

## QCD Calculation of $B \rightarrow \pi l \bar{\nu}_l$ and the matrix element $|V_{bu}|$

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**Abstract.** Using QCD sum rules for a two-point function involving beauty vector currents, together with current algebra-PCAC sum rules, we estimate the hadronic matrix element in  $B \rightarrow \pi l \bar{\nu}_l$ . We find  $\Gamma(\bar{B}^0 \rightarrow \pi^+ l \bar{\nu}_l) = (1.45 \pm 0.59) \times 10^{13} |V_{bu}|^2 \text{s}^{-1}$ . As a byproduct, the vector current contribution to the decay  $B \rightarrow \rho l \bar{\nu}_l$  is also estimated.

The role of the  $B_{13}$  decay  $B \rightarrow \pi l \bar{\nu}_l$  as an important source of information on the quark mixing, Cabibbo–Kobayashi–Maskawa, matrix element  $|V_{bu}|$  [1] has been reappraised recently, following the observation by the ARGUS collaboration at DESY [2] of the  $b \rightarrow u$  non-leptonic transitions  $B \rightarrow p \bar{p} \pi$  and  $B \rightarrow p \bar{p} \pi \pi$ . One of the advantages of semileptonic decay modes is that the relevant hadronic form factors may be estimated from well established general principles such as e.g. PCAC and vector meson dominance (VMD). To this extent  $B_{13}$  should be less affected by the theoretical uncertainties usually encountered in relating quark level dynamics to observable hadronic matrix elements. So far, existing estimates of  $B_{13}$  form factors rely on one form or another of the constituent quark model [3–5], sometimes explicitly supplemented with pion-PCAC [6–7].

In this note we discuss a determination of the  $B \rightarrow \pi l \bar{\nu}_l$  hadronic matrix element in the framework of QCD sum rules [8–9] for a two-point function involving beauty vector currents, combined with current algebra—PCAC sum rules [10]. One virtue of this approach, which is fully relativistic and field-theoretic by construction, is that it naturally accounts for the most significant aspects of QCD dynamics, i.e. perturbative asymptotic freedom and non-

perturbative phenomena such as e.g. chiral symmetry breaking. In a recent analysis [11] of semileptonic charm meson decays within this framework, we determined the matrix elements  $|V_{cs}|$  and  $|V_{cd}|$  by estimating the  $D_{13}$  vector form factor  $f_+(0)$ . The first difficulty one encounters in  $B_{13}$  decays is that the beauty-meson phenomenology is at present rather incomplete. In particular, the observed vector beauty resonance  $B^*(5320)$  [12] does not have enough phase space to decay into a purely hadronic final state; in fact  $B^*(5320) \rightarrow B(5270)\gamma$ . Consequently, it is not feasible to determine  $f_+(0)$  directly and entirely from two-point function QCD sum rules. One must first separate the  $B^*$  leptonic decay constant  $\gamma_{B^*}$  from the  $B^*B\pi$  strong coupling  $g_{B^*B\pi}$  as dictated e.g. by VMD. Two-point function QCD sum rules may then be used to estimate  $\gamma_{B^*}$ , but  $g_{B^*B\pi}$  has to be estimated independently. This contrasts with the  $D_{13}$  case where the phase space permitted decay  $D^* \rightarrow D\pi$  allowed us to write a realistic hadronic spectral function, whose absolute normalization was precisely  $f_+(0)$ , and to relate it to the fundamental QCD parameters through the sum rules [11]. In any case, our results are restrictive enough to allow for a meaningful determination of  $|V_{bu}|$  from data on  $B \rightarrow \pi l \bar{\nu}_l$  (once available). As a side remark we shall comment at the end on the expected vector current contribution to  $B \rightarrow \rho l \bar{\nu}_l$ .

We begin by defining the  $\bar{B}^0 \rightarrow \pi^+ l \bar{\nu}_l$  form factors  $f_{\pm}(t)$  as

$$\begin{aligned} \langle \pi^+(p) | V_{\mu}(0) | \bar{B}^0(p) \rangle \\ = (p + p')_{\mu} f_+(t) + (p - p')_{\mu} f_-(t), \end{aligned} \quad (1)$$

where  $V_{\mu}(x) = \bar{u}(x)\gamma_{\mu}b(x)$ . Concentrating on  $f_+(t)$ , as in the end we shall take the zero lepton mass limit, we can write the following VMD expression

$$f_+(t) = \frac{M_{B^*}^2}{\sqrt{2}\gamma_{B^*}} \frac{g_{B^*B\pi}}{M_{B^*}^2 - t} \mathcal{F}(t), \quad (2)$$

where  $g_{B^*B\pi}$  is the strong  $B^* \rightarrow B\pi$  coupling,  $\gamma_{B^*}$  is the

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leptonic decay constant of the  $B^*$  defined as

$$\langle 0 | V_\mu(0) | B^*(p) \rangle = \sqrt{2} \frac{M_{B^*}^2}{2\gamma_{B^*}} \varepsilon_\mu, \quad (3)$$

and finally  $\mathcal{F}(t)$  accounts for potential corrections to VMD, presumably arising from  $B^*$ -radial excitations. Although according to the non-renormalization theorem [13]  $f_+(0)$  should equal unity in the flavour symmetry limit, one is to expect large deviations on account of the huge  $B-\pi$  mass difference. For instance, in  $D_{13}$ :  $f_+(0) = 0.75 \pm 0.05$  [11].

We proceed next to estimate the leptonic decay constant  $\gamma_{B^*}$  in the framework of QCD sum rules for the two-point function

$$\begin{aligned} \Pi_{\mu\nu}(q) &= i \int d^4x e^{iqx} \langle 0 | T(V_\mu(x) V_\nu^\dagger(0)) | 0 \rangle \\ &= -(g_{\mu\nu} q^2 - q_\mu q_\nu) \Pi^{(1)}(q^2) + q_\mu q_\nu \Pi^{(0)}(q^2). \end{aligned} \quad (4)$$

The appropriate dispersion relations for heavy flavour currents are the Hilbert moments at  $q^2 \equiv -Q^2 = 0$

$$\begin{aligned} \phi_n(0) &\equiv \frac{1}{(n+1)!} \left( -\frac{d}{dQ^2} \right)^{n+1} [-Q^2 \Pi^{(1)}(Q^2)]_{Q^2=0} \\ &= \frac{1}{\pi} \int \frac{ds}{s^{n+1}} \text{Im} \Pi^{(1)}(s) \end{aligned} \quad (5)$$

where  $n \geq 1$ . In this case the  $\phi_n(0)$  admit a well defined short distance QCD expansion in  $\alpha_s$  and in inverse powers of the (current)  $b$ -quark mass  $m_b$ . Neglecting  $m_{u,d}$  in the sequel, one finds at the two-loop level [14]

$$\begin{aligned} \phi_1(0) &= \frac{3}{32\pi^2} \frac{1}{m_b^2} (1 + 1.140\alpha_s) + \frac{1}{m_b^2} \left[ -\frac{C_4 \langle O_4 \rangle}{m_b^4} \right. \\ &\quad \left. + \frac{3}{2} \frac{C_5 \langle O_5 \rangle}{m_b^5} + \frac{5}{3} \frac{C_6 \langle O_6 \rangle}{m_b^6} \right] \end{aligned} \quad (6)$$

$$\begin{aligned} \phi_2(0) &= \frac{1}{40\pi^2} \frac{1}{m_b^4} (1 + 1.582\alpha_s) + \frac{1}{m_b^4} \left[ -\frac{C_4 \langle O_4 \rangle}{m_b^4} \right. \\ &\quad \left. + \frac{5}{2} \frac{C_5 \langle O_5 \rangle}{m_b^5} + 2 \frac{C_6 \langle O_6 \rangle}{m_b^6} \right] \end{aligned} \quad (7)$$

where  $m_b \equiv m_b(Q^2 = m_b^2)$ ,  $\alpha_s \equiv \alpha_s(m_b^2)$ , and

$$C_4 \langle O_4 \rangle = m_b \langle \bar{u}u \rangle + \frac{1}{12} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \quad (8)$$

$$C_5 \langle O_5 \rangle = \langle g_s \bar{q} \sigma_{\mu\nu} G_{\mu\nu}^a \lambda^a q \rangle \quad (9)$$

$$C_6 \langle O_6 \rangle = \pi \alpha_s \langle (\bar{u} \gamma_\mu \lambda u) \sum_q \bar{q} \gamma_\mu \lambda q \rangle \quad (10)$$

Concerning the hadronic spectral function, we follow the established practice and parametrize it by the ground state  $B^*$ -pole plus a smooth continuum identified with the QCD asymptotic freedom expression starting at some threshold  $s_0$ , i.e.

$$\begin{aligned} \frac{1}{\pi} \text{Im} \Pi^{(1)}(s) &= \frac{1}{2} \frac{M_{B^*}^2}{\gamma_{B^*}^2} \delta(s - M_{B^*}^2) \\ &\quad + \Theta(s - s_0) \frac{1}{\pi} \text{Im} \Pi^{(1)}(s)|_{A.F.}, \end{aligned} \quad (11)$$

where [14]

$$\begin{aligned} \frac{1}{\pi} \text{Im} \Pi^{(1)}(x)|_{A.F.} &= \frac{1}{8\pi^2} (1-x)^2 (2+x) \\ &\quad \cdot \left\{ 1 + \frac{4\alpha_s}{3\pi} \left[ \frac{13}{4} + 2l(x) + \ln x \ln(1-x) \right. \right. \\ &\quad \left. \left. + \frac{3}{2} \frac{x}{(2+x)} \ln \left( \frac{x}{1-x} \right) - \ln(1-x) \right. \right. \\ &\quad \left. \left. - \left[ \frac{4-x-x^2}{(2+x)(1-x)^2} \right] x \ln x \right. \right. \\ &\quad \left. \left. - \frac{(5-x-2x^2)}{(2+x)(1-x)} \right] \right\} \end{aligned} \quad (12)$$

with  $x \equiv m_b^2/s$ , and  $l(x)$  the dilogarithm function. Substituting (11) in (5) one obtains the following  $n=1$  and  $n=2$  Hilbert moment QCD sum rules

$$\frac{1}{2\gamma_{B^*}^2} \frac{1}{M_{B^*}^2} = \frac{1}{m_b^2} \int_{x_0}^1 dx \frac{1}{\pi} \text{Im} \Pi^{(1)}(x)|_{A.F.} + \text{N.P.}|_1 \quad (13)$$

$$\frac{1}{2\gamma_{B^*}^2} \frac{1}{M_{B^*}^4} = \frac{1}{m_b^4} \int_{x_0}^1 x dx \frac{1}{\pi} \text{Im} \Pi^{(1)}(x)|_{A.F.} + \text{N.P.}|_2, \quad (14)$$

where  $\text{N.P.}|_{1,2}$  stand for the non-perturbative contributions to the rhs of (6) and (7), respectively. Numerically, these contributions are negligible except for the dimension  $d=4$  condensate which we fix to  $C_4 \langle O_4 \rangle \simeq -0.05 \text{ GeV}^4$  (for a discussion of the values of the condensates in the charm and beauty case see [15]).

The sum rules (13–14) are eigenvalue equations relating  $\gamma_{B^*}$  and  $M_{B^*}$  to the fundamental QCD parameters entering the rhs, including the asymptotic freedom threshold  $s_0$ . The latter is in principle a free parameter, and predictions are meaningful to the extent that they are stable against reasonable changes in  $s_0$ . This can be controlled by inspecting the predictions for  $M_{B^*}$  and comparing with experiment. We find that for  $m_b \simeq (4.6 - 4.8) \text{ GeV}$  there is a duality region of some stability for  $x_0 \simeq 0.65 - 0.75$  within which  $M_{B^*} = 5.32 \text{ GeV}$  is correctly predicted. Inside this region the result for  $\gamma_{B^*}$  is

$$\gamma_{B^*} = 22 \pm 4. \quad (15)$$

It is interesting to compare this prediction with the one following from the constituent quark model [7]

$$\gamma_{B^*} = \frac{M_{B^*}}{2f_\pi} \sqrt{\frac{M_{B^*}}{M_B}} \left( \frac{f_\pi}{f_B} \right), \quad (16)$$

where  $f_B$  is the  $B$ -meson leptonic decay constant normalized such that  $f_\pi = 93.2 \text{ MeV}$ . Using  $f_B/f_\pi = 1.3 \pm 0.2$ , as obtained in recent QCD analyses of the  $B$ -meson channel [15–16], one finds from (16):  $\gamma_{B^*} = 22 \pm 3$ , in excellent agreement with (15). This may indicate that relativistic effects in the  $b$ -flavour channel are small enough so that quark model predictions become reliable.

We now turn to the evaluation of  $g_{B^*B\pi}$  appearing in (2). To this end we exploit current algebra and PCAC following the standard procedure leading to the celebrated Adler–Weisberger sum rule (for a review see e.g. [10]). We begin with the equal-time commutator of axial charges with pionic quantum numbers

$$[Q_A^+, Q_A^-] = 2I_3, \quad (17)$$

and sandwich it between  $|B^+(p)\rangle$  states in the infinite-momentum frame. Using completeness one finds the sum rule

$$\int \frac{d\nu}{\nu^2} t(\nu, q^2 = 0) = 2, \quad (18)$$

where  $\nu = p \cdot q$ , and

$$t(\nu, q^2) = 2f_\pi^2 (2\pi)^3 \left[ \sum_n \delta(p + q - p_n) |\langle B^+(p) | J_{\pi^+} | n \rangle|^2 - \sum_m \delta(p - q - p_m) |\langle B^+(p) | J_{\pi^-} | m \rangle|^2 \right], \quad (19)$$

with  $J_{\pi^\pm}$  representing the massless pion source. In the single particle intermediate state approximation the last term in the rhs of (19) gives no contribution, as it would correspond to doubly charged mesons. Retaining just the  $B^*$ -intermediate state in (19) one obtains from (18)

$$g_{B^*B\pi} \simeq \frac{M_{B^*}}{\sqrt{2}f_\pi}, \quad (20)$$

which is again compatible with the quark model result [7]. However, the advantage of the above derivation is twofold: first, no reference is made to the non-relativistic quark model and second, within the aforementioned approximations  $t(\nu)$  in (18) is positive definite so that the result (20) represents an upper bound. In order to estimate the size of the corrections to the single  $B^*$ -intermediate state approximation, we find it convenient to parametrize them by the first readily excited state  $B^{*'}$  (notice the good convergence of (18) due to the  $\nu^2$  denominator). In this case, the corrected expression for the coupling constant reads

$$g_{B^*B\pi} = \frac{M_{B^*}}{\sqrt{2}f_\pi} \left( 1 + \frac{g_{B^{*'}B\pi}^2 M_{B^*}^2}{g_{B^*B\pi}^2 M_{B^{*'}}^2} \right)^{-1/2}, \quad (21)$$

in an obvious notation. This particular parametrization of higher state contributions leads in turn to the following result for the form factor  $\mathcal{F}(t)$  in (2)

$$\mathcal{F}(t) = 1 - \frac{g_{B^{*'}B\pi} M_{B^*}^2 \gamma_{B^*}}{g_{B^*B\pi} M_{B^*}^2 \gamma_{B^{*'}}} \left( \frac{M_{B^*}^2 - t}{M_{B^{*'}}^2 - t} \right), \quad (22)$$

where a negative sign has been assumed for the correction term, in accord with the expectation that  $f_+(0) < 1$ . To estimate  $M_{B^{*'}}$  we make use of the mass formula proposed in [17] assuming flavour indepen-

dence of the effective quark potential. This gives

$$\frac{M_{B^{*'}}}{M_{B^*}} \simeq 1.14. \quad (23)$$

Likewise, encouraged by the agreement between (15) and (16), we exploit the standard relation between  $\gamma_{B^*}$ ,  $\gamma_{B^{*'}}$  and the respective meson wave functions at the origin and use from [17]

$$\frac{|\psi_{2s}(0)|}{|\psi_{1s}(0)|} \simeq 0.67 - 0.81, \quad (24)$$

which results in

$$\frac{|\gamma_{B^*}|}{|\gamma_{B^{*'}}|} \simeq \frac{|\psi_{2s}(0)|}{|\psi_{1s}(0)|} \left( \frac{M_{B^*}}{M_{B^{*'}}} \right)^{3/2} \simeq 0.55 - 0.66. \quad (25)$$

We feel that these estimates should be reliable to the extent that they are based on phenomenological relations which seem to account fairly well for the beauty-meson spectroscopy (the 1980 prediction [17] of the  $B^*$ -mass was  $M_{B^*} = 5.3$  GeV!). Assuming  $g_{B^{*'B\pi}}/g_{B^*B\pi} \simeq \mathcal{O}(1)$ , (24–25) imply a correction to single VMD at  $t = 0$  of roughly 60%, i.e.

$$\mathcal{F}(0) \simeq 0.40 \pm 0.05. \quad (26)$$

This apparently large correction factor should not come as a surprise if one keeps in mind that in the case of the rho-meson, experiment already gives a 20–25% effect:  $g_{\rho\pi\pi}/f_\rho|_{\text{EXP}} = 1.22 \pm 0.03$  [18]. Using this estimate of  $\mathcal{F}(0)$  together with (15) and (21) in (2) we find at  $t = 0$

$$f_+(0) = 0.4 \pm 0.1, \quad (27)$$

which is in line with expectations. This result should be viewed more as a lower bound than as an absolute prediction, as one would intuitively expect  $g_{B^{*'B\pi}}/g_{B^*B\pi} < 1$ , in which case the corrections would be smaller and  $f_+(0)$  somewhat larger.

It should be clear that these radial excitation corrections to the form factor will also modify the hadronic spectral function (11), and hence could also change in principle the prediction for  $\gamma_{B^*}$ . However, to the extent that the  $B^{*'}$  resonance is treated in a narrow width approximation, we find the net effect to be a shift in the value of  $x_0$  which leads to the correct result for  $M_{B^*}$ , with no appreciable change in  $\gamma_{B^*}$ .

This completes our estimate of the various parameters entering the expression for  $f_+(t)$ , (2), and we proceed to predict the semileptonic decay rate for  $B \rightarrow \pi l \bar{\nu}_l$

$$\Gamma(\bar{B}^0 \rightarrow \pi^+ l \bar{\nu}_l) = \frac{G_F^2 |V_{bu}|^2}{192\pi^3} \frac{1}{M_B^3} I_{ps}, \quad (28)$$

where the phase space integral  $I_{ps}$  (neglecting  $m_l$ ) is given by

$$I_{ps} \equiv \int_0^t dt [(t - t_+)(t - t_-)]^{3/2} |f_+(t)|^2, \quad (29)$$

$t_{\pm} = (M_B \pm \mu_{\pi})^2$ , and  $f_+(t)$  is given by (2) with  $\mathcal{F}(t)$  as in (22). Numerically, we obtain

$$\Gamma(\bar{B}^0 \rightarrow \pi^+ l \bar{\nu}_l) = (1.45 \pm 0.59) \times 10^{13} |V_{bu}|^2 \text{s}^{-1}, \quad (30)$$

which may be compared with the free-quark model inclusive rate [19]

$$\Gamma(b \rightarrow ul \bar{\nu}_l) \simeq 1.4 \times 10^{14} |V_{bu}|^2 \text{s}^{-1}. \quad (31)$$

Hence, the  $B \rightarrow \pi$  semileptonic decay mode saturates about 10% of the inclusive  $b \rightarrow ul \bar{\nu}_l$  rate (or 20% of the rate induced by the vector current). Nevertheless, it should be possible to observe it experimentally since with  $0.25 \geq |V_{bu}|/|V_{bc}| \geq 0.07$ , and  $|V_{bc}| = 0.05$  [1–2] one predicts from (30)

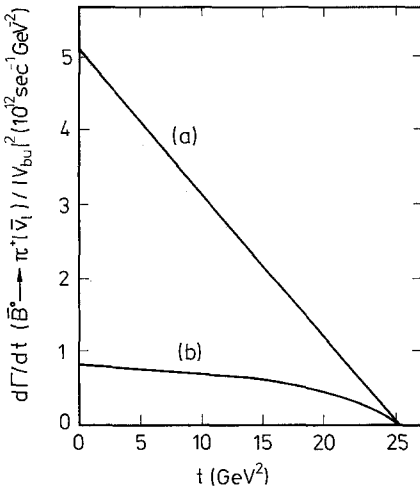
$$BR(B \rightarrow \pi l \bar{\nu}_l) \simeq (2 - 30) \times 10^{-4}. \quad (32)$$

In any case, we must reiterate that on account of the various approximations made in estimating the corrections to pure VMD, the prediction (30) is likely a lower bound. A trivial upper bound is easily obtained by fixing  $f_+(0) = 1$  and  $\mathcal{F}(t) = 1$  in (2). This leads to

$$\Gamma(\bar{B}^0 \rightarrow \pi^+ l \bar{\nu}_l) \leq 6.7 \times 10^{13} |V_{bu}|^2 \text{s}^{-1}. \quad (33)$$

The total rate (30) is larger than some quark model estimates [3–4], but still, consistent within errors. The difference is due to the functional form of the form factor (2), which in fact changes qualitatively and quantitatively the shape of the differential spectrum. This may be appreciated from Fig. 1 where we show  $d\Gamma(t)/dt$  calculated with and without corrections to VMD, i.e. with  $\mathcal{F}(t) = 1$  ( $f_+(0) = 1$ ) and  $\mathcal{F}(t)$  given by (22) ( $f_+(0) = 0.4$ ), respectively. A measurement of the spectrum near the end point would then be very important to clarify this issue.

As a final point we discuss briefly the semileptonic decay  $B \rightarrow \rho l \bar{\nu}_l$ , which proceeds via the vector and the axial- vector current. The latter involves two hadronic



**Fig. 1.** Differential  $B \rightarrow \pi$  semileptonic decay rate for  $f_+(0) = 1$ ,  $\mathcal{F}(t) = 1$  (curve (a)), together with  $f_+(0) = 0.4$  and  $\mathcal{F}(t)$  as in (22) (curve (b))

form factors in addition to an axial leptonic decay constant. Since non-perturbative effects are very small in the beauty sector, we would expect  $M_{B^*} \simeq M_{B^*_A}$ , and  $\gamma_{B^*} \simeq \gamma_{B^*_A}$ . However, in the absence of experimental information on the axial-vector channel it is difficult to estimate the form factors. One should keep in mind the problems already encountered in the light-quark analogue  $A_1 \rho \pi$  vertex. Hence, we concentrate on the vector current induced part

$$\begin{aligned} \frac{d\Gamma}{dt}(B^- \rightarrow \rho^0 l \bar{\nu}_l) \\ = \frac{G_F^2 |V_{bu}|^2 |F_V(t)|^2}{384\pi^3} \frac{1}{M_{B^*}^2} t[(t - t_+)(t - t_-)]^{3/2}, \end{aligned} \quad (34)$$

where the form factor  $F_V(t)$  may be written as

$$F_V(t) = \frac{M_{B^*}^2}{\sqrt{2}\gamma_{B^*}} \frac{g_{B^* \rightarrow B \rho^0}}{M_{B^*}^2 - t} \mathcal{F}(t). \quad (35)$$

Assuming  $g_{B^* B \rho} / g_{B^* B \rho} \simeq \mathcal{O}(1)$ , the correction form factor  $\mathcal{F}(t)$  is the same as before, and thus the only new parameter is the strong coupling constant  $g_{B^* \rightarrow B \rho^0}$ . This can be estimated from the rate  $\Gamma(B^{*-} \rightarrow B^- \gamma)$  through VMD. Using the theoretical value  $\Gamma(B^{*-} \rightarrow B^- \gamma) = 1.7 \text{ keV}$  from [17] together with ideal  $\omega - \phi$  mixing, we obtain:  $g_{B^* \rightarrow B \rho^0} \simeq 11 \text{ GeV}^{-1}$ . This leads e.g. to  $F_V(0) \simeq 0.14 \text{ GeV}^{-1}$ , in line with other estimates [3]. Integrating (34) we finally predict

$$\begin{aligned} \Gamma(B^0 \rightarrow \rho^- l \bar{\nu}_l) &= 2\Gamma(B^- \rightarrow \rho^0 l \bar{\nu}_l) \\ &= (1.0 \pm 0.4) |V_{bu}|^2 \times 10^{13} \text{ s}^{-1}, \end{aligned} \quad (36)$$

which should be about one half of the total rate in this channel.

To summarize, we have estimated the  $B^*(5320)$  leptonic decay constant  $\gamma_{B^*}$  using QCD sum rules for a two-point function involving beauty vector currents, and the strong coupling constant  $g_{B^* B \pi}$  from a current algebra-PCAC sum rule. Both parameters enter the VMD parametrization of the vector form factor  $f_+(t)$  in  $B \rightarrow \pi l \bar{\nu}_l$ . This separation and independent estimate of the above parameters is required by the fact that the hadronic spectral function in this channel has a pole below the physical threshold  $t_+ = (M_B + \mu_{\pi})^2$ . Hence, it is not feasible to determine  $f_+(0)$  entirely from QCD sum rules as in the case of e.g.  $K_{13}$  and  $D_{13}$  [11]. This is unfortunate in the sense that one needs then to estimate the corrections to VMD. These should be large since already in the case of the  $\rho \pi \pi$  vertex they are at the level of 20–25%. We have used a simple parametrization of these corrections in terms of a second pole, corresponding to the first radial excitation of the  $B^*$ , with parameters estimated from the QCD effective potential of [17]. The effect of these corrections, in addition to the renormalization of  $f_+(0)$ , is to flatten the differential decay rate as illustrated in Fig. 1. This semileptonic decay mode should be experimentally observable, and would

clearly provide an important source of information on the value of  $|V_{bu}|$ .

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