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PREDICTIONS OF THE CKM MODEL FOR *CP* ASYMMETRIES IN B DECAY

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We discuss present-day uncertainties for the value of the *CP*-violating phase δ in the CKM matrix and point out how a knowledge of m_t and/or x_s could substantially reduce this uncertainty A model-independent measurement of δ is, in principle, possible by studying certain *CP*-violating asymmetries, involving B^0 mesons decaying into *CP*-conjugate hadronic final states There exist three different classes of these asymmetries and we give estimates for their values, based on our present knowledge of the CKM matrix Some comments on the experimental requirements for detecting these asymmetries are also presented

1. Introduction

In the standard electroweak model, with three generations of quarks and leptons, *CP*-violating phenomena arise simply from the presence of a nontrivial phase δ in the Cabibbo–Kobayashi–Maskawa (CKM) mixing matrix V_{ij} Although the standard model cannot explain the deeper origin for this phase, it is obviously very important to know whether or not the observed *CP* violation in the kaon system arises from the phase δ All the evidence we have at present, including the recent positive signal of a nonvanishing value for ε'/ε [1], is consistent with this hypothesis However, the evidence for *CP* violation being due only to the CKM phase δ is weak, and this phase itself is badly determined

In the coming years this situation is likely to be improved by new experimental observations Of particular importance would be a determination of the top quark mass and of the value of the mixing parameter x_s in the B_s–$\bar{B}_s$ system It may well be that, with these measurements in hand, one will find an inconsistency with the simple CKM mixing scheme However, even if this turns out not to be the case, there is likely to remain considerable uncertainty attached to the value of δ This is because we are still unable to calculate the hadronic matrix elements of weak operators reliably and this, obviously, directly affects δ. The purpose of this note is

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to discuss critically what future experimental information is most likely to provide clear tests of the CKM scheme and, in particular, will allow for a reliable determination of the CP-violating phase δ

The plan of this paper is as follows We shall begin by briefly reviewing the present uncertainties in determining the CP-violating phase in the CKM matrix, which stem both from our ignorance of a precise value for m_t and of the value of the ratio $|V_{ub}/V_{cb}|$, as well as from the unreliability of hadronic-matrix-element calculations We shall show, next, that these latter uncertainties, however, can be largely obviated by studying certain classes of CP-violating asymmetries in B decays, involving decays of neutral B's into CP-conjugate hadronic final states The non-negligible asymmetries of this type, either time integrated or time dependent, measure one of three possible combinations of phases of the matrix elements V_{ij} (in a convenient parametrization), each of them, of course, being a function of δ Using the best information available at present on the CKM matrix, we then present ranges of predictions for these important CP-violating asymmetries in the B system and draw some conclusions on their likely observability

2. The mixing matrix

For our purposes, it is convenient to parametrize the CKM matrix in the form suggested by Maiani [2]

$$V=\begin{pmatrix} c_1c_3 & s_1c_3 & s_3e^{i\delta} \\ -s_1c_2-c_1s_2s_3e^{-i\delta} & c_1c_2-s_1s_2s_3e^{-i\delta} & s_2c_3 \\ s_1s_2-c_1c_2s_3e^{-i\delta} & -c_1s_2-s_1c_2s_3e^{-i\delta} & c_2c_3 \end{pmatrix}, \tag{1}$$

where $s_1 \equiv \sin\theta_1$, $c_1 \equiv \cos\theta_1$, etc Since the angles θ_i are known to have a hierarchical pattern, it is useful to write, following Wolfenstein [3]

$$s_1=\lambda, \qquad s_2=A\lambda^2, \qquad s_3=A\rho\lambda^3, \tag{2}$$

with λ corresponding essentially to the Cabibbo angle, $\lambda \simeq 0\ 22$ Then, to $O(\lambda^4)$, but keeping for the moment the phase information for each of the elements of the CKM matrix, one has

$$V=\begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\rho\lambda^3e^{i\delta} \\ -\lambda(1+A^2\lambda^4\rho\,e^{-i\delta}) & 1-\frac{1}{2}\lambda^2-A^2\rho\lambda^6e^{-i\delta} & A\lambda^2 \\ A\lambda^3(1-\rho\,e^{-i\delta}) & -A\lambda^2(1+\lambda^2\rho\,e^{-i\delta}) & 1 \end{pmatrix} \tag{3}$$

One sees immediately that, with this parametrization, only two elements of V can have a significant imaginary part. V_{ub} and V_{td} It is these phases which will play a crucial role in the B-decay CP asymmetries and which should give rise to rather substantial experimental signals. In evaluating the CP-violating ε and ε' parameters in the K-sector, however, one needs also to keep track of the "small" phase in V_{cd} Of course, all CP-violating phenomena disappear if the CKM phase $\delta = 0$ or π

The parameters A and ρ are, in principle, obtainable from B decay. For the estimate of these quantities, we rely (as is commonly done) on the quark picture supplemented by some hadronization model The ratio*

$$R = \frac{\Gamma(\mathrm{b} \to \mathrm{u})}{\Gamma(\mathrm{b} \to \mathrm{c})} \simeq 2\frac{|V_{\mathrm{bu}}|^2}{|V_{\mathrm{bc}}|^2} \simeq 2(\lambda\rho)^2, \tag{4}$$

measures ρ, while the B lifetime fixes A A recent analysis by Altarelli and Franzini [5] gives

$$|V_{\mathrm{bc}}|^2 = \lambda^4 A^2 = \frac{(2\ 9 \pm 0.6) \times 10^{-3}}{\tau_{\mathrm{B}}(10^{-12}\,\mathrm{s})} \tag{5}$$

Using $\tau_{\mathrm{B}} = (1\ 11 \pm 0\ 16) \times 10^{-12}$ s [6], the above implies

$$A = 1\ 05 \pm 0\ 17 \tag{6}$$

In the analysis which follows, we shall take A equal to its central value. The ratio R is subject to more theoretical uncertainty which is related to problems with the interpretation of the lepton spectra from semileptonic B decays From a study of these decays [7], one infers the rather conservative upper bound $R \leq 0\ 08$, which implies $\rho \leq 0\ 9$** On the other hand, the recent observation of charmless B decays by the ARGUS collaboration [9] shows that $V_{\mathrm{ub}} \neq 0$ and so provides a lower bound for ρ. Another very conservative analysis [9] gives $\rho \geq 0.3$, so that ρ lies in the range

$$0.3 \leq \rho \leq 0\ 9 \tag{7}$$

The phase δ is directly, but far from uniquely, determined by the CP-violating parameter ε in the K system. A further constraint on δ is also provided by the recent observation of $\mathrm{B_d}$–$\overline{\mathrm{B}}_{\mathrm{d}}$ oscillations The mixing parameter x_{d} is proportional to $|V_{\mathrm{td}}|^2$ and hence it is sensitive to the combination $(1 + \rho^2 - 2\rho\cos\delta)$ In principle, the new data on ε'/ε [1] provide a further constraint on δ. However, the theoretical uncertainties are such that this measurement does not restrict δ beyond the range allowed by ε and x_{d} [10] Since m_{t} is not known, and ρ is only fixed to be in the interval of eq (7), we will display, below, the allowed values of δ as a function of

* The factor 2 in eq (4) arises from phase space and QCD correctors See, for example, ref [4]

** Theoretical analysis (for a discussion, see ref [5]), along with the CLEO result [8], indicate that $\rho \leq 0\ 6$ However, in light of uncertainties in extracting R from experiments, and in eqs (4) and (5), we will use the conservative upper bound $\rho \leq 0\ 9$ Since we will plot the allowed region in ρ–δ space, the CLEO bound is easily seen these two parameters, indicating, furthermore, the effects of the uncertainties induced by the hadronic matrix elements Similar analyses have been carried out by a number of different groups recently [11-14]

The standard analysis of Buras et al [15] gives for $|\varepsilon|$ the formula

$$|\varepsilon| = \frac{G_F^2 f_K^2 M_K M_W^2}{6\sqrt{2}\,\pi^2 \Delta M_K} B_K \left(A^2 \rho \lambda^6 \sin\delta\right)\left(y_c\{\eta_3 f_3(y_c, y_t) - \eta_1\}\right.$$
$$\left. + \eta_2 y_t f_2(y_t) A^2 \lambda^2 (1 - \rho\cos\delta)\right) + \sqrt{\tfrac{1}{2}}\,\xi \qquad (8)$$

Here, $y_i = m_i^2/M_W^2$ and f_2 and f_3 are weakly dependent functions of the top and charm masses

$$f_2(y_t) = 1 - \frac{3y_t(1+y_t)}{4(1-y_t)^2}\left(1 + \frac{2y_t}{1-y_t^2}\ln y_t\right), \qquad (9a)$$

$$f_3(y_c, y_t) = \ln\frac{y_t}{y_c} - \frac{3y_t}{4(1-y_t)}\left(1 + \frac{y_t}{1-y_t}\ln y_t\right) \qquad (9b)$$

The η_i are QCD correction factors ($\eta_1 \simeq 0\,7$, $\eta_2 \simeq 0\,6$, $\eta_3 \simeq 0\,4$ [16]), while B_K encapsulates our present ignorance of the matrix element of $(\bar{d}\gamma^\mu(1-\gamma_5)s)^2$ between K^0 and $\bar{K}^0$, with $B_K = 1$ corresponding to the vacuum insertion approximation Finally, ξ is the phase parameter of the $(I=0)$ $K \to 2\pi$ weak amplitudes ($\xi = \mathrm{Im}\,A_0/\mathrm{Re}\,A_0$) In the quark phase convention which we are using, its presence guarantees that $|\varepsilon|$ is actually convention independent ξ is directly related to $|\varepsilon'|$ [15] and one has

$$|\varepsilon'| = \frac{1}{\sqrt{2}}\frac{\mathrm{Re}\,A_2}{\mathrm{Re}\,A_0}|\xi| \simeq 0.035|\xi|, \qquad (10)$$

where the numerical value above uses experimental information for the kaon amplitudes The recent determination of $|\varepsilon'/\varepsilon|$ [1] implies that $|\xi| \simeq 0.1|\varepsilon|$ In view of this, and of the other uncertainties in eq (8), we shall neglect ξ altogether in our analysis

3. B_d–$\bar{B}_d$ mixing

The observation of B_d–$\bar{B}_d$ mixing by the ARGUS collaboration [17] has provided an independent constraint on the parameters of the Cabibbo–Kobayashi–Maskawa matrix. Since for the B_d system one expects [15] $\Delta\Gamma \ll \Delta M$ and the magnitude of the $\Delta B = 2$ CP-violation to be small ($|(1-\varepsilon_d)/(1+\varepsilon_d)| \simeq 1$) the measured ratio,

$$r_d = \frac{\Gamma(B_d \to \ell^- X)}{\Gamma(B_d \to \ell^+ X)} = 0\ 21 \pm 0\ 08\,, \tag{11}$$

directly fixes the mixing parameter $x_d = \Delta M/\Gamma$. Using the expression

$$r_d \simeq \frac{x_d^2}{2 + x_d^2} \tag{12}$$

and the ARGUS measurement [17] yields $x_d = 0\ 73 \pm 0\ 18$ In our analysis, following Ali [11], we shall use for x_d the 90% confidence limit provided jointly by this measurement and the upper bound of the CLEO collaboration [18]

$$0\ 78 \geq x_d \geq 0\ 44\,. \tag{13}$$

Theoretically, x_d receives its dominant contribution by the presence of top quarks in the box diagram and one finds [15, 19]

$$x_d = \tau_B \frac{G_F^2}{6\pi^2} M_B M_W^2 \left(f_{B_d}^2 B_{B_d}\right) \eta_B y_t f_2(y_t) \left\{A^2 \lambda^6 (1 + \rho^2 - 2\rho\cos\delta)\right\} \tag{14}$$

Here, the hadronic uncertainty is hidden in the factor $f_{B_d}^2 B_{B_d}$, whose meaning is analogous to that of the corresponding quantities in the kaon system, except that here also f_B is not measured The parameter η_B is a QCD correction factor, which in refs [15, 19] has been estimated to be $\eta_B \simeq 0.85$. A recent calculation [20], however, including certain higher order QCD effects, obtains a lower value, $\eta_B \simeq 0.63$. We shall adopt this value here, but we note that since $f_{B_d}^2 B_{B_d}$ is quite uncertain, one cannot really tell the difference between these two assumptions Being rather conservative, we shall allow, for $(f_{B_d}^2 B_{B_d})^{1/2}$, the range 100–200 MeV, which is slightly larger than that used in ref. [5].

We have not included in our analysis the MARK II [21] or UA1 [22] results on B–$\overline{B}$ oscillations, since these experiments cannot distinguish B_s–$\overline{B}_s$ from B_d–$\overline{B}_d$ oscillations. As Ali [11] has pointed out, these results can powerfully constrain the CKM matrix, given a knowledge of the relative amount of B_d and B_s produced However, these constraints are very dependent on the B_d/B_s production ratio The experiments measure the quantity χ, which is

$$\chi = P_d \frac{x_d^2}{2(1 + x_d^2)} + P_s \frac{x_s^2}{2(1 + x_s^2)}\,, \tag{15}$$

where P_d (P_s) is the probability that a B_d (B_s) is produced. The MARK II

TABLE 1
The upper bound on x_d from the MARK II data [21], assuming a value of $x_s = 7$, is shown here as a function of the production probabilities P_d and P_s

P_d	P_s	$(x_d)_{max}$
0 4	0 2	0 35
	0 15	0 55
	0 1	0 74
0 375	0 2	0 37
	0 15	0 57
	0 1	0 78

experiment gives a 90% confidence level upper limit on χ

$$\chi \leq 0.12 \tag{16}$$

Thus, for a given P_d, P_s and x_s, this gives an upper limit on x_d In table 1, we have shown this upper limit, for $x_s = 7$ (a typical value), as a function of P_d and P_s As is evident from the table and eq (13), depending on the values of P_d and P_s taken, this data can either rule out the standard model (e g. $P_d = 0\ 4$, $P_s = 0\ 2$) or give no bounds whatsoever (e g $P_d = 0\ 375$, $P_s = 0\ 1$) Given the uncertainty in the information which these experiments provide, we have preferred to be conservative and ignore this information altogether

Assuming some (typical) values for B_K and $f^2_{B_d} B_{B_d}$, eqs (8) and (14) determine δ as a function of m_t and ρ* For example, taking $B_K = 1$ and $f^2_{B_d} B_{B_d} = (150\ \text{MeV})^2$ and letting m_t vary from 40 to 180 GeV** gives the "moon-shaped" allowed region in the ρ–δ plane of fig. 1 (a similar analysis has been carried out in ref [13]) Low values of m_t require that the phase δ be near π [11–14], so as to enhance the $(1 + \rho^2 - 2\rho \cos \delta)$ factor in eq (14) A substantial portion of the allowed δ values is eliminated when one imposes the lower bound of $\rho > 0\ 3$ For this choice of theoretical parameters, the observation of charmless B decays [9] cuts the moon in half, eliminating all δ-values below $\delta \leq 2\ 7$! This effect is still present, although less sharply so, when one allows variations in the theoretically uncertain parameters over sensible ranges $1/3 \leq B_K \leq 1$, $(100\ \text{MeV})^2 \leq f^2_{B_d} B_{B_d} \leq (200\ \text{MeV})^2$*** This is demonstrated in fig 2. We see that for $B_K = 1$, but ranging over $f^2_{B_d} B_{B_d}$, the moon shaped region of fig. 1 expands. This expansion grows further as B_K is lowered to $\frac{2}{3}$ and a considerable portion of the ρ–δ plane is filled if $B_K = \frac{1}{3}$ In the analysis of

* The measurement of ε and the allowed range for ρ constrain $\sin \delta$ to be positive

** While the lower bound on m_t follows from the internal consistency of the presented analysis and refs [11–14], the upper bound results from the study of radiative corrections within the standard model (see ref [23a]), a bound of $m_t \leq 180$ GeV at 90% confidence limit is obtained in ref [23b]

*** These ranges are extensive enough that they compensate any reasonable variation in A

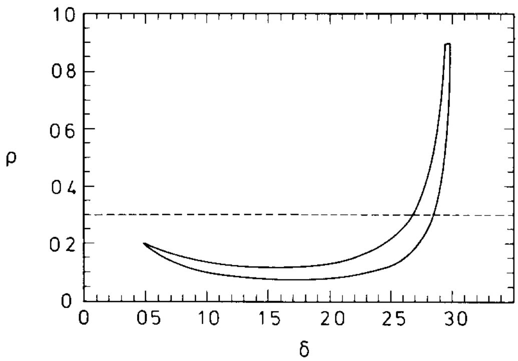

Fig 1 The domain in ρ–δ space (δ in radians), within which the standard model is compatible with the measurements of ε and x_d (90% confidence limit) We vary m_t between 40 GeV and 180 GeV and use $(f^2_{B_d} B_{B_d})^{1/2} = 150$ MeV and $B_K = 1$ The dashed line represents the lower bound $\rho \geq 0\ 3$ inferred from the observation of charmless B decays by ARGUS [9]

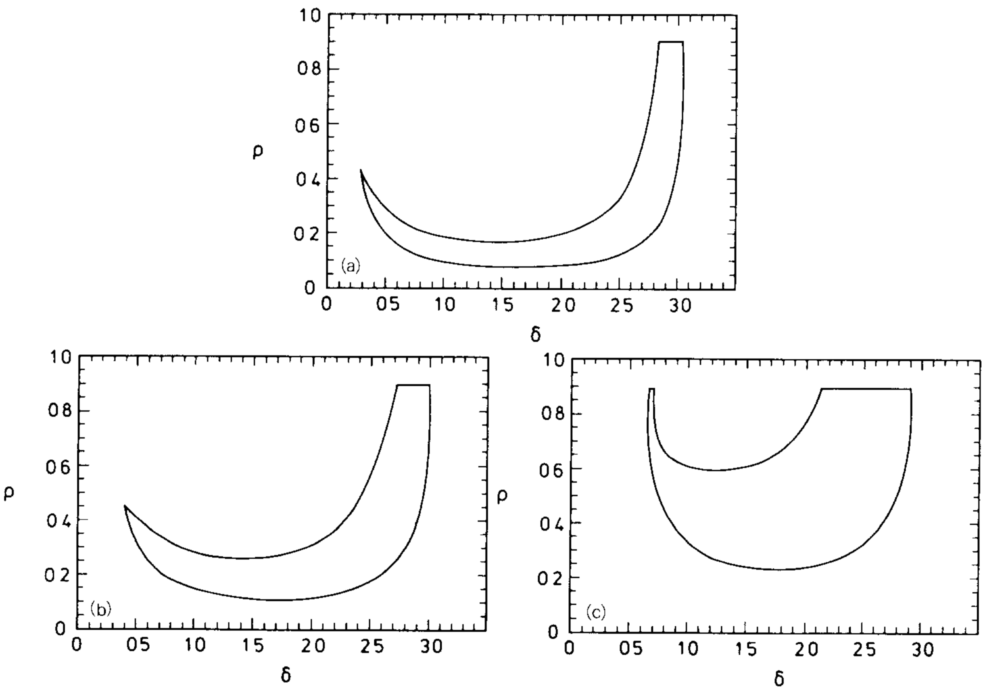

Fig 2 As in fig 1, but now, in addition, $(f^2_{B_d} B_{B_d})^{1/2}$ is varied between 100 MeV and 200 MeV (a) $B_K = 1$, (b) $B_K = \frac{2}{3}$, (c) $B_K = \frac{1}{3}$

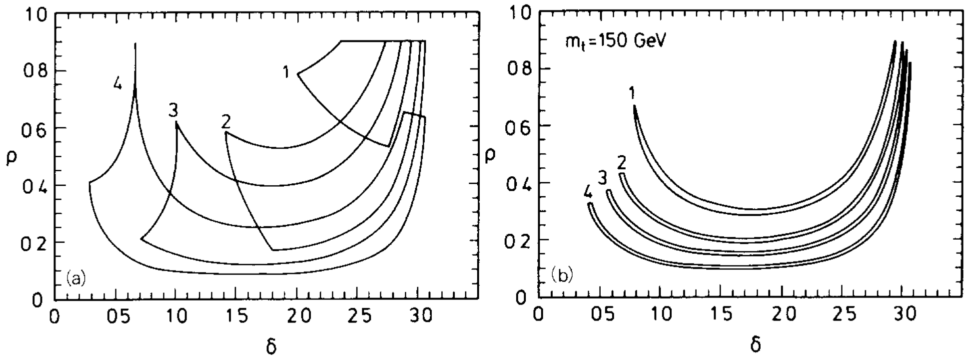

Fig 3 As in fig 1, but now m_t is kept fixed while B_K and $(f_{B_d}^2 B_{B_d})^{1/2}$ are allowed to vary within the ranges $\frac{1}{3} \le B_K \le 1$ and 100 MeV $\le (f_{B_d}^2 B_{B_d})^{1/2} \le$ 200 MeV (a) The areas 1, 2, 3, 4 correspond to fixed values m_t = 60, 90, 120, 180 GeV, respectively (b) Taking m_t = 150 GeV, the strips 1, 2, 3, 4 correspond to $B_K = \frac{1}{3}, \frac{1}{2}, \frac{2}{3}$, 1, respectively

CP-violating phenomena in the B-system, we shall take, for definitiveness, $B_K = \frac{2}{3}$, but shall continue to allow m_t and $f_{B_d}^2 B_{B_d}$ to vary over the ranges indicated above

A direct measurement of m_t and/or of B_s–$\overline{B}_s$ oscillations would do much to clarify the above situation, even if ρ cannot be restricted better than eq (7) This is illustrated in fig 3a, where we show the allowed ρ–δ ranges for four values of m_t (m_t = 60, 90, 120, 180 GeV), for the full uncertain theoretical ranges. This uncertainty comes mainly from our lack of knowledge of B_K, as is demonstrated in fig. 3b. There, we plot the allowed areas corresponding to fixed values of $B_K = \frac{1}{3}, \frac{1}{2}, \frac{2}{3}$ and 1, for m_t = 150 GeV In the SU(3) limit we have $\tau_{B_d} M_{B_d} \eta_{B_d} f_{B_d}^2 B_{B_d} =$

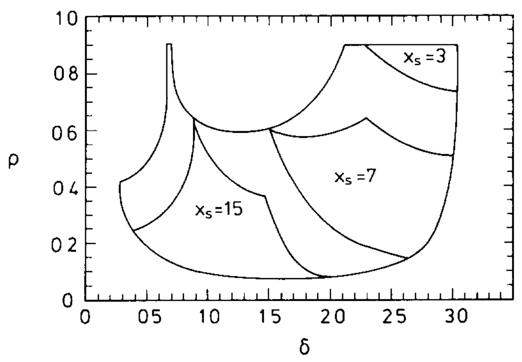

Fig 4 As in fig 1, but now all three parameters m_t, $(f_{B_d}^2 B_{B_d})^{1/2}$ and B_K are varied 40 GeV $\le m_t \le$ 180 GeV, 100 MeV $\le (f_{B_d}^2 B_{B_d})^{1/2} \le$ 200 MeV and $\frac{1}{3} \le B_K \le 1$ In addition, the further restrictions given by fixed x_s = 3, 7, 15 are shown

$\tau_{B_s} M_{B_s} \eta_{B_s} f_{B_s}^2 B_{B_s}$ and thus the ratio x_d/x_s is given by

$$\frac{x_d}{x_s} = \lambda^2 (1 + \rho^2 - 2\rho \cos\delta) . \tag{17}$$

Even if $f_{B_s}^2 B_{B_s}$ should turn out to differ from $f_{B_d}^2 B_{B_d}$ by a factor of 2, a measurement of x_s would still give substantial restrictions in the ρ–δ plane To illustrate this point, in fig. 4, using eq (17), we plot the constraints on the allowed ρ–δ range, for three values of x_s ($x_s = 3, 7, 15$), using the range of eq (13) for x_d Because fixing x_s gives correlated ranges for $f_{B_d}^2 B_{B_d}$ and m_t, the interval (13) for x_d is not always fully allowed. This has been taken into account in fig 4 Thus, a measurement of x_s is particularly constraining for the Cabibbo–Kobayashi–Maskawa model and we very much hope it will be attempted at Cornell (or DESY?).

4. *CP*-violating asymmetries

Because B mesons possess many more decay channels than K mesons, there exist a considerable variety of *CP*-violating phenomena that one can search for experimentally This subject, naturally, has generated intense theoretical interest and has, in some sense, been fully explored [24–27] However, most of the investigations to date have been "broad band", concentrating on the totality of the phenomena, without looking at any one decay, or class of decays, in detail Furthermore, in many instances, predictions are given only for what the *maximum* signal of *CP* violation could be Thus, many optimistic dynamical assumptions are made and the Cabibbo–Kobayashi–Maskawa parameters are stretched to their limits [26, 28] Here, we would like to take a rather more "narrow band" approach, by concentrating on *CP* asymmetries which are essentially independent of theoretical assumptions Furthermore, we want to predict what are reasonable expectations for these asymmetries, based on the constraints which we know today already exist for the CKM matrix elements

To observe *CP*-violating effects in B decays requires that there should be interference between two amplitudes with different phases Because the B and the $\overline{\mathrm{B}}$ states mix, if one looks at decays of B mesons into a final state f which can be reached by both B^0 and $\overline{\mathrm{B}}^0$ decays, then the required interference exists The interesting asymmetry to consider is the difference between the decay probability of a state which at $t = 0$ started as B^0 – denoted here by $\mathrm{B}^0(t)$ – into f, compared to the decay probability of $\overline{\mathrm{B}}^0(t)$ into $\bar{\mathrm{f}}$ (for a recent discussion, see [25, 26]) One finds the time integrated asymmetry

$$A_{\mathrm{f}} = \frac{\Gamma(\mathrm{B}^0(t) \to \mathrm{f}) - \Gamma(\overline{\mathrm{B}}^0(t) \to \bar{\mathrm{f}})}{\Gamma(\mathrm{B}^0(t) \to \mathrm{f}) + \Gamma(\overline{\mathrm{B}}^0(t) \to \bar{\mathrm{f}})} = -\frac{2x \operatorname{Im} \lambda_{\mathrm{f}}}{2 + x^2 + x^2 |\rho_{\mathrm{f}}|^2} . \tag{18}$$

Here, x is the mixing parameter of the B meson, while

$$\rho_f = \frac{A(\overline{B}^0 \to \bar{f})}{A(B^0 \to f)}, \tag{19}$$

$$\lambda_f = \frac{1-\varepsilon_B}{1+\varepsilon_B}\rho_f, \tag{20}$$

where ε_B is the analogue of ε for the B system This asymmetry becomes *independent* of strong interaction effects if f is a *CP* eigenstate ($\bar{f} = \pm f$), and the weak-decay process is dominated by just *one* amplitude* [27]. In this case $|\rho_f| = 1$ and eq (18) reduces to

$$A_f = -\frac{x}{1+x^2}\operatorname{Im}\lambda_f \tag{21}$$

Observe that the sign of A_f depends on the *CP* eigenvalue of f, thus, final states with opposite *CP* properties give rise to asymmetries of opposite sign [25]. Various comments are in order

(i) The asymmetry A_f vanishes, either in the case of no mixing ($x \to 0$) or full mixing ($x \to \infty$) For B_d, eq (13) puts one almost in an ideal situation since $x_d/(1+x_d^2) \simeq 0\,5$ For B_s, on the other hand, the situation is less favourable. Using eq. (17), our analysis suggests x_s extends over the range $3 \leq x_s \leq 20$, so that $0\,3 \geq x_s/(1+x_s^2) \geq 0\,05$

(ii) The magnitude of the factor $(1-\varepsilon_B)/(1+\varepsilon_B)$ in eq (20) is very nearly unity [15]. However, one must be careful about its phase, since only by including this phase information will $\operatorname{Im}\lambda_f$ be independent of the phase convention adopted In the quark phase convention we are using, since the top-quark graph totally dominates, one finds simply that

$$\frac{1-\varepsilon_B}{1+\varepsilon_B} = \begin{cases} \dfrac{V_{tb}^*}{V_{tb}}\dfrac{V_{td}}{V_{td}^*} = \dfrac{V_{td}}{V_{td}^*} \equiv e^{2i\phi}, & (B_d), \\[2ex] \dfrac{V_{tb}^*}{V_{tb}}\dfrac{V_{ts}}{V_{ts}^*} \simeq 1, & (B_s), \end{cases} \tag{22}$$

where the second line follows from the form of our CKM matrix, eq (3), in which only two elements have non-negligible phases, V_{ub} and V_{td}

(iii) Since $|\rho_f| = 1$, ρ_f itself is also a pure phase In fact, since only one weak-decay amplitude enters by assumption, ρ_f is a ratio of two Cabibbo–Kobayashi–Maskawa

* This will happen, in general, if the quark subprocesses in the decay do not contain both a u and a $\bar{c}$ quark Decays where the quark subprocess involves, for instance, $b \to u\bar{u}d$ or $b \to c\bar{c}s$ (e g $B_d \to \pi^+\pi^-$, $B_d \to \Psi K_S$) are examples where one expects $|\rho_f| = 1$

matrix elements (times a CP-sign, which we take to be positive in what follows). The ratio of CKM matrix elements in ρ_f containing only light quarks (u, d, s, c), with the convention of eq. (3), is essentially real and unity Thus, it can be ignored in the following Since V_{cb} is real, only for Cabibbo suppressed decays will ρ_f involve a phase.

$$\rho_f \simeq \begin{cases} \dfrac{V_{ub}}{V_{ub}^*} \equiv e^{2i\delta} & \text{(Cabibbo suppressed)}, \\ \dfrac{V_{cb}}{V_{cb}^*} = 1, & \text{(Cabibbo allowed)}. \end{cases} \tag{23}$$

The above simple considerations tell us that, in B decays, there are three classes of *model-independent* asymmetries which can be sizeable Each of these classes measures a *different* combination of phases of the CKM matrix elements – all, of course, related ultimately to δ

(1) Cabibbo-allowed B_d decays (e.g. $B_d \to \Psi K_S$ [29]),

$$\operatorname{Im} \lambda_1 \simeq \operatorname{Im} \frac{V_{td}}{V_{td}^*} = \sin 2\phi = \frac{2\rho \sin\delta(1-\rho\cos\delta)}{1+\rho^2-2\rho\cos\delta}, \tag{24}$$

(2) Cabibbo-suppressed B_d decays (e.g. $B_d \to \pi^+\pi^-$ [25]),

$$\operatorname{Im} \lambda_2 \simeq \operatorname{Im} \frac{V_{td}}{V_{td}^*}\frac{V_{ub}}{V_{ub}^*} = \sin 2(\phi+\delta) = \frac{2\sin\delta(\cos\delta-\rho)}{1+\rho^2-2\rho\cos\delta}, \tag{25}$$

(3) Cabibbo-suppressed B_s decays (e.g $B_s \to \rho^0 K_S$ [14]),

$$\operatorname{Im} \lambda_3 \simeq \operatorname{Im} \frac{V_{ub}}{V_{ub}^*} = \sin 2\delta = 2\sin\delta\cos\delta \tag{26}$$

It is obviously very interesting to know what ranges of $\operatorname{Im}(\lambda_i)$ are allowed by present data. The relevant plots are presented in fig. 5, where $\operatorname{Im}(\lambda_i)$ ($i=1,2,3$) is plotted against ρ, in the range $0.3 \le \rho \le 0.9$. In these graphs we have let m_t range from 40 to 180 GeV, have fixed $B_K = \frac{2}{3}$ and let 100 MeV $\le (f_{B_d}^2 B_{B_d})^{1/2} \le$ 200 MeV Fig 6 presents the same quantities but now for specific m_t values (m_t = 60, 90, 120, 150, 180 GeV).

One sees from figs. 5b, c that $\operatorname{Im}(\lambda_2)$ and $\operatorname{Im}(\lambda_3)$, for $0.3 \le \rho \le 0.9$, can take on rather large values. For $\operatorname{Im}(\lambda_1)$, on the other hand, values greater than $\sim 0\,4$ appear to be excluded. The actual measured asymmetries are, however, reduced by the mixing factor $x/(1+x^2)$ This is, at least, a factor of 2 for B_d and could be near a factor of 10 for B_s. From this viewpoint, therefore, the most promising processes

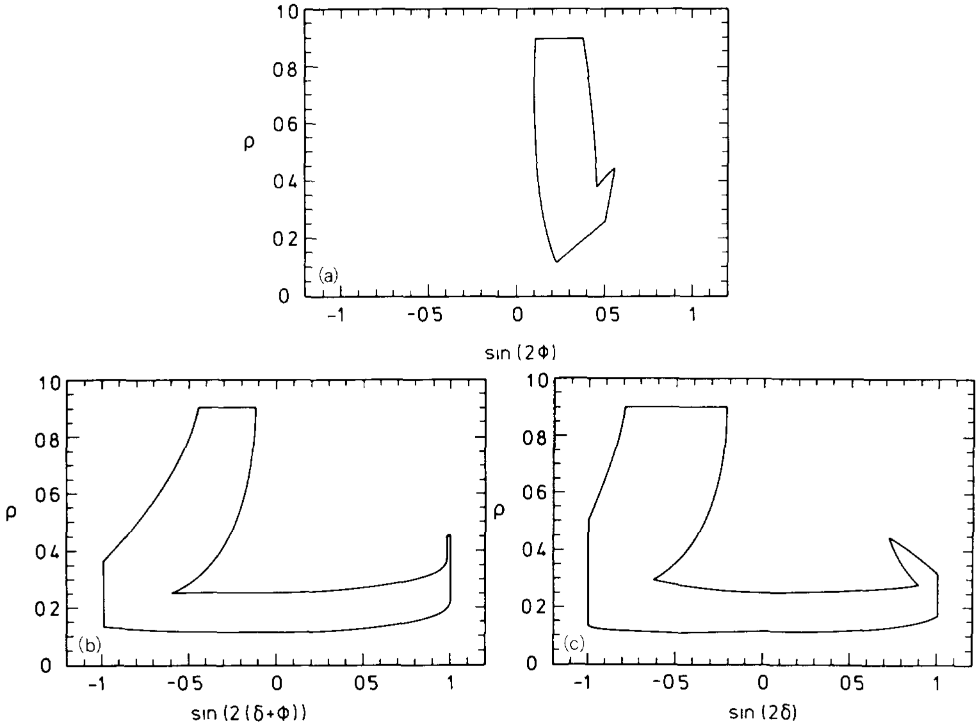

Fig 5 Varying $(f_{B_d}^2 B_{B_d})^{1/2}$ and m_t, 100 MeV $\leq (f_{B_d}^2 B_{B_d})^{1/2} \leq$ 200 MeV and 40 GeV $\leq m_t \leq$ 180 GeV, and fixing $B_K = \frac{2}{3}$, the areas within which the standard model is compatible with the measurements of ε and x_d (90% confidence limit) are shown for the following parameter spaces (a) $(\sin 2\phi, \rho)$, (b) $(\sin 2(\delta + \phi), \rho)$, (c) $(\sin 2\delta, \rho)$

appear to involve $\mathrm{Im}(\lambda_2)$ However, since these asymmetries concern Cabibbo-suppressed B_d decays, this overall rate is going to be considerably smaller For instance, the branching ratio $B_d \to \pi^+\pi^-$ is probably of $O(10^{-5})$, while we know that $BR(B_d \to \Psi K_S)$ is of $O(10^{-3})$ If it is possible to follow the time development of the B decays [24, 27, 30], then one gets rid of the reduction factor $x/(1 + x^2)$, since the probability of obtaining a state f at time t, for a beam which at $t = 0$ was pure B^0, is simply

$$N_f(t) = N_f(0)e^{-\gamma t}[1 - \mathrm{Im}\,\lambda_f \sin \Delta m t]. \tag{27}$$

If one has a large mixing parameter, $x = \Delta m/\gamma$, as is likely to be the case for B_s, then the non-exponential behaviour of eq (27) should be visible, provided of course that one can track the decay at all

It is difficult to estimate the number of B^0 decays needed to perform the CP-violation tests we have discussed First of all, these asymmetries A_f require that one know if the decaying B was originally a B^0 or a $\overline{B}^0$ To determine this, perhaps

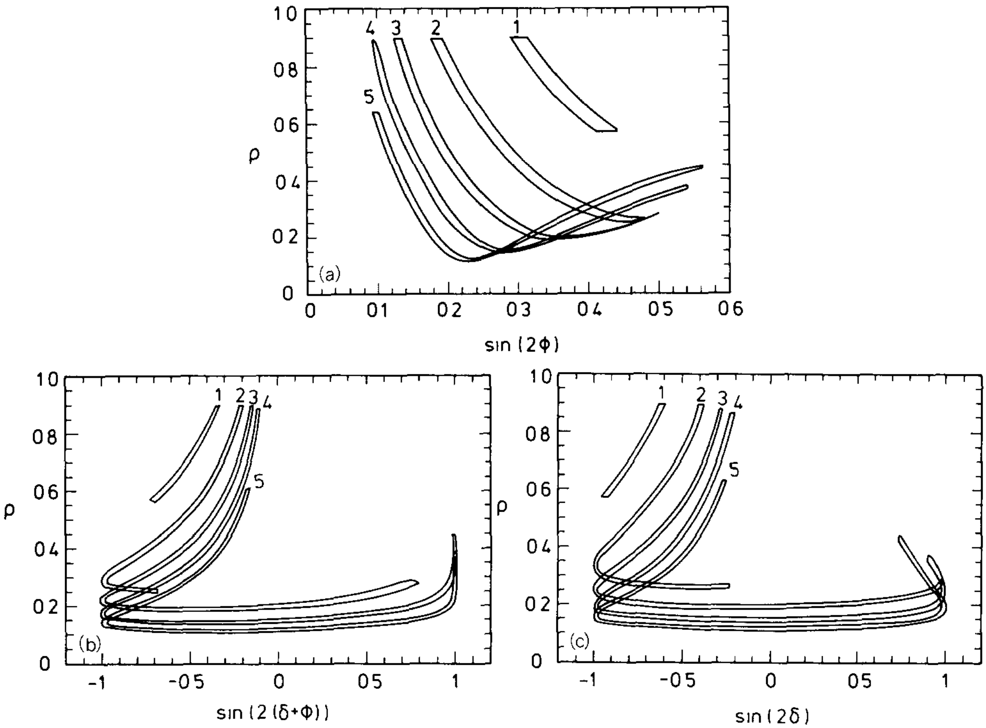

Fig 6 As in figs 5a, b, c, but now the strips 1, 2, 3, 4, 5 correspond to fixed values of m_t 60, 90, 120, 150, 180 GeV, respectively

the best method is to try to establish the charge of the associated B [27] This requires looking for another secondary vertex, besides that of the original decaying B Even being optimistic, this should cost at least a factor of 10 Consider the decay $B_d \to \pi^+\pi^-$ and imagine $\mathrm{Im}(\lambda_2) \simeq -0\,5$, so that $A_{\pi^+\pi^-} \simeq +0.25$ Establishing this asymmetry at the 3σ level requires approximately 150 tagged $B_d(t) \to \pi^+\pi^-$ events, which, with $\mathrm{BR}(B_d \to \pi^+\pi^-) \simeq 10^{-5}$ and a tagging efficiency of 10%, calls for 10^8 B decays. This number is quite typical and appears discouraging* Perhaps it is more important, therefore, to look for final states with clear experimental signals The decay $B_d \to p\bar{p}$**, for instance, whose branching ratio should also be of $O(10^{-5})$, appears very interesting. However, a cautionary remark is in order Since the $p\bar{p}$ pair in the final state can either be in a *p*- or an *s*-wave configuration, which have opposite *CP* eigenvalues, one may expect a large cancellation of the asymmetry,

* One should, of course, do a detailed study for any given process, before quoting a definitive number of B mesons needed However, we are skeptical of optimistic statements in the literature [31]

** This decay was suggested by Harari [32a], but see also ref [32b] The relevance of B decays into baryonic final states has also been stressed (although not in the context of *CP* violation) by Stech [32c] unless there is a dynamical suppression of the p-wave configuration. In this respect, the final state $\pi^+\pi^-$ is safe since it involves only spin-0 particles

Although the number of 10^8 B's is unpleasant to countenance, perhaps we should point out that the situation would be much worse if the predicted asymmetries were below 10%. Fortunately, as fig. 5 shows, $\mathrm{Im}(\lambda_t)$ in the standard model seems to be well away from this unfortunate region Obviously, as fig. 6 shows, a knowledge of m_t would allow a much more restricted prediction for these *CP*-violating asymmetries in B decays.

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