SEMILEPTONIC CHARM MESON DECAYS AND THE MATRIX ELEMENTS $|V_{cs}|$ AND $|V_{cd}|$

C A DOMINGUEZ [1 2]
Deutsches Elektronen-Synchrotron DESY, D-2000 Hamburg 52, Fed Rep Germany

and

N PAVER [3]
Dipartimento di Fisica Teorica, Università di Trieste Scuola Internazionale Superiore di Studi Avanzati, and Istituto Nazionale di Fisica Nucleare, Sezione di Trieste, I-34100 Trieste Italy

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Using QCD sum rules for a two-point function involving charmed vector currents we determine the $D_{\ell 3}$ form factor $f_+(0)=0\ 75\pm0\ 05$ This result, combined with the $D_{\ell 3}$ decay widths, leads to a prediction for the quark mixing matrix elements $|V_{cs}|$ and $|V_{cd}|$ We find $|V_{cs}|=0\ 96\pm0\ 12$ and $\Gamma(D\to\pi\ell\bar{\nu}_\ell)=(0\ 76\pm0\ 24)\times10^{11}\ [|V_{cd}|/(0\ 21\pm0\ 03)]^2\ s^{-1}$ Our estimate is reliable to the extent that employing the same technique for $K_{\ell 3}$ decay we obtain $f_+(0)|_{K_{\ell 3}}=0\ 96\pm0\ 13$

An accurate knowledge of the form factor $f_+(0)$ in $D_{\ell 3}$ decays is quite important as it allows for a determination of the quark mixing, Cabibbo–Kobayashi–Maskawa, matrix elements $|V_{cs}|$ and $|V_{cd}|$, once the corresponding decay rates are known from experiment (for a recent review see e g ref [1]) According to the non-renormalization theorem [2] the deviation of $f_+(0)$ from unity is of second order in flavour symmetry breaking While this feature has allowed for reliable and accurate estimates of $f_+(0)$ in $K_{\ell 3}$ [3] $f_+(0)=0\ 978$, it is not of much help for $D_{\ell 3}$ decays as SU(4) is a badly broken symmetry In the latter case, constituent quark model estimates give $f_+(0)\simeq0\ 75$–$0\ 82$ [4], and $f_+(0)\simeq0\ 58$ [5] Most of the well known caveats of this approach are not present in the formalism of QCD sum rules, which is fully relativistic and field theoretic by construction, and where the basic QCD features, such as asymptotic freedom and non-perturbative spontaneous symmetry beaking, are incorporated in a natural way

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[2] On leave of absence from Facultad de Fisica, Pontificia Universidad Catolica de Chile, Santiago, Chile
[3] Supported by MPI (Ministero della Pubblica Istruzione, Italy)

The value of $f_+(0)$ in $D_{\ell 3}$ has indeed been estimated in this framework using three-point function sum rules, with the result [6] $f_+(0)=0\ 6\pm0\ 1$ This estimate, however, depends among other things on the leptonic decay constant f_D which is affected by some uncertainty [7,8] In addition, it is well known that three-point function QCD sum rules suffer from a systematic, and difficult to assesss, uncertainty due to their complicated structure and lack of positivity of the spectral function As a result, these sum rules cannot in general compete in accuracy with their two-point function counterparts

In this note we discuss a determination of $f_+(0)$ from QCD sum rules for a *two-point function* involving the charmed vector currents $\bar{c}\gamma_\mu d$ and $\bar{c}\gamma_\mu s$ Since $\Gamma(D\to K\ell\bar{\nu}_\ell)$ is known experimentally [1] we are able to estimate the matrix element $|V_{cs}|$ A prediction for $|V_{cd}|$ has to await an experimental measurement of $\Gamma(D\to\pi\ell\bar{\nu}_\ell)$ For the time being, we can use the value of $|V_{cd}|$ as determined from ν and $\bar{\nu}$ charm production [1] and predict $\Gamma(D\to\pi\ell\bar{\nu}_\ell)$

As a preamble, and to gauge the reliability of our main estimate in $D_{\ell 3}$ decays, we discuss a determination of $f_+(0)$ in $K\to\pi\ell\bar{\nu}_\ell$ We consider to this end the following two-point function

$$\Pi_{\mu\nu}(q)=i\int d^4x\,\exp(iqx)\langle 0|T(V_\mu(x)V_\nu^\dagger(0))|0\rangle, \tag{1}$$

where $V_\mu(x)=\bar{Q}(x)\gamma_\mu q(x)$, with q=u, d and Q=s in the case of $K_{\ell 3}$ decays, and

$$\Pi_{\mu\nu}(q^2) = -(g_{\mu\nu}q^2-q_\mu q_\nu)\Pi^{(1)}(q^2)+q_\mu q_\nu \Pi^{(0)}(q^2) \tag{2}$$

The form factors $f_\pm(t)$ in $P(p)\to P'(p')+\ell\bar{\nu}_\ell$ are defined as

$$\langle P'(p')|V_\mu(0)|P(p)\rangle = (p+p')_\mu f_+(t)+(p-p')_\mu f_-(t) \tag{3}$$

Concentrating on the function $\Pi^{(1)}(Q^2\equiv -q^2)$, its QCD expression at the three-loop level in perturbation theory ($\overline{\mathrm{MS}}$-renormalization scheme), and with power corrections à la Shifman, Vainshtein, Zakharov [9] up to dimension $d=6$, reads [10]

$$4\pi^2\Pi^{(1)}(Q^2) = -\ln\frac{Q^2}{\nu^2}+\frac{5}{3}-\frac{3}{Q^2}[m_u^2(\nu^2)+m_s^2(\nu^2)] - \frac{\alpha_s(\nu^2)}{\pi}\ln\frac{Q^2}{\nu^2}+\left(\frac{\alpha_s(\nu^2)}{\pi}\right)^2 \times\left(-\tfrac{1}{4}\beta_1\ln^2\frac{Q^2}{\nu^2}-F_3\ln\frac{Q^2}{\nu^2}\right) + \frac{C_4\langle O_4\rangle}{Q^4}+\frac{C_6\langle O_6\rangle}{Q^6} + O\left(\frac{m_q^4}{Q^4}\right)+O\left(\left(\frac{\alpha_s}{\pi}\right)^3\right)+O\left(\frac{1}{Q^8}\right), \tag{4}$$

where $\beta_1=-\frac{9}{2}$ and $F_3=1\ 756$ for three colours and three flavours The non-perturbative vacuum condensates are given by

$$C_4\langle O_4\rangle = \tfrac{1}{3}\pi\langle\alpha_s G^2\rangle+4\pi^2[m_u\langle\bar{u}u\rangle+m_s\langle\bar{s}s\rangle], \tag{5}$$

$$C_6\langle O_6\rangle = -8\pi^3\alpha_s[\langle(\bar{q}\gamma_\mu\gamma_5\lambda^a q)^2\rangle+\tfrac{2}{9}\langle(\bar{q}\gamma_\mu\lambda^a q)^2\rangle] \tag{6}$$

The Laplace transform QCD sum rules [9], i e

$$L(\sigma)=\int_0^\infty dt\,\exp(-t\sigma)\frac{1}{\pi}\operatorname{Im}\Pi^{(1)}(t) \tag{7}$$

corresponding to eq (4) read

$$4\pi^2\sigma L(\sigma)=1+\frac{\alpha_s(1/\sigma)}{\pi}+\left(\frac{\alpha_s(1/\sigma)}{\pi}\right)^2 \times\left(F_3-\tfrac{1}{2}\beta_1\gamma_E+\frac{\beta_2}{\beta_1}\ln\ln(\sigma\Lambda^2)\right) -3[m_u^2(1/\sigma)+m_s^2(1/\sigma)]\sigma +C_4\langle O_4\rangle\sigma^2+C_6\langle O_6\rangle\sigma^3/2!, \tag{8}$$

where $\gamma_E=0\ 5772$ is the Euler constant, $\beta_2=-8$, and $m_q(1/\sigma)$ are the running quark masses at a scale $M^2\equiv 1/\sigma$

On the hadronic sector, the threshold behaviour of the spectral function can be easily obtained from the lowest two-meson intermediate state contribution $\langle P'\bar{P}|$ with the result

$$\lim_{t\to t_+}\frac{1}{\pi}\operatorname{Im}\Pi^{(1)}(t)\to\frac{\eta^2}{48\pi^2}[f_+(t_+)]^2 \times\left[\left(1-\frac{t_+}{t}\right)\left(1-\frac{t_-}{t}\right)\right]^{3/2} \tag{9}$$

where $t_\pm=(M_P\pm M_{P'})^2$, and η^2 is a Clebsch–Gordan coefficient, e g $\eta^2=\frac{3}{2}$ for $K_L\to\pi e\nu$ Imposing this behaviour on a Breit–Wigner formula for the K*(890) resonance, and adding a continuum identified with the asymptotic freedom expression starting at some threshold $t_0\geqslant M_{K^*}^2$ completes the parametrization of the hadronic spectral function This becomes

$$\frac{1}{\pi}\operatorname{Im}\Pi^{(1)}(t)=\frac{\eta^2}{48\pi^2}[f_+(t_+)]^2 \times\left[\left(1-\frac{t_+}{t}\right)\left(1-\frac{t_-}{t}\right)\right]^{3/2} \times\frac{(t_+-M_{K^*}^2)^2+M_{K^*}^2\Gamma_{K^*}^2}{(t-M_{K^*}^2)^2+M_{K^*}^2\Gamma_{K^*}^2} +\frac{1}{4\pi^2}\left(1+\frac{\alpha_s}{\pi}+O(\alpha_s^2)\right)\Theta(t-t_0) \tag{10}$$

Using eqs (8) and (10) in eq (7) leads to a determination of $f_+(t_+)$ in terms of the QCD parameters, the experimental values of the mass and width of the K*(890), and the asymptotic freedom threshold t_0 The latter is of course a free parameter, but Laplace transform sum rules exponentially depress the t_0-dependence, we choose the wide range $1\ \mathrm{GeV}^2\leqslant t_0\leqslant 1\ 5$

GeV^2 For the QCD parameters we use $\bar{m}_s(1\ \mathrm{GeV})=199\pm33$ MeV [11], $C_4\langle O_4\rangle$ in the range $-0.07\ \mathrm{GeV}^4$ to $+0.12\ \mathrm{GeV}^4$, and $C_6\langle O_6\rangle=-(0.08\text{–}0.16)\ \mathrm{GeV}^6$, which take into account the *standard* values of the gluon and four-quark condensates [9], as well as higher values obtained in recent analyses [12] An inspection of the behaviour of $f_+(t_+)$ as a function of the short-distance Laplace variable σ shows a wide region of remarkable stability for $\sigma\simeq(1/1.5\text{–}1/3)\ \mathrm{GeV}^{-2}$ In fact, inside this sum rule window $f_+(t_+)$ changes by less than 6% for fixed values of the QCD parameters The value of the form factor at $t=0$ can be obtained from $f_+(t_+)$ by using the standard K*-pole dominance approximation We obtain

$$f_+(0)|_{K_{\ell 3}}=0.96\pm0.13, \tag{11}$$

where the error takes into account the uncertainties in all the QCD parameters (the driving one being m_s)

Encouraged by this result, we proceed to $D_{\ell 3}$ decays in which case the vector current in eq (1) is built from q=d, s and Q=c quark fields Laplace transform QCD sum rules suffer from many disadvantages in the case of heavy flavours [7], and hence we shall work with Hilbert power moments at $Q^2=0$ These lead to a well-behaved short-distance expansion in terms of inverse powers of the charm quark mass (neglecting $m_{d,s}$ in the sequel) and read

$$\phi_n(0)\equiv\frac{1}{n!}\left(-\frac{d}{dQ^2}\right)^n \Pi^{(1)}(Q^2)|_{Q^2=0}$$
$$=\frac{1}{\pi}\int\frac{dt}{t^{n+1}}\,\mathrm{Im}\,\Pi^{(1)}(t), \tag{12}$$

where $n\geqslant1$ The two-point function has been calculated in QCD up to two loops in perturbation theory and up to dimension $d=6$ in the Wilson coefficients of the operator product expansion [13] From these results it is straightforward to compute the Hilbert moments (12) For the first two we obtain

$$\phi_1(0)=\frac{3}{32\pi^2}\frac{1}{m_c^2}(1+1.140\alpha_s)$$
$$+\frac{1}{m_c^2}\left(-\frac{C_4\langle O_4\rangle}{m_c^4}+\frac{3}{2}\frac{C_5\langle O_5\rangle}{m_c^5}+\frac{5}{3}\frac{C_6\langle O_6\rangle}{m_c^6}\right), \tag{13}$$

$$\phi_2(0)=\frac{1}{40\pi^2}\frac{1}{m_c^4}(1+1.582\alpha_s)$$
$$+\frac{1}{m_c^4}\left(-\frac{C_4\langle O_4\rangle}{m_c^4}+\frac{5}{2}\frac{C_5\langle O_5\rangle}{m_c^5}+2\frac{C_6\langle O_6\rangle}{m_c^6}\right), \tag{14}$$

where $m_c\equiv m_c(Q^2=m_c^2)$, $\alpha_s\equiv\alpha_s(m_c^2)$, and

$$C_4\langle O_4\rangle=m_c\langle\bar{q}q\rangle+\frac{1}{12}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle, \tag{15}$$

$$C_5\langle O_5\rangle=\langle g_s\bar{q}\sigma_{\mu\nu}G^a_{\mu\nu}\lambda^a q\rangle, \tag{16}$$

$$C_6\langle O_6\rangle=\pi\alpha_s\left\langle(\bar{q}\gamma_\mu\lambda q)^2\sum_q\bar{q}\gamma_\mu\lambda q\right\rangle \tag{17}$$

The hadronic spectral function may be parametrized in analogy with eq (10), except that now K*(890) must be replaced by D*(2009) (notice that $D^*\to D\pi$ is phase-space allowed), and the hadronic continuum is simulated by the asymptotic freedom expression [13]

$$\frac{1}{\pi}\mathrm{Im}\,\Pi^{(1)}(x)=\frac{1}{8\pi^2}(1-x)^2(2+x)$$
$$\times\left\{1+\frac{4\alpha_s}{3\pi}\left[\tfrac{13}{4}+2\ell(x)+\ln x\ln(1-x)\right.\right.$$
$$+\frac{3}{2}\frac{x}{(2+x)}\ln\left(\frac{x}{1-x}\right)-\ln(1-x)$$
$$\left.\left.-\left(\frac{4-x-x^2}{(2+x)(1-x)^2}\right)x\ln x-\frac{(5-x-2x^2)}{(2+x)(1-x)}\right]\right\}, \tag{18}$$

where $x\equiv m_c^2/t$, and $\ell(x)$ is the dilogarithm function The value of the asymptotic freedom threshold t_0 is in principle a free parameter, predictions are meaningful provided they are stable against reasonable changes in t_0 This can be explicitly checked by computing the ratio of the first two Hilbert moments which may be written as

$$M^2_{D^*}=\frac{\phi_1(0)-\phi_1(0)|_{\mathrm{cont}}}{\phi_2(0)-\phi_2(0)|_{\mathrm{cont}}}, \tag{19}$$

where $\phi_n(1)$ are given by eqs (13), (14), and $\phi_n(0)|_{\mathrm{cont}}$ can be calculated using eq (18) in eq (12) and integrating in the interval $t\in(t_0,\infty)$ A comparison with the experimental value of M_{D^*} allows one to find the optimal duality region for t_0, as well as to fine tune the less known vacuum condensates For the latter we use $C_4\langle O_4\rangle = -(0\ 008$–$0\ 012)$ GeV^4, and $C_6\langle O_6\rangle = -(0\ 0017$–$0\ 0029)$ GeV^6 The mixed quark–gluon condensate is usually parametrized as $C_5\langle O_5\rangle \equiv 2M_0^2\langle\bar{q}q\rangle$, where M_0^2 is not well known Baryon sum rules tend to give values on the low side, e g $M_0^2 \simeq 0\ 1$–$0\ 4$ GeV^2 [14], in agreement with charmonium [15], while a lattice estimate suggests $M_0^2 \simeq 1$ GeV^2 [16] In our recent analysis of the charm pseudoscalar channel [7] we pointed out that in order to reproduce the observed D-meson mass, the parameter M_0^2 had to lie on the lower end of the above range In the present case, there is no sensible solution to eq (19) for values $M_0^2 \geqslant 0\ 15$ GeV^2, independently of t_0 and of the values of the other condensates Using $M_0^2 \simeq 0\ 10$–$0\ 15$ GeV^2, together with $m_c = 1\ 3$ GeV and $\alpha_s = 0\ 296$, we obtain after solving eq (19)

$$M_{D^*} = 2\ 03 \pm 0\ 10\ \text{GeV} \tag{20}$$

in the safe and wide range $t_0 = (2$–$3)M_{D^*}^2$ It should be clear from this result that the resolution of the method is at the level of SU(3) breaking in the masses ($M_{D^*} = 2\ 01$ GeV, $M_{D_s^*} = 2\ 1$ GeV), this has already been stressed in ref [7] The form factor $f_+(t_+)$ turns out to be remarkably stable in an even wider region Extrapolating it to $t=0$ through D*-pole dominance we find

$$f_+(0) = 0\ 75 \pm 0\ 05 \tag{21}$$

for $t_0 = (2$–$4)M_{D^*}^2$ The error on the $K_{\ell 3}$ form factor was somewhat larger mainly because of the large uncertainty in the strange quark mass As expected, the deviation of $f_+(0)$ from unity in this case is larger than in $K_{\ell 3}$ It is interesting to notice, however, that the size of this deviation is still moderate, so that the pattern following from the underlying SU(4) symmetry is not totally obscured

This result for $f_+(0)$ can now be used to predict the quark mixing matrix elements in terms of the semileptonic widths

$$\Gamma(D \to X\ell\bar{\nu}_\ell) = G_F^2 |V_{ij}|^2 \frac{[f_+(0)]^2}{192\pi^3} \frac{M_{D^*}^4}{M_D^3} I_{ps}, \tag{22}$$

where $X = \pi$, K and the phase space integral I_{ps} (neglecting m_ℓ) is given by

$$I_{ps} \equiv \int_0^{t_-} \frac{dt}{(t - M_{D^*}^2)^2} [(t-t_+)(t-t_-)]^{3/2} \tag{23}$$

In the case of $D \to K\ell\bar{\nu}_\ell$ one finds, using the average masses $M_{D_s^*} = 2\ 11$ GeV, $M_D = 1\ 867$ GeV, and $M_K = 0\ 495$ GeV,

$$\Gamma(D \to K\ell\bar{\nu}_\ell) = 1\ 53 \times 10^{11} f_+^2(0) |V_{cs}|^2\ \text{s}^{-1}, \tag{24}$$

which when compared with the experimental width $(0\ 79 \pm 0\ 11) \times 10^{11}$ s^{-1} leads to the prediction

$$|V_{cs}| = 0\ 96 \pm 0\ 12, \tag{25}$$

which is consistent with $|V_{ud}| = 0\ 974$ [1] For $D \to \pi\ell\bar{\nu}_\ell$ our result may be cast as

$$\Gamma(D \to \pi\ell\bar{\nu}_\ell) = (0\ 76 \pm 0\ 24) \times 10^{10} \left(\frac{|V_{cd}|}{0\ 21 \pm 0\ 03}\right)^2 \text{s}^{-1}, \tag{26}$$

where the value $|V_{cd}| = 0\ 21 \pm 0\ 03$ from neutrino reactions [1] has been used A measurement of this decay width will then be very important for an independent determination of $|V_{cd}|$

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