x}')$ and $\hat{\phi}^\dagger(\underline{x})\hat{U}(\underline{L_r})\hat{\phi}(\underline{x}')$ are affected by the closed vortex flux surfaces that intersect $\underline{L_m}$ once, but $\hat{U}(\underline{L})$ is not. Arguing as in section 2 we expect (15) to decay exponentially with $|\underline{L_m}| = |\underline{x} - \underline{x}'|$. In the Higgs region $\langle\, 0 \,|\, \hat{\mathcal{E}}_C(\partial^*\underline{\Lambda}) \,|\, 0 \,\rangle$ is relatively small, so there are few closed vortex flux surfaces in the vacuum. In the confinement region $\langle\, 0 \,|\, \hat{\mathcal{E}}_C(\underline{S}) \,|\, 0 \,\rangle$ is relatively large both for closed and open surfaces $\underline{S}$, so the dominant role is played by the open vortex flux surfaces that screen the magnetic flux. Thus in the confinement-Higgs phase we expect the numerator and denominator of (15) to be roughly the same, up to a contribution from the endpoints. To sum up, the criterion for existence of vortex states in a massive phase is that for $|\underline{x} - \underline{x}'| \to \infty$ (15) behaves as:

$$\begin{cases} \exp(-c_1 |\underline{x} - \underline{x}'| & \exists \text{ free vortices} \\ \text{const} & \text{vortices are screened} \end{cases} \quad (16)$$

C. In terms of euclidean expectation values (15) is the square of (6), with the euclidean time and one of the space directions interchanged. Thus *the* VOOP *is the* FCOP *for center-magnetic vortices.*

D. For large β, (15) behaves similarly to a ϕ-two-point function at $\beta = \infty$. In the *confinement region* however, (15) defines the characteristic length R_c, which can be now reinterpreted as the *screening length for the center-magnetic flux* (in the vacuum: for dynamical magnetic flux fluctuations).

E. The spatial VOOP can be defined at finite temperatures too. In the confinement region the vortices are condensed. It would be highly surprising if objects that are condensed at low temperatures exist as excitations at higher temperatures. Thus we expect the spatial VOOP to define the characteristic length R_c in the the deconfinement region too. We can use it to investigate the transition (or crossover) between the confinement-deconfinement region and the Higgs region, since in the latter the vortices are not condensed.

While in the original interpretation the VOOP is on a stronger theoretical footing than the FCOP, in the dual interpretation the situation is reversed.

6. CONCLUSIONS AND OUTLOOK

By investigating properties of center-magnetic vortices we have given a dual interpretation of the VOOP and the FCOP. A nice duality between the confinement and the Higgs region emerged. At finite temperatures, we gained a better understanding of the deconfinement region and of the transitions leading to it.

In 3 dimensions it is the center-magnetic monopoles rather than the vortices that play a role dual to the center-electric charged states. We have [1,3,5] in principle (for discrete G in detail) given a method to construct the monopoles.

Let us mention a few important open problems and tasks for the future:

- Clarify the theoretical situation in massless phases.
- Compute the VOOP in perturbation theory.
- Compute the FCOP in simulations.
- Can the FCOP be computed perturbatively?
- Are vortices in the Higgs region stable excitations, can they be used to detect new particles, or are they purely theoretical string-like objects?
- What is the connection between the characteristic lengths R_H and R_c, and the usual picture of screening in the deconfinement region?

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