ON THE CONNECTION BETWEEN THE SCALES OF WEAK AND STRONG INTERACTIONS

C. WETTERICH

Deutsches Elektronen-Synchrotron DESY, D-2000 Hamburg 52, Fed. Rep. Germany

Received 17 December 1987; revised manuscript received 9 March 1988

We investigate within the standard model the possibility that nonperturbative QCD effects determine the Fermi scale and electroweak symmetry breaking is a consequence of chiral symmetry breaking. In this scenario the ratio between the Fermi scale and the quark condensate $\langle \bar{\psi}\psi \rangle_{0}^{1/3}$ comes out inversely proportional to the Yukawa coupling of the strange quark, consistent with observation. The Higgs particle mass is predicted in the range of 200 keV.

The standard model of electroweak and strong interactions [1,2] has two different mass scales: The Fermi scale $\varphi_0 = 174$ GeV determines the strength of weak interactions and the masses of quarks and leptons. It is given by the vacuum expectation value (VEV) of the scalar doublet which spontaneously breaks weak $SU(2) \times U(1)$ symmetry. The QCD scale $\Lambda_{\rm QCD}$ (a few times hundred MeV) sets the scale for nuclear masses and interactions. It characterizes the scale where the gauge coupling g_s of SU(3) becomes strong. In addition, there are important features of the standard model which depend on a complicated interplay between φ_0 and Λ_{OCD} . For example the pion mass m_{π} is proportional to (φ_0/φ_0) $\Lambda_{\rm QCD}$)^{1/2}. It is obvious that even a moderate change in the ratio

$$\gamma = \Lambda_{\rm QCD} / \varphi_0 \approx 10^{-3} \,, \tag{1}$$

would lead to a very different picture in almost all branches of physics. (For example the electron-to-proton mass ratio is proportional to γ^{-1} .)

It is certainly one of the most important challenges for a fundamental theory to explain why γ is of the order of 10^{-3} . In addition, modern unification is often associated with a huge unification scale: of the order of the Planck mass $M_{\rm Pl}$. This not only leads to the puzzle how to understand the small ratio $\varphi_0/M_{\rm Pl}$ (gauge hierarchy problem [3]) but also to the question why $\Lambda_{\rm QCD}$ and φ_0 are so "near" to each other when looked upon from a characteristic scale $M_{\rm Pl}$. From the viewpoint of the short-distance physics around $M_{\rm Pl}$ the difference between $\Lambda_{\rm QCD}$ and φ_0 appears like a "fine structure" in the effective long-distance physics, similar in size to the structure in the fermion mass matrices [4] reflected by different Yukawa couplings. We may call this the "connection problem" between the scales of weak and strong interactions. What has φ_0 to do with $\Lambda_{\rm QCD}$? The connection problem and the gauge hierarchy problem are of course not unrelated. Whenever the tiny ratio $\Lambda_{\rm QCD}/M_{\rm Pl}$ is explained by the logarithmic evolution of the strong gauge coupling, a solution of the connection problem and thereby an understanding of γ would automatically solve the gauge hierarchy problem.

In perturbation theory the scales $\Lambda_{\rm QCD}$ and φ_0 are essentially unrelated free parameters of the standard model ^{#1}. In the presence of a scale $\Lambda_{\rm QCD}$ emerging from strong interactions, however, a naive perturbative treatment of electroweak symmetry breaking becomes questionable. Nonperturbative QCD effects lead to interactions between the σ field (quark-antiquark bound state) and the Higgs doublet of the type $\sigma^3 \bar{\varphi}, \sigma^2 \bar{\varphi}^2$, etc. The field $\bar{\varphi}$ corresponds to the average value of the weak doublet in a volume $V_{\rm QCD}$ with characteristic length scale $\Lambda_{\rm QCD}^{-1}$. These interactions are local only for momenta below $\Lambda_{\rm QCD}$ but they be-

0370-2693/88/\$ 03.50 © Elsevier Science Publishers B.V. (North-Holland Physics Publishing Division)

^{#1} There is a possible exception for the case of seven or eight generations where the strong gauge coupling increases substantially only for momenta below the Fermi scale.

come nonlocal when considered at length scales smaller than A_{QCD}^{-1} . It requires some thought to compare these nonlocal interactions with the local interactions described by the (classical) potential for the weak doublet. We will see that in analogy to the physics of Weiss' domains in ferromagnets this amounts to a comparison of "surface effects" (from the local interactions) with volume effects from QCD.

To be more precise, let us consider the Higgs model (four-component φ^4 theory) in the spontaneously broken phase. We choose parameters such that the minimum of the perturbative potential $V_{\rm p}$ is at some large scale M. (One may take M in the vicinity of the Planck scale.) We couple this model to QCD through the usual Yukawa couplings h of the quarks. (Global) $SU(2) \times U(1)$ invariance allows quark mass terms only $\sim h\bar{\varphi}$. Imagine now that the quark and gluon degrees of freedom are integrated out, resulting in an effective action for the scalar field $S[\varphi] = S_0[\varphi] +$ $S_{\text{OCD}}[\varphi]$. Here S_0 is the original action of the φ^4 theory (kinetic term + potential) whereas S_{OCD} involves complicated nonlocal interactions. We divide S_{OCD} in a "perturbative part" S_{QCD}^{p} which involves the nonlocalities at length scales $\ll \Lambda_{\rm OCD}^{-1}$ (generated by fluctuations with high momenta) and a "nonperturbative part" where the typical length scale for the nonlocality is $\approx \Lambda_{\rm OCD}^{-1}$. (At length scales $\gg m_{\pi}^{-1}$ the action becomes effectively local.) For a first illustration of the problem we will neglect S_{OCD}^{p} and approximate the nonperturbative part by a potential term $V_{\omega\sigma}(\phi_A)$

$$S_{\rm QCD} = \int_{\Omega} \mathrm{d}x \, V_{\varphi\sigma}(\phi_A(x)) \,. \tag{2}$$

The average field $\phi_k(x)$ is defined by integrating over a volume $V_k \sim k^{-d}$ around x

$$\phi_k(x) = \frac{1}{V_k} \int_{V_k} dy \, \varphi(x+y) , \qquad (3)$$

and we choose $k=\Lambda$ as a typical QCD scale. (We work in a euclidean formulation and the total volume Ω of spacetime should be taken to infinity at the end.)

The long-distance operators relevant for the vacuum properties can be expressed in terms of ϕ_A . It is then convenient to describe the contributions from the local action S_0 in terms of a "finite volume action" $\hat{S}_k[\bar{\phi}]$:

$$\exp(-S_k[\bar{\varphi}]) = \int \mathscr{D}\varphi \prod_x \delta(\phi_k(x) - \bar{\varphi}(x)) \exp(-S_0[\varphi]).$$
(4)

This can be expanded in potential, kinetic and higher derivative terms

$$\hat{S}_{k}[\bar{\varphi}] = \int_{\Omega} \mathrm{d}x \{ U_{k}(\bar{\varphi}) + K_{k}(\bar{\varphi}) (\partial_{\mu}\bar{\varphi})^{+} \partial^{\mu}\bar{\varphi} + ... \}.$$
(5)

In the pure scalar model the mean value of an operator depending only on average fields $\phi_k(x)$ is

$$\langle O(\phi_k(x)) \rangle = \frac{\int \mathscr{D}\bar{\varphi}(x) O(\bar{\varphi}(x)) \exp(-\hat{S}_k[\bar{\varphi}])}{\int \mathscr{D}\bar{\varphi}(x) \exp(-\hat{S}_k[\bar{\varphi}])}.$$
(6)

Inclusion of the long-range QCD interaction (2) simply results in adding $V_{\varphi\sigma}(\bar{\varphi})$ to the "finite-volume potential" $U_A(\bar{\varphi})$ obtained for constant $\bar{\varphi}^{\#2}$

$$U_{k}(\bar{\varphi}) = -\frac{1}{\Omega} \ln \int \mathscr{D}\varphi \prod_{x} \delta(\phi_{k}(x) - \bar{\varphi}) \exp(-S_{0}[\varphi]) .$$
(7)

We therefore have to compare the relative importance of $U_{\mathcal{A}}(\bar{\varphi})$ and $V_{\varphi\sigma}(\bar{\varphi})$.

For Ω large compared to V_k and the correlation length the finite-volume potential (7) becomes independent of Ω . Its functional dependence on $\overline{\phi}$ does also not depend on how dense the points x are chosen (supposing there are many within V_k). One may interpret \hat{S}_k as some type of "block spin" action in the limit where only one x is inside a typical volume V_k . If in addition $\Omega = V_k$ the finite-volume potential $U_k(\bar{\varphi})$ corresponds to the "constraint effective potential" [5] $C_k(\bar{\varphi})$. This is related to the relative probability of finding an average value $\phi_k = \bar{\phi}$, given by exp[$-V_k C_k(\bar{\varphi})$]. For $k \to 0$ the potential $C_k(\bar{\varphi})$ approaches the effective potential $\Gamma(\bar{\varphi})$ which is the convex hull [6] of the perturbative potential V_p obtained by summing over Feynman graphs (compare fig. 1). We expect that $U_k(\bar{\varphi})$ behaves qualitatively similar as $C_k(\bar{\varphi})$ and interpolates between V_p and Γ

^{#2} We need to define $U_k(\bar{\varphi})$ only up to a constant. The δ distribution may be replaced by an appropriate gaussian (large ν): $\prod_x \delta(\phi_k(x) - \bar{\varphi}) \rightarrow \exp\{-\int dx [\nu(\phi_k(x) - \bar{\varphi})^2 + C]\}$. Wavefunction renormalization can be used to bring the kinetic term into standard form $K(\bar{\varphi}) = 1$.



Fig. 1. Effective potential $\Gamma(\bar{\varphi})$ and perturbative potential $V_p(\bar{\varphi})$ in the spontaneously broken phase.

as $k \rightarrow 0$. For a computation of Γ , C_k or U_k in the spontaneously broken phase naive perturbation theory can be trusted ^{#3} only for $|\bar{\varphi}| \ge M$ since otherwise the expansion around one saddlepoint becomes invalid. The relative probability of configurations with $\phi_k = 0$ compared to $\phi_k = M$ is not suppressed by exp[-(potential energy times volume)]. Only surface energy is needed to change in some domain the phase of φ . Even though in every small volume V_M one has $|\varphi| \approx M$ we can arrange the phase to obtain $\phi_k = 0$ for a large volume V_k . The relative probability for $\phi_k = 0$ is expected $\exp(-ck^{-d+\alpha})$. A rough guess gives $\alpha \ge 1$ for models with discrete symmetries (nonvanishing surface energy) whereas for continuous symmetries we expect $\alpha \ge 2^{\#4}$. For $|\bar{\varphi}| \le M$ we therefore expect that the finite-volume potential $U_k(\bar{\varphi})$ converges to the flat effective potential $\Gamma(\bar{\varphi})$ as k^{α} . The volume effects $V_{\sigma\sigma}$ compare with "surface" effects $U_{A}!$

Let us for a moment approximate $U_A(\bar{\varphi})$ by $\Gamma(\bar{\varphi})$ and add the nonperturbative QCD effects. A perturbation linear in $\bar{\varphi}$ pushes the minimum of $\Gamma(\bar{\varphi}) + \tilde{\alpha}\sigma^3\bar{\varphi}$ to a now uniquely selected minimum of V_p at $|\bar{\varphi}| = M$. Here perturbation theory applies and the nonperturbative QCD effects are completely negligible. This situation can change drastically for nonlinear perturbations. It becomes possible that the nonlinear perturbation $V_{\varphi\sigma}(\sigma,\bar{\varphi})$ develops a minimum within the flat region of $\Gamma(\bar{\varphi})$. The minimum of $\Gamma + V_{\varphi\sigma}$ and therefore the expectation value of $\bar{\varphi}$ is then determined by the minimum of $V_{\varphi\sigma}$ and not by V_p ! Intuitively the QCD effects can favour energetically a certain mean value of $\bar{\varphi}$ within a volume $V_{\rm QCD}$. Since $V_{\rm QCD}$ is large compared to a volume with length scale M^{-1} it costs comparatively little "electroweak" energy to arrange the domains within $V_{\rm QCD}$ so that this mean value obtains. The nonperturbative QCD effects could dominate the effective potential!

At long distances strong interactions can be described by an effective (linear) σ model. Quark-antiquark pairs $q_L \bar{q}_R$ form scalar mesons which transform as doublets under SU(2) with hypercharge one – just the same as the weak doublet φ . Chiral symmetry is spontaneously broken by a VEV $\sigma_0 \approx \frac{1}{2} f_{\pi} = 67$ MeV ^{#5}. Due to its Yukawa couplings to quarks the average field $\bar{\varphi}$ will interact with the $\bar{q}q$ condensate and therefore with σ . The interaction linear in $\bar{\varphi}$ contains terms of the form ($\tilde{\sigma} = i\tau_2 \sigma^*$)

$$V_{\varphi\sigma}^{(1)} = \alpha_1(\sigma^+\sigma)(\sigma^+\bar{\varphi}) + \alpha_2(\sigma^+\bar{\sigma})(\bar{\sigma}^+\bar{\varphi}) + \text{h.c.} \quad (8)$$

This has three immediate consequences: First, there will be a vacuum alignment between $\bar{\varphi}$ and σ . If we choose a convention where the lower component of σ has a real VEV σ_0 , we find that a VEV of the lower component of $\bar{\varphi}$ is energetically favoured compared to the upper component. This correlation guarantees that the electromagnetic U(1) symmetry remains unbroken. Similarly, the phase of $\langle \bar{\varphi} \rangle = \varphi_0$ (compared to σ_0) is dictated by the phases of α_1 and α_2 . The correlation between the phases of φ_0 and σ_0 may have implications for the *CP* problem.

Second, effective terms involving $(\sigma^+\bar{\varphi})$ break the global SU $(2N_G)_L \times$ SU $(2N_G)_R$ flavour symmetry which would exist in the absence of electroweak interactions. The VEV of $\bar{\varphi}$ induces masses for the quarks and for $\varphi_0 \neq 0$ the pions (and similarly other mesons) acquire a mass.

Finally, the interaction (8) puts a lower bound on the ratio φ_0/σ_0 if weak interactions are in the spontaneously broken phase. (By this we mean in our context that the potential for $\bar{\varphi}$ – neglecting its interactions with σ – should not contain a positive quadratic term.) A potential of the form $V = -\tilde{\alpha}\sigma_0^3\bar{\varphi} + \frac{1}{2}\lambda_{\varphi}\bar{\varphi}^4$ leads to

$$\varphi_0 = (\tilde{\alpha}/2\lambda_{\varphi})^{1/3}\sigma_0 , \qquad (9)$$

^{#3} There is convincing evidence [7] for the reliability of perturbation theory, even for the case of strong quartic bare couplings in S_0 .

^{#4} Energy density considerations would give $\alpha = 1$ or $\alpha = 2$ respectively, but the effect of entropy may enhance the new critical exponent α .

^{#5} In the absence of φ the W and Z bosons would acquire a mass of the order f_{π} [8].

and a negative quadratic term $-\mu_{\varphi}^2 \bar{\varphi}^2$ only increases φ_0/σ_0 . Additional interactions with σ influence the quantitative value for φ_0/σ_0 but do not change the order of magnitude of the bound. Without the non-perturbative QCD effects bounds for the Higgs mass can be obtained for a given value of φ_0 [9,7], but a lower bound on the VEV φ_0 itself does not exist unless the term (8) is included. All these effects of the linear term are independent of the detailed properties of the "electroweak" potential U_A .

Volume 209, number 1

For the observed value of φ_0 the interaction between σ and $\bar{\varphi}$ is dominated by the Yukawa coupling of the strange quark. We estimate $\tilde{\alpha} \approx 30 h_s$ using the identification

 $V_{\varphi\sigma} = -\tilde{\alpha}\sigma_0^3\bar{\varphi} + \text{terms nonlinear in }\bar{\varphi} + \text{const.}$

$$\approx h_{\rm u}\bar{\varphi}\langle\bar{\rm u}{\rm u}\rangle + h_{\rm d}\bar{\varphi}\langle{\rm d}{\rm d}\rangle + h_{\rm s}\bar{\varphi}\langle\bar{\rm s}{\rm s}\rangle + h_{\rm c}\bar{\varphi}\langle\bar{\rm c}{\rm c}\rangle + \dots.$$
(10)

The contribution of up and down quarks can be neglected and the condensates of heavier quarks are in leading order inversely proportional to the quark mass [10]. Alternatively, we can extract $\tilde{\alpha}$ (and also terms nonlinear in $\bar{\phi}$) from the phenomenological analysis of chiral perturbation theory [11]. (The term $\sim \tilde{\alpha}\sigma^3\bar{\phi}$ leads to the quark mass term in the nonlinear σ model.)

There is no reason why the interactions between σ and $\bar{\varphi}$ should be linear in $\bar{\varphi}$. As a consequence of chiral symmetries the contribution $\sim \bar{\varphi}^N$ is of the order of the Nth power of some Yukawa coupling (compare fig. 2). Using naive dimensional arguments yields a quadratic term of the order $h_s^2 \langle \bar{s}s \rangle^{2/3} \bar{\varphi}^2$. As an alternative approach we may express all QCD condensates in their dependence on $\bar{\varphi}$ and investigate how the QCD vacuum energy density varies as a function of $\bar{\varphi}$. For an illustration we approximate $V_{\varphi\sigma}$ by the term $h_s \langle \bar{s}s \rangle \bar{\varphi}$. We consider $\langle \bar{s}s \rangle$ as a function of $\bar{\varphi}$ and express the resulting $\bar{\varphi}$ dependence of $V_{\varphi\sigma}$ through the dependence of the condensate $\langle \bar{s}s \rangle$ on the strange quark mass m_s

$$\frac{\mathrm{d}V_{\varphi\sigma}(\bar{\varphi})}{\mathrm{d}\bar{\varphi}} = h_{\mathrm{s}}\frac{\mathrm{d}}{\mathrm{d}m_{\mathrm{s}}} \left[m_{\mathrm{s}}\langle\bar{\mathrm{s}}\mathrm{s}\rangle(m_{\mathrm{s}})\right]. \tag{11}$$

For low values of m_s the strange quark is effectively massless and $\langle \bar{s}s \rangle$ becomes independent of m_s . Extrapolating the QCD sum rule estimate [10] for large m_s



Fig. 2. Typical contributions to $V_{\omega\sigma}$.

$$\langle \bar{\mathbf{s}} \mathbf{s} \rangle = -\left(1/12m_{\mathbf{s}}\right) \langle \left(\alpha_{\mathbf{s}}/\pi\right) G^a_{\mu\nu} G^{\mu\nu}_a \rangle , \qquad (12)$$

to a strange quark mass of the order 150-200 MeV gives a value for $\langle \bar{s}s \rangle$ which is a factor 2–3 smaller in size than $\langle \bar{\psi}\psi \rangle_0 \simeq (230 \text{ MeV})^3$. This suggests that (12) is approached from below and therefore $m_{\rm s}\langle \bar{\rm ss} \rangle (m_{\rm s})$ should have a minimum at $\bar{m}_{\rm s}$. This also obtains if $\langle \bar{s}s \rangle$ vanishes faster than m_s for large m_s . The scale for \bar{m}_s is obviously given by $\langle \bar{\psi}\psi \rangle_0^{1/3}$. This is in the vicinity of the physical strange quark mass $m_{\rm s}^{\rm phys}$. The same conclusion is suggested by an expansion in m_s within chiral perturbation theory [11]. Taking $\varepsilon_s = 0.3$ in ref. [11] leads to a minimum value $\bar{m}_{s} \approx m_{s}^{\text{phys}}$ (compare fig. 3). The minimum of $V_{aa}(\bar{\varphi})$ corresponds to a value \bar{m}_{s} where the strange quark changes its role from a light to a heavy quark, $\varphi_0 = \bar{m}_s / h_s$. It is puzzling that our naive estimate of φ_0 fits well the observed value $\varphi_0 = m_s^{\text{phys}}/h_s = 174 \text{ GeV}!$

For a more accurate treatment we have to account for the fact that $\Lambda_{\rm QCD}$ (and therefore the gluon condensate and $\langle \bar{\psi}\psi \rangle_0$) depends on $\bar{\phi}$ via the heavy fermion mass thresholds in the evolution equation for the strong gauge coupling. Also the contributions from kinetic terms and the other quarks have to be included in $V_{\varphi\sigma}$. It is still plausible that $\bar{\phi} = m_s/h_s$ is near a local minimum of $V_{\varphi\sigma}$. The question if it is a global



Fig. 3. $m_s \langle \bar{s}s \rangle$ in dependence on m_s .

minimum seems more complicated. The quadratic term at the minimum ^{#6}, $M_{\rm H}^2 = \frac{1}{2} {\rm d}^2 V_{\varphi\sigma}(\varphi_0)/{\rm d}\bar{\varphi}^2$, depends on the smoothness of the transition between the light quark and the heavy quark regime. For a rough estimate we approximate

$$V_{\varphi\sigma} \approx -h_{\rm s} \langle \bar{\psi}\psi \rangle_0 \bar{\varphi} + \beta \langle \bar{\psi}\psi \rangle_0^{2/3} \bar{\varphi}^2 ,$$

$$\beta = h_{\rm s} \langle \bar{\psi}\psi \rangle_0^{1/3} / 2\varphi_0 = \mathcal{O}(h_{\rm s}^2) , \qquad (13)$$

and obtain

$$M_{\rm H}^2 = h_{\rm s} \langle \bar{\psi}\psi \rangle_0 / 2\varphi_0 \approx (0.2 \,\,{\rm MeV})^2 \,. \tag{14}$$

This is an unusually small value for the Higgs particle mass! It is easy to understand the large value for φ_0/σ_0 intuitively: Although the linear term driving $\bar{\varphi}$ away from the origin at $\bar{\varphi}=0$ is small $\sim h_s$, a minimum can only occur once the restoring force is of equal strength as the driving term. Restoring forces are suppressed by even higher powers of h_s ($\beta \sim h_s^2$). Therefore φ_0 must come out proportional to the inverse of the small coupling h_s . One may also observe that due to the flatness of $U_A(\bar{\varphi})$ the finite-volume action $\hat{S}_A + S_{\rm QCD}$ has an approximate dilatation symmetry, broken explicitly at the scale Λ and spontaneously by the VEV φ_0 . The mode $\bar{\varphi}$ may be considered as an effective pseudodilaton with $M_{\rm H}^2 \sim \Lambda^4/\varphi_0^2$.

The relevance of the nonperturbative QCD effects depends on how fast $U_k(\bar{\varphi})$ converges to $\Gamma(\bar{\varphi})$. Also the short-range QCD fluctuations have to be included. This can be done by a suitable generalization of \hat{S}_k to include quark and gluon degrees of freedom in a way that all fluctuations with momenta greater than k are effectively integrated out. This gives additional contributions to $U_k(\bar{\varphi})$. Leptons and their Yukawa couplings are easily added. Inclusion of the electroweak gauge interactions requires modifications since the average field ϕ_k is not a gauge invariant quantity. There are no massless Goldstone bosons in this case. Nevertheless, the flatness of quantities like $U(\bar{\varphi})$ is not directly related to the existence of propagating massless modes but only to the fact that the coherence in the phase of the mean value of φ becomes weak over large distances. In a gauge fixed version the problem looks at first sight not very different from the global $SU(2) \times U(1)$ model discussed before.

One may wonder if fermion or gauge boson loops with momenta near $\Lambda_{\rm OCD}$ or φ_0 do not induce terms $\mu^2 \bar{\varphi}^2$ or $\lambda \bar{\varphi}^4$ compared to which the nonperturbative QCD effects are completely negligible. Such loops induce effective nonlocal scalar interactions and they influence the way how $U_k(\bar{\varphi})$ approaches $\Gamma(\bar{\varphi})$. The behaviour of $U_k(\bar{\varphi})$, however, cannot be determined by naive perturbation theory. The standard perturbative renormalization group equations for μ^2 and λ are inadequate for $|\bar{\varphi}| < M$. (In contrast, the standard renormalization group analysis remains valid for Yukawa couplings and gauge couplings.) We need a modification of the renormalization group equations which accounts for the fact that the behaviour of $U_k(\bar{\varphi})$ is determined by the physics of "domain adjustment" in the presence of light fermion and gauge boson loops. More precisely we are interested in the quantity $k dU_k(\bar{\varphi})/dk$. For small $\bar{\varphi}$ we may expand

$$U_k(\bar{\varphi}) = \mu^2(k)\bar{\varphi}^+\bar{\varphi} + \lambda(k)(\bar{\varphi}^+\bar{\varphi})^2 + \dots$$

We expect that the evolution of the quadratic term is given by an anomalous dimension

$$k d\mu^{2}(k)/dk = A\mu^{2}(k)$$
,
 $A = \alpha + O(h^{2}(k), g^{2}(k), ...)$. (15)

For A > 2 the mass term $\mu^2(M) \approx M^2$ becomes smaller than Λ^2 at $k = \Lambda^{\#7}$. We do not know if in the presence of Yukawa and gauge couplings the evolution equation for $\lambda(k)$ will have a fixed point for $\lambda = 0$ or not. If not, a radiative symmetry breaking [13] may replace the determination of φ_0 through nonperturbative QCD effects leading to a higher mass $M_{\rm H}$ (in the GeV range?).

One may also ask about the relevance of the QCD σ model for weak processes involving the exchange of W or Z bosons. Naively one could think that at momenta around the Fermi scale the nonperturbative QCD effects should not be relevant. The weak bosons propagate through the vacuum for which $\bar{\varphi} = \varphi_0$. The VEV φ_0 sets the scale for all weak interaction processes. Once we know φ_0 we can forget its origin, use perturbative weak interaction field theory and

^{#6} Strictly speaking there is a mass matrix involving σ and $\bar{\varphi}$. Corrections to the eigenvalues are small $(O(h_s))$ in the present case.

^{*7} This idea for a solution of the gauge hierarchy problem was already proposed in ref. [12] in a somewhat different context.

neglect all nonperturbative QCD effects. The value of φ_0 itself, however, is a property of the surrounding vacuum and related to zero momentum ^{#8} rather than to the momenta of a particular weak scattering process. Its value is therefore sensitive to the long-distance behaviour of the theory where nonperturbative QCD effects become important.

Many questions are left open. Nevertheless it seems not excluded that electroweak symmetry breaking is indeed determined by the chiral symmetry breaking in QCD. The connection between nonperturbative OCD effects and (perturbative) electroweak physics certainly needs and merits a more profound investigation. This shows that there are still important holes in our understanding of spontaneous symmetry breaking within the standard model. Although we are aware of the somewhat speculative character of this hypothesis it seems worthwhile to mention some of the consequences of this version of electroweak symmetry breaking: The gauge hierarchy problem would be solved and φ_0 becomes calculable in terms of $\Lambda_{\rm OCD}$ and Yukawa couplings. There is a possible reduction in the number of parameters since the parameters $\mu_{\varphi}^{2}(M^{2})$ and $\lambda_{\varphi}(M^{2})$ become irrelevant if they correspond to a region in parameter space for which $U_{\mathcal{A}}(\bar{\varphi})$ is effectively flat. This could also have important consequences for the issues of dilatation symmetry, the cosmological constant and a possible new intermediate-range interaction [14] since $\Lambda_{\rm QCD}$ is now the only low energy mass scale #9. The mass of the Higgs scalar is predicted to be very low (≈ 200 keV). The scalar is therefore not expected to decay into e⁺e⁻ and must have a rather long lifetime. A scalar of this type is not excluded experimentally so far. It may be possible to detect it by future precision experiments. There are experimental bounds [15] on the scalar couplings to nucleons. These are not easy to evaluate theoretically. In particular we should mention that the Higgs scalar has residual "strong" interactions since there is a mixing with the σ field of order $\tilde{\alpha} \sim h_s$. The physical Higgs scalar is a sort of collective excitation. At energies below $\Lambda_{\rm QCD}$ it can be treated as a fundamental scalar, but higher energy scales need a detailed investigation. The chiral and weak phase transitions would look quite different from the standard picture. This has possibly important consequences for the early universe. In view of these prospects the questions concerning the connection between the scales of strong and electroweak interactions should be an important task for a deeper field theoretical investigation of symmetry breaking in the standard model.

References

- S.L. Glashow, Nucl. Phys. 22 (1961) 579;
 S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264;
 A. Salam, in: Elementary particle theory, ed. N. Svartholm (Almqvist and Wiksell, Stockholm, 1968).
- [2] H. Fritzsch, M. Gell-Mann and H. Leutwyler, Phys. Lett. B 47 (1973) 365;
 - H.D. Politzer, Phys. Rev. Lett. 30 (1973) 1346;
 - D.J. Gross and F. Wilczek, Phys. Rev. Lett. 30 (1973) 1343.
- [3] E. Gildener and S. Weinberg, Phys. Rev. D 13 (1976) 3333;
- E. Gildener, Phys. Rev. D 14 (1976) 1667;
 S. Weinberg, Phys. Lett. B 82 (1979) 387.
- [4] C. Wetterich, Nucl. Phys B 279 (1987) 711;
 J. Bijnens and C. Wetterich, Nucl. Phys. B 283 (1987) 237;
 Phys. Lett. B 199 (1987) 525.
- [5] R. Fukuda and E. Kyriakopoulos, Nucl. Phys. B 85 (1975) 354;
 - L. O'Raifeartaigh, A. Wipf and H. Yoneyama, Nucl. Phys. B 271 (1986) 653.
- [6] Y. Fujimoto, L. O'Raifeartaigh and G. Parravicini, Nucl. Phys. B 212 (1983) 268.
- [7] M. Lüscher and P. Weisz, Nucl. Phys. B 290 [FS20] (1987)
 25; B 295 (1988) 65.
- [8] L. Susskind, Phys. Rev. D 20 (1979) 2619;
- S. Weinberg, Phys. Rev. D 13 (1976) 974; D 19 (1979) 1277.
- [9] A.D. Linde, JETP Lett. 23 (1976) 64;
 - S. Weinberg, Phys. Rev. Lett. 36 (1976) 294; L. Maiani, G. Parisi and R. Petronzio, Nucl. Phys. B 136 (1978) 115;
 - C. Wetterich, DESY preprint 87-154 (1987).
- [10] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B 147 (1979) 385.
- [11] J. Gasser and H. Leutwyler, Nucl. Phys. B 250 (1985) 465.
- [12] C. Wetterich, Phys. Lett. B 104 (1981) 269; B 140 (1984) 215.
- [13] S. Coleman and E. Weinberg, Phys. Rev. D 7 (1973) 1888.
- [14] R.D. Peccei, J. Solá and C. Wetterich, Phys. Lett. B 195 (1987) 183;
 - C. Wetterich, Nucl. Phys. B 302 (1988) 668.
- [15] R. Barbieri and T.E.O. Ericson, Phys. Lett. B 57 (1975) 270.

^{#8} One may ask if for a weak process the relevant quantity is not the mean value of the doublet in a volume of Fermi size (φ_0^{-1}) . In any case, by simple dimensional arguments, this mean value cannot be very different from φ_0 .

^{#9} If there is a cosmon force its range should be near the kilometer range ($\sim M_{\rm Pl}/\Lambda_{\rm OCD}^2$).