ANALYSIS OF MULTIJET FINAL STATES IN e^+e^- ANNIHILATION

TASSO Collaboration

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Data accumulated by the TASSO detector across the whole range of energies spanned at PETRA, $12 \leqslant \sqrt{s} \leqslant 46.8$ GeV, have been analysed in terms of cluster algorithms. Using parameters optimised at 35 GeV CM energy, three perturbative QCD+fragmentation models were compared with the data. The $O(\alpha_s^2)$ model gives too few 4,5- cluster events, implying that higher order QCD contributions are required to describe the data. The parton cascade model, incorporating many orders in perturbation theory, gives a better description of the rates of $\geqslant 4$ clusters, but shows a lack of hard gluon emission by giving too few 3-, and too many 2-cluster events. When hard gluon emission is taken into account, by the cascade model incorporating the $O(\alpha_s)$ matrix element, all cluster rates are reproduced well. All the models describe the trend of the evolution of the cluster rates between $\langle\sqrt{s}\rangle = 14$ and 43.8 GeV. We find that the rate of 3-jet events seen in the data decreases as s increases in a manner consistent with the Q^2 dependence of α_s as predicted by QCD.

1. Introduction

Perturbative QCD calculations, including four parton final states, complete to second order in α_s have been available for some years [1,2]. These matrix elements have been incorporated, together with models for the fragmentation of the partons into observable final state hadrons, into Monte Carlo computer programs, some of which have been very successful in describing many features of hadronic data from e^+e^- annihilation experiments [3]. However, it had been reported [4] that such $O(\alpha_s^2)$ models were unable to account for the rate of 4-jet events observed in the data, the theoretical predictions being much too low.

The calculation of third, let alone higher order QCD contributions in perturbation theory, is very complex and has not yet been achieved. An alternative perturbative treatment is provided by the leading logarithm approximation (LLA). In this approach the quarks produced in $e^+e^- \to q\bar{q}$ may be well of mass-shell and are allowed to radiate gluons, which may themselves branch into parton pairs. A parton shower or cascade thereby develops, the probability for each

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branching being derived from the Altarelli–Parisi equations [5], and the total cross section for the shower being assumed to be proportional to the product of independent probabilities, one for each branching. The LLA model thereby incorporates many orders in α_s. The latter assumption is a good approximation in the case in which the momentum-transfer-squared, Q^2, at each branching is much less than that at the preceeding branching in the cascade. This approximation is poor when a hard parton is emitted at large angle relative to the parent's trajectory. A decreasing Q^2 at successive parton branchings is built into the formalism, so that the most energetic wide-angle parton radiated by the q or $\bar{q}$ is usually the first gluon emitted. The LLA may therefore be expected to estimate poorly the cross section for partonic states containing a hard gluon, namely "hard 3-jet events ".

In the only detailed study of multi-jet events published to date [6], it was confirmed that an $O(\alpha_s^2)$ QCD model [7] produces too few $\geqslant$4-jet-like events, whereas a LLA model [8] gives a satisfactory description of these rates. This suggests that third (and higher) order terms in perturbative QCD already make a significant contribution to the 4-jet cross section at PETRA energies. However, the LLA model was not able to account for the observed rate of hard 3-jet events, presumably because of the approximations discussed above.

A recent development is the incorporation of the $O(\alpha_s)$ matrix element into the parton cascade formalism in a theoretically satisfying way [9]. All branchings in the cascade are generated according to the usual LLA formalism, though the start-up of the algorithm is such that the rate of hard wide-angle gluon emission is actually *overestimated*. The *first* $q \to q+g$ (or $\bar{q} \to \bar{q}+g$) splitting is then accepted with a probability

$$P = d^2\sigma_{\text{matrix}} / d^2\sigma_{\text{shower}} ,$$

where $d^2\sigma_{\text{matrix}}/dx_1dx_2$, $d^2\sigma_{\text{shower}}/dx_1/dx_2$ are the $O(\alpha_s)$ and LLA cross sections respectively for a 3-parton final state. This ratio lies in the range $0 \leqslant P \leqslant 1$ for all x_1, x_2 (see ref. [9] for full details), and is expected to compensate for the overestimation of the LLA in the hard 3-jet phase space region.

In this paper we shall analyse our hadronic data in terms of cluster multiplicities and compare with the three most successful perturbative QCD+fragmentation models:

(i) The second order perturbative QCD calculations of Gutbrod, Kramer and Schierholz (GKS) [2] incorporated into the Lund Monte Carlo, version 6.3 [10]. We refer to this as the "$O(\alpha_s^2)$ model".

(ii) The LLA cascade model [8], which includes soft gluon interference effects, incorporated into the Monte Carlo program BIGWIG version 4.2. We refer to this as the "LLA model".

(iii) The LLA cascade model (with soft gluon interference) containing the $O(\alpha_s)$ matrix element factor [9], also incorporated into version 6.3 of the Lund Monte Carlo. We refer to this as the "LLA+$O(\alpha_s)$ model".

In cases (i), (iii) hadronization according to the Lund string model [11] is employed, whereas in (ii) hadronisation is via the formation and decay of colourless clusters.

2. Event selection

The data were taken with the TASSO detector at PETRA at centre of mass energies in the range $12.0 \leqslant W \leqslant 46.8$ GeV. Details of the detector may be found in ref. [12]. The bulk of the data is conveniently divided into groups at mean $W = 14.0, 22.0, 35.0$ and 43.8 GeV. The selection of multihadronic final states from e^+e^- annihilation was based upon the information on charged particle momenta measured in the central detector. The selection criteria are described in ref. [13]. In addition, for the multicluster analysis of sections 5–7 three further cuts were made: (a) events were removed for which the sum of the particle momenta $\Sigma|\boldsymbol{p}|$ exceeded $2W$; (b) the angle θ_s between the sphericity axis and the beam direction was required to satisfy $|\cos\theta_s| < 0.85$; (c) the angle θ_N between the normal to the event plane and the beam direction had to satisfy $|\cos\theta_N| > 0.1$ in order to reject badly reconstructed events and events with a hard photon radiated from the e^+ or e^- in the initial state. The numbers of hadronic events before and after these cuts are shown in table 2.

3. QCD fragmentation model parameter tuning at 35 GeV CM energy

This is described in ref. [14] and we give only the main features here. Distributions of sphericity, particle transverse momentum out of the event plane, charged particle multiplicity and momentum were produced for the TASSO data. Values of each of the most important parameters in each model were used to define a tuning grid for that model. Monte Carlo events were generated for each point in the grid and put through the TASSO detector simulation program to yield the same distributions. For every bin in each distribution the χ^2 between data and MC was expressed as a quadratic function of the tuning parameters. The sum of χ^2's over all bins was then minimised to yield the optimised parameters [14]. Note that the models were not tuned directly to the jet rates in the data.

4. The cluster algorithm

There are many procedures and algorithms available for reconstructing jets from hadronic events (see ref. [15] for a comprehensive discussion). In most cases the number of jets required must first be specified, then the event division proceeds by optimising a chosen event variable, e.g. minimising transverse, or maximising longitudinal, momentum (or various powers thereof) relative to jet axes to be determined. Our Monte Carlo studies have shown that momentum-based multi-jet reconstruction algorithms can be sensitive to track reconstruction errors and biases due to loss of particles in the detector, especially where only charged particles are used, such that the vector momentum sum of the particles in an event is typically very far from zero. With such algorithms fair comparison between data and MC thus relies heavily upon a very faithful simulation of particles in the central detector.

An appealing jet-finding algorithm has been used by the JADE collaboration in a similar analysis of their hadronic data [6]. We used an algorithm which operates under the same principles:

The invariant mass-squared m_{ij}^2 is calculated for all pairs of charged particles i, j in an event according to the formula:

$$m_{ij}^2 = 2E_iE_j(1-\cos\theta_{ij}) ,$$

where all particles are assumed to have the charged pion mass. The pair with the lowest m^2 are combined into a "pseudoparticle" by adding their momentum 4-vectors. The procedure is repeated, the pair with the lowest m^2 being combined each time, until all remaining pseudoparticle pairs have invariant masses which satisfy:

$$m_{ij}^2 > m_{ij}^{\mathrm{cut}\,2} = y_c E^2$$

where y_c is a jet resolution parameter. For our analysis using only charged particles, $E = E_{\mathrm{vis}}$ was used, E_{vis} being the visible energy of the event. The resulting number of pseudoparticles is called the cluster multiplicity of the event. Note that there is no a priori specification of the number of clusters to be found, and that every charged particle belongs to a single cluster.

In Monte Carlo studies using the $O(\alpha_s^2)$ model, the above expression for m^2 was found to provide the best agreement between the cluster multiplicity reconstructed from final state charged particles and the initial parton multiplicity for the same values of y_c and $y_{\min}$, where $y_{\min}$ is the QCD (massless) parton resolution parameter used in the matrix element calculations [16]; $y_{\min} = 0.02$ was used throughout this analysis.

The algorithm was found to be robust with respect to the details of the tracking simulation. The invariant mass-squared is, of course, a Lorentz-invariant quantity, and should be unaffected by the boosting of the hadronic rest frame by initial state photon bremsstrahlung, or by "apparent" boosting, which is the net effect of using only charged particles and of particle losses due to detector acceptance and inefficiency, such that the measured vector momentum sum for the event is large.

5. Cluster rates in the data and comparison with QCD models at models at $\sqrt{s} = 35$ GeV

All the QCD model Monte Carlo events, with initial state radiative effects included, were put through the TASSO detector simulation program and underwent the same hadronic selection and analysis cuts as applied to the data.

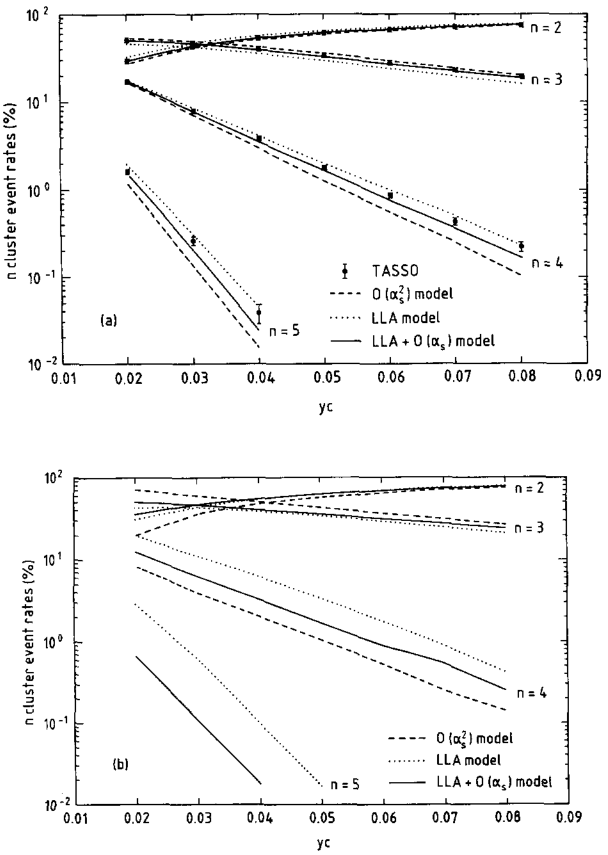

Fig. 1. (a) The n-cluster event rates at 35 GeV CM energy for the data and QCD models as a function of the jet resolution parameter y_c, (b) the n-parton-cluster event rates at 35 GeV CM energy for the QCD models as a function of y_c. For the models the curves consist of straight lines joining points generated at the same values of y_c as for the data.

We show in fig. 1a the rates of 2, 3, 4, ≥5-cluster events for the data and QCD models, as reproduced by the algorithm described above, at 35 GeV CM as a function of the jet resolution parameter y_c; the errors are statistical only. Taking $E=35$ GeV, the cluster-pair mass cut-off range spanned by $0.02 \leqslant y_c \leqslant 0.08$ is $4.9 \leqslant m_{ij}^{cut} \leqslant 9.9$ GeV/c^2. The $O(\alpha_s^2)$ model gives a satisfactory description of the 2- and 3-cluster rates, though at low y_c it slightly underestimates the rate of 2-, and overestimates the rate of 3-, clusters seen in the data. The LLA model gives too many 2-, and too few 3-, clusters for all y_c. The LLA+$O(\alpha_s)$, model matches the 2- and 3- cluster rates of the data extremely well; it gives a comparable or better description than the $O(\alpha_s^2)$ model across the whole y_c range. This model also describes the rates of 4- and 5- clusters well, as does the LLA model, whilst the $O\alpha_s^2$) model shows a serious deficiency.

Such is the large number of events in the data sample that for many of the data points the statistical errors are smaller than the size of the symbols used in fig. 1a; together with the logarithmic scale this masks discrepancies between the data and MC at the level of a few per cent for the 2- and 3- cluster cases. As an illustration, we show in table 1 the cluster rates at $y_c=0.04$, which corresponds to a cluster-pair mass cut-off value of 7 GeV/c^2 taking $E=35$ GeV. The good numerical agreement between the LLA+$O(\alpha_s)$ model and the data for all cluster rates is remarkable.

The results have been checked using different clustering algorithms [7,17]. Whilst the absolute rates of clusters differ between different algorithms, so that it is impossible to compare results numerically, it was

Table 1
The cluster rates (%) observed in the data and for the three QCD models at 35 GeV CM energy and $y_c=0.04$.

	Data	Model		
		LLA+$O(\alpha_s)$	LLA	$O(\alpha_s^2)$
2-cluster	54.7 ±0.5	55.6 ±0.4	58.6 ±0.4	53.1 ±0.4
3-cluster	41.1 ±0.4	40.6 ±0.3	36.9 ±0.3	43.6 ±0.3
≥4-cluster	4.00 ±0.12	3.67 ±0.09	4.34 ±0.10	3.11 ±0.08
>4-cluster / 3-cluster	0.097±0.003	0.090±0.002	0.118±0.003	0.071±0.002

found that the $O(\alpha_s^2)$ model is always seriously deficient in the rates of 4- and 5- clusters, whilst the LLA and LLA + $O(\alpha_s)$ models are in good agreement with the data. Furthermore, the LLA + $O(\alpha_s)$ model gives comparable or better agreement with the 2- and 3-cluster rates than does the $O(\alpha_s^2)$ model, which tends to underestimate and overestimate respectively these rates, whereas the LLA model overestimates and underestimates respectively.

The results were also found to be insensitive to reasonable changes of the model parameters from the values given in ref. [14]. The parameters were varied so as to preserve a reasonable overall description of the global features of the data: one parameter was fixed to a value different from the tuned one and a new fit was performed allowing the remaining parameters to vary as described in section 3. In particular for the LLA model decreasing the QCD scale parameter Λ_{LL} gave lower rates of $\geqslant 4$ clusters, in better agreement with the data, but also a reduced 3-cluster rate falling even further beneath the data than previously. Similarly for the $O(\alpha_s^2)$ model increasing $\Lambda_{\overline{MS}}$ from 0.62 to 1.1 GeV, a change in α_s from 0.17 to 0.20, gave 4, $\geqslant 5$- cluster rates in much better agreement with the data, at the expense of a seriously underestimated 2-cluster rate and over-estimated 3-cluster rate across the whole range of y_c. We note that the $O(\alpha_s^2)$ model employs the second order perturbative calculations of Gutbrod, Kramer and Schierholz [2], where certain second order correction terms are known to have been neglected [16]. It was found in ref. [6] that the effect of including the more detailed second order matrix elements of Gottschalk and Shatz (see references in ref. [6]) resulted in an even greater discrepancy in the 4-cluster rate between the $O(\alpha_s^2)$ model and the data. This detail does not affect our conclusion and will not be considered further here.

By definition, the $O(\alpha_s^2)$ model can produce only 2,3 or 4 partons, whereas around 6 partons are typically produced by the two cascade models, though as many as 10 is not uncommon. For the latter it is hence difficult to relate partons to the hadronic clusters directly, and also to compare parton-level results with the $O(\alpha_s^2)$ model. Therefore, in order to check that the differences between the models shown in fig. 1a arise from the different perturbative QCD treatments, and not from fragmentation effects, the cluster algorithm was applied directly to the partons before hadronization. The resulting "parton-cluster" rates are shown as a function of y_c, for $W = 35$ GeV, in fig. 1b. Note that the cluster-pair mass criterion used was

$$m_{ij}^2 > y_c E_{rad}^2 ,$$

where E_{rad} is the hadronic CM energy after initial state photon radiation from the e^+ or e^-.

The differences between the models already exist at the parton level, though are considerably reduced by hadronisation, and we conclude that the observed hadronic cluster rates essentially reflect the underlying QCD processes.

6. Energy evolution of the cluster rates

We present the cluster rates for data and models at CM energies in the PETRA range and also show the LLA + $O(\alpha_s)$ model extrapolation for W up to 200 GeV. The data were therefore corrected for acceptance, neutral particles detector effects, initial state radiation and the cuts described in section 2 using Monte Carlo simulations; the correction procedure is described in detail in ref. [14]. To make a comparison at different CM energies the mass cut-off m_{ij}^{cut} was fixed at 7 GeV/c^2, which corresponds to $y_c = 0.04$ at $E = W = 35$ GeV. The corresponding values of y_c at $E = 14$, 22 and 43.8 GeV are 0.25, 0.10 and 0.025, respectively. The energy dependence of the cluster rates is shown in fig. 2 and table 2. It is notable that

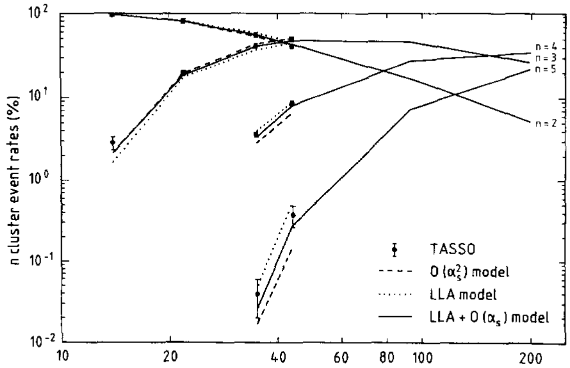

Fig. 2. The n-cluster event rates as a function of CM energy for the data and QCD models. The cluster-pair mass cut-off m_{ij}^{cut} was fixed at 7 GeV/c^2. The curves for the QCD models consist of straight lines joining points generated at the same values of W as for the data; the LLA + $O(\alpha_s)$ model is also shown for $W = 93,200$ GeV.

Table 2
The cluster rates (%) observed in the data, corrected for acceptance, neutral particles, detector and radiative effects, at the four mean CM energy values used in the study, for fixed cluster-pair mass cut-off $m_{ij}^{cut} = 7$ GeV/c^2.

⟨W⟩ (GeV)	Number of events	Number used	2-cluster	3-cluster	4-cluster	⩾5-cluster
14.0	2999	2757	97.1±2.1	2.9±0.5	–	–
22.0	1914	1730	80.3±2.3	19.7±1.2	–	–
35.0	31176	27178	55.2±0.7	41.1±0.6	3.60±0.16	0.04±0.02
43.8	6380	5261	41.0±1.2	49.6±1.1	8.7 ±0.5	0.38±0.11

using parameters extracted from a fit to data at $W=35$ GeV, all models reproduce the trend of the data across the whose energy range spanned. The LLA+O(α_s) model is in good numerical agreement with the data for all cluster rates, whilst the O(α_s^2) model shows a similar deficiency of ⩾4 clusters at 44 GeV and the LLA model underestimates the 3-cluster rate at all energies.

In addition in fig. 2 the cluster rates for the LLA+O)α_s) model are shown at $W=93$ and 200 GeV for u, d, s, c, b production only. At the higher energy events containing ⩾4 jets dominate the cross section and will form a background to multijet final states arising from the decay of heavy particles via quarks, for example in W^+W^- events [18].

7. The Q^2 dependence of α_s

In the language of exact matrix elements the 3-jet cross section in e^+e^- annihilation is proportional to α_s. One way to investigate the Q^2 dependence of α_s is hence to measure the 3-jet rate, R_3, as a function of Q^2. We have followed the procedure of ref. [9] and used the cluster algorithm of section 4 to define the jet multiplicity in events. This is only meaningful provided the reconstructed jets reflect the underlying parton structure and are not strongly influenced by fluctuations due to hadronisation.

In general $R_3=R_3(W, y_c)$. If the ratio $R_3(W_1, y_c)/R_3(W_2, y_c)$ is independent of y_c then hadronisation fluctuations are small at CM energies W_1, W_2. We show in fig. 3 the ratios $R_3(14)/R_3(35)$, $R_3(22)/R_3(35)$, $R_3(43.8)/R_3(35)$ as functions of y_c. At $W=14$ GeV the ratio depends strongly on y_c, implying that the fluctuations are large. At $W=22$ GeV the ratio becomes independent of y_c above $y_c \sim 0.06$, whilst there is virtually no dependence on y_c at all at

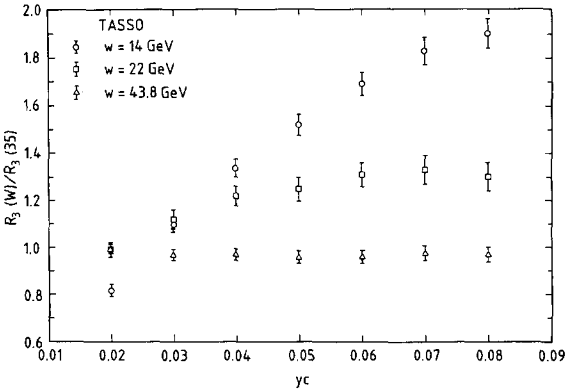

Fig. 3. The ratios $R_3(14)/R_3(35)$, $R_3(22)/R_3(35)$, $R_3(43.8)/R_3(35)$ for the data as a function of y_c.

$W=43.8$ GeV. Therefore, provided y_c is chosen sufficiently large R_3 is insensitive to hadronisation fluctuations for $W>22$ GeV.

We show in fig. 4 and table 3 R_3 for the data, cor-

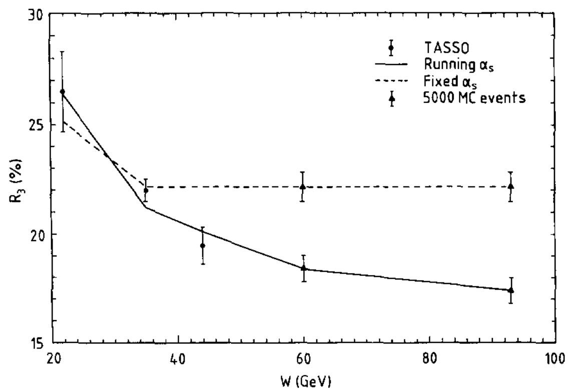

Fig. 4. R_3 defined by $y_c=0.08$ as a function of W for the data and the LLA+O(α_s) model with both a running strong coupling constant and a fixed coupling constant. The error bars at $W=60$, 93 GeV represent the statistical error if R_3 were determined from only 5000 events.

Table 3
The corrected rate of 3-jet events, defined with $y_c=0.08$, for the data.

14.0	41.4±1.7
22.0	26.5±1.8
35.0	22.0±0.5
43.8	19.5±0.8

rected for acceptance, detector and radiative effects, for fixed $y_c=0.08$. Also shown are curves for the LLA+O(α_s) model for the two cases: (i) running α_s with $\Lambda_{LL}=0.26$ GeV, determined from the parameter optimisation at $W=35$ GeV [14]; and (ii) fixed $\alpha_s=0.265$; this value being chosen to be in good agreement with R_3 for the data at $W=35$ GeV. For the data R_3 clearly decreases as W increases: $R_3(22)/R_3(43.8)=1.36\pm0.11$. Note however that a slight decrease is also shown by the LLA+O(α_s) model with fixed α_s, though the data prefer the model with running α_s. The model results are also shown at $W=60$, 93 GeV. In addition, error bars are shown at these higher energies corresponding to the statistical errors on a measurement of R_3 using only 5000 events. Such measurements would demonstrate conclusively whether α_s runs with Q^2.

8. Conclusion

Our analysis of multi-cluster rates has shown that O(α_s^2) QCD cannot reproduce the rates of spherical and 4-jet like events observed in the data. A high value of $\Lambda_{\overline{MS}}$ is required to give sufficient rates of ⩾4 clusters, but such a value causes a severe underestimation of the 2-, and an overestimation of the 3-, cluster rates. For a $\Lambda_{\overline{MS}}$ determined from a best fit to a few global properties of hadronic final states, which are dominated by 2- and 3- jet-like events, this QCD model shows a serious deficiency of ⩾4 clusters. Conversely LLA QCD gives a more satisfactory description of ⩾4-jet rates at the expense of 2, 3-jet rates which are overestimated and underestimated respectively. These results are in agreement with a previous study [6].

We find that the LLA+O(α_s) QCD model is in very good agreement with the data for all cluster rates and conclude the following:

(i) Contributions from higher order, e.g. O(α_s^2), QCD are observable at the highest e^+e^- CM energies analysed to data, and can be expected to be considerable at colliders operating in the 100 GeV range. QCD models accurate only to O(α_s^2) are already inadequate to describe multijet rates, and the discrepancy will increase as the amount of gluon bremsstrahlung increases with energy.

(ii) LLA QCD models incorporating many orders in perturbation theory are, because of the approximations used in the formalism, unable to account for both the 3- and ⩾4-jet rates *simultaneously*. When the ⩾4-jets are well-described there is a deficiency of hard 3-jet events. Whilst both LLA and O(α_s^2) models may give comparable descriptions of many hadronic final state properties at PETRA energies, their predictions will diverge considerably as the CM energy rises [14,20].

(iii) The most satisfactory account of the observed jet rates at CM energies up to 44 GeV is provided by the LLA+O(α_s) model. In the absence of third (and higher order) matrix element calculations, this model provides a sensible basis for future theoretical developments and is a good tool for extrapolation to higher energy scales.

We have also found that the rate of 3-jet events in the data decreases with increasing CM energy, consistent with a running coupling strength α_s as predicted by QCD. The data are in agreement with a Q^2 dependence of the form $\alpha_s \sim 1/\ln(Q^2/\Lambda^2)$, though a fixed α_s cannot be ruled out by the limited statistics and relatively narrow energy range of the PETRA data. Measurements of R_3 at higher energies, in combination with the data presented here and elsewhere [19], should clarify these issues.

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References

[1] R.K. Ellis, D.A. Ross and A.E. Terrano, Nucl. Phys. B 178 (1981) 421;
K. Fabricius, G. Kramer, G. Schierholz and I. Schmitt, Z. Phys. C 11 (1982) 315.

[2] F. Gutbrod, G. Kramer and G. Schierholz, Z. Phys. C 21 (1984) 235.

[3] TASSO Collab., M. Althoff et al., Z. Phys. C 26 (1984) 157; C 29 (1985) 29;
Mark II Collab., A. Petersen et al., Phys. Rev. Lett. 55 (1985) 1954; Phys. Rev. D 37 (1988) 1;
TPC/Two-Gamma Collab., H. Aihara et al., Z. Phys. C 28 (1985) 31;
JADE Collab., W. Bartel et al., Phys Lett. B 157 (1985) 340.

[4] T. Sjöstrand, Z. Phys. C 26 (1984) 93.

[5] G. Altarelli and G. Parisi, Nucl. Phys. B 126 (1977) 298.

[6] JADE Collab., W. Bartel et al., Z. Phys. C 33 (1986) 23.

[7] T. Sjöstrand, Comput. Phys. Commun. 28 (1983) 229.

[8] G. Marchesini and B.R. Webber, Nucl. Phys. B 238 (1984) 1;
B.R. Webber, Nucl. Phys. B 238 (1984) 492.

[9] M. Bengtsson and T. Sjöstrand, Phys. Lett. B 185 (1987) 435.

[10] T. Sjöstrand and M. Bengtsson, Comput. Phys. Commun. 43 (1987) 367.

[11] B. Andersson, G. Gustafson, G. Ingelman and T. Sjöstrand, Phys. Rep 97 (1983) 33.

[12] TASSO Collab., R. Brandelik et al., Phys. Lett. B 83 (1979) 261; Z. Phys. C 4 (1980) 87.

[13] TASSO Collab., M. Althoff et al., Z. Phys. C 22 (1984) 307.

[14] TASSO Collab., in preparation.

[15] S.L. Wu, Phys. Rep. 107 (1984) 59.

[16] G. Kramer and B. Lampe, preprint DESY 86-119 (1986);
F. Gutbrod, G. Kramer, G. Rudolph and G. Schierholz, DESY preprint DESY 87-047 (1987).

[17] F. Barreiro, S. Brandt and M. Gebser, Nucl. Instrum. Methods A 240 (1985) 237.

[18] P. Mättig and M. Dittmar, Z. Phys. C 35 (1987) 221.

[19] S. Bethke, Heidelberg Habilitation Thesis, LBL report LBL 50-208 (1987).

[20] P.N. Burrows and G. Ingelman, Z. Phys. C 34 (1987) 91.