CANONICAL QUANTIZATION OF NONABELIAN GAUGE THEORIES IN TWO DIMENSIONS

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The canonical quantization of vector and chiral nonabelian gauge theories is performed, using the bosonization procedure. The gauge current commutators are computed and a close relation between the vector and chiral interacting gauge theories is shown to exist for a particular value of the regularization dependent parameter *a*.

After the work of Jackiw and Rajaraman [1], where two-dimensional chiral electrodynamics was proved to be a consistent, unitary although non gauge invariant theory, there has been renewed interest in anomalous gauge theories. Such an interest has grown after the proposal [2] of Faddeev and Shatashvili of modifying the canonical quantization by adding new fields in the fundamental representation of the gauge group. If a Wess–Zumino action is chosen for these fields, the resulting theory becomes both equivalent to the chiral one and gauge invariant. Recently it was shown that within the path integral quantization procedure [3], the anomaly is absorbed while the group of gauge transformations appears as a new dynamical field not present at the classical level. The resulting Wess–Zumino action arises naturally and does not need to be introduced in an ad hoc manner. The original chiral theory may be seen as a particular gauge fixing condition, the so called unitary gauge, in this framework [4].

In this letter we analyse the canonical quantization of the vector and chiral gauge theories, using the formalism developed in ref. [5]. We consider theories with a field transforming in the fundamental representation of $U(N)$ and whose actions includes a Wess–Zumino term. For simplicity, let us consider the nonlinear sigma model, with action

$$S(g)=\frac{1}{8\pi}\int d^2x\,\mathrm{Tr}(\partial_\mu g^{-1}\partial^\mu g)-\frac{1}{4\pi}\mathrm{Tr}\left(\int dr\int d^2x\,\epsilon^{\mu\nu}\dot{g}\partial_\mu\dot{g}^{-1}\partial_\nu\dot{g}^{-1}\right), \tag{1}$$

where the second term in eq. (1) is the Wess–Zumino term, that is linear in the time derivatives and can always be rewritten as

$$S_{WZ}=\int d^2x\,\mathrm{Tr}(A(g)\partial_0 g)\ . \tag{2}$$

An important observation made in ref. [5] is that $A(g)$ does not need to be specified. Only its variations are important in the canonical quantization of the theory. More specifically, the tensor

$$F_{ijkl}=\partial A_{ij}/\partial g_{lk}-\partial A_{kl}/\partial g_{ji}\ , \tag{3}$$

is the only relevant quantity for the canonical procedure. This last expression can be easily calculated from variations of S_{WZ}, yielding the result

$$F_{ijkl}=(\partial_1 g^{-1}_{il})g^{-1}_{kj}-g^{-1}_{il}(\partial_1 g^{-1}_{kj})\ . \tag{4}$$

Now, the momenta conjugate to g_{ij} may be calculated from eqs. (1) and (2)

$$\pi_{ij}=\partial_0 g_{ji}^{-1}/4\pi+A_{ji}/4\pi\,. \tag{5}$$

If the hamiltonian is expressed in terms of g_{ij} and π_{ij}, A_{ij} will explicitly appear in this expression. Alternatively, new momenta may be defined

$$\hat{\pi}_{ij}=\pi_{ij}-A_{ji}/4\pi \tag{6}$$

and now the hamiltonian will not depend explicitly on the Wess–Zumino term. It follows from the Poisson brackets of g_{ij} that

$$\{g_{ij}(x),\hat{\pi}_{kl}(y)\}_t=\delta_{ik}\delta_{jl}\delta(x^1-y^1)\,,\quad \{\hat{\pi}_{ij}(x),\hat{\pi}_{kl}(y)\}_t=-F_{jilk}\delta(x^1-y^1)/4\pi\,, \tag{7,8}$$

while the Poisson brackets of $g_{ij}^{-1}(g)$ may be obtained from eq. (7) using the property $\delta(g^{-1}g)=0$ [5]. In principle, eqs. (7),(8) are not restricted to a particular selection of the phase space, and it seems that to apply the formalism explained above, we have to introduce additional constraints. However, this is not necessary since all quantities of physical interest depend on the hermitian operators $(\mathrm{i}\pi^{\mathrm{T}}g)_{ij}$ and $(\mathrm{i}g\pi^{\mathrm{T}})_{ij}$ that generate infinitesimal transformations of g_{ij} and g_{ij}^{-1} tangent to the phase space and then their Poisson brackets with the additional constraints vanish [#1]. Instead of continuing the canonical quantization of the nonlinear model, we will apply this procedure to quantize bosonized vector gauge theories. We will be able to obtain the free case as the limit of vanishing gauge fields of the interacting theory.

The general method of quantization explained above relies on the bosonization procedure [7]. Starting with the fermionic theory, a bosonic theory with the same set of symmetries at the quantum level [8], and the same effective action for the gauge fields may be found while gauging a subgroup of the global $\mathrm{U}(N)\times\mathrm{U}(N)$ classical symmetry of the fermionic free theory [9]. In particular, if we gauge the vector $\mathrm{U}(N)$ symmetry we obtain the following bosonized theory [9]:

$$S=S(g)-\int \mathrm{d}^2x\frac{\mathrm{Tr}(F^{\mu\nu}F_{\mu\nu})}{4}+\frac{1}{4\pi}\int \mathrm{d}^2x\,\mathrm{Tr}(-\mathrm{i}eA_+g\partial_-g^{-1}-\mathrm{i}eA_-g^{-1}\partial_+g-e^2A_+gA_-g^{-1}+e^2A_+A_-)\,, \tag{9}$$

where for a general vector we define $x_\pm=x_0\pm x_1$ and $S(g)$ in the above is given in eq. (1). The equation of motion for A_μ can be easily obtained from here. It reads

$$\mathscr{D}_\mu F^{\mu\nu}+eJ^\nu=0\,, \tag{10}$$

where J_μ is the gauge current given as

$$J_0=(-\mathrm{i}g\partial_-g^{-1}-\mathrm{i}g^{-1}\partial_+g-egA_-g^{-1}-eg^{-1}A_+g+e2A_0)/4\pi\,,$$
$$J_1=(\mathrm{i}g\partial_-g^{-1}-\mathrm{i}g^{-1}\partial_+g+egA_-g^{-1}-eg^{-1}A_+g+e2A_0)/4\pi\,. \tag{11}$$

It follows from eq. (10) that it is covariantly conserved. The momenta conjugate to the fields are easily computed. We find

$$\hat{\pi}_{ij}=[\partial_0 g_{ji}^{-1}+\mathrm{i}e(g^{-1}A_+)_{ji}-\mathrm{i}e(A_-g^{-1})_{ji}]/4\pi\,,\quad \pi_0^a=\omega_1^a=0\,,\quad \pi_1^a=F_{01}^a\,, \tag{12,13,14}$$

and the hamiltonian reads

$$H=\int \mathrm{d}^2x[\tfrac{1}{2}\pi_1^a\pi_1^a-A_0^a(\mathscr{D}_1\pi_1)^a+A_+^a(\mathrm{i}eg\hat{\pi}^{\mathrm{T}}-\mathrm{i}eg\partial_1g^{-1}/4\pi)^a$$
$$+A_-^a(-\mathrm{i}e\hat{\pi}^{\mathrm{T}}g+\mathrm{i}eg^{-1}\partial_1g/4\pi)^a+e^2(A_1^a)^2/2\pi+\mathrm{Tr}(-2\pi(\hat{\pi}^{\mathrm{T}}g\hat{\pi}^{\mathrm{T}}g)+\partial_1g\partial_1g^{-1}/8\pi)+\lambda_1^a\omega_1^a]\,. \tag{15}$$

[#1] For a related and more detailed discussion, for the group $\mathrm{SU}(N)$, see ref. [6].

Following the general Dirac procedure [10], eq. (13) is a set of primary constraints in the theory and their time evolution should weakly vanish, for the consistent quantization of the theory, and λ_1^a are the undetermined Lagrange multipliers. Then, the additional condition

$$\{\pi_0^a, H\}=0\,, \tag{16}$$

must be satisfied. A secondary constraint appears

$$\omega_2^a=(\mathscr{D}_1\pi_1)^a-(\mathrm{i}eg\hat{\pi}^{\mathrm{T}}-\mathrm{i}eg\partial_1 g^{-1}/4\pi)^a-(-\mathrm{i}e\hat{\pi}^{\mathrm{T}}g+\mathrm{i}eg^{-1}\partial_1 g/4\pi)^a=0\,. \tag{17}$$

However, due to the weak cancelation of its Poisson bracket with the hamiltonian, no additional constraint appears. Furthermore, the constraint structure is of first class, that is to say

$$\{\omega_2^a(x), \omega_2^b(y)\}=ef_{abc}\omega_2^c(x)\delta(x^1-y^1)\,, \tag{18}$$

while the other Poisson brackets vanish identically. ω_2^a is the Gauss law constraint, and it must be imposed over physical states in order to obtain a gauge invariant quantum theory. The canonical commutators may then be obtained from the Poisson brackets by the usual substitution, $[A, B]\to\mathrm{i}\{A, B\}$, due to the constraint algebra. The gauge current commutators can be obtained easily while expressing (11) in terms of the conjugate momenta to the fields

$$J_0^a=(-\mathrm{i}g\hat{\pi}^{\mathrm{T}}+\mathrm{i}g\partial_1 g^{-1}/4\pi)^a+(\mathrm{i}\hat{\pi}^{\mathrm{T}}g-\mathrm{i}g^{-1}\partial_1 g/4\pi)^a\,, \tag{19}$$

$$J_1^a=-(-\mathrm{i}g\hat{\pi}^{\mathrm{T}}+\mathrm{i}g\partial_1 g^{-1}/4\pi)^a+(\mathrm{i}\hat{\pi}^{\mathrm{T}}g-\mathrm{i}g^{-1}\partial_1 g/4\pi)^a+eA_1^a/\pi\,. \tag{20}$$

The resulting equal time current algebra is

$$[J_0^a, J_0^b]_t=\mathrm{i}f_{abc}J_0^c(x)\delta(x^1-y^1),\quad [J_0^a(x), J_1^b(y)]_t=\mathrm{i}f_{abc}J_1^c(x)\delta(x^1-y^1)+\mathrm{i}\mathscr{D}_1^{ab}(x^1-y^1)/\pi\,, \tag{21,22}$$

$$[J_1^a(x), J_1^b(y)]_t=\mathrm{i}f_{abc}J_0^c(x)\delta(x^1-y^1)\,, \tag{23}$$

or, defining $J_\pm=(J_0\pm J_1)/2$,

$$[J_\pm^a(x), J_\pm^b(y)]_t=\mathrm{i}f_{abc}J_\pm^c(x)\delta(x^1-y^1)\pm\mathrm{i}\mathscr{D}_1^{ab}\delta(x^1-y^1)/2\pi,\quad [J_+^a(x), J_-^b(y)]=0\,. \tag{24,25}$$

No Schwinger term appears in eq. (21) as is necessary in any consistent theory and the algebra differs from a Kac–Moody algebra only in the appearance of covariant derivatives. One can now easily check that the hamiltonian of the theory can be given as

$$H=\int \mathrm{d}^2x[\pi_1^a\pi_1^a/2-A_0^a\omega_2^a+\pi(J_+^aJ_+^a+J_-^aJ_-^a)+\lambda_1^a\omega_1^a+\lambda_2^a\omega_2^a]\,, \tag{26}$$

that reduces to the Sugawara form in the case of vanishing gauge fields and coupling constant e, and is positive when acting over physical states. Finally, the transformation laws of the gauge currents are obtained

$$[\omega_2^a(x), J_0^b(y)]=\mathrm{i}ef_{abc}J_0^c(x)\delta(x^1-y^1),\quad [\omega_2^a(x), J_1^b(y)]=\mathrm{i}ef_{abc}J_1^c(x)\delta(x^1-y^1)\,. \tag{27}$$

Eqs. (21)–(23) give us information only while acting over physical states, that is to say those cancelled by the first class constraints. However, the Schwinger terms will be preserved in the vacuum expectation value of the current commutators since they are c-numbers. The same Schwinger terms appear in the gauge current algebra of the fermionic theory [11] as expected from the bosonization procedure we applied [9].

The chiral gauge theory is obtained by gauging only the left-handed global classical symmetry of the fermionic free theory. Recently, great effort has been made in understanding its properties [12,13], and the abelian theory has been solved [12,13]. However, the analysis of the nonabelian case is far from being completed. The bosonization procedure has been applied to quantize the theory in the original approach of ref. [1], both in the abelian and nonabelian case [14,15,5]. The canonical quantization of the gauge invariant formulation has been given only in the abelian case [4], and we intend to generalize it to the nonabelian case. The bosonized action is now explicitly gauge invariant, a fact that must be true, due to the invariance of the Haar measures in the bosonized theory. The lagrangian density reads

$$\mathscr{L}=\mathrm{Tr}[-F^{\mu\nu}F_{\mu\nu}/4+ae^2A_\mu A^\mu/8\pi+\partial_\mu g\partial^\mu g^{-1}/8\pi-ieg^{-1}\partial^\mu g(g_{\mu\nu}+\epsilon_{\mu\nu})A^\nu/4\pi+\mathscr{L}_{\mathrm{WZ}}(g)$$
$$+\partial_\mu\tilde{g}\partial^\mu\tilde{g}^{-1}(a-1)/8\pi-ie\epsilon_{\mu\nu}\tilde{g}\partial^\nu\tilde{g}^{-1}A^\mu/4\pi+\mathscr{L}_{\mathrm{WZ}}(\tilde{g})-ie\tilde{g}\partial_\mu\tilde{g}^{-1}A^\mu(a-1)/4\pi]\,, \tag{28}$$

where the first term is the one considered in refs. [15,5], and the second term is the one that comes from the chiral jacobian [16,17] of the bosonized fermions. The second term may be obtained directly from the partition function of the bosonized theory considered in refs. [15,5], while applying the Faddeev–Popov gauge fixing procedure in the same way as in ref. [3]. The arbitrariness in the regularization procedure of the chiral jacobian, leads to the appearance of the dimensionless parameter a [18], and an infinite set of nonequivalent theories is achieved for each value of the coupling constant e.

The canonical momenta of the fields are given by

$$\pi_1^a=F_{01}^a,\quad \pi_0^a=0\,, \tag{29,30}$$

$$\hat{\pi}_{ij}=\partial_0 g_{ji}^{-1}/4\pi-ie(A_-g^{-1})_{ji}/4\pi,\quad \hat{\hat{\pi}}_{ij}=(a-1)\partial_0\tilde{g}_{ji}^{-1}/4\pi+ie\tilde{g}^{-1}(aA_++(a-2)A_-)_{ji}/8\pi\,. \tag{31,32}$$

If $a=1$, eq. (32) does not depend on the time derivatives and must be considered as a primary constraint in the theory. In the following, we will not analyse this case. As in the vector theory, eq. (30) is a set of primary constraints. Now, the hamiltonian of the theory can be given as

$$H=\int d^2x\{\pi_1^a\pi_1^a/2-A_0^a(\mathscr{D}_1\pi_1+eJ_0)^a+\mathrm{Tr}\{-2\pi\hat{\hat{\pi}}^{\mathrm{T}}\tilde{g}\hat{\hat{\pi}}^{\mathrm{T}}\tilde{g}/(a-1)+[(a-1)/8\pi]\partial_1\tilde{g}\partial_1\tilde{g}^{-1}-2\pi(\hat{\pi}^{\mathrm{T}}g\hat{\pi}^{\mathrm{T}}g)$$
$$+\partial_1 g\partial_1 g^{-1}/8\pi+eA_1(\mathrm{i}\hat{\pi}^{\mathrm{T}}g-\mathrm{i}g^{-1}\partial_1 g/4\pi)+eA_1[\mathrm{i}\tilde{g}\hat{\hat{\pi}}^{\mathrm{T}}/(a-1)-\mathrm{i}\tilde{g}\partial_1\tilde{g}^{-1}(a-1)/4\pi]$$
$$+e^2a^2A_1^2/8\pi(a-1)\}+\lambda_1^a\omega_1^a\}\,, \tag{33}$$

where J_μ is the gauge current, that satisfies the same equation of motion, eq. (10), as in the vector theory and is then covariantly conserved. J_μ can be written in terms of the canonical momenta as

$$J_0=(\mathrm{i}\hat{\pi}^{\mathrm{T}}g-\mathrm{i}g^{-1}\partial_1 g/4\pi)+(-\mathrm{i}\tilde{g}\hat{\hat{\pi}}^{\mathrm{T}}+\mathrm{i}\tilde{g}\partial_1\tilde{g}^{-1}/4\pi)\,, \tag{34}$$

$$J_1=(\mathrm{i}\hat{\pi}^{\mathrm{T}}g-\mathrm{i}g^{-1}\partial_1 g/4\pi)-[-\mathrm{i}\tilde{g}\hat{\hat{\pi}}^{\mathrm{T}}/(a-1)+(a-1)\mathrm{i}\tilde{g}\partial_1\tilde{g}^{-1}/4\pi]+ea^2A_1/4\pi(a-1)\,. \tag{35}$$

As in the vector case, for the quantization of the theory the additional constraint

$$\{\pi_0^a, H\}=0\,,$$

or equivalently

$$(\mathscr{D}_1\pi_1+eJ_0)^a=\omega_2^a=0\,, \tag{36}$$

should be satisfied. This is a set of secondary constraints and, following the Dirac procedure, no other constraint appears. The constraint structure is of first class and the constraints satisfy the same Poisson bracket algebra as in the vector theory, eq. (18), for every value of a different from one. In fact, ω_2^a is the Gauss law operator and the first class structure of the constraint is a reflection of the gauge invariance of the theory at the quantum level. The equal time commutators are then obtained through a multiplication of the Poisson brackets by an imaginary constant. In particular, the equal time gauge current algebra is

$$[J_0^a(x), J_0^b(y)]=\mathrm{i}f_{abc}J_0^c(x)\delta(x^1-y^1)\,, \tag{37}$$

$$[J_0^a(x), J_1^b(y)]=\mathrm{i}f_{abc}J_1^c(x)\delta(x^1-y^1)+\mathrm{i}[a^2/4\pi(a-1)]\mathscr{D}_1^{ab}\delta(x^1-y^1)\,, \tag{38}$$

$$[J_1^a(x), J_1^b(y)]=\mathrm{i}f_{abc}J_0^c(x)\delta(x^1-y^1)-[a(a-2)\mathrm{i}f_{abc}/(a-1)^2](-\mathrm{i}\tilde{g}\hat{\hat{\pi}}^{\mathrm{T}}-\mathrm{i}\tilde{g}\partial_1\tilde{g}^{-1}/4\pi)^c(x)\delta(x^1-y^1). \tag{39}$$

This result reduces for the abelian U(1) theory to the same expression found in refs. [12,4]. The expression (38) allows us to draw a first conclusion. The consistency of the theory requires a positive Schwinger term [19]. Then $(a-1)$ should take on positive values. Let us compare this current algebra with the one obtained in the vector theory. We see that both coincide for the case $a=2$. In fact, the similarity is even greater, as may be seen by computing the complete effective action in the vector theory, and in the chiral, $a=2$, theory. One can easily integrate out the scalar fields while using the transformation properties of $S(g)$, eq. (1),

$$S(gU)=S(g)+S(U)+\mathrm{Tr}\int \mathrm{d}^2x(g^{-1}\partial_+g)(U\partial_-U^{-1})/4\pi\,. \tag{40}$$

The resulting effective action is

$$S_{\mathrm{eff}}(A)=\mathrm{Tr}\int \mathrm{d}^2x(-F_{\mu\nu}F^{\mu\nu}/4)-S(V^{-1}U^{-1})\,, \tag{41}$$

where U and V obey $A_-=(\mathrm{i}/e)U^{-1}\partial_-U$ and $A_+=(\mathrm{i}/e)V\partial_+V^{-1}$ respectively. This result coincides with the one found for the same quantity in the vector theory [8].

One can also calculate, for $a=2$, those quantities that give information about the confinement behaviour of the theory. For static charged particles, the relevant quantity is the Wilson loop, that only depends on the gauge fields and then must coincide with the one of the vector case. However, if one wants to get information on the behaviour of the dynamical left handed particles the behaviour of the Wilson loop fails in giving it [20]. More involved quantities may be defined [20] as a function of the gauge invariant propagator of the matter fields and the Wilson loop, that effectively characterize the confinement phase. If one returns to the original fermionic theory, the behaviour of the left handed fermions may be analysed in terms of these quantities. However, since the effective action is the same for both theories, one obtains the same expression for the gauge invariant propagator of the left fermions in terms of the gauge fields and the free left-handed fermion propagator, once the fermions are integrated out. Then, the confinement properties of the left handed fermions should be the same as in the nonabelian Schwinger model, a relation that holds, for every value of $a>1$ in the abelian case [21].

In analogy to what has been done previously for the vector case, the transformation laws of J_μ can now be easily obtained. We find

$$[\omega_2^a(x),J_0^b(y)]=\mathrm{i}ef_{abc}J_0^c(x)\delta(x^1-y^1),\quad [\omega_2^a(x),J_1^b(y)]=\mathrm{i}ef_{abc}J_1^c(x)\delta(x^1-y^1)\,, \tag{42,43}$$

which coincides with the relations found in the vector case.

An alternative procedure to that applied above is to introduce gauge fixing conditions in such a manner that the complete set of constraints becomes of second class. The advantage is that one gets information directly from the operators of the theory. As we have already elaborated earlier, the original theory analysed in refs. [15,5] can be easily obtained in the path integral formulation by going to the unitary gauge $\tilde{g}=I$. The equivalence between the canonical and the path integral formulation requires this to hold in the present formalism. This has been first shown to be true in the abelian case, in ref. [4]. In the nonabelian case the additional constraints

$$(\mathrm{i}\tilde{g}\partial_1\tilde{g}^{-1})^a=(\omega_3)^a=0\,,\quad [-\mathrm{i}\tilde{g}\hat{\tilde{\pi}}^{\mathrm{T}}-e(a-1)A_0/4\pi-eA_1/4\pi]^a=(\omega_4)^a=0\,, \tag{44,45}$$

must be imposed in the theory in order to get the unitary gauge. The set of constraints is now of second class. After computing the nontrivial matrix of Poisson brackets the same commutation relations as in ref. [5] appear if the Dirac procedure is applied. One should note that the gauge currents are not gauge invariant as in the abelian case and the gauge current algebra in this gauge reads

$$[J_0^a(x),J_0^b(y)]=-\mathrm{i}f_{abc}J_0^c(x)\delta(x^1-y^1)\,, \tag{46}$$

$$[J_0^a(x),J_1^b(y)]=\mathrm{i}f_{abc}\frac{J_0^c(x)}{(a-1)}\delta(x^1-y^1)+\frac{\mathrm{i}a^2}{4\pi(a-1)}\mathscr{D}_1^{ab}\delta(x^1-y^1)\,, \tag{47}$$

$$[J_1^a(x), J_1^b(y)] = i f_{abc}\left(J_0^c(x) + 2\frac{J_1^c(x)}{(a-1)} - \frac{a(a-2)}{(a-1)^2}\{[(a-1)eA_0^c(x) + eA_1^c(x)]/4\pi\}\right)\delta(x^1 - y^1) . \qquad (48)$$

The same Schwinger terms as in eq. (38) appear as we expected, but the close relation with the vector theory cannot be inferred from here. Let us mention that the unitary gauge is special in a sense. If we study the theory with fermions, one can always define physical gauge invariant operators, in the same way as has been done in the standard model [22]

$$\psi_L^{\text{phys.}} = \tilde{g}^{-1}\psi_L, \quad \psi_R^{\text{phys.}} = \psi_R . \qquad (49,50)$$

The unitary gauge provides us then with an appropiate framework to get information about the physical excitations in the theory [13]. In the abelian case, there have been claims that physical asymptotic fermion states exist in the chiral theory in contradistinction to the vector case [14]. This is not in contradiction with the confinement properties of the fundamental fermions. In fact, since the operators eqs. (49),(50) are gauge invariant the physical fermions carry no colour quantum number. The quantization in the nonabelian case may be followed in the way of refs. [23,5], and the positiveness of the hamiltonian may be demonstrated, for values of a greater than one [23].

A final comment is in order. We have studied a complete set of $U(N)$ gauge theories, for general N. Our results can be easily generalized to the case of $SU(N)$ if we do not gauge the $U(1)$ invariant subgroup of the original $U(N)$. Note that now, $\tilde{g}$ is not a field in the fundamental representation of $U(N)$, but of $SU(N)$. However, we can still consider $\tilde{g}$ transforming in the fundamental representation of $U(N)$, due to the fact that the additional degree of freedom can always be integrated away and has no consequences in the interacting theory. Then the current algebra in both the vector and chiral theory may be obtained from eqs. (21),(23) and (37)–(39) respectively, by choosing the subset of the Lie algebra that corresponds to the subgroup $SU(N)$.

In conclusion, we have discussed the canonical quantization of bosonized vector and chiral nonabelian gauge theories. We have shown that the constraint structure is of first class and that both theories are gauge invariant at the quantum level. The gauge current algebra commutators have been computed and the existence of a consistent algebra has been shown for values of a greater than one. From the current algebra we have also noted a close relation between the gauge field sectors of both quantized theories for a special choice of the arbitrary parameter a. From this relation, the shielding of the fundamental fermions in the theory has been demonstrated, by analogy with the vector theory, a behaviour we expect to hold also for values of a different from 2. Finally, the equivalence of the gauge invariant formulation with the gauge noninvariant formulation of refs. [23,5] has been proved.

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Note added. After completion of this manuscript, we saw a preprint of Ramallo [24], where he arrived at similar results for the chiral theory. However, the gauge current algebra reported in that paper disagrees with eqs. (37)–(39) and we believe it to be erroneous. As a result of this, the relation with the vector theory is not noticed by Ramallo.

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