Commun. Math. Phys. 121, 255–270 (1989)

Communications in Mathematical Physics
© Springer-Verlag 1989

On the Existence of Equilibrium States in Local Quantum Field Theory

Detlev Buchholz[1] and Peter Junglas[2]

[1] II. Institut für Theoretische Physik, Universität Hamburg, D-2000 Hamburg 50, Federal Republic of Germany
[2] Institut für Theoretische Physik, Universität Göttingen, D-3400 Göttingen, Federal Republic of Germany

Dedicated to E. H. Wichmann on the occasion of his 60th birthday

Abstract. It is shown that any local quantum field theory admits thermodynamical equilibrium states (KMS-states) for all positive temperatures provided it satisfies a "nuclearity condition," proposed by Wichmann and one of the authors, which restricts the admissible number of local degrees of freedom.

1. Introduction

Although the requirement of a reasonable thermodynamical interpretation is an indispensible condition on any fundamental physical theory it has not found an expression in the generally accepted postulates of local quantum field theory [1, 2]. There may be two reasons for this omission: first, quantum field theory is regarded primarily as a framework for the description of elementary systems (particles), even though its thermodynamical aspects are receiving increasing attention in recent years [3, 4]. And second, there is the common belief that the thermodynamical features of a theory can be derived from its microscopic properties by applying the rules of statistical mechanics. It seems therefore unnecessary to amend the basic postulates regulating these microscopic properties by further conditions.

Taking this pragmatic view one misses, however, the point that the rules of statistical mechanics are not always applicable: there exist quantum field theories which do not admit any thermodynamical equilibrium states [5]. Conversely, presuming a decent thermodynamical behaviour, one can establish interesting structural properties of the underlying elementary systems [6]. Thus a closer examination of the relation between the framework of quantum field theory and of quantum statistical mechanics seems to be of interest.

As a step in this direction we establish in the present article the existence of thermodynamical equilibrium states (KMS-states [7, 8]) for all positive temperatures in quantum field theories satisfying a nuclearity condition proposed in [6]. This crucial input restricts the number of local degrees of freedom of a theory in a physically sensible manner. It was argued in [5, 6] that the nuclearity condition distinguishes theories with a realistic thermodynamical behaviour. The present results substantiate this conjecture.

Our construction of equilibrium states is quite explicit and may be of use also in applications: in the first step we exhibit subspaces $\mathscr{H}(\Lambda)$ of the physical Hilbert space $\mathscr{H}$ representing all local excitations of the vacuum in a given region. The relevant properties of these spaces are discussed in Sect. 2. We will show in particular that the operators $e^{-\beta H}E(\Lambda)$, $\beta>0$, where H is the Hamiltonian and $E(\Lambda)$ the projection onto $\mathscr{H}(\Lambda)$, are of trace class.

This result puts us into the position to define (Sect. 3) "quasi-Gibbs states"

$$\omega_{\beta,\Lambda}(A)=\frac{1}{Z}\cdot \operatorname{Tr} E(\Lambda)e^{-\beta H}E(\Lambda)A\,, \tag{1.1}$$

where Z is a normalization constant and A any (bounded) observable. These states differ from the standard Gibbs-ensembles, but they describe a situation which is close to equilibrium in a certain specific sense. (The states satisfy a local version of the KMS-condition.)

It is favourable to work with the states (1.1) instead of the Gibbs ensembles since the thermodynamical limit, where $E(\Lambda)\to 1$, can be controlled more easily. Our main result is that in this limit all limit points of the above states satisfy the KMS-condition. So they describe systems in thermodynamical equilibrium [9, 10].

For the derivation of this result we have to assume that the time translations act strongly continuously on the observables, i.e. we are working in the framework of C^*-dynamical systems. This assumption is necessary since the equilibrium states may be locally singular relative to the vacuum due to infrared problems. We illustrate this phenomenon in Sect. 4, where we also discuss some related questions.

We conclude this introduction with a list of assumptions and definitions.

1. *(Net structure)* We consider a system of (concrete) C^*-algebras $\mathfrak{A}(\mathcal{O})$ labeled by the open, bounded space-time regions $\mathcal{O}\subset\mathbb{R}^d$ and acting on a Hilbert space $\mathscr{H}$. These algebras are subject to the condition of isotony,

$$\mathfrak{A}(\mathcal{O}_1)\subset\mathfrak{A}(\mathcal{O}_2) \quad \text{if} \quad \mathcal{O}_1\subset\mathcal{O}_2\,. \tag{1.2}$$

The smallest C^*-algebra containing all the algebras $\mathfrak{A}(\mathcal{O})$ is denoted by $\mathfrak{A}$ and assumed to act irreducibly on $\mathscr{H}$. (Note that we do not assume that $\mathfrak{A}$ is separable.)

2. *(Time-translations)* The time translations are represented by a group of strongly continuous automorphisms α_t, $t\in\mathbb{R}$, of $\mathfrak{A}$ which act in the geometrically obvious manner

$$\alpha_t(\mathfrak{A}(\mathcal{O}))=\mathfrak{A}(\mathcal{O}+t\cdot n)\,. \tag{1.3}$$

Here n is a unit vector denoting the time direction with respect to some Lorentz-frame which will be kept fixed in the following. We assume that the time-translations α_t are unitarily implemented on $\mathscr{H}$ by

$$\alpha_t(A)=e^{itH}A\,e^{-itH}\,, \qquad A\in\mathfrak{A}\,, \tag{1.4}$$

where H is a positive selfadjoint operator (Hamiltonian) with the simple eigenvalue 0. The corresponding normalized eigenvector is denoted by Ω (vacuum) and we assume that Ω is cyclic for the algebras $\mathfrak{A}(\mathcal{O})$ and $\mathfrak{A}(\mathcal{O})'$, where the prime indicates the commutant of the respective algebra in $\mathscr{B}(\mathscr{H})$. We recall that this property of Ω is a consequence of the standard postulates of local quantum field theory (Reeh-Schlieder theorem [1]).

3. *(Nuclearity)* Besides these familiar properties we assume that the algebras $\mathfrak{A}(\mathcal{O})$ satisfy the nuclearity condition proposed in [6]. We state this assumption in a form, used in [11], which is equivalent to the original condition but more convenient: the maps $\Theta_{\beta,\mathcal{O}}$: $\mathfrak{A}(\mathcal{O})\rightarrow\mathscr{H}$ given by

$$\Theta_{\beta,\mathcal{O}}(A)=e^{-\beta H}A\Omega\,,\qquad A\in\mathfrak{A}(\mathcal{O}),\tag{1.5}$$

are nuclear for any $\beta>0$. This means that for fixed β and $\mathcal{O}$ there exists a sequence of vectors $\Phi_i\in\mathscr{H}$ and of linear functionals $\phi_i\in\mathfrak{A}(\mathcal{O})^*$ (which can be chosen to be continuous with respect to the ultra-weak topology induced by $\mathfrak{A}(\mathcal{O})\subset\mathscr{B}(\mathscr{H})$ [11]) such that $\sum_i \|\phi_i\|\cdot\|\Phi_i\|<\infty$ and

$$\Theta_{\beta,\mathcal{O}}(A)=\sum_i \phi_i(A)\cdot\Phi_i\,,\qquad A\in\mathfrak{A}(\mathcal{O}).\tag{1.6}$$

The trace norm of $\Theta_{\beta,\mathcal{O}}$ is defined by

$$\|\Theta_{\beta,\mathcal{O}}\|_1=\inf\sum_i \|\phi_i\|\cdot\|\Phi_i\|\,,\tag{1.7}$$

where the infimum is to be taken with respect to all vectors and functionals complying with the above condition. We assume that for small $\beta>0$ and large diameters r of $\mathcal{O}$ there holds the bound

$$\|\Theta_{\beta,\mathcal{O}}\|_1\leqq e^{cr^m\beta^{-n}}\,,\tag{1.8}$$

where c, m, n are positive constants which neither depend on r nor on β.

As was discussed in [5, 6] the quantity $\|\Theta_{\beta,\mathcal{O}}\|_1$ may be regarded as a substitute for the partition function in quantum statistical mechanics. Thus the nuclearity condition may be understood as the requirement that the partition function exists and that it exhibits in the limit of large volumes and temperatures a behaviour which complies with the physically motivated bound (1.8). In fact one expects that the constants m, n in this bound can be put equal to the dimension $(d-1)$ of space in realistic theories [6, 12]. But we do not need to make such an assumption here.

We emphasize that we also make no use of spacelike commutation relations (locality) in our discussion. The only specific input from local quantum field theory is the assumption that the time-translations act locally on $\mathfrak{A}$, i.e. the image of any algebra $\mathfrak{A}(\mathcal{O}_1)$ under the action of α_t is, for limited times t, contained in some fixed algebra $\mathfrak{A}(\mathcal{O}_2)$ which is still "small" in the sense of the nuclearity condition. Our reasoning can be extended to any net of C^*-algebras on which a dynamics acts in a similar manner.

2. Spaces of Localized States

In this section we will construct the spaces of localized states, mentioned in the Introduction, and show that they can be used to define a certain analogue of the partition function.

As an essential ingredient we use the fact that systems, which are localized in space-like separated regions, are strongly decoupled, as a consequence of the nuclearity assumption [6, 11]. In particular there exist states, in which all measurements in two such regions are completely uncorrelated. This is made precise in

Lemma 2.1. *Let $\mathcal{O}, \hat{\mathcal{O}}$ be bounded open regions such that the closure of $\mathcal{O}$ is contained in $\hat{\mathcal{O}}$. Then there exists a unique vector $\eta \in \mathcal{H}$ – called the canonical product vector – with the following properties:*

i) $(\eta, AB'\eta) = (\Omega, A\Omega) \cdot (\Omega, B'\Omega)$ (2.1)

for all $A \in \mathfrak{A}(\mathcal{O})$, $B' \in \mathfrak{A}(\hat{\mathcal{O}})'$.

ii) *η is cyclic for $\mathfrak{A}(\mathcal{O}) \vee \mathfrak{A}(\hat{\mathcal{O}})'$.*[1]

iii) *η is an element of the natural cone $P^\natural$ (cf. [8]) associated with the von Neumann algebra $\mathfrak{A}(\mathcal{O}) \vee \mathfrak{A}(\hat{\mathcal{O}})'$ and the cyclic and separating vector Ω.*

Proof. The existence of η has been proven in [6, 11], the uniqueness is a direct consequence of standard properties of the natural cone $P^\natural$ (Theorem 2.5.31 of [8]). □

The significance of the canonical product vector η was uncovered in [25]. Here we use this vector in the following

Definition. Let $\Lambda = (\mathcal{O}, \hat{\mathcal{O}})$ be any pair of space-time regions as in Lemma 2.1. For given Λ we define the Hilbert space $\mathcal{H}(\Lambda) \subset \mathcal{H}$ by

$$\mathcal{H}(\Lambda) := \overline{\mathfrak{A}(\mathcal{O})\eta}, \tag{2.2}$$

where η is the canonical product vector associated with Λ. The projection onto $\mathcal{H}(\Lambda)$ is denoted by $E(\Lambda)$.

The interpretation of $\mathcal{H}(\Lambda)$ as a space of vectors representing local excitations of the vacuum is based on

Lemma 2.2. *Let $\Lambda = (\mathcal{O}, \hat{\mathcal{O}})$. Then*

i) *$\mathcal{H}(\Lambda)$ is invariant under local operations in $\mathcal{O}$:*

$$\mathfrak{A}(\mathcal{O})\mathcal{H}(\Lambda) = \mathcal{H}(\Lambda). \tag{2.3}$$

ii) *The vectors of $\mathcal{H}(\Lambda)$ induce product states on $\mathfrak{A}(\mathcal{O}) \vee \mathfrak{A}(\hat{\mathcal{O}})'$ which are equal to the vacuum state on $\mathfrak{A}(\hat{\mathcal{O}})'$, i.e.:*

$$(\Psi, AB'\Psi) = (\Psi, A\Psi) \cdot (\Omega, B'\Omega) \tag{2.4}$$

for all $\Psi \in \mathcal{H}(\Lambda)$, $A \in \mathfrak{A}(\mathcal{O})$, $B' \in \mathfrak{A}(\hat{\mathcal{O}})'$.

[1] By $M \vee N$ we denote the von-Neumann algebra generated by M and N

iii) $\mathscr{H}(\Lambda)$ *is complete in the following sense: to every normal state* ϕ *on* $\mathfrak{A}(\mathcal{O})$ *there exists a vector* $\Psi \in \mathscr{H}(\Lambda)$ *with*

$$(\Psi, A\Psi) = \phi(A)$$

for all $A \in \mathfrak{A}(\mathcal{O})$.

Proof.

i) is an immediate consequence of the definition,

ii) follows from (2.1).

iii) As in [13] we define a linear operator $W: \mathscr{H} \to \mathscr{H} \otimes \mathscr{H}$ by linear extension from

$$W AB'\eta = A\Omega \otimes B'\Omega, \qquad A \in \mathfrak{A}(\mathcal{O}), \qquad B' \in \mathfrak{A}(\hat{\mathcal{O}})'. \tag{2.5}$$

One easily shows that W is unitary and fulfills

$$WAB'W^* = A \otimes B' \tag{2.6}$$

for A, B' as above. Using W, we can write

$$\mathscr{H}(\Lambda) = W^* \overline{\mathfrak{A}(\mathcal{O})\Omega \otimes \Omega} = W^*(\mathscr{H} \otimes \Omega). \tag{2.7}$$

Since $\mathfrak{A}(\mathcal{O})'$ has a cyclic vector, there exists a vector $\tilde{\Psi} \in \mathscr{H}$ inducing the given normal state ϕ on $\mathfrak{A}(\mathcal{O})$. Hence $\Psi := W^*(\tilde{\Psi} \otimes \Omega) \in \mathscr{H}(\Lambda)$ fulfills iii). □

For an increasing sequence Λ_n of pairs of double cones eventually exhausting all of space-time, one expects the corresponding spaces $\mathscr{H}(\Lambda_n)$ to tend to the whole Hilbert space $\mathscr{H}$. It is, however, necessary to adjust the relative sizes of the regions $\mathcal{O}, \hat{\mathcal{O}}$ in this limit in order to control the surface effects. The precise conditions are given in

Lemma 2.3. *Let* $\mathcal{O}_r$ *be the double cone of radius r about the origin* 0 *of Minkowski space and let* $\Lambda_i = (\mathcal{O}_{r_i}, \mathcal{O}_{R_i})$ *for sequences* $r_i < R_i$, $r_i \to \infty$. *If m, n are the constants in the nuclearity condition* (1.8), *and*

$$\frac{R_i}{r_i} \to \infty \quad \text{for} \quad m \geqq n,$$

respectively

$$\frac{R_i}{r_i^{n/m}} \to \infty \quad \text{for} \quad m < n,$$

then

$$E(\Lambda_i) \xrightarrow{i \to \infty} 1 \tag{2.8}$$

in the strong operator topology.

Proof. It has been shown in [14] that under the above assumptions the canonical product vectors η_i, associated with the Λ_i by Lemma 2.1, converge to the vacuum vector[2] and that, as an easy consequence, the unitary operators W_i defined by (2.5) fulfill

$$W_i^*(\Phi\otimes\Omega)\xrightarrow{i\to\infty}\Phi \quad \text{for all } \Phi\in\mathscr{H}.$$

By (2.7) we have $E(\Lambda_i)=W_i^*(1\otimes P_\Omega)W_i$, where P_Ω is the projection onto $\mathbb{C}\cdot\Omega$. Therefore

$$E(\Lambda_i)\Phi=W_i^*(1\otimes P_\Omega)W_i(\Phi-W_i^*(\Phi\otimes\Omega))+W_i^*(\Phi\otimes\Omega)\xrightarrow{i\to\infty}\Phi$$

for all $\Phi\in\mathscr{H}$. □

Further properties of the $\mathscr{H}(\Lambda)$, which support their interpretation as spaces of vectors describing excitations of the vacuum localized in a finite volume will be discussed in a separate publication [15]. For our present task of constructing equilibrium states, the most important feature of $\mathscr{H}(\Lambda)$ is the fact that the restriction of the exponential function of the Hamiltonian H to $\mathscr{H}(\Lambda)$ has the spectral properties, which are necessary for the Gibbs construction.

Proposition 2.4. *Let $\Lambda=(\mathcal{O},\hat{\mathcal{O}})$, $\beta>0$. The operator $e^{-\beta H}E(\Lambda)$ is of trace-class, and*

$$\|e^{-\beta H}E(\Lambda)\|_1\leqq\|\Theta_{\beta,\hat{\mathcal{O}}}\|_1, \tag{2.9}$$

where $\Theta_{\beta,\hat{\mathcal{O}}}$ is the nuclear map defined in (1.5).

Proof. In a first step we construct a convenient orthonormal basis of $\mathscr{H}(\Lambda)$: let $\{\Psi_i\}$ be an arbitrary orthonormal basis of $\mathscr{H}$ with $\Psi_1=\Omega$. We define

$$U_{ij}:=W^*M_{ij}\otimes 1W,$$

where $M_{ij}\in\mathscr{B}(\mathscr{H})$ are matrix units given by

$$M_{ij}\Psi=(\Psi_j,\Psi)\cdot\Psi_i\ \forall\Psi\in\mathscr{H}.$$

By (2.6) we have $U_{ij}\in\mathfrak{A}(\hat{\mathcal{O}})''$. Furthermore

$$U_{ij}U_{kl}=\delta_{jk}\cdot U_{il},\ U_{ij}^*=U_{ji},\ s\text{-}\lim_{N\to\infty}\sum_{i=1}^{N}U_{ii}=1. \tag{2.10}$$

Then

$$U_{i1}\eta=W^*(\Psi_i\otimes\Omega)$$

is the desired orthonormal basis of $\mathscr{H}(\Lambda)$. Introducing an isometry $V\in\mathfrak{A}(\hat{\mathcal{O}})''$ by

$$VB'\Omega=B'\eta,\qquad B'\in\mathfrak{A}(\hat{\mathcal{O}})',$$

we can also represent this orthonormal basis by

$$U_{i1}\eta=U_{i1}V\Omega, \tag{2.11}$$

[2] Actually this has been proven in [14] for the massive case only. One easily gets the general result by combining Lemma 3.1 and Eq. (3.16) of [14] with the assumed bound on $\|\Theta_{\beta,\mathcal{O}}\|_1$ in (1.8)

where $U_{i1} \cdot V \in \mathfrak{A}(\hat{\mathcal{O}})''$. Next we note that for any ultraweakly continuous linear functional ψ on $\mathfrak{A}(\mathcal{O})$ one has

$$|\psi(A)|^2 \leqq \|\psi\| \cdot |\psi|(AA^*), \qquad A \in \mathfrak{A}(\mathcal{O}),$$

where $|\psi|$ denotes the absolute value of ψ [16]. Consequently there hold the inequalities

$$\begin{aligned}\sum_i |\psi(U_{i1})| \cdot \|U_{1i}\Psi\| &\leqq \sum_i (\|\psi\| \cdot |\psi|(U_{ii}) \cdot (\Psi, U_{ii}\Psi))^{1/2} \\ &\leqq \inf_{\lambda > 0} \sum_i \left(\frac{\lambda}{2} \|\psi\| \cdot |\psi|(U_{ii}) + \frac{1}{2\lambda}(\Psi, U_{ii}\Psi) \right) \\ &\leqq \|\psi\| \cdot \|\Psi\|\end{aligned}$$

for any vector $\Psi \in \mathcal{H}$ and any functional ψ as above.

We are now in a position to estimate the trace norm of $e^{-\beta H}E(\Lambda)$. By polar decomposition we get

$$|e^{-\beta H}E(\Lambda)| = F \cdot e^{-\beta H}E(\Lambda),$$

where F is a partial isometry with range in $\mathcal{H}(\Lambda)$. Making use of the previous remarks as well as the nuclearity condition (1.6) and (1.7), we thus obtain the estimate

$$\begin{aligned}\operatorname{Tr}|e^{-\beta H}E(\Lambda)| &= \sum_i (U_{i1}V\Omega, Fe^{-\beta H}U_{i1}V\Omega) \\ &= \sum_i (U_{i1}V\Omega, F \cdot \Theta_{\beta,\hat{\mathcal{O}}}(U_{i1}V)) \\ &\leqq \sum_i \sum_n |\phi_n(U_{i1}V)| \cdot |(U_{i1}V\Omega, F\Phi_n)| \\ &\leqq \sum_n \sum_i |\phi_n(U_{i1}V)| \cdot \|U_{1i}F\Phi_n\| \\ &\leqq \sum_n \|\phi_n\| \cdot \|\Phi_n\| < \infty .\end{aligned}$$

The statement of the proposition now follows from this bound by taking the infimum over all admissible vectors Φ_n and functionals ϕ_n. □

This result allows the following

Definition. The quantity

$$Z(\beta, \Lambda) := \operatorname{Tr} E(\Lambda)e^{-\beta H}E(\Lambda) \tag{2.12}$$

is called quasi-partition function. (This terminology is suggested by the fact that $Z(\beta, \Lambda)$ shares many properties with the partition function of the canonical ensemble occupying a finite volume at temperature β^{-1} [17].)

In the present context we only need the subsequent elementary result on the quasi-partition function. In physical terms it says that the effect of the surface energy of the states in $\mathcal{H}(\Lambda)$, which lessens the eigenvalues of $E(\Lambda)e^{-\beta H}E(\Lambda)$, does not outrun the increase in the number of states contributing to the trace in the thermodynamical limit. For the derivation of this result it is again necessary to adjust the relative sizes of the regions $\mathcal{O}$ and $\hat{\mathcal{O}}$.

Lemma 2.5. *Let Λ_i be a sequence of pairs of double cones as in Lemma 2.3. Then*

$$\liminf_i Z(\beta, \Lambda_i) > 0 . \tag{2.13}$$

If $e^{-\beta H}$ is not a trace-class operator (as is the case in quantum field theory) then the sequence $Z(\beta, \Lambda_i)$ diverges.

Proof. Let Ψ_j, $j=1, \ldots, N$ be any set of mutually orthogonal and normalized vectors. Then it follows from Lemma 2.3 that

$$\liminf_i Z(\beta, \Lambda_i) = \liminf_i \operatorname{Tr} E(\Lambda_i) e^{-\beta H} E(\Lambda_i)$$

$$\geqq \liminf_i \sum_{j=1}^{N} (E(\Lambda_i)\Psi_j, e^{-\beta H} E(\Lambda_i)\Psi_j) = \sum_{j=1}^{N} (\Psi_j, e^{-\beta H}\Psi_j) > 0 ,$$

where the last inequality follows from the fact that $e^{-\beta H}$ is invertible. The second half of the statement follows likewise from this estimate. □

3. Local and Global Equilibrium

We will now show that under the assumptions stated in the Introduction, there exist thermodynamical equilibrium states for every (positive) temperature. To begin with we recall how one distinguishes the equilibrium states within the set of all states by means of the KMS-condition [7, 8]. We also introduce a notion of local KMS-states.

Definition. A state ω is called KMS-state at inverse temperature $\beta > 0$, if for every pair of operators $A, B \in \mathfrak{A}$ there is a function F_{AB}, which is holomorphic on $S_\beta := \{t + is | 0 < s < \beta\}$ and continuous on $\bar{S}_\beta$, such that

$$\begin{aligned} F_{AB}(t) &= \omega(A\alpha_t(B)) , \\ F_{AB}(t+i\beta) &= \omega(\alpha_t(B)A) \end{aligned} \tag{3.1}$$

for all $t \in \mathbb{R}$.

A state ω is called local KMS-state in $\mathcal{O}$ if for every pair of operators $A, B \in \mathfrak{A}(\mathcal{O})$ there exists a function F_{AB} with analyticity and continuity properties as above such that the boundary condition (3.1) holds for t in some neighbourhood of 0.

The interpretation of KMS-states as equilibrium states is justified by their stability and passivity properties [8]. We also note that on physical grounds the KMS condition refers to a particular Lorentz system: the definition of temperature requires the introduction of a heat bath which distinguishes a rest frame. In fact it is a consequence of the KMS-condition that the Lorentz transformations cannot unitarily be implemented in the GNS-representation induced by a KMS-state [18].

Using Proposition 2.4 we can define for given $\Lambda = (\mathcal{O}, \hat{\mathcal{O}})$ and $\beta > 0$ a "quasi-Gibbs state" by

$$\omega_{\beta,\Lambda}(A) := \frac{1}{Z(\beta, \Lambda)} \cdot \operatorname{Tr} E(\Lambda) e^{-\beta H} E(\Lambda) A \tag{3.2}$$

for $A \in \mathfrak{A}$. This state has the following characteristic properties:

Proposition 3.1.

i) $\omega_{\beta,\Lambda}(A \cdot B') = \omega_{\beta,\Lambda}(A) \cdot \omega_0(B')$ (3.3)

for $A \in \mathfrak{A}(\mathcal{O})$ *and* $B' \in \mathfrak{A}(\hat{\mathcal{O}})'$.

ii) $\omega_{\beta,\Lambda}$ *is a local KMS state in any subregion* $\mathcal{O}_0 \subset \mathcal{O}$ *whose closure is contained in the interior of* $\mathcal{O}$.

Proof. i) As a consequence of Lemma 2.2(i) $E(\Lambda)$ commutes with $\mathfrak{A}(\mathcal{O})$. Furthermore, by (2.6) and (2.7) we have

$$E(\Lambda)B'E(\Lambda) = \omega_0(B')E(\Lambda)$$

for all $B' \in \mathfrak{A}(\hat{\mathcal{O}})'$. This immediately implies (3.3).

ii) Let $A, B \in \mathfrak{A}(\mathcal{O}_0)$, where $\mathcal{O}_0$ is any subregion of $\mathcal{O}$ as above. We set

$$F_{AB}(t+is) := \frac{1}{Z(\beta,\Lambda)} \operatorname{Tr} AE(\Lambda)e^{-sH}B(t)e^{-(\beta-s)H}E(\Lambda)$$

with $t+is \in \bar{S}_\beta$. The analyticity and continuity properties of F_{AB} then follow from Proposition 2.4 and the continuity of $t \to U(t)$. Since by Lemma 2.2 $E(\Lambda) \in \mathfrak{A}(\mathcal{O})'$, and since for some $\delta > 0$ $AB(t) \in \mathfrak{A}(\mathcal{O})$ if $|t| < \delta$, it is also clear that F_{AB} satisfies the boundary condition (3.1) for small t. □

Up to this point the strong continuity of α_t was not essential. Yet this assumption becomes crucial in the proof that in the thermodynamical limit all limit points of the quasi-Gibbs states $\omega_{\beta,\Lambda}$ are (global) KMS-states.

The control of this limit requires a careful choice of operators which, on the one hand, are analytic with respect to the time-translations, such that the analyticity part of the KMS condition needs no further considerations. On the other hand the operators have to have good localization properties since we want to exploit the fact that the KMS boundary condition is locally satisfied by the states $\omega_{\beta,\Lambda}$. These two almost conflicting requirements are met by the operators introduced in the subsequent lemma.

Lemma 3.2. *Let* $p \in \mathbb{N}$ *be fixed and let* $\mathfrak{A}_p \subset \mathfrak{A}$ *be the *-algebra generated by all finite sums and products of operators of the form*

$$A(f) = \int dt\, f(t)\alpha_t(A),$$

where f *is any one of the functions*

$$f(t) = \text{const}\, e^{-\kappa(t+w)^{2p}} \tag{3.4}$$

(with $\kappa > 0$, $w \in \mathbb{C}$*) and* $A \in \bigcup_{\mathcal{O}} \mathfrak{A}(\mathcal{O})$ *any local operator. Then*

i) *Each* $B \in \mathfrak{A}_p$ *is an analytic element with respect to* α, *i.e. the operator-valued function* $t \to \alpha_t(B)$ *can be extended to a holomorphic function on* $\mathbb{C}$.

ii) *Each* $B \in \mathfrak{A}_p$ *is almost local in the sense that for any* $r > 0$ *there is a local operator* $B_r \in \mathfrak{A}(\mathcal{O}_r)$ *such that*

$$\|B - B_r\| \leq c \cdot e^{-\kappa(r/2)^{2p}},$$

where c *is a constant which does not depend on* r.

iii) *The algebra* $\mathfrak{A}_p$ *is invariant under* α_z, $z \in \mathbb{C}$, *and norm dense in* $\mathfrak{A}$.

Proof. i) Since each function f is entire analytic and since $|f(t+z)|$ is, for z varying in any compact subset of $\mathbb{C}$, bounded by some integrable function of t, the analyticity of the generating elements $A(f)$ of $\mathfrak{A}_p$ follows from Vitali's theorem. But finite sums and products of analytic operators are again analytic, thus the first statement follows.

ii) Again it suffices to establish this statement for the generating elements $A(f)$ of $\mathfrak{A}_p$. Setting $A(f)_r=0$ if r is small, we can furthermore restrict our attention to the cases (since A is local) where $\alpha_t(A)\subset\mathfrak{A}(\mathcal{O}_r)$ for $|t|\leqq r/2$. With

$$A(f)_r=\int_{|t|\leqq r/2} dt\, f(t)\alpha_t(A)\in\mathfrak{A}(\mathcal{O}_r)$$

the second statement then follows from the decay properties of $f(t)$ for large $|t|$ and the fact that A is bounded.

iii) The set of functions f is invariant under complex translations, hence it is clear that $\mathfrak{A}_p$ is invariant under α_z. Since the set of local operators $\bigcup_{\mathcal{O}}\mathfrak{A}(\mathcal{O})$ is norm dense in $\mathfrak{A}$ and since the time translations act norm-continuously on the elements of $\mathfrak{A}$, the last part of the statement follows by choosing in $A(f)$ a suitable sequence of functions f approximating the δ-function. □

Making use of the restrictions imposed by relation (1.8) on the volume dependence of $\|\Theta_{\beta,\mathcal{O}}\|_1$ we are now in a position to establish the KMS-property for all weak limit points of ω_{β,A_i}, provided the regions $\mathcal{O}_i$, $\hat{\mathcal{O}}_i$ tend to the whole space-time in an appropriate manner.

Proposition 3.3. *Let* $A_i=(\mathcal{O}_{r_i},\mathcal{O}_{R_i})$ *be a sequence of pairs of double cones where*

$$R_i=r_i\cdot(1+r_i^{n/m}),\qquad r_i\to\infty \tag{3.5}$$

and m,n *are the constants appearing in the nuclearity condition* (1.8). *Then every weak limit point of the sequence* ω_{β,A_i} *is a KMS state at inverse temperature* β.

Proof. Let ω_β be any weak limit point of the sequence of states ω_{β,A_i}. For the proof of the statement it suffices to show that ω_β satisfies the KMS-boundary condition (3.1) on one of the norm-dense and α-invariant subalgebras $\mathfrak{A}_p$ of $\mathfrak{A}$ introduced in the previous lemma (cf. Definition 5.3.1 and Proposition 5.3.7 in [8]). In view of the fact that the elements $A,B\in\mathfrak{A}_p$ are analytic we can rewrite this boundary condition in the form

$$\omega_\beta(A\alpha_{i\beta}(B))=\omega_\beta(BA). \tag{3.6}$$

For the proof of this relation we choose $p>(n+m)/2$. Then we have for $A,B\in\mathfrak{A}_p$,

$$\lim_i \frac{1}{Z(\beta,A_i)}\operatorname{Tr}[B,E(A_i)]\cdot e^{-\beta H}E(A_i)A=0, \tag{3.7}$$

which can be seen as follows: with our choice of the regions $\mathcal{O}_i$, $\hat{\mathcal{O}}_i$ we know from Lemma 2.5 that $Z(\beta,A_i)\geqq c$ for some $c>0$. Moreover, if $B_{r_i}\in\mathfrak{A}(\mathcal{O}_{r_i})$ is the local approximation of B constructed in the previous lemma we have $[E(A_i),B_{r_i}]=0$

according to relation (2.3). Hence we obtain the estimate

$$\frac{1}{Z(\beta, A_i)} |\mathrm{Tr}[B, E(A_i)]\, e^{-\beta H} E(A_i) A|$$
$$\leqq \frac{1}{c} \|[B - B_{r_i}, E(A_i)]\| \cdot \mathrm{Tr}|e^{-\beta H} E(A_i)| \cdot \|A\|$$
$$\leqq \frac{2\|A\|}{c} \cdot \|B - B_{r_i}\| \cdot \mathrm{Tr}|e^{-\beta H} E(A_i)| \leqq c_1 e^{-c_2 \cdot r_i^{2p}} \cdot e^{c_3 R_i^m}$$

for certain constants c_1, c_2, and c_3. Here we made use (in the last inequality) of Proposition 2.4, the nuclearity condition (1.8) and the second part of Lemma 3.2. Taking into account the particular choice of R_i and p we thus arrive at relation (3.7).

Now since $B \in \mathfrak{A}_p$, there also holds $\alpha_{i\beta}(B) \in \mathfrak{A}_p$. Hence we get for $\varepsilon > 0$ and suitable (large) $j \in \mathbb{N}$,

$$|\omega_\beta(A\alpha_{i\beta}(B) - BA)| \leqq |\omega_{\beta, A_j}(A\alpha_{i\beta}(B) - BA)| + \varepsilon$$
$$= \frac{1}{Z(\beta, A_j)} |\mathrm{Tr}\, E(A_j) e^{-\beta H} E(A_j) (A\alpha_{i\beta}(B) - BA)| + \varepsilon$$
$$\leqq \frac{1}{Z(\beta, A_j)} |\mathrm{Tr}\, E(A_j) (\alpha_{i\beta}(B) e^{-\beta H} - e^{-\beta H} B) E(A_j) A| + 3\varepsilon,$$

where, in the last step, we twice made use of relation (3.7) and the symmetry and hermiticity properties of the trace. But the bracket under the trace in the final expression vanishes, hence relation (3.6) follows. □

Since the value $\beta > 0$ was completely arbitrary in our discussion we obtain as an immediate consequence

Theorem 3.4. *Let $\mathfrak{A}, \alpha_t$ be a C^*-dynamical system with properties given in the Introduction. Then there exist KMS-states for all inverse temperatures $\beta > 0$.*

We conclude this section with a remark on the construction of equilibrium states with non-vanishing chemical potential. To this end let us assume that $\mathfrak{A}$ is an algebra of charged fields on which there acts continuously a global gauge group which is generated by a charge-operator Q commuting with the Hamiltonian H. If the corresponding charge is tied to massive particles one expects that for sufficiently small numbers μ and $\varepsilon > 0$,

$$H + \mu \cdot Q \geqq \varepsilon \cdot H .$$

Setting $H' = H + \mu \cdot Q$ and

$$\alpha'_t(A) = e^{itH'} A\, e^{-itH'}, \qquad A \in \mathfrak{A},$$

it is then obvious that $\mathfrak{A}, \alpha'_t$ is again a C^*-dynamical system with properties given in the Introduction. Hence there exist KMS-states for chemical potential μ and all $\beta > 0$.

An alternative approach to the construction of these states, which does not rely on charged fields, but only on local observables, could proceed as follows: instead of choosing in the definition of the subspaces $\mathscr{H}(A)$ a product-state vector η representing the vacuum in $\mathcal{O}$ as well as in $\hat{\mathcal{O}}'$ [3], one should take a vector $\eta_q \in \mathscr{H}$ inducing a product state, which has a fixed charge-density q in $\mathcal{O}$, and which coincides with the vacuum in $\hat{\mathcal{O}}'$. Then there must also sit a compensating charge in the boundary region $\mathcal{O}' \cap \hat{\mathcal{O}}$ if the vector η_q is to have total charge 0.

It is clear that the vectors η_q will not converge in the thermodynamical limit, thus there is no analogue of Lemma 2.3 under these circumstances. But Lemma 2.5 should hold nevertheless, and the construction of KMS-states could then be performed as in the present case.

It would be of some interest to convert this heuristic argument into a rigorous proof. More generally, it would be desirable to know whether all KMS-states on $\mathfrak{A}, \alpha_t$ can be approximated by "quasi-Gibbs states" of the form (3.2) for a suitable choice of product-state vectors η.

4. Conditions for Normality

The general results of the preceding analysis have been established within the framework of C^*-dynamical systems $\mathfrak{A}, \alpha_t$. It is the aim of the present section to discuss under which circumstances the assumption of a strongly continuous action of α_t on $\mathfrak{A}$ can be relaxed. (For a similar discussion, cf. [26].)

Let $\mathfrak{A}$ be a C^*-algebra with the properties given in the Introduction, apart from the strong continuity requirements on α_t. Since relation (1.4) implies that $\alpha_t(A)$ is weakly continuous with respect to t, we can still define for any $A \in \mathfrak{A}$ and any real test function f

$$A(f) = \int dt\, f(t)\alpha_t(A) \tag{4.1}$$

as a weak integral. In order to simplify the subsequent discussion we assume that all operators $A(f)$ are elements of $\mathfrak{A}$, and we denote the C^*-algebra generated by these operators by $\mathfrak{A}_0$. The local algebras generating $\mathfrak{A}_0$ are given by $\mathfrak{A}_0(\mathcal{O}) = \mathfrak{A}(\mathcal{O}) \cap \mathfrak{A}_0$, and these algebras are weakly dense in the original local algebras generating $\mathfrak{A}$. It also follows from relation (4.1) that the automorphisms α_t act strongly continuous on $\mathfrak{A}_0$. Hence the preceding results can be applied, showing that there exist KMS-states ω_β on the C^*-dynamical system $\mathfrak{A}_0, \alpha_t$ for any $\beta > 0$.

The question now arises whether these states can be extended to $\mathfrak{A}$ without violating the KMS-condition. This is always possible if the states ω_β are locally normal relative to the vacuum representation of $\mathfrak{A}$ on $\mathscr{H}$. In fact, this condition is also necessary if the local algebras $\mathfrak{A}(\mathcal{O})$ are von Neumann algebras [19].

In general, however, a little less is needed. We first note that one can extend in a canonical manner any KMS-state ω_β on $\mathfrak{A}_0$ to an α_t-invariant state $\tilde{\omega}_\beta$ on $\mathfrak{A}$. This extension is given by

$$\tilde{\omega}_\beta(A) = \omega_\beta(A(f)), \qquad A \in \mathfrak{A}, \tag{4.2}$$

where f is any test function with $\int dt\, f(t) = 1$. Making use of the fact that ω_β is an α_t-invariant state on $\mathfrak{A}_0$, it is easy to verify that the functional $\tilde{\omega}_\beta$ does not depend

[3] Note that if $\mathfrak{A}$ is an algebra of local observables, then $\mathfrak{A}(\hat{\mathcal{O}}') \subseteq \mathfrak{A}(\hat{\mathcal{O}})'$, where $\hat{\mathcal{O}}'$ is the causal complement of $\hat{\mathcal{O}}$

on the particular choice of the function f. From this it follows at once that $\tilde{\omega}_\beta$ is positive and invariant under α_t.

We will see that $\tilde{\omega}_\beta$ is a KMS-state on $\mathfrak{A}$ if ω_β is regular in the sense of the following

Definition. The state $\tilde{\omega}_\beta$ is said to be α-normal (with respect to the given dynamics α_t) if

$$\lim_n \tilde{\omega}_\beta(B^* [A - A(\delta_n)]) = 0 \tag{4.3}$$

for all $A, B \in \mathfrak{A}$ and all sequences δ_n of real test functions (with uniformly bounded L^1-norm) which approximate the δ-function.

It is clear that $\tilde{\omega}_\beta$ is α-normal if it is locally normal relative to the vacuum representation of $\mathfrak{A}$. The converse needs, however, not be the case. As already mentioned, there holds the

Proposition 4.1. *Let $\tilde{\omega}_\beta$ be the extension of ω_β given by relation* (4.2). *If $\tilde{\omega}_\beta$ is α-normal it is a KMS-state.*

Proof. Let $\mathcal{H}_\beta$, π_β, Ω_β be the GNS-Hilbert space, the representation, and the cyclic vector, respectively, fixed by $\tilde{\omega}_\beta$. Since $\tilde{\omega}_\beta$ is α_t-invariant we can define a unitary representation $U_\beta(t)$ of the time translations by

$$U_\beta(t)\pi_\beta(A)\Omega_\beta = \pi_\beta(\alpha_t(A))\Omega_\beta, \qquad A \in \mathfrak{A}.$$

The operators $U_\beta(t)$ are continuous with respect to t on the subspace $\pi_\beta(\mathfrak{A}_0)\Omega_\beta \subset \mathcal{H}_\beta$ because α_t acts strongly continuous on $\mathfrak{A}_0$. We will show below that this subspace is dense in $\mathcal{H}_\beta$. Hence there exists by Stone's theorem a selfadjoint operator H_β such that

$$U_\beta(t) = e^{iH_\beta t}.$$

For the proof that $\pi_\beta(\mathfrak{A}_0)\Omega_\beta$ is dense we make use of the assumption that $\tilde{\omega}_\beta$ is α-normal. Namely if $A, B \in \mathfrak{A}$ and δ_n is any δ-sequence as in the preceding definition we have

$$\lim_n (\pi_\beta(B)\Omega_\beta, [\pi_\beta(A) - \pi_\beta(A(\delta_n))]\Omega_\beta) = \lim_n \tilde{\omega}_\beta(B^* [A - A(\delta_n)]) = 0.$$

Since $\pi_\beta(A(\delta_n))\Omega_\beta$ is uniformly bounded and $\pi_\beta(\mathfrak{A})\Omega_\beta$ is dense in $\mathcal{H}_\beta$, it thus follows that

$$w - \lim_n \pi_\beta(A(\delta_n))\Omega_\beta = \pi_\beta(A)\Omega_\beta,$$

showing that

$$\overline{\pi_\beta(\mathfrak{A}_0)\Omega_\beta} = \overline{\pi_\beta(\mathfrak{A})\Omega_\beta} = \mathcal{H}_\beta.$$

The proof that $\tilde{\omega}_\beta$ fulfills the KMS-condition can now be accomplished in a standard manner: let $A(\delta_n)$ be any sequence of operators as in the preceding step. Since convex combinations of δ-sequences are again δ-sequences we may assume without loss of generality that $\pi_\beta(A(\delta_n))\Omega_\beta$ as well as $\pi_\beta(A^*(\delta_n))\Omega_\beta$ converge strongly to $\pi_\beta(A)\Omega_\beta$ and $\pi_\beta(A^*)\Omega_\beta$, respectively. Moreover, since $\tilde{\omega}_\beta \upharpoonright \mathfrak{A}_0$ satisfies the KMS-condition, we have

$$\|e^{-\beta H_\beta/2}\pi_\beta(A(\delta_n))\Omega_\beta\| = \|\pi_\beta(A^*(\delta_n))\Omega_\beta\|.$$

From this equality and the fact that $e^{-\beta H_\beta/2}$ is closed it readily follows that all vectors in $\pi_\beta(\mathfrak{A})\Omega_\beta$ are in the domain of $e^{-\beta H_\beta/2}$ and

$$\|e^{-\beta H_\beta/2}\pi_\beta(A)\Omega_\beta\| = \|\pi_\beta(A^*)\Omega_\beta\|, \qquad A \in \mathfrak{A}.$$

But the latter relation implies that $e^{-\beta H_\beta}$ is the modular operator [8] corresponding to $\pi_\beta(\mathfrak{A})''$, Ω_β. Hence $\tilde{\omega}_\beta$ is a KMS-state. □

This result makes plain how $\tilde{\omega}_\beta$ might fail to be a KMS-state: the functions $t \to \tilde{\omega}_\beta(B^*\alpha_t(A))$ may not be continuous in the sense of the above definition. In the case of C^*-dynamical systems this possibility is ruled out from the outset. But if one relaxes the requirement of strong continuity of α_t there do exist models fitting into our framework where this does happen.

An instructive example of this kind is the theory of a free massless scalar particle in $d=3$ space-time dimensions. The structure of this model relevant for our purposes (in an arbitrary number $d \geqq 3$ of dimensions) can be summarized as follows: the basic building blocks are the symmetric Fock-space $\mathscr{H}$ over the single-particle space $\mathscr{K} = L^2(\mathbb{R}^{d-1})$ and the unitary Weyl-operators

$$W(f) = e^{i(a^*(f)+a(f))^-}, \tag{4.4}$$

where $f \in \mathscr{K}$ and $a^*(\,\cdot\,)$, $a(\,\cdot\,)$ are the familiar creation and annihilation operators. The dynamics is fixed by

$$\alpha_t(W(f)) = W(e^{it\omega}f) = e^{itH}W(f)e^{-itH}, \tag{4.5}$$

where ω is the single particle Hamiltonian which, in momentum space, acts according to

$$(\widetilde{\omega f})(\mathbf{p}) = |\mathbf{p}| \cdot \tilde{f}(\mathbf{p}), \tag{4.6}$$

and H is the "second quantization" of ω. The (weakly closed) local algebras are defined by

$$\mathfrak{A}(\mathcal{O}) = \{W(f) | f \in \mathscr{L}(\mathcal{O})\}'', \tag{4.7}$$

where $\mathscr{L}(\mathcal{O})$ are certain specific real linear subspaces of $\mathscr{K}$. If, for example, $\mathcal{O} \subset \mathbb{R}^d$ is a double cone centered at 0 and $\mathbf{O} \subset \mathbb{R}^{d-1}$ is its base, then

$$\mathscr{L}(\mathcal{O}) = \omega^{-1/2}\mathscr{D}(\mathbf{O}) + i\omega^{+1/2}\mathscr{D}(\mathbf{O}), \tag{4.8}$$

where $\mathscr{D}(\mathbf{O})$ is the space of real test functions with support in $\mathbf{O}$.

It is well-known [20] that the algebras $\mathfrak{A}(\mathcal{O})$ and the time translations e^{itH} have, up to the strong continuity of α_t, the properties listed in the first two conditions given in the Introduction. Following the reasoning in [12] it is also straightforward to show that these theories satisfy for any $d \geqq 3$ the nuclearity condition. In fact one can put $m=n=d-1$ in the bound (1.8), in agreement with Stefan-Boltzmann's law and our interpretation of the quasi-partition function.

In spite of this structure there does not exist any KMS-state in this model if $d=3$. This can be seen as follows: assuming that there is a KMS-state ω_β for some $\beta > 0$ one finds, by making use of the KMS-condition as in [21] that $\omega_\beta(W(f)) = 0$ whenever $f \in \mathscr{L}(\mathcal{O})$ is not in the domain of $\omega^{-1/2}$. At this point the dimension of space-time enters: only for $d=3$ there exist such elements f in $\mathscr{L}(\mathcal{O})$. For any such

f the function $\lambda \to \omega_\beta(W(\lambda f))$ is discontinuous at $\lambda = 0$, and consequently ω_β cannot be a normal state on the von Neumann algebras $\mathfrak{A}(\mathcal{O})$. But this is in conflict with the general results in [19] according to which any KMS-state ω_β on $\mathfrak{A}$ is necessarily normal on each $\mathfrak{A}(\mathcal{O})$. Hence there is no such state in the theory. (For a more detailed discussion of this model cf. [17].)

The reason for the absence of KMS-states in this model can be explained in physical terms by comparing it with the corresponding models in $d > 3$ space-time dimensions. There spatially homogenous, locally normal and primary KMS-states exist and are given by

$$\omega_{\beta,c}(W(f)) = e^{ic\widetilde{(\omega^{1/2}f)}(0)} \cdot e^{-(f,\coth\beta\omega/2 \cdot f)/2} \tag{4.9}$$

for arbitrary $c \in \mathbb{R}$. These states correspond to ensembles in thermodynamical equilibrium with a mean background field c which is produced by the collective effect of zero-energy massless particles (Bose-condensation).

The limit-points of $\omega_{\beta,c}$ for $c \nearrow \infty$ describe states with infinite field strength which are no longer locally normal. These states do not contribute to the Gibbs-ensemble if $d > 3$, since there the creation of an infinite field requires already locally an infinite amount of energy.

The situation is different, however, if $d = 3$. There one can create states with arbitrarily large field strength c from the vacuum without needing much energy. The important point is that in three dimensions the surface energy between regions of field strength c and of zero field strength can be kept arbitrarily small by making the surface layer between these regions sufficiently large. Hence states with infinite field-strength contribute to the Gibbs-ensemble in the thermodynamical limit if $d = 3$, and this explains why the corresponding states are not locally normal.

This simple model illustrates the fact that the nuclearity condition (1.8) does not impose stringent conditions on the infrared properties of a theory. It essentially restricts only the high energy behaviour. It seems, however, that infrared problems of the above type are absent in theories satisfying a compactness condition proposed by Fredenhagen and Hertel: instead of putting limitations on the number of states which are localized in $\mathcal{O}$ and at the same time contribute substantially to a given spectral subspace of the Hamiltonian, this in a sense dual condition restricts the number of states of fixed total energy on a given local algebra $\mathfrak{A}(\mathcal{O})$. (For the precise condition cf. [22].)

It is noteworthy that the above model violates this condition if $d = 3$, but for $d \geqq 4$ the condition is satisfied [23]. This fact as well as further partial results [24] seem to indicate that the states $\tilde{\omega}_\beta$ constructed above are KMS-states in all theories satisfying the Fredenhagen-Hertel condition.

Acknowledgements. The second author (P.J.) wishes to thank Klaus Fredenhagen and João C. A. Barata for many helpful discussions and the "Studienstiftung des deutschen Volkes" for financial support.

References

1. Streater, R.F., Wightman, A.S.: PCT, spin and statistics and all that. New York: Benjamin 1964
2. Haag, R., Kastler, D.: An algebraic approach to quantum field theory. J. Math. Phys. **5**, 848 (1964)

3. Satz, H. (ed.): Statistical mechanics of quarks and leptons. Amsterdam, New York, Oxford: North-Holland 1981
4. Piran, T., Weinberg, S. (ed.): Intersection between particle physics and cosmology. Singapore: World Scientific 1986
5. Buchholz, D., Junglas, P.: Local properties of equilibrium states and the particle spectrum in quantum field theory. Lett. Math. Phys. **11**, 51 (1986)
6. Buchholz, D., Wichmann, E.H.: Causal independence and the energy-level density of states in local quantum field theory. Commun. Math. Phys. **106**, 321 (1986)
7. Haag, R., Hugenholtz, N.H., Winnink, M.: On the equilibrium states in quantum statistical mechanics. Commun. Math. Phys. **5**, 215 (1967)
8. Bratteli, O., Robinson, D.W.: Operator algebras and quantum statistical mechanics. I, II. Berlin, Heidelberg, New York: Springer 1979, 1981
9. Haag, R., Kastler, D., Trych-Pohlmeyer, E.B.: Stability and equilibrium states. Commun. Math. Phys. **38**, 173 (1974)
10. Pusz, W., Woronowicz, S.L.: Passive states and KMS states for general quantum systems. Commun. Math. Phys. **58**, 273 (1978)
11. Buchholz, D., D'Antoni, C., Fredenhagen, K.: The universal structure of local algebras. Commun. Math. Phys. **111**, 123 (1987)
12. Buchholz, D., Jacobi, P.: On the nuclearity condition for massless fields. Lett. Math. Phys. **13**, 313 (1987)
13. Buchholz, D., Doplicher, S., Longo, R.: On Noether's theorem in quantum field theory. Ann. Phys. **170**, 1 (1986)
14. D'Antoni, C., Doplicher, S., Fredenhagen, K., Longo, R.: Convergence of local charges and continuity properties of W^*-inclusions. Commun. Math. Phys. **110**, 325 (1987)
15. Junglas, P.: In preparation
16. Sakai, S.: C^*-Algebras and W^*-algebras. Berlin, Heidelberg, New York: Springer 1971
17. Junglas, P.: Thermodynamisches Gleichgewicht und Energiespektrum in der Quantenfeldtheorie. Dissertation, Hamburg 1987
18. Ojima, I.: Lorentz invariance vs. temperature in QFT. Lett. Math. Phys. **11**, 73 (1986)
19. Takesaki, M., Winnink, M.: Local normality in quantum statistical mechanics. Commun. Math. Phys. **30**, 129 (1973)
20. Araki, H.: Von Neumann algebras of observables for the free scalar field. J. Math. Phys. **5**, 1 (1964)
21. Rocca, F., Sirugue, M., Testard, D.: On a class of equilibrium states under the Kubo-Martin-Schwinger condition. Commun. Math. Phys. **19**, 119 (1970)
22. Haag, R.: Local relativistic quantum physics. Physica **124**A, 357 (1984)
23. Buchholz, D., Porrmann, M.: In preparation
24. Buchholz, D., Junglas, P.: Work in progress
25. Doplicher, S., Longo, R.: Standard and split inclusions of von Neumann algebras. Invent. Math. **75**, 493 (1984)
26. ten Brinke, G., Winnink, M.: Spectra of Liouville operators. Commun. Math. Phys. **51**, 135 (1976)

Communicated by H. Araki

Received June 20, 1988