

THREE MIRROR PAIRS OF FERMION FAMILIES

I. MONTVAY

Deutsches Elektronen-Synchrotron DESY, D-2000 Hamburg, Fed. Rep. Germany

Received 7 January 1988; revised manuscript received 6 February 1988

A simple model with three mirror pairs of fermion families is considered which allows for a substantial mixing between the mirror fermion partners without conflicting with known phenomenology.

The standard model of electroweak interactions [1] is based on a spontaneously broken $SU(2) \otimes U(1)$ gauge theory with chiral fermions: the left-handed and right-handed components of the fermions have different transformation properties with respect to the gauge group. For instance, with respect to $SU(2)$ the left-handed fermions are in doublets, the right-handed ones in singlets. In order to have a more symmetric description it is possible to duplicate the fermion fields by introducing for every fermion a "mirror partner" with exchanged left- and right-handed transformation properties. If a consistent asymmetric theory without the mirror partners does exist, the duplication of the spectrum is just a technical tool, because in this case the mirror partners can be removed in some way, for instance by giving them an infinitely large mass. (The assumption that a theory without the mirror fermions can be obtained as some limit of its mirror symmetric extension is a rather weak one.) In the framework of perturbation theory the mass ratios are considered to be free parameters, therefore the mirror partners can be removed by an infinite mass without any apparent consistency problem. Since the extensions of the standard model were formulated up to now typically within perturbation theory, the mirror fermions were never considered to be a particularly appealing possibility, although they occur in connection with different theoretical ideas. To mention a few typical examples, mirror fermions were introduced in order to cancel anomalies [2,3], they occur in grand unified theories with large orthogonal groups [4], in modern Kaluza-Klein theo-

ries [5] and in extended supersymmetry [6]. The question of the presence or absence of the mirror fermions in the physical spectrum has a different status in a non-perturbative regularization scheme as, for instance, in lattice regularization. In this case the introduction of the mirror partners is essential, because it allows for a formulation with exact local chiral symmetry [7,8]. In a non-perturbative framework the restrictions on the possible values of the physical parameters, like e.g. mass ratios, can also be manifested. Such constraints imposed by the requirement of consistency can, in principle, imply the impossibility to remove the mirror partners from the spectrum. In this case the mirror symmetric spectrum introduced originally as a technical tool is becoming a physical reality.

At present there are at least two different kinds of non-perturbative constraints known: the first kind can occur in spontaneously broken theories if the phase transition separating the broken phase from the symmetric one is of first order [9,10]. In this case, due to the jump in the vacuum expectation value of the scalar field, lower limits arise for some masses created by spontaneous symmetry breaking. The second kind of constraints appear in the case of non asymptotically free couplings if one tries to remove the regularizing cut-off from the theory ("continuum limit" on the lattice). This limit is governed by the infrared structure of the Callan-Symanzik renormalization group equations. For instance, if there is an infrared fixed point for the couplings or for some coupling ratios, cut-off dependent bounds on the renormalized

couplings arise. In the spontaneously broken phase these bounds imply bounds on the mass ratios. (For a review see ref. [11].)

Returning to the question of the mirror fermion partners, besides the advantage of an explicit local chiral symmetry there is also another very important aspect of the mirror doubling of the fermion spectrum. In order to be more specific, let us now consider a simplified prototype version of the standard electroweak model, namely the Yukawa interaction of a fermion doublet with a scalar doublet field. The $SU(2)_L$ gauge interaction is weak, therefore it can be considered as a small perturbation and the $U(1)_Y$ interaction is neglected altogether in order to have no problem with the triangle anomaly (because $SU(2)$ is anomaly free [12]). For zero fermion mass the model has a chiral $SU(2)_L \otimes SU(2)_R$ symmetry. The mirror fermion is defined in this case in such a way that the right-handed component of it is a doublet under $SU(2)_L$ and the left-handed component a doublet under $SU(2)_R$. At this point a very important aspect of the introduction of the mirror fermion partner becomes apparent: since the spatial reflection does not commute with the chiral symmetry, but transforms $SU(2)_L$ into $SU(2)_R$ and vice versa, the massive representations of $SU(2)_L \otimes SU(2)_R$ always contain degenerate pairs of particles with opposite parity. The mass terms in the action connect left-handed with right-handed components, therefore in the chiral symmetric case they are allowed only between mirror partners but not between the components of the same fermion. In the symmetric case this corresponds to a mass matrix with opposite eigenvalues, but the sign of a fermion mass is unimportant. Therefore the physical states with definite parity have, indeed, degenerate masses. In the case of spontaneous symmetry breaking, when the fermion mass terms are produced by the vacuum expectation value of the scalar field, the masses of the original fermion and of its mirror partner can be different. The space reflection symmetry is broken and the physical states are mixtures of the mirror fermion pair [7]. The consequence of this is that if the mirror fermions are not introduced a priori then either the symmetric phase is not represented at all, hence the description of the model is incomplete, or if the symmetric phase is present, then the parity partners have to appear dynamically as bound states of the fields in the lagran-

gian. A description where all the important states of the model are represented by "elementary" fields can obviously be expected to be simpler than an incomplete description with only a subset of the fields.

The question is whether a chirally asymmetric physical spectrum without the mirror fermion partners can be realized as a limit of the complete theory or not? The answer to this question in renormalized perturbation theory is yes if the remaining fermion set is anomaly free. (The theory including the mirror partners is always anomaly free.) In a non-perturbative framework an impasse for removing the mirror partners by a very large mass would be if there were some infrared fixed point at some definite value of the ratio of the renormalized Yukawa couplings of the fermion and of its mirror partner. This is, however, not the case. On the contrary, according to the one-loop β -functions an arbitrary ratio of the Yukawa couplings is infrared stable [8]. According to this it would seem possible to answer the above question about a chirally asymmetric physical spectrum in an affirmative way. Nevertheless, all explicit attempts to remove the mirror partners from the spectrum encounter enormous difficulties, at least in a lattice formulation. (For some proposals on the lattice see ref. [13] and the review [14].) Without considering the quarks and leptons together, it is certainly impossible to remove the mirror partners due to the non-vanishing anomaly. To remove a complete mirror fermion family by a more complicated Higgs sector (possibly in an extended, say, grand unified framework) seems also impossible, because of the necessary occurrence of large scalar doublet expectation values which imply a very large W-boson mass too. The naive way of just taking the limit of infinitely large bare Yukawa couplings for the mirror partners has a good chance not to work either. The tree level relation between the mass and Yukawa coupling becomes unreliable as soon as the renormalized Yukawa coupling corresponds to a strong interaction. This occurs near the unitarity bound at about 500 GeV [15]. Since the Yukawa coupling is not asymptotically free, similarly to the quartic coupling, it is plausible that there is a relatively low upper bound for the renormalized Yukawa coupling and therefore an upper bound also for the fermion masses produced by spontaneous symmetry breaking, similarly to the upper bound for the Higgs-boson mass. (For recent non-perturbative

upper bounds on the Higgs mass see ref. [16].) Although the chiral symmetry does not imply the naturalness of small fermion masses, an arbitrarily large fermion mass hierarchy is possible *downwards* from the scale of the vacuum expectation value.

In summary: in lattice regularization the mirror partners of the fermions cannot be completely removed from the spectrum. Therefore the possibility of the existence of mirror pairs of fermions has to be considered very seriously. The first step is, of course, to find the limitations imposed on the mirror partners by known phenomenology. In the present letter a simple model with three mirror pairs of standard fermion families is considered which is consistent with experiments and still has a non-negligible mixing among mirror fermion partners.

The simplest kind of mirror fermion models consistent with phenomenology is when the mirror partners of the known light fermions are all heavy, say above 100 GeV, and the mixing between mirror partners is zero. Due to the limited accuracy of the experiments there is some finite neighbourhood of this point in the parameter space where the mixing is small and all known experimental constraints are satisfied. The question is whether there are other more general points with larger mixing angles where the precision constraints (as light lepton number conservation, absence of flavour changing neutral quark currents etc.) are satisfied?

The mixing pattern of the three mirror pairs of fermion families can be specified by a $6 \otimes 6$ mass matrix for each fermion species [7,8]. In a $3 \otimes 3$ block matrix notation we assume

$$\begin{pmatrix} \mu_{\psi}^{Ac} & \mu_{\psi\chi}^c \\ \mu_{\psi\chi}^c & \mu_{\chi}^{Ac} \end{pmatrix}. \tag{1}$$

The index convention in this paper will be as follows: $A=1,2$ will be used for the SU(2) weak isospin index, $c=l,q$ to distinguish leptons and quarks and $K=1,2,3$ for the family index. The block-diagonal elements in eq. (1) arise due to spontaneous symmetry breaking and are assumed to be hermitean here. Moreover it is assumed that they both can be diagonalized by the same unitary matrix F_{Ac} depending on the indices A and c . The chiral invariant off-diagonal elements are taken to be proportional to the unit matrix (and are assumed to be A -independent). The

consequence of these assumptions is that the $3 \otimes 3$ unitary matrix F_{Ac} simultaneously diagonalizes all the entries in the mass matrix (1), therefore, with respect to mixing there is a one-to-one correspondence between the fermions and the mirror fermions. This fact can be expressed by calling such mixing schemes "monogamous". It is also true that the $3 \otimes 3$ Kobayashi-Maskawa matrix

$$M_{c,K_1K_2} \equiv \sum_K F_{2c,K_1K}^{-1} F_{1c,KK_2} \tag{2}$$

is the same in both the fermion and mirror fermion sectors.

Denoting the original fermion fields in the lagrangian by ψ^{AcK} and the corresponding mirror fermion fields by χ^{AcK} , the complete diagonalization of the mass matrix is achieved by the fields

$$\begin{aligned} \xi^{AcK_1} &= \sum_K F_{Ac,K_1K}^{-1} (\cos \alpha_{AcK_1} \psi^{AcK} - \sin \alpha_{AcK_1} \chi^{AcK}), \\ \eta^{AcK_1} &= \sum_K F_{Ac,K_1K}^{-1} (\sin \alpha_{AcK_1} \psi^{AcK} + \cos \alpha_{AcK_1} \chi^{AcK}). \end{aligned} \tag{3}$$

Note that because of the hermicity of $\mu_{\psi,\chi}^{Ac}$ the mixing is the same for left- and right-handed components.

The $SU(2)_L \otimes U(1)_Y$ electroweak interaction of the fermions can be written as

$$\begin{aligned} g[J_-(x)_\mu W^+(x)^\mu + J_+(x)_\mu W^-(x)^\mu] \\ + eJ_{em}(x)_\mu A(x)^\mu \\ + \sqrt{g^2 + g'^2} [\sin^2 \theta_W J_{em}(x)_\mu - J_0(x)_\mu] Z(x)^\mu. \end{aligned} \tag{4}$$

The vector bosons are in the usual notation W,A,Z . θ_W is the Weinberg angle with $\sin \theta_W = g' / \sqrt{g^2 + g'^2}$ and the electromagnetic coupling is $e = gg' / \sqrt{g^2 + g'^2}$. The vector-like electromagnetic current of the fermions J_{em} is defined by the electric charges, whereas the chiral weak currents $J_a (a = +, -, 0)$ can be written as

$$J_a(x)_\mu = \bar{\xi}(x) \Gamma_{\xi\xi,\mu}^a \xi(x) + \bar{\eta}(x) \Gamma_{\eta\eta,\mu}^a \eta(x) + \dots \tag{5}$$

Here the matrix Γ^+ is given by

$$\begin{aligned} \Gamma_{\xi\xi,\mu}^{+c,K_1K_2} &= \frac{\tau^+}{\sqrt{8}} M_{c,K_1K_2}^+ [\gamma_\mu \cos(\alpha_{1cK_1} - \alpha_{2cK_2}) \\ &+ \gamma_\mu \gamma_5 \cos(\alpha_{1cK_1} + \alpha_{2cK_2})], \end{aligned} \tag{6}$$

$$\begin{aligned} \Gamma_{\xi_1, \mu}^{+c, K_1 K_2} &= \frac{\tau^+}{\sqrt{8}} M_{c, K_1 K_2}^+ [\gamma_\mu \sin(\alpha_{2cK_2} - \alpha_{1cK_1}) \\ &+ \gamma_\mu \gamma_5 \sin(\alpha_{2cK_2} + \alpha_{1cK_1})], \\ \Gamma_{\eta_{\xi, \mu}^+}^{+c, K_1 K_2} &= \frac{\tau^+}{\sqrt{8}} M_{c, K_1 K_2}^+ [\gamma_\mu \sin(\alpha_{1cK_1} - \alpha_{2cK_2}) \\ &+ \gamma_\mu \gamma_5 \sin(\alpha_{1cK_1} + \alpha_{2cK_2})], \\ \Gamma_{\eta_{\mu}^+}^{+c, K_1 K_2} &= \frac{\tau^+}{\sqrt{8}} M_{c, K_1 K_2}^+ [\gamma_\mu \cos(\alpha_{2cK_2} - \alpha_{1cK_1}) \\ &- \gamma_\mu \gamma_5 \cos(\alpha_{2cK_2} + \alpha_{1cK_1})], \end{aligned} \quad (6 \text{ cont'd})$$

with $\tau^\pm \equiv \frac{1}{2}(\tau_1 \pm i\tau_2)$. Γ^- is obtained from here by $\tau^+ \rightarrow \tau^-$, $M^+ \rightarrow M$ and $(A=1) \leftrightarrow (A=2)$. Γ^0 is diagonal in the SU(2) index A and family index K :

$$\begin{aligned} \Gamma_{\xi_{\mu}^0}^{0, AcK} &= \frac{1}{4} \tau_{3, AA} [\gamma_\mu + \gamma_\mu \gamma_5 \cos(2\alpha_{AcK})], \\ \Gamma_{\eta_{\mu}^0}^{0, AcK} &= \frac{1}{4} \phi \tau_{3, AA} [\gamma_\mu - \gamma_\mu \gamma_5 \cos(2\alpha_{AcK})], \\ \Gamma_{\xi_{\mu}^0}^{0, AcK} &= \Gamma_{\eta_{\mu}^0}^{0, AcK} = \frac{1}{4} \tau_{3, AA} \gamma_\mu \gamma_5 \sin(2\alpha_{AcK}). \end{aligned} \quad (7)$$

This shows that at the tree level are no flavour changing neutral currents, and in the neutral current mirror mixing occurs only in the axial-vector part.

Since the experimental limits on flavour changing neutral currents (as for instance $d\bar{s} \rightarrow \bar{s}d$) are very stringent, a cancellation mechanism has to be provided also at the one-loop level. This requires to suppress the two-W transitions shown in fig. 1, which is proportional to

$$\sum_K f(K) M_{K_1 K}^+ M_{K K_2}. \quad (8)$$

The function $f(K)$ depends on the sums and differences of the mixing angles α_{AcK} and, due to the fermion propagator, also on the heavy fermion masses. There are two ways to make the two-W transition ex-

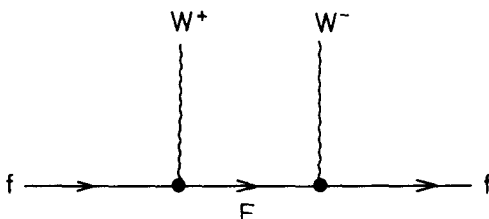


Fig. 1. The flavour changing neutral process by two W-boson emission, which occurs in one-loop graphs for $\bar{s}d \rightarrow \bar{d}s$ or $\mu \rightarrow e\gamma$, etc.

actly diagonal in the family index K : either the Kobayashi–Maskawa matrix M has to be diagonal, or the function $f(K)$ has to be a constant. ($f(K)$ is K -independent if the mirror mixing and the mass of the mirror fermions are K -independent.) The first possibility is viable for the leptons, the second one corresponds to the GIM-mechanism [1] which is operating for quarks. Of course, if only an approximate vanishing of the non-diagonal elements in eq. (8) is required, then nearly diagonality and nearly K -independence can collaborate to make the restrictions on the mass matrix parameters less severe. A simple model yielding nearly K -independent mirror mixing is defined by the mass matrix

$$\begin{pmatrix} m_{Ac} & \delta_c \\ \delta_c & m_{Ac} + \Delta_{Ac} \end{pmatrix} + \epsilon. \quad (9)$$

Here m_{Ac} denotes a hermitean $3 \otimes 3$ matrix and δ_c, Δ_{Ac} are proportional to the unit $3 \otimes 3$ matrix. The elements of the $6 \otimes 6$ matrix ϵ are assumed to be small and will be neglected in what follows. For $\epsilon=0$ the mirror mixing angle α_{Ac} is given by

$$\sin \alpha_{Ac} = \frac{\delta_c \sqrt{2}}{\sqrt{\Delta_{Ac}^2 + 4\delta_c^2 + \Delta_{Ac} \sqrt{\Delta_{Ac}^2 + 4\delta_c^2}}} \rightarrow \frac{\delta_c}{\Delta_{Ac}}. \quad (10)$$

Here also the limiting case $\Delta_{Ac} \gg \delta_c$ is given. The masses of the two physical states are

$$\begin{aligned} \mu_1^{AcK} &= \frac{1}{2} (\Delta_{Ac} + 2m_{AcK} - \sqrt{\Delta_{Ac}^2 + 4\delta_c^2}) \\ &\rightarrow m_{AcK} - \delta_c^2 / \Delta_{Ac}, \\ \mu_2^{AcK} &= \frac{1}{2} (\Delta_{Ac} + 2m_{AcK} + \sqrt{\Delta_{Ac}^2 + 4\delta_c^2}) \\ &\rightarrow \Delta_{Ac} + m_{AcK} + \delta_c^2 / \Delta_{Ac}. \end{aligned} \quad (11)$$

The eigenvalues of the matrix m_{Ac} are denoted here by m_{AcK} . The mass matrix in eq. (9) has altogether six new parameters ($\delta_c, \Delta_{Ac}; A=1,2; c=q,\ell$) for the masses and mixings of the three mirror fermion families. Taking the eigenvalues m_{AcK} to be of the order 1, one can have for instance Δ_{Ac} of the order of 100 and δ_c of the order of 10. In this case the heavy masses are of the order of 100, the light masses of the order of 1 and the mixing angles of the order of 1/10. The very light masses of the first family and especially of the three light neutrinos can be reproduced by a cancellation in the lower eigenvalue μ_1^{AcK} . The peculiarity of this mirror fermion mass pattern is that the

mixing angles α_{Ac} are independent from the family index K and the mass splittings between the heavy (mirror) families are relatively small (they are the same as between the light families). The addition of the small matrix ϵ can somewhat modify the K -independence of the mixing, but the small mass splittings qualitatively remain.

The χ -components of the light physical fermions have $V+A$ couplings to the $SU(2)_L \otimes U(1)_Y$ vector bosons. This gives at present the most important limits on the mirror mixings, because the weak currents of the known fermions are to a good approximation of $V-A$ -type. The present upper limits on the mixings are, however, not very strong. Most of the limits for the mirror lepton mixings can be inferred from different papers of Enqvist, Maalampi, Mursula and Roos [17]. The best limits are typically of the order of 5–10%. These authors did not consider the possibility of suppressing the flavour changing neutral currents by a family independent mixing scheme, therefore they concluded that the mirror mixings for the quarks are below 10^{-4} . Given the present scheme, the limits for the mirror quark mixings are typically less stringent than for charged leptons [18]. The typical range of the bounds for quark mirror mixing can also be inferred from ref. [19], but there the question of the flavour changing neutral currents was not discussed.

The mirror partners of the known leptons and quarks can be produced by the next generation of accelerators, if their masses are not very large. In e^+e^- collisions the heavy mirror states can be pair-produced by the electromagnetic and/or neutral weak current. The associated production of a heavy–light fermion pair has more phase space but it is suppressed by the small mixing angle. For instance, the decay width of $Z \rightarrow E^+e^-$ is

$$\Gamma_{Z \rightarrow E^+e^-} = M_Z \sin^2(2\alpha_{2\ell}) \frac{(g^2 + g'^2)}{384\pi} \times \left(1 - \frac{M_E^2}{M_Z^2}\right) \left(2 - \frac{M_E^2}{M_Z^2} - \frac{M_E^4}{M_Z^4}\right). \quad (12)$$

The mirror partners will generally be denoted by capital letters (for instance, E for electron, N for neutrino, U for u -quark, etc.). In the above formula M_E is the mass of E^+ and the electron mass is neglected. In the case of $M_E = M_Z/2$ and $\sin^2(\dots) = 10^{-3}$ this

corresponds to a $Z \rightarrow E^+e^-$ branching ratio of $\simeq 2 \times 10^{-5}$. (The electroweak parameters are taken here from the review of Langacker [20].) Together with the other leptonic channels this gives a branching ratio in the order of 10^{-4} , therefore if the mirror leptons are below the Z and if the mixing is not extremely small, they will be seen in the e^+e^- “ Z -factories”. In high-energy ep collisions single mirror fermions can be produced by W - or Z -exchange via the mirror mixing. The pair production of the mirror quarks is similar to the usual heavy quark pair production by the boson–gluon fusion (see ref. [21] and references therein).

The decay signature of a heavy mirror fermion is quite spectacular: the mirror leptons can decay to three leptons or to a lepton plus two jets, the mirror quarks to three jets or to a jet plus a lepton pair. For masses larger than $\simeq 100$ GeV the decay to a light fermion plus a vector boson is important. For instance, the decay width of $E^- \rightarrow \nu_e W^-$ is given by

$$\Gamma_{E^- \rightarrow \nu_e W^-} = M_E [\sin^2(\alpha_{2\ell} - \alpha_{1\ell}) + \sin^2(\alpha_{2\ell} + \alpha_{1\ell})] \times \frac{g^2}{128\pi} \left(1 - \frac{M_W^2}{M_E^2}\right) \left(1 - 2 \frac{M_W^2}{M_E^2} + \frac{M_E^2}{M_W^2}\right). \quad (13)$$

For $M_E = 2M_W$ and $[\dots] \equiv s_E^2 = 10^{-3}$ this gives $\simeq 0.6$ MeV. The formula for $N \rightarrow eW$ is the same as eq. (13), with the appropriate mixing angle combinations. The decays to Z can be obtained by $g^2 \rightarrow (g^2 + g'^2)/2$. If the splittings in the doublets are large enough there will be also decays to another mirror fermion, as for instance $E \rightarrow N\nu_e$, which are not suppressed by the mirror mixing but have a smaller phase space. As an example, the ratio of these three-body decays to the decay $E \rightarrow \nu W$ is shown in fig. 2, as a function of the mass splitting in the lepton doublet, for $M_E = 2M_W$ and again $s_E^2 = 10^{-3}$. In the three-body decays the third generation quarks (bt) were omitted, therefore f_1/f_2 stands for nine different light fermion pairs. As it can be seen from the figure, in this particular situation the direct two-body decay is more important.

Finally, there are also some hints in known experimental data which can be interpreted as possible evidence for mirror fermions. The two-muon event observed in e^+e^- annihilation by the CELLO Collaboration at PETRA [22] is a candidate for the associated production of a mirror muon: $e^+e^- \rightarrow \mu M$.

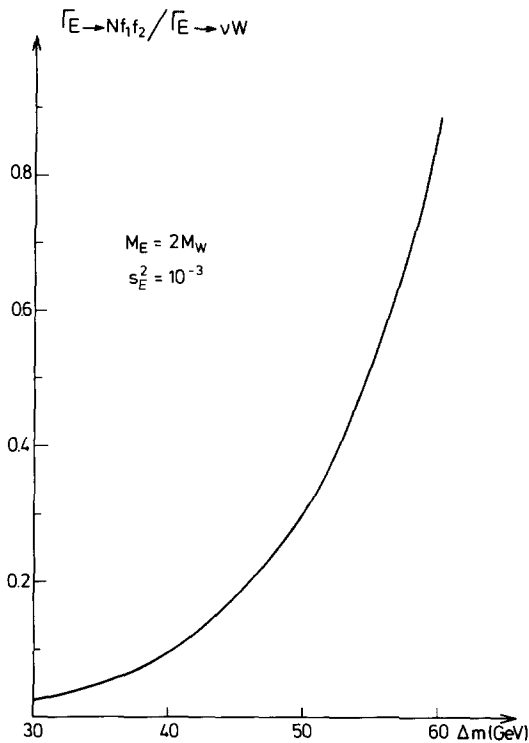


Fig. 2. The ratio of the three-body to two-body decays of the mirror electron E as a function of the mass splitting in the mirror lepton doublet, for $M_E = 2M_W$ and $s_E^2 = 10^{-3}$.

In this event the mirror muon mass is either 30.5 GeV or 28.2 GeV, depending on the charge assignment. Some of the low thrust hadron events with isolated muon observed by the MARK-J and JADE Collaborations [23] can have a similar origin. If this explanation of the CELLO event is indeed correct, then the mirror muons will be copiously produced in e^+e^- annihilation above 60 GeV. The slight discrepancies in $e-\mu-\tau$ -universality (see ref. [24] and references therein) could also be due to the mixing of the leptons with its mirror partners.

It is a pleasure to thank Hans Joos, Paul Langacker and Roberto Peccei for discussions on different questions concerning mirror fermions.

References

- [1] S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264;
- [2] D.J. Gross and R. Jackiw, Phys. Rev. D 6 (1972) 477.
- [3] P. Fayet, in: Proc. 17th Rencontre de Moriond on Elementary particles (Gif-sur-Yvette), ed. J. Tran Thanh Van (Editions Frontières, Dreux, 1982) p. 483.
- [4] G. Senjanovic, F. Wilczek and A. Zee, Phys. Lett. B 141 (1984) 389.
- [5] E. Witten, Nucl. Phys. B 186 (1981) 412.
- [6] F. de Aquila, M. Dugan, B. Grinstein, L. Hall, G.G. Ross and P. West, Nucl. Phys. B 250 (1985) 225.
- [7] I. Montvay, Phys. Lett. B 199 (1987) 89.
- [8] I. Montvay, DESY preprint 87-147 (1987), in: Proc. 1987 Seillac Conf. on Lattice gauge theory, to be published.
- [9] S. Coleman and E. Weinberg, Phys. Rev. D 7 (1973) 1888.
- [10] S. Weinberg, Phys. Rev. Lett. 36 (1976) 294; A.D. Linde, Sov. Phys. JETP 23 (1976) 64.
- [11] I. Montvay, in: Proc. 1987 Uppsala EPS Conf. on High energy physics, Vol. I, ed. O. Botner (Uppsala University, 1987) p. 298.
- [12] H. Georgi and S.L. Glashow, Phys. Rev. D 6 (1972) 429.
- [13] E. Eichten and J. Preskill, Nucl. Phys. B 268 (1986) 179.
- [14] J. Smit, Amsterdam preprint ITFA-87-21 (1987), in: Proc. 1987 Seillac Conf. on Lattice gauge theory, to be published.
- [15] M.S. Chanowitz, M.A. Furman and I. Hinchliffe, Phys. Lett. B 78 (1978) 285; Nucl. Phys. B 153 (1979) 402.
- [16] W. Langguth and I. Montvay, Z. Phys. C 36 (1987) 725; A. Hasenfratz and T. Neuhaus, Tallahassee preprint FSU-SCRI-98-29 (1987); M. Lüscher and P. Weisz, Nucl. Phys. B 290 [FS20] (1987) 25; DESY preprint 87-075; J. Kuti and Y. Shen, San Diego preprint UCSD/PTH-87/14; P. Hasenfratz and J. Nager, in: Nonperturbative methods in quantum field theory, eds. Z. Horváth et al. (World Scientific, Singapore, 1987) p. 89; Bern preprint BUTP-87/18.
- [17] J. Maalampi, K. Mursula and M. Roos, Nucl. Phys. B 207 (1982) 233; K. Enqvist, K. Mursula and M. Roos, Nucl. Phys. B 226 (1983) 121; J. Maalampi and K. Mursula, Nucl. Phys. B 269 (1986) 109.
- [18] P. Langacker, private communication.
- [19] J. Maalampi and K. Mursula, Z. Phys. C 16 (1982) 83.
- [20] P. Langacker, DESY preprint 87-153 (1987).
- [21] G.A. Schuler, DESY preprint 87-114 (1987).
- [22] CELLO Collab., H.J. Behrend et al., Phys. Lett. B 141 (1984) 145.
- [23] J.G. Branson, in: Proc. 1986 Berkeley Conf. on High energy physics, Vol. I, ed. S.C. Loken (World Scientific, Singapore, 1987) p. 591.
- [24] ARGUS Collab., H. Albrecht et al., Phys. Lett. B 199 (1987) 580.