Renormalization of Yang-Mills theories in axial gauges within a uniform prescription

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We investigate the renormalization properties of Yang-Mills theories in axial gauges within the generalized Leibbrandt-Mandelstam prescription which unifies all axial gauges, including the light-cone gauge, within one uniform prescription. We calculate the divergent part of the gluon self-energy in a planar-type gauge fixing. All known results on the planar gauge and on the light-cone gauge are recovered as special cases. At variance to the light-cone gauge for $n^2 \neq 0$ only the S matrix, but not the Green's functions, can be renormalized by local counterterms.

I. INTRODUCTION

In spite of the simplicity of covariant gauge conditions, noncovariant gauges have been employed time and again due to various advantages in certain situations. In particular, much interest has been devoted in the past 15 years to the quantization of Yang-Mills (YM) theories in the ghost-free axial-type gauges, i.e., noncovariant gauges specified by $n^\mu A_\mu = 0$ (or their nonhomogeneous extensions), n_μ being an arbitrary constant four-vector[1] (frequently the case $n^2 > 0$ is distinguished from the pure axial gauge $n^2 < 0$ by the name temporal gauge; the gauge $n^\mu A_\mu = 0, n^2 = 0$ is called the light-cone gauge or the *physical gauge*). Two well-known members of this class of gauges are the *homogeneous axial* gauge and the *planar* gauge, the latter having been employed in perturbative QCD (Ref. 2) because of its relatively simple gluon propagator. Although the light-cone gauge was first studied in quantum field theory even before (or roughly at the same time as) the axial ($n^2 \neq 0$) gauges,[3] it came into prominence only in the last five years, mainly for two reasons. It was in this gauge that the $N = 4$ supersymmetric YM theories have been proved to be finite for the first time.[4] Further, the absence of unphysical degrees of freedom in this gauge facilitated the proof of supersymmetry of the fermionic string.[5]

However, all the merits of these gauges (all axial-type gauges are ghost-free, the planar gauge is devoid of Gribov gauge copies, . . .) are partly compensated for by the fact that in associated Feynman integrals factors $1/(qn)^\beta$ occur leading to new singularities when $qn = 0$. The central question is how to deal with these unphysical poles. For the axial-type gauges the principal-value (PV) prescription, which, however, lacks deeper justification, has proved to resolve almost all problems connected with these gauge artifacts.[6] It amounts to setting

$$P\frac{1}{(qn)^\beta} \equiv \lim_{\epsilon \to +0} \frac{1}{2} \left[\frac{1}{(qn + i\epsilon)^\beta} + \frac{1}{(qn - i\epsilon)^\beta} \right] . \tag{1.1}$$

Although tested successfully in one-loop calculations, its validity for all orders in perturbation theory is undecided: For the temporal gauge the PV technique fails (at least at the two-loop order), as has been shown in Wilson-loop calculations.[7] Hence a few alternative pole prescriptions have been suggested,[8] some of which even give up translation invariance. The situation looked even worse for the light-cone gauge: it was only in 1983 that Mandelstam[4] and independently Leibbrandt[9] proposed new prescriptions (turning out to be equivalent later), denoted by LM, which obeyed power counting, as well as other basic criteria and implemented them in the framework of dimensional regularization:

$$\frac{1}{(qn)_M^\beta} = \frac{1}{[qn + i\epsilon\, \mathrm{sgn}(qn^*)]^\beta}, \quad \epsilon > 0 , \tag{1.2a}$$

where $n_\mu = (n_0, \mathbf{n})$ and $n_\mu^* = (n_0, -\mathbf{n})$, or equivalently,

$$\frac{1}{(qn)_L^\beta} = \frac{(qn^*)^\beta}{(qnqn^* + i\epsilon)^\beta}, \quad \epsilon > 0 . \tag{1.2b}$$

This new prescription was strongly vindicated afterwards by Bassetto and collaborators, who demonstrated that the LM prescription naturally emerges quantizing light-cone YM theories by means of the usual equal-time commutator algebra.[10]

Apparently, this variety of different ways of handling the spurious gauge poles is not very attractive, both from a theoretical and a computational point of view and motivated us to search for a pole prescription, which unifies all axial gauges *including the light-cone gauge.* In a recent Letter[11] two of the authors (P.G. and M.K.) proved that a generalized LM prescription achieves this object provided certain conditions on the vectors n_μ and n_μ^* are satisfied. Recently, Leibbrandt[12] has also analyzed the possibility of unifying all axial gauges using completely different methods. Burnel[13] and Nyeo[14] presented a unification of the Coulomb gauge and the temporal gauge considering them as critical limits of well-defined gauges to which the same quantization methods apply as for covariant gauges.

In Sec. II we apply the new technique to the evaluation of the divergent part of the gluon self-energy within the one-parameter family of axial gauges comprising the homogeneous axial gauge and the planar gauge. Our findings contain previous results as special cases and respect the usual Ward identities. We observe that off the light cone Green's functions cannot be renormalized by local counterterms. In Sec. III we compare our results with the predictions of extended Becchi-Rouet-Stora (BRS) symmetries and comment on the renormalization of YM theories in these gauges.

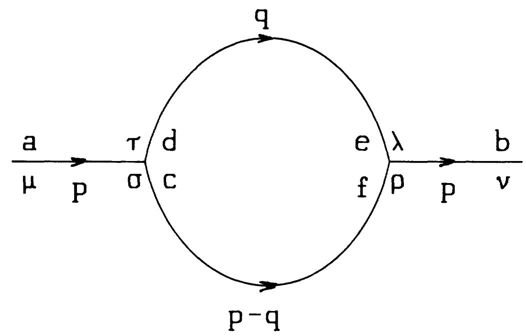

FIG. 1. The gluon self-energy.

II. YANG-MILLS THEORIES IN AXIAL GAUGES

The ghost-free action for YM theories including sources reads

$$S=S_{\rm YM}+S_{\rm gf}+S_s,\quad S_{\rm YM}=-\tfrac{1}{4}{\rm tr}\int d^4x\, F^{\mu\nu}F_{\mu\nu}\ ,$$
$$S_{\rm gf}={\rm tr}\int d^4x\frac{1}{2\alpha}nAOnA,\quad S_s={\rm tr}\int d^4x\, jA\ , \tag{2.1}$$

where $A_\mu=A_\mu^a\tau^a$ and $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu+g[A_\mu,A_\nu]$. The operator O may depend on both the space-time derivative ∂_μ and the vector n_μ. For $O=\partial^2/n^2$ and $\alpha=-1$ the name *planar gauge* has become common upon an application in QCD (Ref. 2). $O=1$ characterizes the usual axial gauge; the limit $\alpha\to 0$ yields the homogeneous case $n^\mu A_\mu=0$. Remarkably enough, all the different axial gauges reduce to the homogeneous axial gauge in the limit $\alpha\to 0$ (Ref. 15). The action Eq. (2.1) in the generating functional $Z(j_\mu)$ for Green's functions leads to the Slavnov-Taylor identity:[16]

$$\left[\frac{1}{\alpha}n^\mu n^\nu O\frac{\delta}{\delta j^\nu}-ij^\mu\right]D_\mu\left[\frac{1}{i}\frac{\delta}{\delta j}\right]Z=0\ . \tag{2.2}$$

The Legendre transform from the generating functional of connected Green's functions

$$Z_c=-i\ln Z \tag{2.3a}$$

to a generating functional

$$\Gamma(A)=Z_c-j^\mu A_\mu \tag{2.3b}$$

of one-particle-irreducible (1PI) functions yields

$$D^\mu(A)\Gamma_{,\mu}=F,\quad F=\frac{ig}{\alpha}On^\mu n^\lambda\Gamma^{-1}_{,\mu\lambda}\ , \tag{2.4}$$

where a subscript comma means differentiation with respect to A_μ. Differentiating Eq. (2.4) with respect to A_ν we obtain the relevant Ward identity the two-point function has to obey

$$D^\mu\Gamma_{,\mu\nu}=\frac{ig}{\alpha}On^\mu n^\lambda\Delta_\mu{}^\rho\Gamma_{,\rho\sigma\nu}\Delta^\sigma{}_\lambda\ . \tag{2.5}$$

Of course, the $1/\alpha$ dependence is spurious because the homogeneous limit $\alpha\to 0$ should exist [in this case the right-hand side (RHS) of Eq. (2.5) has to vanish]. For all further calculations we use the nonhomogeneous planar gauge, where $O=\partial^2/n^2$. In this gauge the propagator $\Delta^{ab}_{\mu\nu}$ and the three-vertex $V^{abc}_{\rho\sigma\lambda}$ read (cf. Fig. 1)

$$\Delta^{ab}_{\mu\nu}(q)=\frac{-i\delta^{ab}}{q^2+i\epsilon}\left[g_{\mu\nu}-\frac{q_\mu n_\nu+q_\nu n_\mu}{qn}+\frac{(1+\alpha)n^2q_\mu q_\nu}{(qn)^2}\right]\ , \tag{2.6}$$

$$V^{abc}_{\mu\nu\lambda}(p_1,p_2,p_3)=(2\pi)^{2\omega}gf^{abc}[g_{\mu\nu}(p_2-p_1)_\lambda+g_{\nu\lambda}(p_3-p_2)_\mu+g_{\lambda\mu}(p_1-p_3)_\nu]\ . \tag{2.7}$$

The gluon self-energy $\Pi^{ab}_{\mu\nu}(p)$ as given in Fig. 1 is of the form

$$\Pi^{ab}_{\mu\nu}(p)=\int d^{2\omega}q\ V^{acd}_{\mu\sigma\tau}(p,p-q,q)V^{bef}_{\nu\lambda\rho}(-p,-q,q-p)\Delta^{de}_{\tau\lambda}(q)\Delta^{cf}_{\rho\sigma}(p-q)\ . \tag{2.8}$$

In the following calculations we applied the generalized LM prescription as proposed in Ref. 11. Some of the Feynman integrals have been simplified by means of the so-called *splitting formula*

$$\frac{1}{qn\,(p+q)n}=\frac{1}{pn}\left[\frac{1}{qn}-\frac{1}{(p+q)n}\right]\ , \tag{2.9}$$

which, at variance with the PV technique, does not pick up additional δ functions.[17] After some tedious calculations, using the program package for symbolic manipulation REDUCE, we obtain, for the divergent part of $\Pi^{ab}_{\mu\nu}$,

$$
\begin{aligned}
\Pi^{ab}_{\mu\nu}(p) = & -g^2 f^{acd} f^{cdb} \frac{i\pi^2}{2-\omega} \\
& \times \Bigg[\Bigg\{ (p^2 g_{\mu\nu} - p_\mu p_\nu) \left[\frac{11}{3} - 2\frac{n^2}{R}\frac{pn^*}{pn} \right] \\
& \quad - p^2 \left[\hat{n}_\mu \left[\hat{n}^*_\mu - \frac{pn^*}{pn}\hat{n}_\nu \right] + \hat{n}_\nu \left[\hat{n}^*_\mu - \frac{pn^*}{pn}\hat{n}_\mu \right] \right] \left[\frac{2}{R} + \frac{n^2(n^*)^2}{R^3} - \frac{n^2(nn^*)}{R^3}\frac{pn^*}{pn} \right] \\
& \quad + p^2 \left[\hat{n}^*_\mu \left[\hat{n}^*_\nu - \frac{pn^*}{pn}\hat{n}_\nu \right] + \hat{n}^*_\nu \left[\hat{n}^*_\mu - \frac{pn^*}{pn}\hat{n}_\mu \right] \right] \left[\frac{n^2(nn^*)}{R^3} - \frac{(n^2)^2}{R^3}\frac{pn^*}{pn} \right] \Bigg\} \\
& \quad + \alpha \Bigg\{ (p^2 g_{\mu\nu} - p_\mu p_\nu) \left[2 - 2\frac{nn^*}{R} \right] \\
& \qquad + p^2 (n_\mu \hat{n}^*_\nu + n_\nu \hat{n}^*_\mu) \left[\frac{n^2(n^*)^2}{R^3} - 2\frac{n^2(nn^*)}{R^3}\frac{pn^*}{pn} + \frac{(n^2)^2}{R^3}\left[\frac{pn^*}{pn}\right]^2 \right] \\
& \qquad + p^2 (n_\mu \hat{n}_\nu + n_\nu \hat{n}_\mu) \Bigg[2\frac{nn^* - R}{n^2 R} - \frac{(n^*)^2(nn^*)}{R^3} \\
& \qquad\qquad + 2\frac{n^2(n^*)^2}{R^3}\frac{pn^*}{pn} - \frac{n^2(nn^*)}{R^3}\left[\frac{pn^*}{pn}\right]^2 \Bigg] \Bigg\} \\
& \quad + (1+\alpha)^2 \frac{(n^2)^2}{R}\frac{p^2 pn^*}{(pn)^3} \left[(p^2 g_{\mu\nu} - p_\mu p_\nu) + p^2 \left[\frac{n^2}{R^2}\hat{n}^*_\mu \hat{n}^*_\nu - \frac{nn^*}{R^2}(\hat{n}_\mu \hat{n}^*_\nu + \hat{n}_\nu \hat{n}^*_\mu) + \frac{(n^*)^2}{R^2}\hat{n}_\mu \hat{n}_\nu \right] \right] \Bigg] ,
\end{aligned}
\tag{2.10}
$$

where

$$R = [(nn^*)^2 - n^2(n^*)^2]^{1/2} , \tag{2.11a}$$

$$\hat{n}_\mu = n_\mu - \frac{pn}{p^2} p_\mu , \tag{2.11b}$$

$$\hat{n}^*_\mu = n^*_\mu - \frac{pn^*}{p^2} p_\mu . \tag{2.11c}$$

The integrals we used have been derived in Refs. 11, 17, and 18 and are given, for the sake of completeness, in the Appendix. In order to write down the final (and in our view the most elegant) form of $\Pi^{ab}_{\mu\nu}(p)$ we first define the following seven tensors:

$$S^1_{\mu\nu}(p) = (p^2 g_{\mu\nu} - p_\mu p_\nu) , \tag{2.12a}$$

$$S^2_{\mu\nu}(p) = p_\mu p_\nu - \frac{p^2}{pn}(p_\mu n_\nu + p_\nu n_\mu) + \left[\frac{p^2}{pn}\right]^2 n_\mu n_\nu , \tag{2.12b}$$

$$S^3_{\mu\nu}(p) = \frac{1}{R}\left[pn(p_\mu n^*_\nu + p_\nu n^*_\mu) - pn^*(p_\mu n_\nu + p_\nu n_\mu) - p^2(n_\mu n^*_\nu + n_\nu n^*_\mu) + 2p^2 \frac{pn^*}{pn} n_\mu n_\nu \right] , \tag{2.12c}$$

$$S^4_{\mu\nu}(p) = \frac{(n^2)^2}{R^2}\frac{1}{(pn)^2}[(pn^*)^2 p_\mu p_\nu - p^2 pn^*(p_\mu n^*_\nu + p_\nu n^*_\mu) + (p^2)^2 n^*_\mu n^*_\nu] , \tag{2.12d}$$

$$S^5_{\mu\nu}(p) = \frac{n^2}{R^2}\left[2p^2 n^*_\mu n^*_\nu - pn^*(p_\mu n^*_\nu + p_\nu n^*_\mu) + \frac{(pn^*)^2}{pn}(p_\mu n_\nu + p_\nu n_\mu) - p^2\frac{pn^*}{pn}(n_\mu n^*_\nu + n_\nu n^*_\mu) \right] , \tag{2.12e}$$

$$N^1_{\mu\nu}(p) = \frac{1}{R}[p^2(n_\mu n^*_\nu + n_\nu n^*_\mu) - pn^*(p_\mu n_\nu + p_\nu n_\mu)] , \tag{2.12f}$$

$$N^2_{\mu\nu}(p) = \frac{1}{n^2}[2p^2 n_\mu n_\nu - pn(p_\mu n_\nu + p_\nu n_\mu)] . \tag{2.12g}$$

The first five tensors are transverse,

$$p^\mu S^i_{\mu\nu}(p) = 0 , \tag{2.13a}$$

but only four of them are linearly independent. We prefer to use these five tensors in order to avoid artificial nonlocalities and to facilitate the interpretation of counterterms. The last two tensors only satisfy

$$p^\mu p^\nu N^i_{\mu\nu}(p)=0 \ . \tag{2.13b}$$

The expression for the gluon self-energy Eq. (2.10) is now given by

$$\begin{aligned}\Pi^{ab}_{\mu\nu}(p)= c^{ab}\frac{i\pi^2}{2-\omega}\\ \times\Bigg[\Bigg[S^1_{\mu\nu}(p)\left[\frac{11}{3}-2\frac{n^2}{R}\frac{pn^*}{pn}\right]+S^3_{\mu\nu}(p)\left[2+\frac{n^2(n^*)^2}{R^2}-\frac{n^2(nn^*)}{R^2}\frac{pn^*}{pn}\right]+S^5_{\mu\nu}(p)\left[\frac{nn^*}{R}-\frac{n^2}{R}\frac{pn^*}{pn}\right]\Bigg]\\ +\alpha\Bigg\{S^1_{\mu\nu}(p)\left[2-2\frac{nn^*}{R}\right]+N^1_{\mu\nu}(p)\left[\frac{n^2(n^*)^2}{R^2}-2\frac{n^2(nn^*)}{R^2}\frac{pn^*}{pn}+\frac{(n^2)^2}{R^2}\left[\frac{pn^*}{pn}\right]^2\right]\\ +N^2_{\mu\nu}(p)\left[2\frac{nn^*-R}{R}-\frac{n^2(n^*)^2(nn^*)}{R^3}+2\frac{(n^2)^2(n^*)^2}{R^3}\frac{pn^*}{pn}-\frac{(n^2)^2(nn^*)}{R^3}\left[\frac{pn^*}{pn}\right]^2\right]\Bigg\}\\ +(1+\alpha)^2\Bigg[S^1_{\mu\nu}(p)\frac{(n^2)^2}{R}\frac{p^2pn^*}{(pn)^3}+S^2_{\mu\nu}(p)\frac{n^2(nn^*)^2}{R^3}\frac{pn^*}{pn}\\ +S^4_{\mu\nu}(p)\frac{n^2}{R}\frac{pn^*}{pn}+\left[S^5_{\mu\nu}(p)\frac{n^2p^2}{(pn)^2}-2S^4_{\mu\nu}(p)\right]\frac{nn^*}{R}\Bigg]\Bigg]\ ,\end{aligned}\tag{2.14}$$

where $c^{ab}=-g^2 f^{acd}f^{cdb}$. The first consistency check to be done is whether the light-cone gauge results are included in Eq. (2.14). Restricting the vectors n_μ and n^*_μ both to the forward (backward) light cone Eq. (2.14) simplifies to

$$\Pi^{ab}_{\mu\nu}(p)=c^{ab}\frac{i\pi^2}{2-\omega}[\tfrac{11}{3}S^1_{\mu\nu}(p)+2S^3_{\mu\nu}(p)]\ , \tag{2.15}$$

which is in perfect agreement with the results of Refs. 9 and 19. Note, that this result is valid for general n^*_μ (Ref. 11) and independent of α. In Ref. 11 a fundamental relation between the PV prescription and the generalized LM prescription has been given:

$$P\frac{1}{(qn)^\beta}=\frac{1}{2}\left[\left[\frac{1}{(qn)^\beta}\right]_{\rm LM}+\left[\frac{1}{(qn)^\beta}\right]_{\rm LM}\Bigg|_{n^*\to -n^*}\right]. \tag{2.16}$$

Performing that substitution in Eq. (2.14) and adding up we obtain

$$\Pi^{ab}_{\mu\nu}(p)=c^{ab}\frac{i\pi^2}{2-\omega}[\tfrac{11}{3}S^1_{\mu\nu}(p)+2\alpha S^1_{\mu\nu}(p)-2\alpha N^2_{\mu\nu}(p)]\ , \tag{2.17}$$

reproducing the well-known results of Ref. 15. To check the Ward identity we have to evaluate the Pincer diagram,[15] yielding

$$p^\mu\Pi^{ab}_{\mu\nu}(p)=-c^{ab}\frac{\alpha n^2}{pn}I^\mu(1,1)S^1_{\mu\nu}(p)\ , \tag{2.18}$$

where $I^\mu(1,1)$ is given in the Appendix. This result concurs completely with $p^\mu\Pi^{ab}_{\mu\nu}$ taken from Eq. (2.10). We want to remark that we do not have to define a singular integral to zero in order to obtain consistent results, as is done in Ref. 12. As to be expected $\Pi^{ab}_{\mu\nu}(p)$ vanishes on-shell between physical states, because all terms are proportional to p_μ, p_ν, or p^2. In contrast with the light cone gauge[20] the Green's functions off the light cone cannot be made finite by *local* counterterms. This can be seen from the nonlocal coefficients of $n^*_\mu n^*_\nu$ in Eq. (2.10). Thus, in general, nonlocal divergences are expected to vanish only for the S matrix.

III. SOME COMMENTS ON THE RENORMALIZATION

The most efficient methods to prove renormalizability of a gauged quantum field-theory have been developed by Becchi, Rouet, and Stora[21] and Slavnov and Taylor[16] and extended by Piguet and Sibold.[22] To achieve control on the gauge (parameter) dependence of physical quantities and to gain some additional information on the structure of possible counterterms the authors of Ref. 22 enlarged the usual BRS methods by transforming the gauge parameters into Grassmann variables which are assigned a Faddeev-Popov ($\phi\Pi$) charge, also. But this technical trick reveals its full power only when one uses a Lagrange multiplier field B to fix the gauge. Only then the Slavnov-Taylor identities are homogeneous owing to the nilpotency of the BRS transformations. Hence, in order to use the power of these methods, we have to change our action Eq. (2.1). Our new (equivalent) action reads

$$\begin{aligned}S&=S_{\rm YM}+S_{\phi\Pi}+S_s\ ,\\ S_{\rm YM}&=-\tfrac{1}{4}{\rm tr}\int d^4x\, F^{\mu\nu}F_{\mu\nu}\ ,\\ S_{\phi\Pi}&={\rm tr}\int d^4x(-\tfrac{1}{2}\alpha BO_1B+Bn^\mu O_2A_\mu-c_-n^\mu O_2D_\mu c_+),\\ S_s&={\rm tr}\int d^4x\,(\rho^\mu sA_\mu+\sigma sc_+)\ ,\end{aligned}\tag{3.1}$$

where $sA_\mu = D_\mu c_+$, $sc_+ = i[c_+, c_+]$, $sc_- = B$, $sB = 0$ are the BRS transformations. ρ^μ and σ are the external sources for the composite operators occurring in these transformations. $S_{\phi\text{II}}$ contains the gauge-fixing term and the ghost action. Different choices of O_1 and O_2 lead to different axial gauges; $O_1 = O_2 = \partial^2/n^2$ characterizes the planar gauge. The usual gauge-fixing term S_{gf} of Eq. (2.1) is regained by inserting the equations of motion for the B field into the action Eq. (3.1). With the help of the extended BRS symmetry it has been shown, in accordance with Ref. 23, that the one-loop counterterm Γ_{ct} must obey $\Delta\Gamma_{\text{ct}} = 0$, where

$$\Delta = \frac{\delta S}{\delta A^\mu}\frac{\delta}{\delta\rho_\mu} + \frac{\delta S}{\delta\rho_\mu}\frac{\delta}{\delta A^\mu} + \frac{\delta S}{\delta c_+}\frac{\delta}{\delta\sigma} + \frac{\delta S}{\delta\sigma}\frac{\delta}{\delta c_+} + B\frac{\delta}{\delta c_-} + \chi^\mu\frac{\delta}{\delta n^\mu} + \varphi\frac{\delta}{\delta\alpha} \tag{3.2}$$

is the extended Slavnov-Taylor operator, φ and χ_μ, an anticommuting constant four-vector, are the BRS variation of α, viz., n_μ. Δ can be written as $\Delta_{\text{BRS}} + \Delta_\chi + \Delta_\alpha$ with

$$\Delta_\chi = \chi^\mu \frac{\delta}{\delta n^\mu} . \tag{3.3}$$

All three operators and their sum are nilpotent, but for Δ_χ the cohomology is trivial:[24]

$$\Delta_\chi \omega = 0 \Longrightarrow \omega = \Delta_\chi \omega_1 . \tag{3.4}$$

Solving the cohomology problem yields Γ as a BRS variation plus a term that contains no χ's at all.[22] The latter must also be independent of n_μ. The gauge vector n_μ can be varied independently of n^*_μ within an open subset of Minkowski space,[11] so there is no need of Lagrange multipliers, as used in the first paper of Ref. 25. As our result must be generated by a one-loop counterterm that contains neither ghosts nor sources, since it is a ΔX, it must have the form

$$\Delta X = \frac{\delta S_{\text{YM}}}{\delta A^\mu}\frac{\delta}{\delta\rho_\mu}(\rho^\nu M_\nu) , \tag{3.5}$$

which is equivalent to $D^\mu F^\nu_\mu M_\nu$. In M_μ all also nonlocal counterterms are included. We now come back to the tensor decomposition we did for $\Pi^{ab}_{\mu\nu}(p)$. As already explained previously we have to interpret our results as generated by counterterms. This is readily achieved for $S^i_{\mu\nu}$ $(i = 2, \ldots, 5)$ and $N^i_{\mu\nu}$ $(i = 1, 2)$:

$$S^2_{\mu\nu}(p) \simeq (DFn)\frac{1}{(nD)^2}(DFn) , \tag{3.6a}$$

$$S^3_{\mu\nu}(p) \simeq (DFn)\frac{1}{nD}(nFn^*) , \tag{3.6b}$$

$$S^4_{\mu\nu}(p) \simeq (DFn^*)\frac{1}{(nD)^2}(DFn^*) , \tag{3.6c}$$

$$S^5_{\mu\nu}(p) \simeq (DFn^*)\frac{1}{nD}(nFn^*) , \tag{3.6d}$$

$$N^1_{\mu\nu}(p) \simeq (DFn^*)(nA) , \tag{3.6e}$$

$$N^2_{\mu\nu}(p) \simeq (DFn)(nA) . \tag{3.6f}$$

It is less obvious for $S^1_{\mu\nu}(p)$ which also might have n_μ-dependent coefficients:

$$S^1_{\mu\nu}(p) \simeq F^2 \simeq (DF)^\mu \frac{1}{nD}(Fn)_\mu \simeq (DF)^\mu A_\mu , \tag{3.7a}$$

$$\frac{pn^*}{pn} S^1_{\mu\nu}(p) \simeq (DF)^\mu \frac{1}{nD}(Fn^*)^\mu . \tag{3.7b}$$

(Of course, for the two-point function all the D_μ's reduce to ∂_μ's.) Now it should become clear why we have chosen five transverse tensors: There are more counterterms than linearly independent transverse tensors. In accordance with Ref. 20, for the light-cone gauge $n^2 = 0$ only $S^3_{\mu\nu}(p)$ appears. Further, this counterterm should be the only one needed to render YM theories in the light-cone gauge finite.[20]

IV. CONCLUSIONS

Using a generalized LM prescription for $n^2 \neq 0$ we have calculated the divergent part of the one-loop gluon self-energy. We are able to reproduce all results hitherto published. At variance with the light-cone gauge, nonlocalities are present not only in the vertex but also in the Green's functions and they cannot be compensated by a renormalization with local counterterms only. Their contribution to the S matrix, however, vanishes. We have also accomplished the interpretation of all n_μ-dependent nonlocal divergent structures as generated by the extended Slavnov-Taylor operator.[22–25] This is a strong argument that all these terms decouple from the physical sector of the theory. They certainly are artifacts of the gauge and the integral prescription. Decoupling from the physical sector is less obvious than in the physical gauge, where they disappear completely if one uses the two-component formalism.[26]

Note added in proof. After this paper had been submitted for publication we received a paper by Nardelli and Soldati,[27] who independently calculated the gluon self-energy in the planar gauge $\alpha = 1$ within the generalized LM prescription. Their results agree with ours if two misprints in their Eqs. (7) and (10) are corrected. Also the recent results of Leibbrandt and Nyeo[28] turned out to be special cases of our Eq. (2.10) for $(n^*)^2 = 0$.

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APPENDIX

Here we list the UV-divergent parts of the integrals necessary to compute the divergent terms of the YM self-energy Eq. (2.10) and give a brief derivation (n_μ and n^*_μ are entirely independent constant four-vectors):

$$I(1,2)=\int d^{2\omega}q\frac{1}{(q-p)^2(qn)^2}\bigg|_{\rm div}=\frac{2}{R}\left[\frac{nn^*-R}{n^2}\right]\bar{I}\ , \tag{A1}$$

$$\begin{aligned}I_\mu(1,1)&=\int d^{2\omega}q\frac{q_\mu}{(p-q)^2qn}\bigg|_{\rm div}\\&=\frac{1}{R}\left[2\left[pn^*-pn\frac{nn^*-R}{n^2}\right]p_\mu+\frac{(pn^*)^2n^2-2(pn)(pn^*)(nn^*)+(pn)^2(n^*)^2}{R^2}n^*_\mu\right.\\&\qquad\left.+\left[\frac{-(pn^*)^2(nn^*)n^2+2(pn)(pn^*)n^2(n^*)^2-(pn)^2(nn^*)(n^*)^2}{n^2R^2}+2(pn)^2\frac{nn^*-R}{(n^2)^2R}\right]n_\mu\right]\bar{I}\ ,\end{aligned} \tag{A2}$$

$$I_\mu(2,1)=\int d^{2\omega}q\frac{q_\mu}{q^2(p-q)^2(qn)}\bigg|_{\rm div}=\frac{1}{R}\left[n^*_\mu-\frac{nn^*-R}{n^2}n_\mu\right]\bar{I}\ , \tag{A3}$$

$$\begin{aligned}I_{\mu\nu}(2,2)&=\int d^{2\omega}q\frac{q_\mu q_\nu}{q^2(p-q)^2(qn)^2}\bigg|_{\rm div}\\&=\frac{1}{R}\left[\frac{1}{R^2}\left[nn^*n^*_\mu n^*_\nu-(n^*)^2(n_\mu n^*_\nu+n_\nu n^*_\mu)+\frac{nn^*(n^*)^2}{n^2}n_\mu n_\nu\right]+\frac{nn^*-R}{n^2}\left[g_{\mu\nu}-\frac{2}{n^2}n_\mu n_\nu\right]\right]\bar{I}\ ,\end{aligned} \tag{A4}$$

$$\begin{aligned}I_{\mu\nu}(2,1)&=\int d^{2\omega}q\frac{q_\mu q_\nu}{q^2(p-q)^2(qn)}\bigg|_{\rm div}\\&=\frac{1}{2R}\left\{\frac{1}{R^2}\left[(pn^*n^2-pn\,nn^*)n^*_\mu n^*_\nu-[pn^*nn^*-pn\,(n^*)^2](n_\mu n^*_\nu+n_\nu n^*_\mu)+\left[pn^*(n^*)^2-\frac{pn\,nn^*(n^*)^2}{n^2}\right]n_\mu n_\nu\right]\right.\\&\qquad\left.+(g_{\mu\nu}pn^*+p_\mu n^*_\nu+p_\nu n^*_\mu)-\frac{nn^*-R}{n^2}\left[p_\mu n_\nu+p_\nu n_\mu+pn\left[g_{\mu\nu}-\frac{2}{n^2}n_\mu n_\nu\right]\right]\right\}\bar{I}\ ,\end{aligned} \tag{A5}$$

where

$$R=\sqrt{(nn^*)^2-n^2(n^*)^2},\quad R^2>0\ , \tag{A6a}$$

$$\bar{I}=\frac{i\pi^2}{2-\omega}\ . \tag{A6b}$$

R always denotes the positive root. Note that the singularity at $n^2=0$ in these integrals is only fictitious. Evaluation of axial gauge integrals in LM prescription has been explained in Refs. 17 and 18, but for the sake of self-consistency, we recall the essential steps. From now on, $n_\mu=(n_0,\mathbf{n})$ and $n^*_\mu=(n_0,-\mathbf{n})$. (How to deal with independent n^*_μ and n_μ is shown in Ref. 11.) Using Feynman parameters, we first compute

$$\int d^{2\omega}q\,(q^2+2pq-L+i\epsilon)^{\beta-\alpha}(qn^*qn+i\eta)^{-\beta}\ , \tag{A7}$$

and apply β times the differential operator

$$D=-\tfrac{1}{2}(n^*)^\mu\left[\frac{\partial}{\partial p^\mu}\right]\ . \tag{A8}$$

This yields

$$\begin{aligned}I(\alpha,\beta)&\equiv\int d^{2\omega}q\,(q^2+2pq-L+i\epsilon)^{-\alpha}\left[\frac{qn^*}{qn^*qn+i\eta}\right]^\beta\\&=i\pi^\omega\sum_{j=0}^{[\beta/2]}(-1)^{\alpha+j}\frac{\Gamma(\alpha-\omega+\beta-j)}{\Gamma(\alpha)\Gamma(\beta)}\frac{\beta!}{(\beta-2j)!j!2^j}\\&\qquad\times\int_0^1dx\,x^{-\beta}(1-x)^{\beta-1}(A\bar{A})^{-1/2}\mathcal{L}^{\omega-\alpha-\beta+j}(D\mathcal{L})^{\beta-2j}(D^2\mathcal{L})^j\ ,\end{aligned} \tag{A9}$$

where

$$\begin{aligned}&A=x+n_0^2(1-x),\quad \bar{A}=x+\mathbf{n}^2(1-x),\quad \mathcal{L}=L+p^2+(1-x)\left[\frac{(\mathbf{p}\cdot\mathbf{n})^2}{\bar{A}}-\frac{(p_0n_0)^2}{A}\right]\ ,\\&D\mathcal{L}=x\left[\frac{p_0n_0}{A}+\frac{\mathbf{p}\cdot\mathbf{n}}{\bar{A}}\right],\quad D^2\mathcal{L}=\frac{1}{2}\frac{x^2n^2}{A\bar{A}}\ .\end{aligned} \tag{A10}$$

Differentiation with respect to p_μ yields

$$I_{\mu_1\cdots\mu_n}(\alpha,\beta)\equiv\int d^{2\omega}q\frac{q_{\mu_1}\cdots q_{\mu_n}}{(q^2+2pq-L)^\alpha}\left[\frac{qn^*}{qn^*qn+i\epsilon}\right]^\beta$$
$$=\left[-\frac{1}{2}\right]^n\frac{\Gamma(\alpha-n)}{\Gamma(\alpha)}\frac{\partial}{\partial p^{\mu_1}}\cdots\frac{\partial}{\partial p^{\mu_n}}I(\alpha-n,\beta)\ , \tag{A11}$$

where the RHS is well defined for noninteger ω even if $\alpha-n$ is a negative integer. Multiplying n_μ and n_μ^* with the real number $1/n_0$ in (A9) just gives an overall factor. Therefore we can set $n_0=1$, implying $A=1$. Now we can substitute $\overline{A}=z^2$ and compute the divergent parts of (A11) explicitly:

$$\int d^{2\omega}q\frac{q_{\mu_1}\cdots q_{\mu_n}}{(q^2+2pq-L)^\alpha(qn)^\beta}\bigg|_{\text{div}}=\overline{I}\frac{\partial}{\partial p^{\mu_1}}\cdots\frac{\partial}{\partial p^{\mu_n}}\left[n_0^{2(\alpha-n-2)}\sum_{j=\max\{0,\alpha+\beta-n-2\}}^{[\beta/2]}c(n,\alpha,\beta,j)I(n,\alpha,\beta,j)\right]\ , \tag{A12}$$

where

$$c(n,\alpha,\beta,j)=\frac{(-1)^\beta\beta}{(\beta-2j)!j!(n+2-\alpha-\beta+j)!(\alpha-1)!2^{n+j-1}}\ , \tag{A13}$$

$$I(n,\alpha,\beta,j)=\int_{|r|}^1\frac{dz}{(2z^2)^j}\frac{(1-z^2)^{\beta-1}}{(1-r^2)^{\beta-j}}\left[n_0p_0-\frac{\mathbf{p}\cdot\mathbf{n}}{z^2}\right]^{\beta-2j}\mathcal{L}_z^{n+2-\alpha-\beta+j}\ , \tag{A14}$$

$$\overline{I}=\frac{i\pi^2}{2-\omega},\quad r\equiv\frac{|\mathbf{n}|}{n_0},\quad \mathcal{L}_z=n_0^2(L+p^2)+\frac{1-z^2}{1-r^2}\left[\frac{(\mathbf{p}\cdot\mathbf{n})^2}{z^2}-p_0^2n_0^2\right]\ . \tag{A15}$$

This depends only on p_0n_0, $\mathbf{p}\cdot\mathbf{n}$, $|n_0|$, and $|\mathbf{n}|$ [note the lower limit of integration in (A14)]. Hence, R always denotes the positive root, and does not depend on the sign of n_0.

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