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Quantization of restricted gravity

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Abstract. We investigate the quantization of a gravitational theory with restricted coordinate invariance. The gauge fixing and the corresponding ghost system follow from an analysis of the BRS algebra. In addition to the graviton the metric contains a dilaton.

The known gravitational phenomena are successfully described by Einstein's equations. These equations do not only follow from an action which is invariant under general coordinate transformations but emerge already if one restricts the transformations to the ones which do not change the scale set by the volume element [1]. In this note we show that these restricted coordinate transformations are an acceptable gauge symmetry of a quantized theory. In particular one can introduce a gauge fixing and ghosts to allow for perturbation theory. The physical subspace with positive definite norm can be defined such that scattering leads from physical states again to physical states with amplitudes which are independent of the gauge fixing parameters. The metric tensor $g_{\mu\nu}$ turns out to generate 3 physical degrees of freedom: the helicity 2 states of the graviton and one massive scalar particle, the dilaton, in agreement with the analysis of the classical theory [1].

To establish all these properties we require invariance of the action under BRS-transformations [2] s and $\bar{s}$, which are linear, real, anticommuting operators satisfying

$$s^2 = \bar{s}^2 = \{s, \bar{s}\} = [s, \partial_\mu] = [\bar{s}, \partial_\mu] = 0, \tag{1a}$$

$$s(AB) = (sA)B + (-)^{|A|} A sB,$$
$$\bar{s}(AB) = (\bar{s}A)B + (-)^{|A|} A \bar{s}B, \tag{1b}$$

$$(sA)^* = (-)^{|A|} sA^*, \quad (\bar{s}A)^* = (-)^{|A|} \bar{s}A^*, \tag{1c}$$

with $|A| = 1(0)$ for anticommuting (commuting) fields. Consistently with the algebra one attributes to $s(\bar{s})$ the ghost number 1 (-1), physical fields are taken to have vanishing ghost number.

On physical fields s and $\bar{s}$ generate coordinate transformations which for a scalar field ϕ "generally" take the form

$$s\phi = c^\mu \partial_\mu \phi, \quad \bar{s}\phi = \bar{c}^\mu \partial_\mu \phi. \tag{2}$$

This is the classical transformation law where the infinitesimal gauge parameters $c^\mu(x)$, $\bar{c}^\mu(x)$ have become real anticommuting ghost fields. More generally tensors transform with the Lie-derivative. For the metric one therefore has

$$s g_{\mu\nu} = c^\lambda \partial_\lambda g_{\mu\nu} + \partial_\mu c^\lambda g_{\lambda\nu} + \partial_\nu c^\lambda g_{\lambda\mu}. \tag{3}$$

In the analogous relation for $\bar{s} g_{\mu\nu}$ the antighost $\bar{c}^\rho$ replaces c^ρ. The implied change of the volume element

$$\sqrt{-g} \equiv (-\det g_{\mu\nu})^{1/2}$$

$$s\sqrt{-g} = \partial_\lambda (c^\lambda \sqrt{-g}), \quad \bar{s}\sqrt{-g} = \partial_\lambda (\bar{c}^\lambda \sqrt{-g}) \tag{4}$$

coincides with the transformation of a scalar field (2) if one restricts the ghost fields by

$$\partial_\mu c^\mu = 0 = \partial_\mu \bar{c}^\mu. \tag{5}$$

We solve these restrictions by

$$\begin{aligned} c^\mu &= \partial_\nu A^{\nu\mu}, \quad A^{\mu\nu} = -A^{\nu\mu}, \\ \bar{c}^\mu &= \partial_\nu \bar{A}^{\nu\mu}, \quad \bar{A}^{\mu\nu} = -\bar{A}^{\nu\mu}, \end{aligned} \tag{6}$$

and define restricted coordinate transformations by (2) and (3) where the ghosts c^μ and $\bar{c}^\mu$ are given by (6). The algebra (1) then completely determines the ghost system and the action of s and $\bar{s}$ on the ghosts. For example $s^2\phi$ and $s^2 g_{\mu\nu}$ vanish if and only if

$$s\partial_\nu A^{\nu\mu} = \partial_\nu (sA^{\nu\mu}) = \partial_\lambda A^{\lambda\nu} \partial_\nu \partial_\rho A^{\rho\mu} = \partial_\nu (\partial_\lambda A^{\lambda\nu} \partial_\rho A^{\rho\mu}). \tag{7}$$

It is nontrivial that at this stage (as in the subsequent steps) the differential operators match so that (7) can be integrated with local fields*. From (7) one deduces

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* The counterexample $c^\mu = \partial_\nu A^{\nu\mu} + \partial^\mu A$ (6′) shows that $s^2 = 0$ together with a local transformation law for the ghosts is a nontrivial requirement

$sA^{\nu\mu} = \partial A^\nu \partial A^\mu$ up to terms with vanishing divergence, so (7) implies

$$sA^{\nu\mu} = \partial A^\nu \partial A^\mu - \partial_\lambda A^{\lambda\nu\mu}, \quad \partial A^\nu \equiv \partial_\lambda A^{\lambda\nu}. \tag{8}$$

Here $A^{\lambda\nu\mu}$ is a completely antisymmetric tensor. If one had not included $A^{\lambda\nu\mu}$ then $s^2 A^{\nu\mu}$ could not be made vanish. $A^{\lambda\nu\mu}$ are ghosts for the ghosts $A^{\nu\mu}$ which one can change by $\partial_\lambda A^{\lambda\nu\mu}$ without changing c^μ (6). Iterating the sketched arguments one deduces the complete ghost multiplet: All ghosts are completely antisymmetric tensors $(A^I_G)^{\mu_1\cdots\mu_I}$ of rank I, $I \geqq 2$, and ghost number G which ranges from $1-I$ to $I-1$. (We identify $A^{\nu\mu}$ with $(A^2_1)^{\nu\mu}$ and $\bar{A}^{\nu\mu}$ with $(A^2_{-1})^{\nu\mu}$.) The rank I and the ghost number G with

$$2 \leqq I, \quad 1-I \leqq G \leqq I-1 \tag{9}$$

uniquely characterize the ghost fields. They are commuting (anticommuting) if G is even (odd) and are real or purely imaginary

$$(A^I_G)^* = (A^I_G)\cdot(-)^{I(I-1)/2+G(I-1)}. \tag{10}$$

In 4 space-time dimensions $I \leqq 4$ and one has altogether 45 ghosts of which 24 anticommute and 21 commute. They are necessary to compensate for the 3 gauge degrees of freedom of restricted coordinate transformations (5).

The action of s and $\bar{s}$ on the ghosts contains nonlinear terms which only appear in the completely antisymmetric combinations

$$(S^{N+\bar{N}}_{N-\bar{N}})^{\mu_1\ldots\mu_{N+\bar{N}}} \equiv \frac{1}{N!\bar{N}!} \cdot\sum_\pi \operatorname{sign}(\pi) c^{\mu_{\pi(1)}}\ldots c^{\mu_{\pi(N)}}\bar{c}^{\mu_{\pi(N+1)}}\ldots\bar{c}^{\mu_{\pi(N+\bar{N})}}. \tag{11}$$

The sum runs over all permutations π of $N+\bar{N}$ indices, c^μ and $\bar{c}^\mu$ are given by (6) and $G = N - \bar{N}$ is the ghost number. It is convenient to denote the ghost fields A^I_G by B^I_G if $G+I$ is even, they will turn out to be auxiliary fields. The action of s and $\bar{s}$ is given by

$$s(B^I_G)^{\mu_1\ldots\mu_I} = 0, \quad I+G \text{ even}, \tag{12a}$$

$$s(A^I_G)^{\mu_1\ldots\mu_I} = (B^I_{G+1})^{\mu_1\ldots\mu_I}, \quad I+G \text{ odd}, \quad G \leqq I-3, \tag{12b}$$

$$s(A^I_{I-1})^{\mu_1\ldots\mu_I} = -\partial_\nu(A^{I+1}_I)^{\nu\mu_1\ldots\mu_I} + (S^I_I)^{\mu_1\ldots\mu_I}, \tag{12c}$$

$$\bar{s}(B^I_G)^{\mu_1\ldots\mu_I} = \partial_\nu(B^{I+1}_{G-1})^{\nu\mu_1\ldots\mu_I} - s(S^I_{G-2})^{\mu_1\ldots\mu_I}, \quad I+G \text{ even}, \tag{13a}$$

$$\bar{s}(A^I_G)^{\mu_1\ldots\mu_I} = -(B^I_{G-1})^{\mu_1\ldots\mu_I} - \partial_\nu(A^{I+1}_{G-1})^{\nu\mu_1\ldots\mu_I} + (S^I_{G-1})^{\mu_1\ldots\mu_I}, \quad I+G \text{ odd}. \tag{13b}$$

The algebra (1a) may easily be verified once one has established the relation

$$s(S^I_{G-1})^{\mu_1\ldots\mu_I} + \bar{s}(S^I_{G+1})^{\mu_1\ldots\mu_I} = \partial_\nu(S^{I+1}_G)^{\nu\mu_1\ldots\mu_I}. \tag{14}$$

The most general local action which conserves ghost number and parity and is invariant under this BRS algebra (3, 6, 12, 13) is—at least in the quadratic approximation*—specified by a scalar Lagrangian $\mathscr{L}_{\text{inv}}$ composed out of the metric (for simplicity we consider pure gravity), and a s-$\bar{s}$-invariant part of the form $s\bar{s}X$, which is the only way the Lagrangian can depend on ghosts [3]

$$\mathscr{L} = \mathscr{L}_{\text{inv}}(g) + s\bar{s}X. \tag{15}$$

$\mathscr{L}_{\text{inv}}$ is required to contain two derivatives at most and therefore consists of Einstein's action and a kinetic energy and a potential for $\sqrt{-g}$ [1] (recall that the coordinate transformations are restricted).

$$\mathscr{L}_{\text{inv}} = \tfrac{1}{2}\sqrt{-g}R + k(\sqrt{-g})g^{\mu\nu}\partial_\mu\sqrt{-g}\,\partial_\nu\sqrt{-g} - V(\sqrt{-g}). \tag{15}$$

The last 2 terms are responsible for the fact that for $k>0$ the metric contains a massive physical particle, the dilaton, in addition to the two helicity states of the graviton. The gauge fixing and the ghost Lagrangian enter the action via $s\bar{s}X$ (15) where X is a Lorentz-scalar with vanishing ghost number. X can be suitably chosen such that the propagators of all fields become (algebraically) well-defined. We require X to contain no derivatives in order to avoid higher derivatives and to keep the ghosts B^I_G ($I+G$ even) auxiliary. If in addition X consists only of the lowest powers of the fields it is given by

$$X = -\alpha\eta^{\mu\nu}g_{\mu\nu} + \sum_{\substack{0\leqq G\leqq I-1\\ I+G\,\text{odd}}} \alpha^I_G(A^I_{-G}|A^I_G) \tag{17}$$

with real or imaginary parameters α, α^G_I according to

$$\alpha^* = -\alpha, \quad (\alpha^I_G)^* = (-)^{G+1}\alpha^I_G, \tag{18}$$

$\eta_{\mu\nu}$ is the Lorentz metric diag$(1,-1,-1,-1)$ and the scalar product $(|)$ is introduced as a shorthand for

$$(A^I_{-G}|A^I_G) \equiv \frac{1}{I!}(A^I_{-G})^{\mu_1\cdots\mu_I}(A^I_G)^{\nu_1\cdots\nu_I}\eta_{\mu_1\nu_1}\cdots\eta_{\mu_I\nu_I}. \tag{19}$$

X (17) does not contain terms $(B^I_{-G}|B^I_G)$, $I+G$ even, because they are annihilated by s and would not contribute to $\mathscr{L}$(15).

Because of the reality properties (1c, 10, 18) the action is real and the S-matrix (pseudo-) unitary. Changing the parameters α and α^I_G does not influence physical amplitudes $\langle\phi|\psi\rangle$. By definition [4] they involve only states satisfying

$$s|\phi\rangle = 0. \tag{20}$$

So the amplitudes are invariant

$$\delta\langle\phi|\chi\rangle = \langle\phi|s\bar{s}\delta X|\chi\rangle = \langle s\phi|\bar{s}\delta X|\chi\rangle = 0 \tag{21}$$

as long as δX perturbes analytically.

To exhibit the particle content and to establish positive norm of the physical states it is sufficient to

* The general case is under study

investigate the linearized theory. We expand $\mathscr{L}(15,16,17)$ to second order in

$$h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu} \tag{22}$$

and the ghosts A^I_G, B^I_G. Ordered according to ghost number one has

$$\mathscr{L} = \mathscr{L}_0 + \mathscr{L}_1 + \mathscr{L}_2 + \mathscr{L}_3, \tag{23}$$

$$\begin{aligned}\mathscr{L}_0 = {} & \tfrac{1}{8}(\partial^\mu h^{\rho\sigma}\partial_\mu h_{\rho\sigma} - \partial^\mu h^\lambda_\lambda \partial_\mu h^\rho_\rho - 2\partial_\rho h^{\rho\mu}(\partial^\lambda h_{\lambda\mu} - \partial_\mu h^\lambda_\lambda)) \\ & + \frac{k}{4}(\partial_\mu h^\rho_\rho \partial^\mu h^\lambda_\lambda - M_D^2 (h^\lambda_\lambda)^2) + 2\alpha\partial_\mu\partial^\lambda h_{\lambda\nu}(B^2_0)^{\mu\nu} \\ & + \alpha^2_1(B^2_0|B^2_0 + \partial A^3_0) + 2\alpha^3_0(B^4_0|dA^3_0) \\ & + \alpha^4_1(B^4_0|B^4_0),\end{aligned} \tag{24}$$

$$\begin{aligned}\mathscr{L}_1 = {} & -2\alpha\partial^\mu(\partial A^2_{-1})^\nu\partial_\mu(\partial A^2_1)_\nu + \alpha^2_1(dA^2_{-1}|B^3_1) \\ & + \alpha^2_1(B^3_{-1}|dA^2_1) + (2\alpha^3_0 - \alpha^3_2)(B^3_{-1}|B^3_1) \\ & - \alpha^3_2(B^3_{-1}|\partial A^4_1) + 2\alpha^3_0(\partial A^4_{-1}|B^3_1),\end{aligned} \tag{25}$$

$$\begin{aligned}\mathscr{L}_2 = {} & \alpha^2_1(\partial A^3_{-2}|\partial A^3_2) + \alpha^3_2(B^4_{-2}|dA^3_2) + \alpha^3_2(dA^3_{-2}|B^4_2) \\ & + (\alpha^4_3 - \alpha^4_1)(B^4_{-2}|B^4_2),\end{aligned} \tag{26}$$

$$\mathscr{L}_3 = -\alpha^3_2(\partial A^4_{-3}|\partial A^4_3). \tag{27}$$

d denotes the exterior derivative, i.e.

$$(B^{I+1}|dA^I) = \frac{1}{I!}(B^{I+1})^{\mu_1\cdots\mu_{I+1}}\partial_{\mu_1}(A^I)_{\mu_2\cdots\mu_{I+1}}, \tag{28}$$

and $(\partial A)^{\mu_2\cdots\mu_I} = \partial_{\mu_1}(A^I)^{\mu_1\cdots\mu_I}$. The mass M_D of the dilaton $\sqrt{k}h^\lambda_\lambda$ is given by

$$M_D^2 = \frac{1}{2k}\left(\frac{\partial}{\partial\sqrt{-g}}\right)^2 V|_{\sqrt{-g}=1}. \tag{29}$$

(We have chosen $\partial V/\partial\sqrt{-g}|_{\sqrt{-g}=1} = 0$ without loss of generality to make the expansion (22) consistent.) Note that only the curl*

$$\partial_\mu C_\nu - \partial_\nu C_\mu, \quad C_\mu = \partial^\lambda h_{\lambda\mu} - \tfrac{1}{2}\partial_\mu h^\lambda_\lambda, \tag{30}$$

of the familiar harmonic gauge term C_μ couples to the auxiliary field $B^{\mu\nu} = (B^2_0)^{\mu\nu}$. Therefore the gauge fixing leads to higher derivatives of the metric. In addition the kinetic energy of the ghosts $A^2_{\pm 1}$ contains higher derivatives. Usually higher derivatives are unacceptable because they lead to negative norm states. In our case, however, these negative norms are harmless as they do not occur for physical states (20). Actually the higher derivatives are essential to yield an odd number, namely 3, of physical degrees of freedom, as we shall see shortly.

If a Lagrangian contains higher derivatives $\mathscr{L}(q, \dot{q}, \ddot{q})$ one introduces [6] new coordinates $\hat{q}$ via the equation $\dot{q} = \hat{q}$ which can be enforced by a Lagrange multiplier. Then the Lagrangian contains first order derivatives only with constraints among coordinates and canonically conjugate momenta. Working out the Dirac-brackets [7] one obtains two sets of canonical phase space coordinates $q, p, \hat{q}, \hat{p}$ where

$$\mathscr{H}(q, p, \hat{q}, \hat{p}) = p\hat{q} + \hat{p}\dot{\hat{q}} - \mathscr{L}(q, \hat{q}, \dot{\hat{q}}), \tag{31}$$

$$\hat{p} = \frac{\partial\mathscr{L}(q, \hat{q}, \dot{\hat{q}})}{\partial\dot{\hat{q}}}. \tag{32}$$

If the Lagrangian has a continuous symmetry

$$s\mathscr{L}(\phi, \partial_\mu\phi, \partial_\mu\partial_\nu\phi) = \partial_\mu \Lambda^\mu, \tag{33}$$

the conserved Noether current is given by

$$\begin{aligned} j^\mu = {} & (s\phi)\left(\frac{\partial\mathscr{L}}{\partial(\partial_\mu\phi)} - \partial_\nu\frac{\partial\mathscr{L}}{\partial(\partial_\mu\partial_\nu\phi)}\right) \\ & + (s\partial_\nu\phi)\frac{\partial\mathscr{L}}{\partial(\partial_\mu\partial_\nu\phi)} - \Lambda^\mu \end{aligned} \tag{34}$$

(which reduces to the better known current if $\partial\mathscr{L}/\partial(\partial\partial\phi) = 0$). The Lagrangian (23–27) yields a constrained phase space where all constraints are second class (confirming that the gauge fixing is complete). Quantization proceeds by imposing the commutation relations corresponding to the Dirac brackets. The BRS-transformation is then generated by the charge

$$\begin{aligned} S = {} & \int d^3x j^0 = \int d^3x \tfrac{1}{2}\dot{h}_{\mu\nu}\partial^\mu\partial A^\nu \\ & + \tfrac{1}{4}(\partial\dot{A}^\mu + \partial^\mu\partial A^0)(\partial_\mu h^\lambda_\lambda - 2\partial^\lambda h_{\lambda\mu}) \\ & - \tfrac{1}{2}h^{\mu\nu}\partial_\mu\partial\dot{A}_\nu + \tfrac{1}{2}h^0_\mu\Box\partial A^\mu - \tfrac{1}{4}h^\lambda_\lambda\Box\partial A^0 \\ & - 2\alpha\partial B^\mu\partial_\mu\partial A^0 + \alpha(B^{\mu\nu}\partial^\lambda - \partial^\lambda B^{\mu\nu}) \\ & \cdot(\eta_{\mu\lambda}\partial\dot{A}_\nu + \eta_{\mu 0}\partial_\lambda\partial A_\nu) + \alpha((\partial A^3_2)^{\mu\nu}\partial_\mu(\partial\dot{A}^2_{-1})_\nu \\ & + (\partial A^3_2)^{0\mu}\Box(\partial A^2_{-1})_\mu - \partial^\lambda(\partial A^3_2)^{0\mu}\partial_\lambda(\partial A^2_{-1})_\mu) \\ & + \frac{\alpha^2_1}{2!}(B^3_1)_{0ij}B^{ij} + \frac{\alpha^2_1}{2!}(B^3_{-1})_{0ij}(\partial A^3_2)^{ij} \\ & - \frac{\alpha^2_1}{2!}(\partial A^3_{-2})^{ij}(\partial A^4_3)_{0ij} + \frac{\alpha^3_2}{3!}(B^4_2)_{0ijk}(B^3_{-1})^{ijk} \\ & + \frac{2\alpha^3_0}{3!}(B^4_0)_{0ijk}(B^3_1)^{ijk} - \frac{\alpha^3_2}{3!}(B^4_{-2})_{0ijk}(\partial A^4_3)^{ijk}. \end{aligned} \tag{35}$$

An analogous formula holds for $\bar{S}$.

The ghosts compensate for the unphysical degrees of freedom which propagate after gauge fixing. The effective number of degrees of freedom is therefore obtained by counting negative (positive) all pairs of anticommuting (commuting) phase space variables [8]. Disregarding higher derivatives one counts 10 from the metric and 12 from commuting ghosts minus 16 from anticommuting ghosts resulting in 6. However 3 of the 10 components of the metric and the 2×3 components $(A^2_{\pm 1})^{0i}i = 1,2,3$ of the anticommuting ghosts $A^2_{\pm 1}$ enter the Lagrangian with second time derivatives. In toto there are 3 degrees of freedom in agreement with the analysis of the classical theory [1]. This simple counting argument is confirmed by the mode expansion of the asymptotic fields in terms of 47 creation and annihilation operators.

* Subsequently this curl has been used for gauge fixing also in general relativity [5]

To show that physical states have positive norm we group the creation operators into BRS-multiplets [4]. There are 11 multiplets consisting of 4 creation operators $(a^{\dagger}, sa^{\dagger}, \bar{s}a^{\dagger}, s\bar{s}a^{\dagger})$ each. The BRS-singlets $s\bar{s}a^{\dagger}$ generate zero norm contributions to the physical states and drop out if one defines physical states modulo states of the form $s|\chi\rangle + \bar{s}|\psi\rangle$. There remain 3 BRS singlets: the 2 transverse graviton creation operators and one massive spin 0 creation operator. All 3 generate positive norm states provided the parameter k in the Lagrangian (16) is positive. This establishes that physical states have positive norm.

The propagators of the metric

$$\begin{aligned}\Delta_{\alpha\beta,\mu\nu} &= \frac{2}{\Box}(\eta_{\alpha\mu}\eta_{\beta\nu} + \eta_{\alpha\nu}\eta_{\beta\mu} - \eta_{\alpha\beta}\eta_{\mu\nu})\\ &+ \frac{4}{\Box^2}(\partial_\alpha\partial_\beta\eta_{\mu\nu} + \partial_\mu\partial_\nu\eta_{\alpha\beta})\\ &- \frac{2}{\Box^2}(\partial_\alpha\partial_\mu\eta_{\beta\nu} + \partial_\alpha\partial_\nu\eta_{\beta\mu} + \partial_\beta\partial_\mu\eta_{\alpha\nu} + \partial_\beta\partial_\nu\eta_{\alpha\mu})\\ &- \frac{8}{\Box^3}\partial_\alpha\partial_\beta\partial_\mu\partial_\nu + \frac{\alpha_1^2}{2(\alpha)^2}\left(\frac{1}{\Box^3}(\partial_\alpha\partial_\mu\eta_{\beta\nu}\right.\\ &+ \partial_\alpha\partial_\nu\eta_{\beta\mu} + \partial_\beta\partial_\mu\eta_{\alpha\nu} + \partial_\beta\partial_\nu\eta_{\alpha\mu})\\ &\left.- \frac{4}{\Box^4}\partial_\alpha\partial_\beta\partial_\mu\partial_\nu\right) + \frac{2}{k}\frac{\partial_\alpha\partial_\beta\partial_\mu\partial_\nu}{\Box^2(\Box + M_D^2)} \end{aligned} \tag{36}$$

and of the ghosts $A^2_{\pm 1}$

$$\begin{aligned}\Delta_{\pm 1}^{\alpha\beta,\mu\nu} &= \left(\frac{1}{4\alpha\Box^3} - \frac{\alpha_2^3 - 2\alpha_0^3}{2(\alpha_1^2)^2\Box^2}\right)\\ &\cdot(\partial^\alpha\partial^\mu\eta^{\beta\nu} - \partial^\alpha\partial^\nu\eta^{\beta\mu} - \partial^\beta\partial^\mu\eta^{\alpha\nu} + \partial^\beta\partial^\nu\eta^{\alpha\mu})\\ &+ \frac{\alpha_2^3 - 2\alpha_0^3}{2(\alpha_1^2)^2\Box}(\eta^{\alpha\mu}\eta^{\beta\nu} - \eta^{\alpha\nu}\eta^{\beta\mu}) \end{aligned} \tag{37}$$

have infrared divergent contributions, i.e. are not the Fourier transform of a well defined time-ordered vacuum expectation value in configuration space. These divergences occur in the gauge sector and for commuting and anticommuting fields. So there is a hope that they cancel in the loop amplitudes.

In this note we have described the quantization of restricted gravity, i.e. established tree-level unitarity of the S-matrix and positivity of the norm of physical states. Classically this theory reproduces Einstein's equations apart from the problematic relation between energy density of the vacuum and curvature of the ground state [1]. The gauge fixing introduces higher derivatives into the action, but does not spoil unitarity. The metric automatically contains a massive dilaton, which coincides with the trace of the metric up to a BRS-variation of some antighost creation operator:

$$\begin{aligned}h_\lambda^\lambda(x) = \int d^4k\,\theta(k_0)e^{ikx}(&\delta(k^2 - M_D^2)h_D^\dagger(k)\\ &+ \delta(k^2)[S, a_{-1}^\dagger(k)]_+) + \text{h.c.}\end{aligned} \tag{38}$$

This makes restricted gravity an interesting alternative to Einstein's theory.

We are indebted to Wilfried Buchmüller for numerous and helpful discussions and to Friedemann Brandt for communicating his result [3] prior to publication.

Note added. After this paper was submitted we became aware of related work by J.J. van der Bij, H. van Dam, and Y.J. Ng [9]. These authors also investigate gravity with restricted coordinate invariance. They use, however, noncovariant gauge fixing. One of us (M.K.) is grateful to J.J. van der Bij for valuable discussion.

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