HIGGS AND W MASS ON THE LATTICE AT STRONG GAUGE COUPLING

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At infinite gauge coupling the gauge fields in the fundamental lattice SU(N) Higgs model can be integrated out exactly. In the resulting effective theory of the radial Higgs field we derive a string-like correlation function that represents the leading behavior of the W two-point function at small β. For large N we then compute the W-mass and the Higgs mass. These analytical results are qualitatively similar to what has been found in Monte Carlo simulations of the SU(2) model.

Evidence for the confining area law at strong coupling was one of the main triggers for the explosion of interest in lattice gauge theory [1]. An appealing physical picture could also be uncovered with coupled staggered fermions at strong coupling [2]: spontaneous chiral symmetry breaking with a baryon mass given in terms of the nonvanishing chiral condensate. In ref. [3] QCD and the Higgs model have been analysed side by side, and at strong coupling and large N some analogies between Higgs radius and chiral condensate, staggered pion and Higgs particle, and the chiral Goldstone limit and the critical endpoint of the confinement to Higgs transition line have emerged. In the present note we augment this type of analysis for the Higgs model by including the W-meson field. In some sense the W completes the analogies by matching the baryon.

The fundamental SU(N) lattice Higgs model can be characterized by the partition function [3]

$$Z=\int \mathrm{D}\varphi\,\mathrm{D}\varphi^\dagger\,\mathrm{D}U \exp\Big(\frac{\beta}{N}\sum_\square \mathrm{Re\,tr}\,U_\square - N\sum_x\left[\tfrac{1}{2}A(\varphi_x^\dagger\varphi_x)^2+B\varphi_x^\dagger\varphi_x\right] + N\sum_{x\mu}\mathrm{Re}(\varphi_x^\dagger U_{x\mu}\varphi_{x+\hat\mu})\Big). \quad (1)$$

Gauge fields are integrated over SU(N) for each link and weighted with the standard Wilson action. On each site there is a scalar N-component Higgs field $\varphi_x \in \mathbb{C}^N$ with integration measure

$$\mathrm{D}\varphi\,\mathrm{D}\varphi^\dagger + \prod_x \mathrm{d\,Re}\,\varphi_x\,\mathrm{d\,Im}\,\varphi_x\,. \quad (2)$$

In the much studied [4–7] $N=2$ model the Higgs field is usually parameterized by a radial field $\rho_x \in \mathbb{R}_+$ and an angular part $\alpha_x \in \mathrm{SU}(2)$, and the action by the quartic coupling λ, and the hopping parameter κ. We prefer (1) for studying the N-dependence, and for $N=2$ complete equivalence is established by the identifications

$$\varphi_x=\kappa^{1/2}\rho_x\alpha_x\begin{pmatrix}1\\0\end{pmatrix}\in\mathbb{C}^2\,,$$

$$A=\frac{\lambda}{\kappa^2}\,,\quad B=\frac{1-2\lambda}{2\kappa}\,. \quad (3)$$

Excitations of the radial field are associated with the Higgs particle. The angular field α_x is gauge transformed by left multiplication. Under the additional global symmetry of SU(2) multiplications from the right there is a gauge invariant triplet vector field

$$\boldsymbol{W}_{x\mu}=(\kappa/2\mathrm{i})\rho_x\rho_{x+\hat\mu}\,\mathrm{Tr}(\boldsymbol{\tau}\alpha_x^\dagger U_{x\mu}\alpha_{x+\hat\mu})\,, \quad (4)$$

where the $\boldsymbol{t}$ are the Pauli matrices. This field may be taken to define the W-meson in the standard SU(2) lattice Higgs system. Its counterpart in the φ-language is

$$W^1_{x\mu}+\mathrm{i}W^2_{x\mu}=\mathrm{i}\varphi_x^{\mathrm{c}\dagger}U_{x\mu}\varphi_{x+\hat\mu}\,, \quad (5)$$

and

$$W^3_{x\mu}=\mathrm{Im}(\varphi_x^\dagger U_{x\mu}\varphi_{x+\hat\mu})\,, \quad (6)$$

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with the charge conjugate

$$\varphi_x^c = i\tau^2 \varphi_x^* \tag{7}$$

possessing the same gauge behavior as φ_x. While its existence is a special feature of SU(2), the component $W^3_{x\mu}$ in (6) exists for general N. In the sequel we omit the superscript "3" and use it to probe W for general N including the standard case $N=2$.

For $\beta=0$ we can perform the independent gauge field integrations exactly [3] by using the one-link integral [#1]

$$\int_{SU(N)} dU \exp[N \operatorname{Re}(\varphi_1^\dagger U \varphi_2)] = \exp[N K_N(\varphi_1^\dagger \varphi_1 \varphi_2^\dagger \varphi_2)] , \tag{8}$$

$$K_N(z^2) = (1/N) \log[I_{N-1}(Nz)/z^{N-1}] + \text{const.} \tag{9}$$

As a result we have an effective theory for the squared radius $R_x = \varphi_x^\dagger \varphi_x$ given by

$$Z = \prod_x \int_0^\infty dR_x \exp\Big(-N \sum_x [\tfrac{1}{2} A R_x^2 + B R_x - (1-1/N) \log R_x] + N \sum_{x\mu} K_N(R_x R_{x+\hat\mu}) \Big) . \tag{10}$$

This is a self-coupled scalar field theory for the Higgs degree of freedom which still cannot be solved. In ref. [3] its phase diagram was analysed in the $N\to\infty$ limit with the asymptotic form of K_N

$$K_\infty(z^2) = \sqrt{1+z^2} - \log(1+\sqrt{1+z^2}) + \text{const.} \tag{11}$$

The integral (10) is then dominated by a saddle point of constant R_x, and $1/N$ is a loop counting parameter. In the A–B plane a first-order transition line emerges where R jumps, and it ends in a critical point of vanishing Higgs mass in lattice units (see fig. 1 in ref. [3]). Alternatively one may perform a saddle point expansion [4] with the $N=2$ action in (10). The result may then either be regarded as the leading term in a mean field $1/D$ expansion or as the leading contribution for $N\to\infty$ together with part of the $1/N$ corrections evaluated at $N=2$. In this letter we shall follow the second procedure.

To evaluate correlations of $W_{x\mu}$ we need some simple extensions of the one-link integral (8). With $R_i = \varphi_i^\dagger \varphi_i$ we introduce a one-link expectation value

$$\langle O \rangle^{1\ell} = \exp[-N K_N(R_1 R_2)] \times \int dU \exp[N \operatorname{Re}(\varphi_1^\dagger U \varphi_2)] O(\varphi_1, \varphi_2, U) . \tag{12}$$

Using the SU(N) invariance of the measure and scale and phase transformations of φ_i in (8) it is easy to show that

$$\langle \operatorname{Im}(\varphi_1^\dagger U \varphi_2) \rangle^{1\ell} = 0 , \tag{13}$$

$$\langle \operatorname{Re}(\varphi_1^\dagger U \varphi_2) \rangle^{1\ell} = N \langle [\operatorname{Im}(\varphi_1^\dagger U \varphi_2)]^2 \rangle^{1\ell} = 2K'_N(R_1 R_2) R_1 R_2 , \tag{14}$$

$$\langle U \rangle^{1\ell} = 2K'_N(R_1 R_2)(\varphi_1 \varphi_2^\dagger + \delta_{N,2} \varphi_1^c \varphi_2^{c\dagger}) . \tag{15}$$

Note that (15) is a matrix equation. From (13) and (14) it follows that

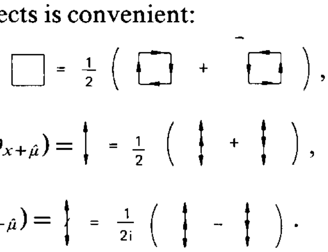
$$\langle W_{x\mu} W_{y\nu} \rangle = \delta_{\mu\nu} \delta_{xy} (2/N) \langle K'_N(R_x R_{x+\hat\mu}) R_x R_{x+\hat\mu} \rangle , \tag{16}$$

i.e. the correlation is ultralocal, and at $\beta=0$ the W has infinite mass. The expectation value on the RHS may be evaluated in the effective Higgs theory. By translation invariance it is a constant independent of x and μ. Clearly $W_{x\mu}$ is odd under charge conjugation $U_{x\mu} \leftrightarrow U^*_{x\mu}$, $\varphi_x \leftrightarrow \varphi_x^*$ (i.e. $i \leftrightarrow -i$), hence (13). In the absence of the plaquette term ($\beta=0$) there is just no way for the C-quantum number to move through the lattice. This interpretation suggests how to obtain the leading nonvanishing contribution to the W hopping parameter (i.e. a finite mass). At this point the following graphical notation for a number of gauge invariant objects is convenient:

$\operatorname{Re}\operatorname{tr} U_\Box =$

$\operatorname{Re}(\varphi_x^\dagger U_{x\mu} \varphi_{x+\hat\mu}) =$ (17)

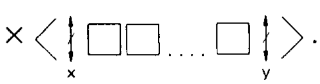
$\operatorname{Im}(\varphi_x^\dagger U \varphi_{x+\hat\mu}) =$



Then an on-axis correlation with $y = x + n\hat\nu$ is given to leading order in β by

$$\langle W_{x\mu} W_{y\mu} \rangle \simeq (\beta/N)^n$$

$\times$ (18)



[#1] K_N equals $-W$ in ref. [3].

The two link/vector fields have to be connected by a string (ladder) of plaquettes, a mechanism reminiscent of the string tension at strong coupling. The average on the RHS of (18) can again be taken in the $\beta=0$ theory. The observable still contains $U_{x\mu}$, and we shall now reduce it to an equivalent quantity in the R-theory.

In a first step we integrate over gauge fields on the "horizontal" plaquette links in (18). On each plaquette we get symbolically ($N>2$)

$$\left\langle \cdots \underset{1\quad 2}{\overset{4\quad 3}{\Box}} \cdots \right\rangle = 4\Big\langle K'_N(R_1R_2)K'_N(R_3R_4)\ldots \times \left\{ \underset{1\quad 2}{\overset{4\quad 3}{\Box}} + \underset{1\quad 2}{\overset{4\quad 3}{\Box}} \right\} \cdots \Big\rangle . \qquad (19)$$

The one-link expectation value (15) has been used here. Only the second part in (19) contributes, and using (14) on the remaining links with $| \; |$ we arrive at

$$\langle W_{x\mu}W_{y\mu}\rangle = (2/N)\,[1+\delta_{N,2})8\beta/N^2]^n \times \Big\langle \prod_{z=x}^{z=y-\hat\nu} K'_N(R_zR_{z+\hat\nu})K'_N(R_{z+\hat\mu}R_{z+\hat\mu+\hat\nu}) \times \prod_{z=x}^{z=y} K'_N(R_zR_{z+\hat\mu})R_zR_{z+\hat\mu}\Big\rangle , \qquad (20)$$

which is now a pure R-observable with a string structure. Also included in (20) is the result for SU(2) where due to the extra term in (15) involving φ^c there appears an extra factor of two for each plaquette. In the saddle point approximation $R_x=\bar R$ to the effective Higgs theory the correlation decays exponentially

$$\langle W_{x\mu}W_{y\mu}\rangle \propto h_W^{|x-y|} , \qquad (21)$$

with a hopping parameter

$$h_W = \exp(-m_W) = (1+\delta_{N,2})(\beta/N^2)\,[2K'_N(\bar R^2)\bar R]^3/\bar R . \qquad (22)$$

The bracketed factor ist bounded as it follows from (14) with $\varphi_1=\varphi_2$ that

$$O<\langle\varphi^+U\varphi\rangle^{1\Omega}/R = 2K'_N(R^2)R<1. \qquad (23)$$

The Higgs mass m_H is read off from the action for quadratic fluctuations of R_x around the saddle point $\bar R$ yielding [3]

$$m_H^2 = [K'_N(\bar R^2)+\bar R^2K''_N(\bar R^2)]^{-1}U''_N(\bar R), \qquad (24)$$

with the effective potential

$$U_N(R) = \tfrac{1}{2}AR^2+BR-(1-1/N)\log R - DK_N(R^2) \qquad (25)$$

in D spacetime dimensions. The prefactor in (24) stems from the normalization of the lattice derivative term. Expanding everything for small and large κ at fixed λ we find the asymptotic behavior deeply in the confinement phase ($\kappa, R\to 0$)

$$m_H^2 \simeq 4[\lambda/\kappa^2+(1-1/N)/R^2] , \quad h_W \simeq (1+\delta_{N,2})(\beta/N^2)R^2/8 , \qquad (26)$$

with

$$R \simeq \tfrac{1}{4}\kappa(|1-2\lambda|/\lambda) \times [\sqrt{1+(1-1/N)16\lambda/(1-2\lambda)^2}-1] .$$

Deeply in the Higgs phase ($\kappa R\to\infty$) we have

$$m_H^2 \simeq 4D , \quad h_W \simeq (1+\delta_{N,2})(\beta/N^2)/R , \qquad (27)$$

with

$$R \simeq D\kappa^2/\lambda .$$

For the general case we solve the saddle point equation $U'_2=0$ numerically. In fig. 1 we display m_H and m_H as functions of κ at $\lambda=0.08$. At this value one passes beyond the phase transition line that ends [#2] at $\lambda=0.071836$, $\kappa=0.29930$. To quote m_W we use $\beta=2.4$, a value popular in Monte Carlo calculations that may easily be converted to other values. Passing right through the endpoint is similar to fig. 1, only the Higgs mass dips down to zero. In fig. 2 at $\lambda=0.055$ the transition line is crossed, and the masses change discontinuously. The dashed lines show continuations into metastable regions.

Our results for Higgs and W masses in the mean

[#2] The endpoint coordinates differ from those in ref. [8]. There an expansion is performed in ρ while we use $R\propto\rho^2$. In the first case the coefficient in front of the log in (25) is $(1-1/2N)$. This amounts to just another choice of the partial $1/N$ corrections included. The qualitative picture does not change.

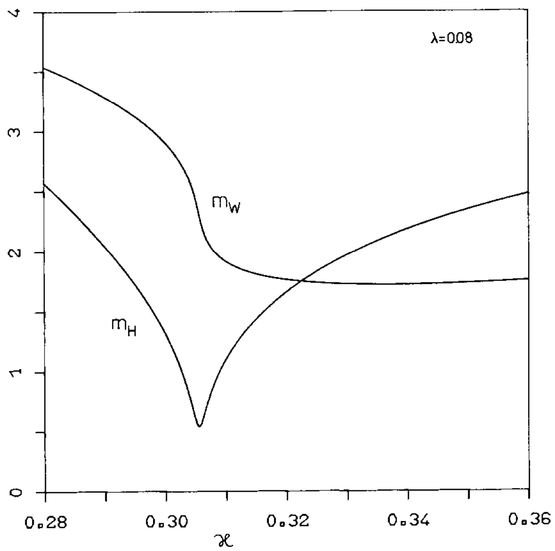


Fig. 1. Higgs and W-masses as functions of κ. The dip in m_H occurs close to the endpoint of the confinement to the Higgs transition line which is not crossed however.

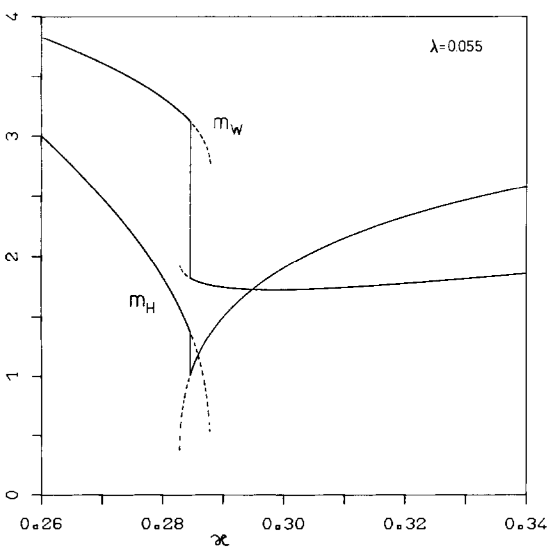


Fig. 2. Same as fig. 1 but crossing the transition line. The dashed lines correspond to metastable phases.

field or $N \to \infty$ approximation to the small-β theory look qualitatively similar to Monte Carlo results [5,7] albeit the mass values are systematically higher. This is of course to be expected or even necessary for consistency at strong coupling. Once again this limit has proved its value in elucidating the structure of nonperturbative physics. Needless to say it is not possible to expose any features of the scaling limit as e.g. effects of triviality or "λ-independence". We conclude by noting that the β-dependence of masses determined in ref. [7] on their largest lattices, namely $m_W = 0.48$, 0.39, 0.26 for $\beta = 2.4$, 2.7, 3.0, follows the strong coupling picture with W hopping parameter $\propto \beta$; they are described within errors by $m = 1.37 - \log\beta$. Clearly finite β has "renormalized" the phase diagram, but our strong coupling picture may possibly hold for the W mass. This however cannot be extended to the more realistic value $\beta \simeq 10$ suggested by weak interactions [6].

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